EDSL Documentation

A-Tas Theory Manual

By EDSL

Theory

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Contents

About This Manual

General

This manual describes the theoretical basis of A-Tas, the building thermal analysis program.

An overview of simulation principles is provided in Section **Simulation Principles**. Sections: **Conduction** ; **Convection** ;**Advection** ; **Psychometrics** ; **Long-wave Radiation** ; **Solar Radiation** ; **Casual Gains** and **Plant** describe the mathematical models used in Tas to represent the various heat and moisture transfer processes occurring within a building, and Section **Zone Heat Balance** describes how these models are combined to simulate the performance of the building as a whole. The sections: **Zone Temperature Outputs** and **Sensible Load Breakdown** describe how subsidiary outputs from the simulation, such as zone mean radiant temperature, are calculated.

Simulation Principles

General

A-Tas is a software tool which simulates the thermal performance of buildings. The main applications of the program are in assessment of environmental performance, prediction of energy consumption, plant sizing, analysis of energy conservation options and energy targeting.

The fundamental approach adopted by A-Tas is dynamic simulation. This technique traces the thermal state of the building through a series of hourly snapshots, providing the user with a detailed picture of the way the building will perform, not only under extreme design conditions, but throughout a typical year. This approach allows the influences of the numerous thermal processes occurring in the building, their timing, location and interaction, to be properly accounted for.

These processes are illustrated schematically in Figure 2.1, which shows the movement of heat in various forms as it is conveyed into, out of and around the building by a variety of heat transfer mechanisms.

Conduction in the fabric of the building is treated dynamically using a method derived from the ASHRAE *response factor* technique. This efficient computational procedure calculates conductive heat flows at the surfaces of walls and other building elements as functions of the temperature histories at those surfaces. Constructions of up to 12 layers may be treated, where each layer may be composed of an opaque material (e.g. brick), a transparent material (e.g. glass) or a gas (e.g. air). Databases of materials and constructions are available.

Convection at building surfaces is treated using a combination of empirical and theoretical relationships relating convective heat flow to temperature difference, surface orientation, and, in the case of external convection, wind speed.

Long-wave radiation exchange is modelled using the Stefan-Boltzmann law, using surface emissivities from the materials database. Long-wave radiation from the sky and the ground is treated using empirical relationships.

Solar radiation absorbed, reflected and transmitted by each element of the building is computed from solar data on the weather file. The calculation entails resolving the radiation into direct and diffuse components and calculating the incident fluxes using a knowledge of sun position and empirical models of sky radiation. Absorption, reflection and transmission is then computed from the thermophysical properties of the building elements. External shading and the tracking of sun patches around room surfaces may be included at the user's option.

Internal Conditions, which include room gains from lights, equipment and occupants as well as infiltration rates and plant operation specifications are grouped together in profiles which are applied to the various zones of the building. Internal Conditions profiles may be stored in a database for later retrieval.

Gains are modelled by resolving them into radiant and convective portions. The convective portion is injected into the zone air, whilst the radiant gains are distributed amongst the zone's surfaces.

Infiltration, ventilation and air movement between the various zones of the building causes a transfer of heat between the appropriate air masses which is represented by terms involving the mass flow, the temperature difference and the heat capacity of air.

Solar radiation entering a zone through transparent building components falls on internal surfaces, where it may be absorbed, reflected or transmitted depending on the surfaces' properties. Distribution of reflected and transmitted solar radiation continues until all the radiation has been accounted for.

Heating and cooling plant is represented by plant capacities, setpoints and control bands. Like gains, plant inputs may have both radiant and convective portions.

Tas solves the sensible heat balance for a zone by setting up equations representing the individual energy balances for the air and each of the surrounding surfaces. These equations are then combined with further equations representing the energy balances at the external surfaces, and the whole equation set is solved simultaneously to generate air temperatures, surface temperatures and room loads. This procedure is repeated for each hour of the simulation.

A latent balance is also performed for each zone which takes account of latent gains, moisture transfer by air movement and the operation of humidification and dehumidifcation plant.

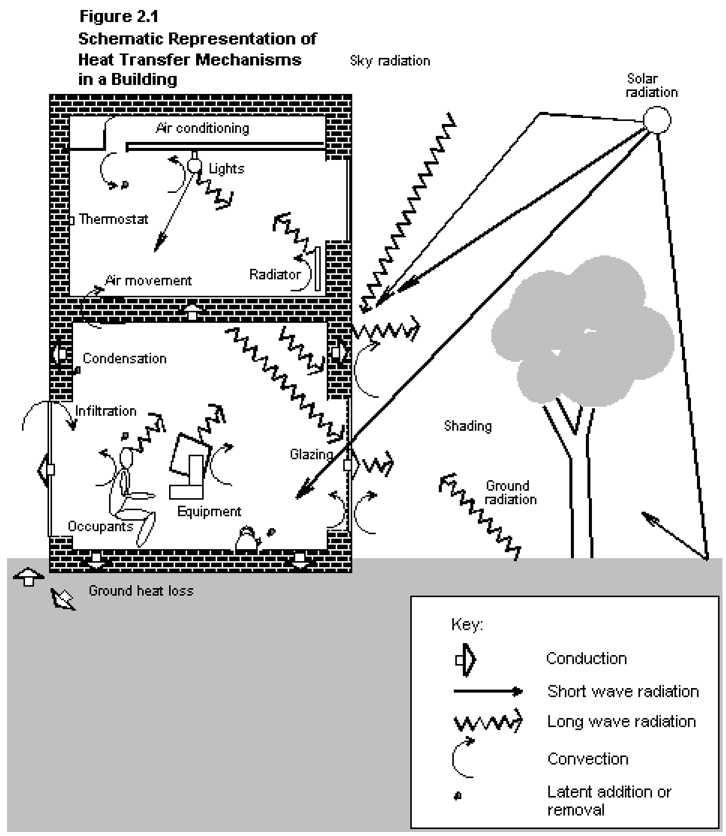
The following are some of the factors which influence the thermal behaviour of a building and whose influences A-Tas allows the user to investigate:

- Thermal insulation
- Thermal capacity ("thermal mass")
- Glazing properties
- Built form and orientation
- Climate
- Shading from nearby buildings and self-shading
- Infiltration
- Natural ventilation
- Mechanical ventilation
- Solar gain
- Gains from lights, occupants and equipment (both sensible and latent)
- Control setpoints & bands, optimum start, frost protection
- Available plant capacities for heating and cooling
- Plant schedules
- Plant radiant/convective characteristics
- Performance of boilers and heat pumps

A-Tas provides output in graphical and tabular form showing the effects of these factors on:

- Air temperature
- Mean radiant temperature
- Resultant temperature
- Surface temperatures
- Humidity
- Condensation risk
- Sensible and latent loads
- Energy consumption
- Required plant size

Schematic Representation of Heat Transfer mechanism in a Building



Conduction

Conduction Algorithm

Outline

Time-varying conduction heat transfer and heat storage in the building fabric is modelled in Tas using the normal co-ordinate method[1] with a time-step of 1 hour. The method is closely related to methods based on Response Factors and Conduction Transfer Functions[3] but offers substantial run-time and storage savings relative to these methods. The following summary is taken from Ref. 1:

The basis of the method is a description of the thermal state of the wall in terms of a set of normal co-ordinate variables. These variables, which define a decomposition of the temperature and flux distributions in the wall in terms of eigenfunctions, are updated at each time increment, and are used, in combination with recent input data, to calculate all output quantities. The method thus possesses elements in common with the two most widely used of existing methods for the analysis of wall heat flows, combining an economical state-representation resembling that employed by finite difference methods with fast and accurate techniques borrowed from response factor theory.

The method assumes linear, uni-dimensional heat flow, and relies on analytical techniques developed by Pipes[2] and Stephenson and Mitalas[3]. Data is accepted in the form of regularly sampled surface temperatures, and values are generated for the temperature or heat flux at any desired point within the wall. Minor modifications, reported elsewhere[4], allow for boundary conditions involving the specification of heat flux in place of temperature at either surface. Other extensions of the method, described in reference 4, include provision for alternative interpolation procedures, the calculation of time integrals of output variables, and methods for dealing with non-linear surface characteristics. The method can also be adapted to model varying time-steps.

Calculation Procedure

The normal co-ordinate method calculates heat flows and temperatures within a wall or other building component (wall, ceiling or floor) from the regularly sampled history of the component's surface temperatures. In common with Response Factor and Conduction Transfer Function methods, the normal co-ordinate method assumes that the behaviour of the surface temperatures over a time-step can be described by linearly interpolating the sample values.

In Tas, the main quantities of interest are the heat fluxes at the two surfaces of the component. These are calculated from recent surface temperatures and a set of normal co-ordinate variables:

Internal surface condition heat flux

$$W^{cond,int} = X^0 T^{int} + X^1 \left\langle T^{int} \right\rangle - Y^o T^{ext} - Y^1 \left\langle T^{ext} \right\rangle + \sum_{n=1}^{N} V_n^{int} \nu_n \quad (3-1)$$

External surface condition heat flux

$$W^{cond,ext} = Z^0 T^{ext} + Z^1 \left\langle T^{ext} \right\rangle - Y^o T^{\text{int}} - Y^1 \left\langle T^{\text{int}} \right\rangle + \sum_{n=1}^{N} V_n^{ext} \nu_n \quad (3-2)$$

where:

$W^{cond,int}$ is the internal surface conduction heat flux (W/m^2) (positive into the wall);
$W^{cond,ext}$ is the external surface conduction heat flux (W/m^2) (positive into the wall);
T^{int} is the internal surface temperature (C);
T^{ext} is the external surface temperature (C);
$\left\langle \chi \right\rangle$ (where χ is any variable) denotes the value of Χ at the previous time-step;
$X^0, X^1, Y^0, Y^1, Z^0, Z^1$ are response factors (W/m^{2}K) - constants which characterise the wall's response to recent surface temperature history;
$\nu_n \quad (n = 1,2....N)$ are a set of normal co-ordinate variables (with the dimensions of heat flux (W/m^2)) which describe the thermal state of the wall at time-step t in relation to a set of eigenfunctions;
$V_n^{\text{int}} \quad V_n^{ext} \quad (n = 1,2....N)$ are dimensionless constants which characterise the relationship between the surface fluxes and the normal co-ordinate variables.

The normal co-ordinate variables ν_n are updated at each time step using the previous time-step's surface temperatures:

$$\nu_n = \delta_n^{\text{int}} \left\langle \left\langle T^{\text{int}} \right\rangle \right\rangle + \delta_n^{ext} \left\langle \left\langle T^{ext} \right\rangle \right\rangle + \eta_n \left\langle \nu_n \right\rangle \quad (n = 1,2,...N) \quad (3-3)$$

where $\left\langle \left\langle \chi \right\rangle \right\rangle$ (where χ is any variable) denotes the value of Χ two time-steps before;
$\delta_n^{\text{int}} \quad \delta_n^{ext} \quad \eta_n \quad (n = 1,2,....N)$ are constants.

