Week 3 Class Note

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Recall any Random Single Variables, Random Vectors, Random Matrix

$$\vec{X} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix} \text{ In Vector X, Each of them(x) is a random variable}$$

$$\text{Note if } i = j, Cov(x_i\ x_j) = Var(x_i)$$

$$Cov(x_i, x_j) = \mathbb{E}[(X_i - \mathbb{E}[X_i])(X_j - \mathbb{E}[X_j])]$$

$$\text{Covariance Matrix } \sum = (Cov(x_i, x_j))_{n\times n} = \begin{pmatrix} Var(x_1) & Cov(x_1, x_2) & \dots & Cov(x_i, x_n) \\ Cov(x_2, x_1) & Var(x_2) & \dots & Cov(x_2, x_n) \\ \vdots & \vdots & \dots & \vdots \\ Cov(x_n, x_1) & Cov(x_n, x_2) & \dots & Var(x_n) \end{pmatrix}$$

$Key : 1) \sum$ is symmetric. $\sum$ can be orthogonal diagonalization,
i,e $\exists P$ $\mathrm{P}^T P = I$ s.t $P^T \sum P = D$

2) $\sum$ is semi-positive definite $i.e \lambda \geq 0, i = 1, 2, \vdots m$
$P = (v_1, v_2, \dots v_n)$
Then V_1 is the first principle direction and V_2 is the second, so on and so forth.

$$\mathrm{E}[\vec{X}] = \mathrm{E} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} \mathbb{E}[X_1] \\ \mathbb{E}[X_2] \\ \vdots \\ \mathbb{E}[X_n] \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = \vec{u}$$

$$\mathbb{E}\left[(\vec{X} - \mathbb{E}[\vec{X}])(\vec{X} - \mathbb{E}[\vec{X}])^T\right] = \left[\begin{pmatrix} x_1 - \mathbb{E}[x_1] \\ x_2 - \mathbb{E}[x_2] \\ \vdots \\ x_n - \mathbb{E}[x_n] \end{pmatrix} (x_1 - \mathbb{E}[x_1], x_2 - \mathbb{E}[x_2] \quad \dots \quad x_n - \mathbb{E}[x_n])\right]$$

$$\mathbb{E}(X) = x_1 p_1 + x_2 p_2 \dots x_n p_n \text{ if } p_1 = p_n$$

$$= \tfrac{1}{n}(x_1 + x_2 + \dots + x_n)$$