At the start of a simulation, the surface temperature histories and the normal co-ordinate variables are initialised to a steady state condition. The process of preconditioning ensures that the initial condition does not have a significant influence on simulation results. As the simulation proceeds, equations 3-1 and 3-2 are used at each time-step to establish linear relationships between the current temperatures and heat fluxes at the component's surfaces, and these relationships are incorporated in the matrix equations representing the overall heat balance for each zone (as described in Section **Zone Heat Balance**).

The constants in the above equations

$$X^0, X^1, Y^0, Y^1, Z^0, Z^1, V_n^{\text{int}}, V_n^{ext}, \delta_n^{\text{int}}, \delta_n^{ext}, \eta_n, N$$

are characteristics of the wall calculated in advance of simulation for each component type (or Building Element). This process involves the following stages:

1. Identify the thermal capacitance and resistance of each layer of the wall (see Section **Conduction Characteristics of Building Elements**).
2. Represent the thermal characteristics of each layer as a transmission matrix in the Laplace Transform domain, and obtain the transmission matrix for the

building component as a whole as the product of the layer transmission matrices.

3. Identify the eigenvalues of the system, which are the values of the Laplace Transform variable (s) at which a particular matrix element becomes zero. The eigenvalues, s_n, are always real and negative and form an infinite series. The series is truncated at $n = N$, where N is chosen in accordance with the following criterion which ensures that the truncation of the eigenvalue series does not lead to errors greater than interpolation errors arising from the use of a finite time-step:

$$-S_{N+1}\Delta > 1.6 \quad (3-4)$$

where Δ is the time-step (1 hour, or 3600 seconds, in Tas).

The value of N is 0 for lightweight components and may be as large as 12 for heavyweight structures such as ground floors.

4. Using the eigenvalues and certain other constants derived from the transmission matrix, calculate the constants for use in the calculation procedure. The constants η_n have a simple relationship to the eigenvalues:

$$\eta_n \exp(S_n\Delta) \quad (n = 1,2,..N) \quad (3-5)$$

For massless components (a class which in Tas is assumed to include all transparent components) the method is greatly simplified. N is zero and the response factors are given by:

$$X^0 = Y^0 = Z^0 = G \quad (3-6)$$

$$X^1 = Y^1 = Z^1 = 0 \quad (3-7)$$

where

G is the steady state conductance of the component (W/m^2/K), as measured between its surfaces (not to be confused with the U-value, which incorporates radiative/convective surface resistances).

For lightweight but not completely massless elements, N may be zero but the response factors X^1, Y^1, Z^1 retain small values. The presence of these response factors ensures that thermal mass is not neglected in cases where a significant amount of mass is present in lightweight components of large area.

The conduction heat flows into the surfaces of the component, $q^{cond,\text{int}}$ and $q^{cond,ext}$ (Watts), are calculated by multiplying the surface fluxes by the component area, A, which is the mean of the internal and external surface areas provided by 3D-Tas. Thus

$$q^{cond,\text{int}} = AW^{cond,\text{int}} \quad (3-8)$$

$$= A(X^0T^{\text{int}} + X^1\langle T^{\text{int}}\rangle - Y^0T^{ext} - Y^1\langle T^{ext}\rangle + \sum_{n=1}^{N} V_n^{\text{int}}\nu_n) \quad (3-9)$$

$$= AX^0T^{\text{int}} - AY^0T^{ext} + q^{hist\_cond,\text{int}} \quad (3-10)$$

where

$q^{hist\_cond,\text{int}}$ is the sum of all the *historical* terms in equation 3-9; these terms can all be evaluated from results from previous time-steps.

Likewise

$$q^{cond,ext} = AW^{cond,ext} \quad (3-11)$$

$$= AZ^0T^{ext} - AY^0T^{int} + q^{hist\_cond,ext} \quad (3-12)$$

Conduction Characteristics of Building Elements

Building Elements and Constructions

A *Building Element* describes the construction of a wall, window, ceiling, floor (etc.) in terms of a set of ordered layers, each layer having given thermophysical properties. Tas provides a database of materials and facilities for the creation of databases of constructions.

In Tas there are two classes of construction and four classes of material:

Construction classes	**Material classes**
Opaque	Opaque Material
Transparent	Opaque Layer
	Transparent Layer
	Gas Layer

Opaque Layers and Transparent Layers incorporate a width as part of the material data. In the case of an Opaque Material, the width must be added when the material is inserted in a construction.

Opaque constructions may contain Opaque Materials, Opaque Layers and Gas Layers, but not Transparent Layers. Transparent constructions may contain Transparent Layers and Gas Layers, but not Opaque Materials or Layers. Transparent constructions are assumed to have negligible thermal mass.

The ordering of the layers of a construction is important. When defined in the constructions database layers are ordered from the *internal* surface to the *external* surface **-** a labelling which, in the case of building components exposed to the outside environment, and ground floors, indicates which surface faces into the zone and which faces the outside. In the case of a component which forms a partition between two zones, or which lies entirely within a single zone, the concept of *internal* depends on the viewpoint, so a further indicator of orientation is required. This is provided by the sign of the *Building Element Number* in the *Surface Geometry* table. Zone surfaces for which the labelling is reversed (from the viewpoint of the zone under inspection) are assigned a negative *Building Element Number*.

Conduction Characteristics

The Tas conduction algorithm requires, for each layer of the construction, a thermal capacitance and a thermal resistance.

The thermal capacitance of a opaque layer (that is, Opaque Material plus width or Opaque Layer) is calculated from

$$c = d\rho C \quad (3-13)$$

where

c is the thermal capacitance of the layer per unit area (J/m^{2}K);

d is the width of the layer (m);
ρ is the density of the material (kg/m^3);
C is the specific heat capacity of the material (J/kgK).

For Transparent Layers and Gas Layers the thermal capacitance is neglected:

$$c = 0 \quad (3-14)$$

With the exception of Gas Layers, the layer thermal resistance is calculated from

$$r = \frac{d}{\lambda} \quad (3-15)$$

where

r is the thermal resistance of the layer (m^{2}K/W);
d is the width of the layer (m);
λ is the thermal conductivity of the material (W/mK).

For Gas Layers, the thermal resistance is calculated by linearising the convective and radiative transfer across the layer:

$$r = \frac{1}{h^{conv} + h^{rad}} \quad (3-16)$$

where h^{conv} and h^{rad} are the convective and radiative heat transfer coefficients for the cavity (see sections **Internal Convection** and **Internal Long-wave Radiation**).

Zone divides (fictitious surfaces used to partition zones where there is no physical barrier) are modelled with a thermal resistance of zero.

References

1. M. Gough. A new method for the calculation of heat transfer in walls and roofs. CIB International Symposium on System Simulation in Buildings, Liege, Belgium, Dec 6-9 1982.
2. L.A. Pipes. Matrix analysis of heat transfer problems. *Franklin Institute Journal*, Vol. 263(3), March 1957, 195-205.
3. D.G. Stephenson, G.P. Mitalas. Calculation of heat conduction transfer functions for multi-layer slabs. *ASHRAE Trans.* 1971, Vol. 77, II, 117-126.
4. M.C.B. Gough. Modelling heat flow in buildings: an eigenfunction approach. PhD dissertation, University of Cambridge, 1982.

Convection

External Convection

The convection coefficient at external building surfaces, h^{ext}, is calculated from hourly values of the wind speed provided on the weather file.

Tas uses the following expression provided by CIBSE[1]:

$$h^{ext} = 5.8 + 4.1 v_m \quad (4-1)$$

where

h^{ext} is the external convection coefficient (W/m^2);

v_m is the wind speed measured at the meteorological station at a height of 10m.

The external convective heat flux, $W^{conv,ext}$ (W/m^2), from the outside air to an exposed building surface is

$$W^{conv,ext} = h^{ext}(T_0^{air} - T^{ext}) \quad (4-2)$$

where

T_0^{air} is the outside air temperature;

T^{ext} is the external surface temperature.

The external convective heat flow, $q^{conv,ext}$ (Watts), is calculated by multiplying $W^{conv,ext}$ by the *effective component surface area*, *A* (the mean of the internal and external surface areas provided by 3D-Tas):

$$q^{conv,ext} = AW^{conv,ext} \quad (4-3)$$

$$= Ah^{ext}(T_0^{air} - T^{ext}) \quad (4-4)$$

Internal Convection

Convection at Internal Zone Surfaces

With the exception of zone divides (see section **Zone Divides** below), the calculation of internal convection coefficients in Tas follows the procedures set out by Alamdari and Hammond[2] for the calculation of free convection heat transfer in rooms.

For vertical surfaces, and for vertical surfaces when the heat flow is in the upward direction, Alamdari and Hammond recommend the use of the formula

$$h^{\text{int}} = \left[\left\{ a\left(\frac{\Delta T}{L}\right)^{1/4} \right\}^{6} + \left\{ b(\Delta T)^{1/3} \right\}^{6} \right]^{1/6} \qquad (4-5)$$

where

h^{int} is the convective heat transfer coefficient (W/m^{2}K) applying between a surface and the room air;

$\Delta T = \left| T^{air} - T^{\text{int}} \right|$ is the absolute temperature difference (K) between the room air (temperature T^{air}) and the surface (temperature T^{int});

L is a characteristic length (m) of the heat transfer surface (height of wall or hydraulic diameter of floor or ceiling);

a and b are taken from the following table:

	A	***b***
Vertical surface (horizontal heat flow)	1.50	1.23
Horizontal surface upward heat flow	1.40	1.63

Equation 4-5 is used in Tas for windows, with L set to the window height as defined in 3D-Tas.

For walls, Tas uses the following simpler formulae, which give a very good approximation to Equation 4-5:

Vertical surface (horizontal heat flow):

$$h^{\text{int}} = h^{hor} = h^{H} + 1.23(\Delta T)^{1/3} \qquad (4-6)$$

Horizontal surface, upward heat flow:

$$h^{\text{int}} = h^{up} = 1.63(\Delta T)^{1/3} \qquad (4-7)$$

where the height-dependent term in equation 4-5 is calculated from the zone height, *H*, using

$$h^{H} = \frac{1}{(670.656H^{6} + 120.43H^{8.7})^{1/6}} \qquad (4-8)$$

For horizontal surfaces when the heat flow is downward, Tas uses the following formula from Alamdari and Hammond's paper:

Horizontal surface, downward heat flow:

$$h^{\text{int}} = h^{down} = 0.6\left(\frac{\Delta T}{L^{2}}\right)^{1/5} \qquad (4-9)$$

For sloping walls, Tas interpolates between the expressions given above, using an angular dependence derived from the work of Fujii and Imura[3]:

Sloping wall, upward heat flow:

$$h^{\text{int}} = h^{up} + (h^{hor} - h^{up})(\sin\gamma)^{1/4} \qquad (4-10)$$

Sloping wall, downward heat flow:

$$h^{\text{int}} = h^{down} + (h^{hor} - h^{down})(\sin\gamma)^{1/4} \quad (4-11)$$

Where

γ is the angle between the inward-facing surface normal and the vertical.

The dependence of h^{int} on the temperature difference, ΔT, means that the internal convection varies from hour to hour.

The convection formulae given above are modified when the surface has an internal blind, as indicated by the *Internal Blind* flag being set to 'Yes' for its construction. In such cases, to make allowance for the fact that convection occurs at the surfaces of both the construction and the blind, h_i is increased by a factor 4/3.

The user may override these convection formulae by setting the parameter *Optional Fixed Convection Coefficient* for one or more zones. If this is done, h^{int} is set to the value entered for this parameter for all internal surfaces of the zone (with the exception of **zone divides**) and for all times.

The internal convective heat flux, $W^{conv,\text{int}}$ (W/m^2), from the zone air to a zone surface is given by

$$W^{conv,\text{int}} = h^{\text{int}}(T^{air} - T^{\text{int}}) \quad (4-12)$$

where

T^{air} is the zone air temperature;

T^{int} is the internal surface temperature.

The internal convective heat flow, $q^{conv,\text{int}}$ (Watts), from the air to the surface is calculated by multiplying $W^{conv,\text{int}}$ by the *effective component surface area, A*:

$$q^{conv,\text{int}} = AW^{conv,\text{int}} \quad (4-13)$$

$$= Ah^{\text{int}}(T^{air} - T^{\text{int}}) \quad (4-14)$$

Zone Divides

A zone divide is a special surface used in Tas to represent the boundary between two zones where there is no physical barrier. A zone divide is indicated by a building element for which the construction code (C-Code) is set to '/'. The function of a zone divide is to model the transmission of solar and long-wave radiation between the zones either side of the surface. Accordingly such surfaces do not convect at all.

Convection in Cavities

Convection in a cavity (or *gas layer*) which forms part of a construction is treated in Tas by a fixed convection coefficient. This coefficient forms part of the input data characterising the cavity.

If a blind exists within a cavity (in which case there will be a gas layer on each side of the blind), the convection coefficients for both gas layers are increased by a factor 4/3 to allow for the fact that air can circulate around or through the blind. The same factor is applied to the convection coefficient for the gas layer adjacent to an internal blind.

The convection coefficient for a gas layer adjacent to an external blind is automatically set to a high value (50 W/m^2K). This is consistent with Tas's approach to the treatment of external blinds, which is limited to modelling the blocking of solar radiation.

References

1. CIBSE Guide, Part C3. The Chartered Institution of Building Services Engineers. Delta House, 222 Balham High Road, London SW12 9BS.

2. F. Alamdari & G. P. Hammond. Improved data correlations for buoyancy-driven convection in rooms. *BSER&T.* Vol 4, No. 3 1983.

3. T.Fujii and H. Imura. Natural convection heat transfer from a plate with arbitrary inclination. *Int. J. Heat Mass Transfer*, Vol. 15, p.755, 1972. As reported in:
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Advection

General

Advection is the transfer of heat via the bodily movement of air. In Tas, air flow rates may either be specified in advance or calculated by the program. Prescribed air flow rates may be specified under three headings: **Infiltration**, **Ventilation** and **Air Movement**. Tas will calculate air flow rates by natural ventilation (**Aperture Air Flows**) given data on the characteristics of apertures in the building fabric.

Infiltration

The term infiltration is used in Tas to describe the user-specified exchange of air between a building zone and the exterior by natural ventilation. Infiltration carries both sensible and latent heat into or out of the zone.

Infiltration rates for each Tas zone may be specified, and scheduled to vary with time, by setting air change rates in **Internal Conditions**. These air change rates are converted to air mass flow rates on the basis of an air density of $\rho^{air} = 1.210$ Kg/m^3(the density of air at standard atmospheric pressure and a temperature of 20C):

$$m^{\inf} = \frac{\rho^{air} aV}{3600} \quad (5-1)$$

where

$m^{\inf}$ is the infiltration air mass flow rate (kg/s),
a is the air change rate (air changes per hour), and
V is the zone volume as displayed in the Tas *Surface Geometry* display facility (the volume of air in the zone).

During the simulation the sensible heat gain due to infiltration into a zone is modelled as

$$Q^{\inf} = m^{\inf} c_p (T_0^{air} - T^{air}) \quad (5-2)$$

where

c_p is the specific heat capacity of air at constant pressure, for which the value $c_p = 1012$ J/kgK is taken (corresponding to a humidity ratio of about 0.003 kg/kg);

T_0^{air} is the outside air temperature;

T^{air} is the zone air temperature.

The net moisture gain, $w^{\inf}$ kg/s), is modelled as

$$w^{\inf} = m^{\inf}(x_0 - x) \quad (5-3)$$

where

x_0 is the outside air humidity ratio (kg/kg);

x is the humidity ratio in the zone (kg/kg).

Ventilation

Tas provides an option to specify mechanical ventilation rate for a zone in addition to the infiltration. The purpose of this is to allow estimates to be made of air conditioning loads in simple cases.

Mechanical ventilation rates are specified in *Internal Conditions* as scheduled input parameters, expressed in kg/s.

The ventilation air is assumed to be drawn into the zone direct from outside, and its effect on the zone sensible and latent balances is modelled by equations of the same form as equations 5-2 and 5-3:

$$Q^{vent} = m^{vent} c_p (T_0^{air} - T^{air}) \quad (5-4)$$

$$w^{vent} = m^{vent}(x_0 - x) \quad (5-5)$$

where m^{vent} is the ventilation air mass flow rate (kg/s),

Air Movement

Air moving between zones in a building carries with it sensible heat and water vapour, and thus affects the thermal balance in the zones concerned. Air Movement may represent either natural or mechanical ventilation.

In Tas inter-zone air movement may be specified as a scheduled input parameter for up to 12 source zones feeding air into each zone. Air movement from the outside may also be specified (in addition to any infiltration and ventilation). The air movements are entered as mass flow rates (kg/s).

The total sensible heat gain, Q^{am} (Watts), into zone *z* due to air movement is modelled as

$$Q^{am} = \sum_{s=0}^{Z} m_{sz} c_p (T_s^{air} - T^{air}) \quad (5-6)$$

where

m_{sz} is the mass flow rate from source zone *s* (with zone 0 representing the outside air) to zone *z;*

Z is the number of zones in the model;

T_s^{air} is the air temperature in source zone *s* (or the outside temperature in the case $s = 0$);

T^{air} is the air temperature in zone *z*.

The total moisture gain for the zone due to inter-zone air movement, w^{am} (kg/s), is modelled as

$$w^{am} = \sum_{s=0}^{Z} m_{sz} (x_s - x) \quad (5-7)$$

where

x_s is the humidity ratio in source zone *s* (or the outside humidity ratio in the case $s = 0$);

x is the humidity in zone *z*.

Aperture Air Flows

The Tas interface allows the user to specify apertures in the building fabric through which air may flow. Each aperture, which may be a window, a door or a portion of a floor, has an area, a mean altitude, an orientation and a plan hydraulic diameter derived from the 3D-Tas geometric model. It also has a time-varying *aperture factor* which is specified in A-Tas. In addition, there is an option to indicate that the aperture is sheltered from the wind.

The *aperture factor* is a number, usually in the range (0,1), specifying the area of the aperture as a fraction of the area of the surface it is associated with.

Aperture air flows are calculated by Tas using a model which takes account of the pressure-flow characteristics of the apertures, wind and stack pressures, and any prescribed air flows.

Pressure-Flow Characteristics of Apertures

The basic relation used to model the pressure-flow characteristics of an aperture is

$$Q = C_F \left|\Delta p\right|^{1/2} \operatorname{sgn}(\Delta p) \quad (5-8)$$

where

Q is the volume flow rate (m^3/s) through the aperture,

C_F is the *flow coefficient* ($m^3/s\ Pa^{1/2}$) of the aperture,

Δp is the pressure drop across the aperture (Pa), and

sgn() is the signum function:

$$\operatorname{sgn}(x) = \begin{cases} 1 & (x > 0) \\ 0 & (x = 0) \\ -1 & (x < 0) \end{cases}$$

The flow coefficient, C_F, is calculated from

$$C_F = 0.62 \left(\frac{2}{\rho} \right)^{1/2} fA \quad (5-9)$$

where

0.62 is the assumed *discharge coefficient*,

ρ is the air density on the inlet side of the aperture,

fA is the area of the aperture (the product of the *aperture factor*, *f*, and the Tas surface area, *A*).

It follows that the mass flow rate through the aperture, *m*, is given by

$$m = \rho Q = 0.62(2\rho)^{1/2}\ fA|\Delta p|^{1/2}\ \mathrm{sgn}(\Delta p) \quad (5-10)$$

Equation 5-10 applies to apertures across which the pressure difference may assumed to be constant across the aperture. This is often a good approximation for small apertures and horizontal apertures.

In the general case it is necessary to consider the possibility of pressure gradients on both sides of the aperture, which can give rise to two-way flow. Pressure gradients arise both from the wind pressure profile and the effects of gravity. To deal with this case we consider small height increments, d*h*, and obtain the total mass flow through the aperture, or any part thereof, as the integral

$$m = \int 0.62(2\rho(h))^{1/2}\ fw|\Delta p(h)|^{1/2}\ \mathrm{sgn}(\Delta p(h))\mathrm{d}h \quad (5-11)$$

where

w is the width of the Tas surface (assumed constant with height).

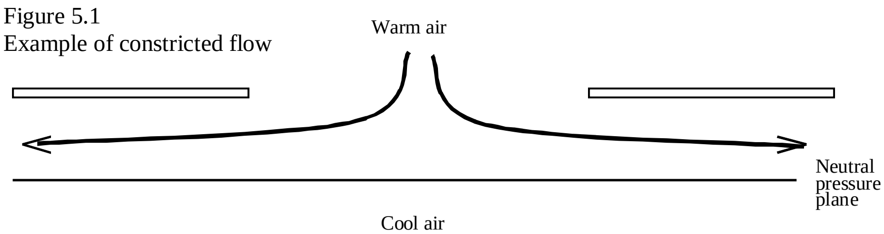
Equation 5-11 is integrated under the assumption that the pressure on each side of the aperture is a linear function of height (*h*) to yield the total flow through the aperture in both directions.

This is the theory which is applied in Tas for the modelling of flow through vertical apertures. For horizontal and sloping apertures, refinements are applied to the basic theory to allow for effects relating to the behaviour of the air flow as it approaches and leaves the aperture. The modifications take different forms depending on whether the air above the aperture is warmer or cooler than that below it, and fall under the headings **Constricted Flow** and **Plume Flow**.

Constricted Flow

Consider the situation drawn in Figure 5.1, which shows a vertical section through a horizontal aperture with warm air above it and cool air below.

Figure 5.1
Example of constricted flow



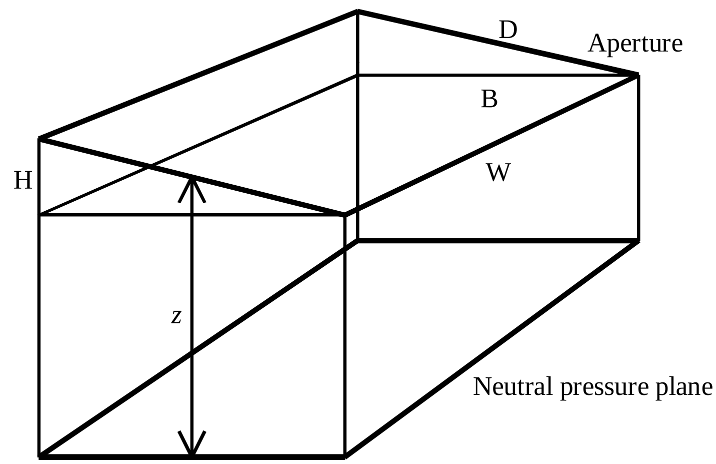
The neutral pressure plane (NPP) is also shown. This is the horizontal plane at which the static pressure in the warm and cool air masses is the same. The height of the neutral pressure plane depends on the pressure and flow boundary conditions applying elsewhere in the system. In the case illustrated, it lies below the aperture.

It is clear from this picture that the effective area for flow through the aperture is less than the area of the aperture itself. The flow is constricted by the neutral pressure plane. This happens for non-vertical apertures under *stratified* conditions (warmer air above cooler air) when the NPP lies close to the aperture.

Figure 5.2 shows a three-dimensional example involving a sloping aperture which will be used to derive an *effective aperture area* which can be applied in Tas to cater for this situation. This effective area will be derived under the assumption that the flow at the neutral pressure plane is horizontal.

Figure 5.2

Constricted flow in 3 dimensions



In this diagram:

Variable	Description
W	the width of the aperture
D	the height of the aperture in its own plane
B	the height of the aperture projected onto the horizontal plane
H	the vertical extent of the aperture
Z	the vertical distance between the centre of the aperture and the NPP

As the air flows down through the aperture it spreads out horizontally as it approaches the NPP, passing through the vertical sides of the polyhedron which connects the aperture with the NPP. The constriction caused by the NPP can be estimated by considering the area of the walls of this polyhedron. The total area of the walls is

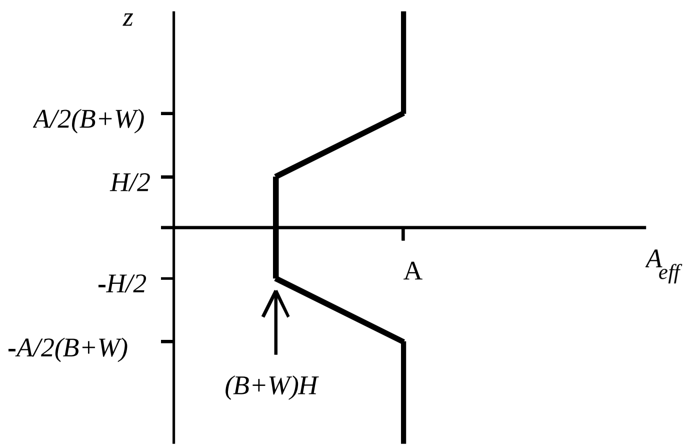
$$A_c = \begin{cases} 2(B+W)|z| & \left(|z| \geq \frac{H}{2}\right) \\ (B+W)H & \left(|z| < \frac{H}{2}\right) \end{cases} \quad (5-12)$$

We say that the flow is constricted if this area is less than the aperture area, A. The effective *aperture area,* A_{eff}, is then the smaller of the two areas A_c and A:

$$A_{eff} = \min(A_c, A) \quad (5-13)$$

The variation of A_{eff} with z described by equations 5-12 and 5-13 is shown in Figure 5.3. Note that for the special case of a horizontal aperture (H = 0), A_{eff} shrinks to zero at z = 0.

Figure 5.3
Variation of effective aperture area
with relative height of NPP



The effective aperture area, A_{eff} , is used by Tas in place of the aperture area, A, in stratified cases where equation 5-13 shows that flow constriction applies. The term $2(B+W)$ which appears in equation 5-12 is the plan-perimeter of the aperture. The value of this parameter is obtained from the closely related *plan hydraulic diameter* parameter displayed in the Surface Geometry table.

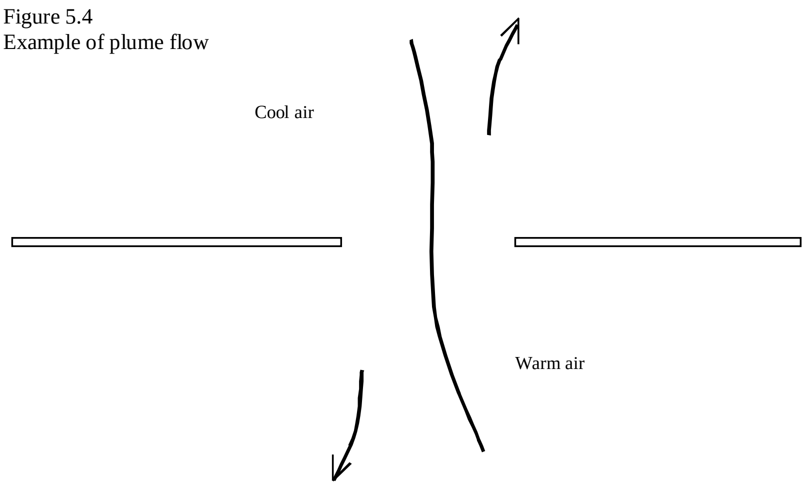
The theory presented above, whilst simplifying the reality somewhat, offers a significant improvement to the basic theory in those (fairly rare) cases when the flow is constricted.

Plume Flow

When the air above a horizontal or sloping aperture is cooler than the air below it, the situation shown in Figure 5.4 may arise.

In this situation, there is no static pressure difference across the aperture, and the standard theory therefore predicts that no flow will occur. The theoretical equilibrium is, however, unstable, and gives way in practice to a flow pattern in which a plume of warm air rises into the upper space while a plume of cool air falls into the lower space.

Figure 5.4
Example of plume flow



Plume flow through a horizontal aperture has been studied with a numerical model based on a theory of turbulent point-source plumes described by

Burmeister[1]. Burmeister's account draws on the theoretical work of Morton[2] and Turner[3] and experiments by Rouse et al.[4].

Results from the theoretical model indicate that the two-way flow through a horizontal circular aperture under non-stratified conditions can be approximated by the flow through a vertical aperture with the same area and a height of 1.2 times the diameter of the horizontal aperture.

Tas adopts this *equivalent vertical aperture* approach for plume flow through horizontal apertures, generalising by using the hydraulic diameter in place of the diameter of the circular aperture.

To cover the case of sloping apertures a further extension is required. The approach adopted by Tas for the general sloping aperture under non-stratified conditions is to define an *effective aperture height*,

$$H_{eff} = \sqrt{H^2 + (1.2d_{hyd})^2} \quad (5-14)$$

where

H is the true aperture height (as in Figure 5.2), and

d_{hyd} is the plan hydraulic diameter of the aperture, defined as four times the

Limits are put on the value of H_{eff}, preventing it exceeding twice the floor-to-floor height of either of the spaces adjacent to the aperture.

The basic theory is then applied with H replaced by H_{eff}. The rationale behind equation 5-14 is that this equation gives the correct value of H_{eff} for vertical apertures ($H_{eff} = H$) and for horizontal apertures ($H_{eff} = 1.2d_{hyd}$), and interpolates smoothly between these cases.

Limitations of Pressure-Flow Model

The theory as presented above applies strictly to sharp-edged orifice apertures which are small in relation to the spaces they connect. Care is needed when using this modelling approach in cases where the apertures are shaped to reduce turbulence at entry or exit, or where they have an area comparable with the cross-sectional areas of the adjacent spaces.

In cases where the aperture is shaped (for example a bell-mouthed duct) the discharge coefficient can be much increased. The user is referred to manufacturers' data for appropriate values of the discharge coefficient in such cases. The effect of an increased discharge coefficient can be modelled by increasing the *openable proportion* in the Aperture Types table.

In cases where an aperture is large in relation to its adjacent spaces it is necessary to consider through-flow and recirculating flow separately. We will take as an example an atrium space extending through the full height of a building with apertures connecting with the exterior at ground and roof level. We assume that the space is divided into two zones by a horizontal, full-width aperture at mid-height.

As regards through-flow, the resistance of the horizontal aperture can be assumed to be negligible in comparison with the resistances of the ground and roof-level apertures. The discharge coefficient for this aperture is therefore largely irrelevant (although we note in passing that for a narrower space, such as a double facade, it might be necessary to consider the resistance offered by friction with the walls).

Recirculating flow - air exchange between spaces connected by large apertures - is considerably more difficult to analyse. The accurate prediction of such flow where large apertures are concerned is beyond the scope of the zonal model adopted by Tas, and requires instead an approach based on computational fluid dynamics (CFD). However, it can be said that recirculating flows tend to be over-estimated by the basic (small aperture) theory when *openable proportions* for the large apertures are set to one. To correct for this, as a rule of thumb, the *openable proportion* should be set to 0.5 for a better representation of such flows.

Wind Pressure

The wind pressure on a building facade depends in a complicated way on the speed and direction of the wind, the geometry of the building and any nearby obstructions, and the nature of the terrain in the building's vicinity.

Possible approaches to the problem of estimating wind pressures include:

1) 3-dimensional computational fluid dynamics (CFD) modelling,
2) use of specific experimental data (from the building itself or a wind tunnel model of it),
3) use of generic correlations based on wind tunnel measurements.

The first two approaches are catered for in Tas by the option of specifying, on a *Wind Pressure Coefficient File*, calculated or measured wind pressure coefficients for each aperture. In the absence of such data, Tas also offers the third approach, using correlations based on wind pressure coefficients derived from wind tunnel experiments.

Whichever approach is adopted, Tas bases its analysis on the concept of a wind pressure coefficient.

The wind pressure coefficient, c_w, for a particular aperture is defined in terms of the wind speed, h, at *the building reference height*, h_b. The wind pressure, p_w, on the aperture is assumed to vary as the square of this wind speed, according to the relation:

$$p_w = \frac{F_s c_w \rho v(h_b)^2}{2} \quad (5-16)$$

where

$v(h_b)$ is the wind speed at the building reference height, h_b,

ρ is the density of the outside air, and

F_s is an adjustment factor applied to certain apertures in the case of pressure coefficients based on generic correlations (approach 3), to allow for the shielding effect of surrounding objects.

The building reference height is the *building height* returned by 3D-Tas optionally modified by the user:

$$h_b = f_r h_{b3d} \quad (5-14)$$

where

h_{b3d} is the height of the highest zoned point in the building,

f_r is the *building height adjustment factor*, f_r, specified by the user in General Details.

On the windward side of the building the wind pressure coefficients are usually positive and their values tend to increase with height. On the leeward side they are usually negative and roughly constant with height. Negative wind pressure coefficients also tend to apply where the wind blows parallel to the building surface.

Since the wind pressure correlations are expressed in terms of the wind speed at the building reference height, Tas requires a method for estimating this from the wind speed on the weather file, which is measured at an altitude of 10m. The method used is set out in British Standard BS5925:1991[5]. The wind speed as a function of altitude is assumed to follow a power law function:

$$v(h) = v_m K h^a \quad (5-17)$$

where

$v(h)$ is the wind speed at altitude *h*,

v_m is the wind speed measured at an altitude of 10m at the meteorological station (which is assumed to be in open flat country),

K and a are constants which depend on the terrain in the vicinity of the building, as set out in the following table:

Terrain Type (as described in BS59255:1991)	**Terrain Type (Tas abbreviation)**	***K***	***a***
Open flat country	*Open*	0.676*	0.17
Country with scattered windbreaks	*Rural*	0.52	0.20
Urban	*Town*	0.35	0.25
City	*City*	0.21	0.33

*BS5925 value 0.68; decimal place added to ensure correct wind speed at height 10m.

Wind Pressure Coefficients Read from File

A *Wind Pressure Coefficient File* may be specified, at the user's option, in the General Details facility. The file specifies pressure coefficients for each aperture for a range of wind directions. The coefficients are applied to the wind speed at the building height, $v(h_b)$, in accordance with equation 5-13 with F_s set to one. If no *Wind Pressure Coefficient File* is specified, Tas reverts to its default wind pressure calculation procedure based on generic correlations, as described in the next section.

The *Wind Pressure Coefficient File* contains the following data in ASCII format:

On the first line:
No. of wind directions
The number of wind directions for which pressure coefficients are specified (in the range 1 to 32).

Wind directions
The wind directions for which pressure coefficients are specified, in degrees clockwise from north in the range 0 to 359.9. The directions must be in clockwise order, and no two wind directions may be closer than 0.1 degrees.
On each subsequent line:

Aperture number
The number of the aperture for which wind pressure coefficients are specified. There must be a line on the file for each aperture in the building, including internal apertures (for which the wind pressure coefficients are ignored).

Wind pressure
The wind pressure coefficients for the specified aperture, for each wind direction specified on the first line.

During simulation the pressure coefficient for each aperture is obtained by linear interpolation from the given coefficients for the nearest wind directions.

The flexibility in specifying the wind directions for which the pressure coefficients apply allows for alignment with the orientations of the principal building facades.

If a *Wind Pressure Coefficient File* is specified, the *mean height of surroundings* parameter, and any *sheltered* flags set in the Aperture Types table, are ignored. However, the *building height* and *building height adjustment* parameters play a part in the calculation of wind speed at the building reference height.

Generic Wind Pressure Coefficients

If no *Wind Pressure Coefficient File* is specified, Tas estimates the wind pressure coefficients using correlations based on wind tunnel experiments carried out at the National Research Council Canada[6].

The tests were performed on a 1:400 scale model of a tall building with a flat roof, located in a rectangular array of flat-roof low rise buildings of 1/6 its height. These low rise buildings were of the same plan dimensions as the tall building (with width to length ratio 1.5:1) and were placed at a spacing of twice their height. The high rise building had actual dimensions 0.11m × 0.076m × 0.23m (width × length × height). Its height was 91m to scale. The wind pressure coefficients for the tall building were referenced to the wind velocity at the same height as the top of the building, but in the undisturbed stream[7].

Results from the tests are in the form of pressure coefficients for an array of rectangles on the building facade, and for several wind directions. For the Tas correlations, these coefficients are averaged across the building facade at each level, and coefficients from different facades are averaged to produce coefficients dependent only on altitude (relative to the building reference height) and the angle of attack of the wind.

The resulting pressure coefficients, c_w, are set out in the table below. The apply to a range of altitudes, h (expressed as a fraction of the building reference height, h_b), and angles of attack, θ (radians). Pressure coefficients for intermediate heights and angles of attack are calculated by linear interpolation.

Generic Wind Pressure Coefficients, c_w

$12\theta/\pi$	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
25 h/h_b													
0	.295	.260	.205	.125	.000	-.105	-.230	-.400	-.475	-.430	-.365	-.290	-.235
1	.306	.261	.212	.125	-.013	-.134	-.271	-.431	-.483	-.437	-.358	-.303	-.240
2	.311	.262	.216	.124	-.021	-.159	-.313	-.458	-.490	-.441	-.358	-.309	-.243
3	.312	.265	.219	.124	-.027	-.181	-.354	-.480	-.496	-.443	-.360	-.312	-.246
4	.313	.269	.222	.124	-.032	-.201	-.393	-.499	-.501	-.442	-.365	-.312	-.248
5	.315	.275	.225	.125	-.035	-.220	-.430	-.515	-.505	-.440	-.370	-.310	-.250
6	.320	.284	.230	.127	-.038	-.237	-.463	-.529	-.508	-.437	-.374	-.308	-.253
7	.329	.295	.237	.131	-.040	-.254	-.493	-.540	-.510	-.434	-.377	-.306	-.256
8	.343	.310	.246	.137	-.042	-.269	-.519	-.550	-.511	-.431	-.378	-.305	-.260
9	.361	.328	.259	.145	-.044	-.283	-.541	-.558	-.511	-.428	-.377	-.304	-.265
10	.385	.350	.275	.155	-.045	-.295	-.560	-.565	-.510	-.425	-.375	-.305	-.270
11	.413	.375	.295	.168	-.045	-.306	-.576	-.570	-.508	-.422	-.371	-.306	-.275
12	.445	.403	.318	.183	-.044	-.315	-.588	-.574	-.506	-.419	-.366	-.308	-.280

13	.479	.434	.343	.201	-.041	-.322	-.599	-.577	-.503	-.416	-.361	-.311	-.284
14	.515	.467	.371	.220	-.036	-.327	-.608	-.579	-.499	-.413	-.355	-.313	-.288
15	.550	.500	.400	.240	-.030	-.330	-.615	-.580	-.495	-.410	-.350	-.315	-.290
16	.583	.532	.429	.260	-.022	-.331	-.622	-.580	-.491	-.407	-.346	-.316	-.291
17	.612	.563	.455	.280	-.014	-.330	-.628	-.579	-.486	-.403	-.342	-.317	-.291
18	.634	.588	.478	.296	-.005	-.327	-.634	-.578	-.482	-.399	-.341	-.316	-.288
19	.648	.606	.494	.309	.004	-.324	-.639	-.576	-.478	-.394	-.340	-.314	-.285
20	.650	.615	.500	.315	.010	-.320	-.645	-.575	-.475	-.390	-.340	-.310	-.280
21	.638	.610	.494	.312	.012	-.317	-.650	-.574	-.472	-.386	-.340	-.305	-.274
22	.610	.589	.470	.298	.008	-.317	-.654	-.575	-.471	-.382	-.340	-.300	-.268
23	.563	.545	.426	.269	-.006	-.321	-.656	-.577	-.471	-.380	-.338	-.294	-.262
24	.494	.476	.356	.221	-.032	-.331	-.655	-.582	-.472	-.379	-.332	-.289	-.257
25	.400	.375	.255	.150	-.075	-.350	-.650	-.590	-.475	-.380	-.320	-.285	-.255

The shielding effect of surrounding buildings and other nearby objects may be taken into account by setting the parameter *mean height of surroundings*. This applies a factor

$$F_s = 1.2435 \exp\left(\frac{-1.30773 h_s}{h_b}\right) \quad (5-18)$$

to all the pressure coefficients, where

h_b is the building reference height,

h_s is the *mean height of surroundings*.

The factor F_s takes the value 1 when the surroundings have a height 1/6 of that of the building (the conditions which applied in the wind tunnel experiment).

An option is provided in Tas to designate all apertures of a given type *sheltered*, which has the effect of treating the apertures as though they were on the leeward side of the building. This feature may be used to treat apertures which face into an enclosed courtyard, and are thus not subject to the wind pressures experienced by an exposed aperture.

Stack Pressure

The stack effect - pressure differences arising from gravity forces - is modelled on the assumption that within each Tas zone, and outside, the air temperature is uniform. The air density, ρ, is assumed to be inversely proportional to absolute temperature:

$$\rho(\Theta) = \frac{\rho(\Theta_0)\Theta_0}{\Theta} \quad (5-19)$$

where

Θ is the absolute zone air temperature, and

Θ_0 is a reference temperature (taken to be 300K).

It follows that the air pressure at altitude *h* above ground level is

$$
\begin{aligned}
p &= p_0 - gh\rho \\
&= p_0 - \frac{gh\rho(\Theta_0)\Theta_0}{\Theta} \qquad (5-20)
\end{aligned}
$$

where

g is the acceleration due to gravity (9.81 m/s^2) and

p_0 is the zone air pressure extrapolated to ground level, or in the case of the exposed side of an external aperture, the wind pressure at the appropriate height.

Solution of Flow and Thermal Equations

The flow equations are solved iteratively. At each time step, wind pressures and wind pressure gradients are calculated for all exposed apertures. Then at each iteration step, air densities in all zones are calculated from the zone temperatures, and these are used (together with the wind pressures if appropriate) to calculate stack pressures and stack pressure gradients for both sides of each aperture. A set of equations is then set up describing the balance of mass flow into and out of each zone. This balance takes into account any air flows prescribed in the *Air Movement* facility. Specified *Infiltration* and *Ventilation* flows play no part in these equations, however, since they are assumed to be take the form of balanced inflows and outflows. The equations are solved using a gradient-based method to yield zone pressures ($p0$) and flow rates in both directions through each aperture. These flows are then fed back to the Tas thermal analysis where they are used to generate updated zone temperatures. The iterative process continues until zone temperatures (both air and mean radiant) converge to an accuracy of 0.01K and flow rates converge to an accuracy of 0.0005 kg/s.

Heat and Moisture Transfer

The heat and moisture transfers due to aperture air flows are treated in the same way as those for prescribed air flows.

Advection Accounting in The Sensible Load Breakdown

Zone heat gains (losses if negative) arising from Infiltration and Ventilation, together with those associated with Air Movement and Aperture Flows from the outside are accounted for in the Sensible Load Breakdown under the heading *Infiltration/Ventilation Heat Gain*. Heat gains due to Air Movement and Aperture Flows from other zones are accounted for under *Air Movement*.

References

1. Louis C. Burmeister. Convective Heat Transfer. 2nd Ed. John Wiley & Sons, Inc. 1993.
2. B. R. Morton, *J. Fluid Mech.* **2**, 127-144 (1957).
3. J. S. Turner, Buoyancy Effects in Fluids, Cambridge University Press, 1973.
4. H. Rouse, Y. S. Yih and H. W. Humphreys, *Tellus* **4**, 201-210 (1952).
5. British Standard BS5925:1991. British Standards Institution.

6. A.J. Bowen. A wind tunnel investigation using simple building models to obtain mean surface wind pressure coefficients for air infiltration estimates. National Aeronautical Establishment, National Research Council Canada, report LTR-LA-209.

7. Technical Note AIVC 44. An Analysis and Data Summary of the AIVC's Numerical Database. Air Infiltration and Ventilation Centre, March 1994.

Psychrometrics

Psychrometric Conversions

For converting between humidity ratio, relative humidity, vapour pressure and other psychrometric quantities, Tas uses the routines provided in Ref. 1. Atmospheric pressure is assumed to be 101325 Pa.

Air Heat Capacity

To raise the temperature of the air in a zone at a rate $\mathrm{d}T^{air} / \mathrm{d}t$ (K/s) requires a power input

$$Q^{air} = \rho^{air} V c_p \frac{\mathrm{d}T^{air}}{\mathrm{d}t} \quad (6-1)$$

Where

ρ^{air} is the density of the air, for which Tas assumes the value 1.21 kg/m^3 (the density of dry air at standard atmospheric pressure and a temperature of 19C);
V is the zone volume as displayed in the Tas *Surface Geometry* display facility (the volume of air in the zone);
c_p is the specific heat capacity of air at constant pressure, for which the value $c_p = 1012$ J/kgK is taken (corresponding to a humidity ratio of about 0.003 kg/kg).

The time derivative in equation 6-1 is calculated on the basis of the air temperature increment since the last time-step, leading to the following expression for the power input associated with the air heat storage:

$$Q^{air} = \frac{\left(p^{air} V c_p\right)\left(T^{air} - \left\langle T^{air} \right\rangle\right)}{\Delta} \quad (6-2)$$

where Δ is the time-step (1 hour, or 3600 seconds, in Tas).
$\langle \chi \rangle$ (where χ is any variable) denotes the value of χ at the previous time-step;

The influence of Q^{air} on the zone heat balance is usually slight, but becomes significant in cases of low ventilation rates in well insulated zones.

Heat storage in the air contained in construction cavities is neglected.

Air Moisture Content

Air moisture content or humidity ratio, x (kg/kg), is defined as the mass of water vapour present in the air per unit mass of dry air. Tas treats the storage of moisture in zone air in similar way to the storage of heat.

To raise the humidity ratio of the air at a rate $\mathrm{d}x/\mathrm{d}t$ (kg/kg s) requires an input of water vapour (kg/s) of

$$w^{air} = \rho^{air} V \frac{\mathrm{d}x}{\mathrm{d}t} \quad (6\text{-}3)$$

Discretising the time derivative as in section **Air Heat Capacity** we obtain

$$w^{air} = \frac{(\rho^{air} V)(x - \langle x \rangle)}{\Delta} \quad (6-4)$$

w^{air} normally plays a relatively minor role in the zone moisture balance, but this quantity can become significant at low ventilation rates.

References

1. R.W. Hyland, A. Wexler, R.B. Stewart, Thermodynamic properties of dry, moist air and water, and SI psychrometric charts. ASHRAE 1984.

Long-Wave Radiation

External Long-Wave Radiation

Long-Wave Radiation from The Sky

Radiation from a Clear Sky

Long-wave radiation from a clear sky originates principally from water vapour and carbon dioxide in the lower atmosphere, with additional contributions from atmospheric ozone, dust and aerosols. The clear sky radiation covers a broad spectrum, but is relatively weak in a wavelength band from 8 to 14µm - the so-called IR *atmospheric window*.

Idso[1], using data from a large number of measurements and a theory of atmospheric humidity which takes into account the presence of water dimers, gives the following formula for the effective hemispheric emissivity of a clear sky:

$$\varepsilon_{hem}^{air} = 0.70 + 5.95 \times 10^{-5} p_w \exp\left(\frac{1500}{\Theta_0^{\mathrm{air}}}\right) \quad (7-1)$$

where:

Θ_0^{air} is the screen dry-bulb absolute temperature (K);

p_w is the screen water vapour pressure (mb).

In using this expression to calculate clear sky long-wave flux the horizontal, R_{hem}^{air}, the emissivity ε_{ah} is applied to radiation from a body at screen air temperature:

$$R_{hem}^{air} = \varepsilon_{hem}^{air} \sigma {\Theta_0^{air}}^4 \quad (7-2)$$

where

$\sigma = 5.6697 \times 10^{-8}$ W/m^{2}K^4 is the Stefan-Boltzmann constant.

Tas adopts equation 7-1, and adds a check so that if the expression gives a value greater than 1 (which can happen if both the temperature and humidity are high), the value 1 is used. Values of ε_{hem}^{air} are typically in the range 0.75 to 0.85.

Effect of Cloud

Unsworth and Monteith[2] have shown that radiation from clouds can be treated by assuming an effective cloud emissivity of

$$\varepsilon_{hem}^{cloud} = 1 - \varepsilon_{hem}^{air} \quad (7-3)$$

This expression reflects the fact that most *clear sky* radiation originates from a layer of air lying within a few hundred metres of the ground, and the cloud radiation must pass through this layer to reach the ground.

The long-wave flux on the horizontal from an overcast sky is

$$E_{hem}^{cloud} = \varepsilon_{hem}^{cloud} \sigma \Theta^{cloud\,4} \quad (7-4)$$

where Θ^{cloud} is the absolute temperature of the cloud base (K).

The cloud base temperature Θ^{cloud} is not usually known accurately, and must therefore be estimated from measurements on the ground. Tas assumes that the cloud temperature is equal to the dewpoint temperature of the air at screen height, adjusted for the drop in atmospheric pressure with height. In calculating this pressure drop we assume an adiabatic lapse conditions (which produce a temperature gradient or lapse rate of about -10K/km).

The horizontal plane long-wave flux for a partially cloudy sky is assumed to be given by

$$E_{hem}^{sky} = E_{hem}^{air} + cE_{hem}^{cloud} = \varepsilon_{hem}^{air} \sigma \Theta_0^{air\,4} + c\varepsilon_{hem}^{cloud} \sigma \Theta^{cloud\,4} \quad (7-5)$$

$$= \varepsilon_{hem}^{air} \sigma \Theta_0^{air\,4} + c(1 - \varepsilon_{hem}^{air}) \sigma \Theta^{cloud\,4} \quad (7-6)$$

using equation 7-3, where c is the fractional cloud amount.

Values of c are derived from sunshine durations data on the Tas weather file. During the day,

$$c = 1 - S \quad (7-7)$$

where S is the sunshine duration (0-1), and during the night c is set to the average for the previous day.

From equation 7-6 it may be seen that for an overcast sky $c = 1$ with low cloud $\left(\Theta^{cloud} \cong \Theta_0^{air}\right)$, the effective sky emissivity tends to unity.

Integration over Solid Angle

The long-wave irradiance from a small solid angle $\mathrm{d}\Omega$ of sky onto an inclined surface is given by

$$\mathrm{d}R^{sky} = \frac{E^{sky}}{\pi} \cos\varphi \,\mathrm{d}\Omega \quad (7-8)$$

where E^{sky} is the emission from the sky patch and φ is the angle of incidence.

Assuming that the emission is isotropic over the sky $E^{sky} = E_{hem}^{sky}$, we integrate this expression over solid angle to obtain the total irradiance on a sloping surface. The irradiance on a surface with outward-facing normal making an angle γ with the zenith is found to be

$$R^{sky}(\gamma) = E_{hem}^{sky} \cos^2\left(\frac{\gamma}{2}\right) \quad (7-9)$$

Long-Wave Radiation from The Ground

For the purpose of calculating long-wave radiation, Tas assumes that the ground is at screen dry-bulb temperature. Its long-wave emission, E^{gnd}, has two components, a radiated component and a scattered component:

$$E^{gnd} = \varepsilon^{gnd} \sigma {\Theta_0^{air}}^4 + (1-\varepsilon^{gnd}) R^{sky}(0) \quad (7-10)$$

where;

ε^{gnd} is the emissivity of the ground (assumed to have the value 0.95);

$R^{sky}(0)$ is the long-wave sky irradiance at the ground surface.

Integrating this over the solid angle of ground visible from an inclined surface we obtain the ground long-wave irradiance on the surface:

$$R^{gnd}(\gamma) = E^{gnd} \sin^2\left(\frac{\gamma}{2}\right) \quad (7-11)$$

Long-Wave Radiation Balance at External Building Surfaces

The total long-wave flux incident on an external building surface from its environment is

$$R^{env} = R^{sky}(\gamma) + R^{gnd}(\gamma) \quad (7-12)$$

The surface absorbs a fraction ε^{ext} of this radiation, where ε^{ext} is the surface emissivity. If its area is A, the total long-wave radiation absorbed by the surface is therefore

$$\varepsilon^{ext} A R^{env} \quad (7-13)$$

The surface area, A, used in Tas is the *component area, A* (the mean of the internal and external surface areas provided by 3D-Tas):

The surface also radiates, by virtue of its absolute temperature, a long-wave radiant flux

$$R^{ext} = \varepsilon^{ext} {\Theta^{ext}}^4 \quad (7-14)$$

where:

ε^{ext} is the emissivity of the surface;

Θ^{ext} is the absolute temperature of the surface (K).

In Tas this expression is linearised as follows:

$$R^{ext} = \varepsilon^{ext} {\Theta_{(ref)}^{ext}}^4 + 4\varepsilon^{ext} {\Theta_{(ref)}^{ext}}^3 \left(\Theta^{ext} - \Theta_{(ref)}^{ext}\right) \quad (7-15)$$

where:

$\Theta_{(ref)}^{ext}$ is the reference absolute temperature (Kelvin) on which the linearisation is based, which Tas takes to be the absolute surface temperature at the previous time-step.

In terms of the Celsius surface temperature the linearisation can be written

$$R^{ext} = \varepsilon^{ext}\Theta_{(ref)}^{ext\ 4} + 4\varepsilon^{ext}\Theta_{ref}^{ext\ 3}(273.15 + T^{ext} - \Theta_{(ref)}^{ext}) \quad (7-16)$$

$$= h^{rad,ext}(273.15 + T^{ext}) - 3\varepsilon^{ext}\Theta_{(ref)}^{ext\ 4} \quad (7-17)$$

$$= R^{(ref)} + h^{rad,ext}T^{ext} \quad (7-18)$$

where:

T^{ext} is the external surface temperature (Celsius);

$$h^{rad,ext} = 4\varepsilon^{ext}\Theta_{(ref)}^{ext\ 3} \quad (7-19)$$

$$R^{(ref)} = 273.15h^{rad,ext} - 3\varepsilon^{ext}\Theta_{(ref)}^{ext\ 4} \quad (7-20)$$

The net radiation gain to the surface, $q^{rad,ext}$ (Watts), is therefore (from 7-13 and 7-18)

$$q^{rad,ext} = \varepsilon^{ext}AR^{env} - AR^{ext} \quad (7-21)$$

$$= \varepsilon^{ext}AR^{env} - A(R^{(ref)} + h^{rad,ext}T^{ext}) \quad (7-22)$$

Internal Long-Wave Radiation

Long-wave Radiation Exchange in Zones

Radiant exchange between room surfaces is modelled in Tas using Carroll's MRT method[3]. This is one of a class of methods which simplify the complex radiant exchanges between the surfaces of a room by coupling each surface to a fictitious *MRT (mean radiant temperature) node*. This vastly reduces the amount of computation involved with little loss of accuracy.

In Carroll's method, the surfaces are first treated as black body radiators. The *i*'th surface is assumed to be coupled to the MRT node with a coupling coefficient

$$G_i = h^{rad,\mathrm{int}}A_iF_i \quad (7-23)$$

where:

A_i is the *component area* of surface *i* (the mean of the internal and external surface areas provided by 3D-Tas);

F_i is the view factor (as yet undetermined) between surface *i* and the MRT node;

$h^{rad,\mathrm{int}}$ is a linearised radiative heat transfer coefficient derived by differentiating the Stefan-Boltzmann radiation equation:

$$h^{rad,\mathrm{int}} = 4\sigma\Theta_{(ref)}^{zone\ 3} \quad (4-24)$$

(where $\sigma = 5.6697\times10^{-8}$ W/m^{2}K^4 is the Stefan-Boltzmann constant; $\Theta_{(ref)}^{zone}$ is a reference absolute temperature which should be close to the mean temperature of the surfaces. In Tas, we assume $\Theta_{(ref)}^{zone} = 288.15$ K (or 15C).)

Under this assumption, the temperature of the MRT node is

$$T^{MRT(C)} = \frac{\sum\left(G_k T_k^{\text{int}}\right)}{\sum G_k} \quad (7-25)$$

where

$T^{MRT(C)}$ is the mean radiant temperature in Carroll's method;

T_k^{int} is the temperature of internal surface k,

and the radiant heat transfer (Watts) from surface *i* to the MRT node is

$$-q_i^{rad,\text{int}} = G_i(T_i^{\text{int}} - T^{MRT(C)}) = G_i\left(T_i - \frac{\sum G_K T_K}{\sum G_K}\right) \quad (7-26)$$

(Note the minus sign on the left of this equation: we adopt the convention that radiant and convective surface heat transfer quantities, *q*, are positive in the direction from the environment to the surface.) The F_i are determined from the requirement that the self shape factor for each surface must be zero. This means that all the radiation from surface *i* must be intercepted by other surfaces, which in turn translates to the mathematical requirement:

$$\frac{\mathrm{d}(-q_i^{rad,\text{int}})}{dT_i^{\text{int}}} = h^{rad,\text{int}} A_i \quad (7-27)$$

So from equation 7-26,

$$G_i - \frac{G_i^2}{\sum G_k} = h^{rad,\text{int}} A_i \quad (7-28)$$

Expressing this in terms of F_i using equation 7-23 we find

$$F_i = \frac{1}{1 - \frac{A_i F_i}{\sum A_k F_k}} \quad (7-29)$$

Carroll's method derives F_i from the surface areas A_j by applying equation 7-29 iteratively, starting with all F_i set to 1. The iterative process converges provided that no surface accounts for more than half the total surface area of the zone (which is impossible in practice). Tas flags an error if one surface accounts for more than 49.5% of zone surface area. The final F_i are all greater than 1, which reflects the fact that a component of T_i is present in $T^{MRT(C)}$.

Using equation 7-26 it is a simple matter to derive the effective shape factor, F_{ij}, mediating thermal exchange between surface i and surface j in this approximation:

$$h^{rad,\text{int}} A_i F_{ij} = \frac{\mathrm{d}(-q_i^{rad,\text{int}})}{\mathrm{d}T_j^{\text{int}}} = \frac{G_i G_j}{\sum G_K} \quad (i <> j) \quad (7-30)$$

which can be expressed (using equation 7-23) as

$$A_i F_{ij} = \beta_i \beta_j \quad (i <> j) \quad (7-31)$$

where $\beta_i = \dfrac{A_i F_i}{\sqrt{\sum A_K F_K}} \quad (7-32)$

The self shape factor for all surfaces is zero:

$$F_{ii} = 0 \quad (7-33)$$

We shall refer to the approximate shape factors F_{ij} as *area-based shape factors*. Unlike F_i , F_{ij} are always less than 1. Equation 7-31 provides an efficient means for storing these coefficients, and from this equation it is clear that they satisfy the reciprocity theorem for shape factors:

$$A_i F_{ij} = A_j F_{ji} \quad (7-34)$$

Area-based shape factors provide an excellent approximation to exact shape factors for rooms with aspect ratios approximating to unity. The approximation is somewhat less good in the case of the end-walls of long, narrow rooms such as corridors. Such walls are, however, seldom important thermally. Another use is found for area-based shape factors in the Tas algorithm for the distribution of diffuse solar radiation.

Having thus established the coupling coefficients G_i for *black* surfaces, Carroll deals with the case of 'grey' surfaces using the Oppenheim *surface resistance* concept. Oppenheim[4] has shown that in radiation networks, surface emissivity can be correctly modelled simply by inserting at each surface (*i*) a resistor with coupling coefficient

$$G_i^{emis} = \frac{h^{rad,\mathrm{int}} A_i \varepsilon_i^{\mathrm{int}}}{1-\varepsilon_i^{\mathrm{int}}} \quad (7-35)$$

where:

$\varepsilon_i^{\mathrm{int}}$ is the emissivity of internal surface i .

With this modification the Carroll coupling coefficient G_i' is replaced by G_i ', where

$$\frac{1}{G_i'} = \frac{1}{G_i} + \frac{1}{G_i^{emis}} \quad (7-36)$$

In summary, the Carroll MRT method achieves an approximation to the true radiant exchange between surfaces in which:

1. Energy balance is assured.
2. No surface radiates to itself.
3. Reciprocity is satisfied.
4. The shape factors provide a good approximation to reality in most cases.
5. The effects of emissivity are correctly accounted for.
6. Radiant exchange is calculated very efficiently.

There are three approximations in the method:

1. Radiant exchange is linearised.
2. Shape factors are calculated solely on the basis of surface area, and are therefore approximations to the true shape factors.
3. The emissivity of the room air is neglected.

A paper comparing radiant interchange algorithms[5] has concluded:
In general the [Carroll] MRT method is subject to smaller errors than other methods of computing radiant interchange in buildings even when coplanar surfaces get no special treatment. As a result, the MRT Network method is

simpler to use than other methods because it does not require as detailed a building description.

It should be noted that the mean radiant temperature used in Carroll's method for zone surface radiation exchange is defined in a different way from the *Mean Radiant Temperature* available as an output from Tas - see Section **Mean Radiant Temperature**.

Long-Wave Radiation Exchange in a Cavity

Radiation across a construction cavity is calculated using the linearised radiant heat transfer coefficient

$$h^{rad} = \frac{4\sigma {\Theta_{(ref)}^{cav}}^{3}}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1} \quad (7-37)$$

where

$\sigma = 5.6697\times10^{-8}$ W/m²K⁴ is the Stefan-Boltzmann constant

$\Theta_{(ref)}^{cav} = 283$ K is a reference absolute temperature for the linearisation of the Stefan- Boltzmann equation (the assumed mean absolute temperature of the surfaces),

$\varepsilon_1, \varepsilon_2$ are the emissivities of the surfaces facing into the cavity (which are attributes of the neighbouring solid layers).

The validity of this expression can readily be checked using the Oppenheim *surface resistance* concept (equation 7-35).

Long-wave Surface Characteristics

The long-wave characteristics of building materials are embodied in the *emissivity* parameter. The emissivities of the internal and external surfaces of a construction are set to the emissivities of its surface layers. Material emissivities are also used, as explained above, to calculate the radiant exchange characteristics of cavities within a construction.

Which surface of a construction is *internal* and which *external* is set in the construction definition. As explained in Section **Conduction Characteristics of Building Elements**, this labelling is reversed for zone surfaces with negative *Building Element Number*.

Zone Divides

The surfaces known as zone divides, which are used in Tas to partition zones where there is no physical barrier, are modelled as highly conducting, non-convecting surfaces which are perfectly transparent to solar radiation and which are assigned an emissivity of one.

References

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3. J.A. Carroll. An MRT method of computing radiant energy exchange in rooms. Proceedings of the 2nd Systems Simulation & Economic Analysis Conference, San Diego, 1980, pp. 343-348.

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Solar Radiation

Solar Radiation Incident on The Building Exterior

Direct And Diffuse Solar Radiation

The Tas weather file provides hourly values for global $\left(I_{hor}^{glob}\right)$ and diffuse $\left(I_{hor}^{dif}\right)$ solar radiation measured on the horizontal (in W/m^2). The direct radiation on the horizontal $\left(I_{hor}^{dir}\right)$ is the difference between these quantities:

$$I_{hor}^{dir} = I_{hor}^{glob} - I_{hor}^{dif} \quad (8-1)$$

To calculate the direct normal (beam) radiation intensity, a knowledge of sun position is required. This is calculated at hourly intervals using the Astronomical Almanac's algorithm[1]. Sun position is derived in the form of a unit vector pointing towards the sun:

$$\mathbf{u} = \left(u_1, u_2, u_3\right) \quad (8-2)$$

The direct normal (beam) solar intensity is then obtained as

$$I^{beam} = \frac{I_{hor}^{dir}}{u_3} \quad (8-3)$$

From this the solar beam vector can then be calculated:

$$\mathbf{I}^{\mathbf{beam}} = -I^{beam}\mathbf{u} \quad (8-4)$$

This vector points in the direction of the solar beam and its magnitude is the direct normal solar radiation flux.

Direct Solar Radiation Incident on a Surface

The direct solar radiation incident on a surface, q^{dir} (Watts), is calculated as minus the inner product of the direct beam solar vector with the outward-pointing area vector of the surface $\left(\mathbf{A}_i\right)$, negative values being replaced by zero:

$$q^{dir} = p\left(-\mathbf{I}^{\mathbf{beam}} \cdot \mathbf{A}_i\right) \quad (8-5)$$

where

$$p(x) = \begin{cases} x & (x > 0) \\ 0 & (x \le 0) \end{cases} \quad (8-6)$$

The magnitude of the surface area vector $\mathbf{A}_i$ is taken to be the *component surface area* (A_i) which is the mean of the internal and external surface areas provided by 3D-Tas.

In appropriate cases the direct incident radiation is reduced by a shading factor read from a shading file or by calculations of the effects shading features (overhangs, side-fins or recesses). These shading calculations may be applied to both transparent and opaque elements. For transparent elements the cosine of the angle of incidence is also obtained from the inner product and used in the calculation of transmission and absorption in the element.

Diffuse & Ground-Reflected Solar Radiation Incident on a Surface

The diffuse component of sky radiation is assumed to be isotropically distributed over solid angle. The diffuse solar radiant power incident on a surface from the sky, $\mathscr{I}^{sky}$ (Watts), is consequently the following function of the surface tilt:

$$\mathscr{I}^{sky} = A_i I_{hof}^{dif} \cos^2\left(\frac{\gamma}{2}\right) \quad (8-7)$$

where γ is the tilt angle from the horizontal.

The ground-reflected radiation is also assumed to be isotropic. The incident ground-reflected radiation, $\mathscr{I}^{gnd}$, is the product of the component area (A_i), the global radiation incident radiation on the horizontal plane, the ground reflectance ρ^{gnd} (which the user may set), and an angle factor arising from integration over solid angle:

$$\mathscr{I}^{gnd} = A_i \rho^{gnd} I_{hor}^{glob} \sin^2\left(\frac{\gamma}{2}\right) \quad (8-8)$$

Tas does not account for diffuse radiation shading.

Solar Transmission & Absorption

Solar Transmission & Absorption Characteristics

Normal Incidence

The solar transmission and absorption characteristics of transparent constructions are derived in advance of simulation from the properties of its constituent layers. Each of these layers is either a Transparent Layer or a Gas Layer. Transparent Layers include glass panes, layers of transparent insulation and blinds. Gas Layers are usually air gaps but may be Argon filled cavities etc. For the purposes of solar radiation calculations Transparent Layers are represented by a transmittance and two reflectances, all applying at normal incidence. The two reflectances relate to radiation from the two sides of the element, to allow for glass types with special coatings and blinds with different finishes on each side. Gas layers are assumed to be totally transparent.

To calculate the characteristics of a glazing construction a normal ray is beamed into the construction from each side in turn. Reflections and absorptions are calculated and successive reflected rays are traced until residuals become small. This calculation yields an overall transmittance, and (for rays from each side) a reflectance and a distribution of absorptance through the construction, all applying at normal incidence.

Using the assumed linearity of heat transfer in the glazing construction, the absorptance distributions are then reduced to equivalent absorptances on the outside and inside surfaces. Absorptance α_p occurring at a point within the construction having thermal resistances r_p^{ext} and r_p^{int} to the inside and outside surfaces, respectively, can be shown to be equivalent to absorptance α_p^{ext} at the outside surface and absorptance α_p^{int} at the inside surface, where

$$\alpha_p^{ext} = \frac{\alpha_p r_p^{\text{int}}}{r_p^{ext} + r_p^{\text{int}}} \qquad (8-9)$$

$$\alpha_p^{\text{int}} = \frac{\alpha_p r_p^{ext}}{r_p^{ext} + r_p^{\text{int}}} \qquad (8-10)$$

This reduction is made possible by a variant of Thevenin’s theorem in electrical circuit theory. It provides a complete characterisation, in terms of a small number of parameters, of the construction’s behaviour as it affects its environment.

The normal incidence transmittances and absorptances of a construction are calculated whenever a construction is created or edited, and may be viewed in Tas as *derived parameters* of the construction. Absorptances are given for rays from both directions. Transmittance is always the same for the two directions, so a single value is given for this parameter.

The directional absorption characteristics are identified with reference to the *internal* and *external* surfaces defined in the construction definition. As explained in Section **Conduction Characteristics of Building Elements**, this labelling is reversed for zone surfaces with negative *Building Element Number*.

Angular Characteristics - Fresnel Equations

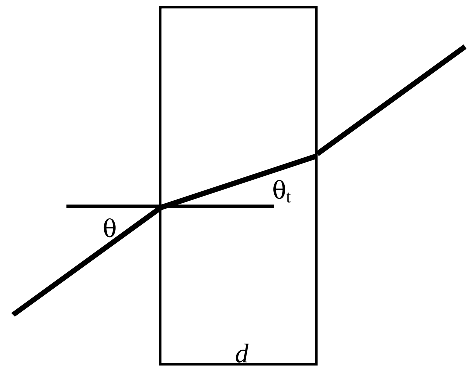
Transmission and absorption characteristics of transparent constructions at non-normal incidence are calculated from the Fresnel equations[2].

Figure 8.1 shows a ray passing through a single layer of material. The transmission angle θ_t is related to the incidence angle θ by Snell’s Law:

$$\sin\theta_t = \frac{\sin\theta}{n} \qquad (8-11)$$

where n is the refractive index of the layer.

Figure 8.1 **Transmission through a single-layer transparent construction**



The transmittance at each of the two interfaces is given by the Fresnel equations for polarisation perpendicular and parallel to the plane of incidence:

$$t_{perp}(\theta) = \frac{4nc(\theta)}{(1+nc(\theta))^2} \quad (8-12)$$

$$t_{para}(\theta) = \frac{4nc(\theta)}{(n+c(\theta))^2} \quad (8-13)$$

where:

$$c(\theta) = \frac{\cos\theta_t}{\cos\theta} \quad (8-14)$$

and, from equation 8-11,

$$\cos\theta_t = \sqrt{1-\frac{\sin^2\theta}{n^2}} \quad (8-15)$$

If the layer is absorbing, the factor by which the ray is attenuated on passing through it will be

$$b(\theta) \quad = \exp\left(\frac{-xd}{\cos\theta_t}\right) \quad (8-16)$$

$$= b_0^{1/\cos\theta_t} \quad (8-17)$$

where:

d is the thickness of the layer (m);
x is the extinction coefficient for the material (m^{-1});
$b_0 = b(0)$ is the normal incidence attenuation factor.

Analysis of an infinite number of internal reflections shows that the transmittance and absorptance of the complete layer, for the two polarisations, are given by

$$\tau_{perp}(\theta) = \frac{bt_{perp}^2}{1-r_{perp}^2 b^2} \quad (8-18)$$

$$\tau_{para}(\theta) = \frac{bt_{para}^2}{1-r_{para}^2 b^2} \quad (8-19)$$

$$\alpha_{perp}(\theta) = \frac{(1-b)t_{perp}}{1-r_{perp}b} \quad (8-20)$$

$$\alpha_{para}(\theta) = \frac{(1-b)t_{para}}{1-r_{para}b} \quad (8-21)$$

where

$$r_{perp} = 1 - t_{perp} \quad (8-22)$$

$$r_{para} = 1 - t_{para} \quad (8-23)$$

The transmittance and absorptance of the layer for randomly polarised radiation are found be averaging over the two polarisations:

$$\tau(\theta) = \frac{\tau_{perp}(\theta) + \tau_{para}(\theta)}{2} \quad (8-24)$$

$$\alpha(\theta) = \frac{\alpha_{perp}(\theta) + \alpha_{para}(\theta)}{2} \quad (8-25)$$

Applying these results to normal incidence, using a subscript *o* to denote evaluation at $\theta = 0$, we find

$$\tau_0 = \frac{b_0 t_0^2}{1 - r_0^2 b_0^2} \quad (8-26)$$

$$\alpha = \frac{(1-b_0)t_0}{1-r_0 b_0} \quad (8-27)$$

where:

$$t_0 = \frac{4n}{(n+1)^2} \quad (8-28)$$

$$r_0 = 1 - t_0 \quad (8-29)$$

Owing to a lack of detailed refractive index data for all layers, the Fresnel equations are applied not to the glazing construction itself but to an approximately equivalent single-layer construction.

Two alternative strategies are possible for achieving an approximate match between the single-layer construction and the actual construction. Both are based on matching the normal transmittance and absorptance.

In Strategy 1 the refractive index, *n*, is set to that of glass $(n = 1.52)$, on the basis that this is the most common glazing material. The interface transmittances, $t_{perp}(\theta), t_{para}(\theta)$, are then fixed by equations 8-12 and 8-13, and the normal interface transmittance, t_0, by equation 8-28 $(t_0 = 0.9574)$. We can now use equation 8-26 to select b_0 to match the layer normal transmittance, τ_0. Solving the quadratic we find

$$b_0 = \left(\frac{t_0^2}{2r_0^2\tau_0}\right)\left(-1 + \sqrt{1 + \frac{4\tau_0^2 r_0^2}{t_0^4}}\right) \quad (8-30)$$

$$\cong \left(\frac{\tau_0}{t_0^2}\right)\left(1 - \frac{\tau_0^2 r_0^2}{t_0^4} + \frac{2\tau_0^4 r_0^4}{t_0^8}\right) \quad (8-31)$$

Equation 8-31 is an excellent approximation which avoids certain numerical problems inherent in equation 8-30. Using this value of b_0 in equation 16 allows us to obtain the angular transmittance function from equations 8-18, 8-19 and 8-24.

In this strategy we use a *different* value of b_0 to match the layer normal absorptance, α_0, using equation 8-27. This value of b_0 is then used in equation 8-17 to obtain the angular absorptance function from equations 8-20, 8-21 and 8-25.

Strategy 2 matches both τ_0 and α_0 with a single construction, by allowing both b_0 and the refractive index n to take appropriate values. Eliminating b_0 from equations 8-26 and 8-27 we find

$$(t_0 - \alpha_0)(t_0 - \alpha_0 r_0) = (t_0(1 + r_0) - 2\alpha_0 r_0)\tau_0 \quad (8-32)$$

and using equation 8-29 we solve the resulting quadratic to obtain an expression for t_0. The refractive index, n *n*, is then obtained from equation 8-28 as the solution to another quadratic, and b_0 is obtained from equation 8-27.

Tas adopts Strategy 1 wherever possible, as this gives the best match to the true angular characteristics for a range of double glazing constructions. Strategy 2 is used where Strategy 1 yields a value of b_0 (either for transmittance or absorptance) which is outside the range (0,1) and which is thus physically impossible. In both cases, the strategy is applied in turn to rays entering from each side of the construction. The distribution of absorptance through the construction is assumed to be the same as at normal incidence.

This procedure produces accurate angular characteristics for the most common glazing systems, and acceptable angular characteristics in all cases.

The angular characteristics are calculated and stored for 7 sample values of cosq spaced at intervals of 1/6 between 0 and 1. During simulation the angular characteristics are applied to the transmission and absorption of direct radiation, both when it enters the building and when it passes through an inter-zone partition. Linear interpolation is applied to values of $\cos\theta$ between the 7 sample $\cos\theta$ values, a procedure which achieves a more accurate interpolation in typical cases than interpolation in the θ domain.

Hemispherical Characteristics

For the transmission and absorption of diffuse radiation, hemispherical characteristics are calculated by numerically integrating the angular characteristics:

$$\chi^{hem} = \int_0^1 \chi(\cos\theta)\cos\theta \mathrm{d}(\cos\theta) \quad (8-32).$$

where χ denotes either or transmittance (τ) or absorptance (α). The integral is evaluated on the basis of linear interpolation over $\cos\theta$ between the 7 sample values.

Zone Divides

The surfaces known as *zone divides*, which are used in Tas to partition zones where there is no physical barrier, are treated differently from other transparent surfaces. Zone divides are assumed to be perfectly transparent to solar radiation at all angles of incidence.

Opaque Building Components

Opaque building components are assumed to have isotropic solar absorptance characteristics: the absorptance is assumed to be equal at all angles.

Pilkington & CIBSE Parameters

Options are provided in Tas for calculating and displaying CIBSE glazing parameters (U values, solar gain factors etc.) and Pilkington glazing parameters, as defined in their literature. These calculations are described in the A-Tas User Manual, Section **Constructions**.

Solar Transmission & Absorption Calculations

During simulation, solar radiation entering the building through windows and other transparent building components is distributed over the building surfaces as described in Section *8.3 Solar Distribution Inside The Building*. In the distribution process, the radiation absorbed by the surfaces of each zone is totalled and stored in the quantities $q_i^{sol,\mathrm{int}}$ and $q_i^{sol,ext}$, where

$q_i^{sol,\mathrm{int}}$ is the solar radiation (Watts) absorbed on internal surface i of the zone;

$q_i^{sol,ext}$ is the solar radiation (Watts) absorbed on external surface i of the zone;.

In the case of transparent components the *internal* and *external* labels on these quantities refer to the *effective* absorption on the two surfaces in the Thevenin equivalent circuit.

In the case of the external surfaces of opaque building components which are exposed to the external environment, $q_i^{sol,ext}$ is given by the simple expression

$$q_i^{sol,ext} = \alpha_i^{ext}(\mathcal{I}_i^{dir} + \mathcal{I}_i^{sky} + \mathcal{I}_i^{gnd}) \quad (8-33)$$

where α_i^{ext} is the external solar absorptance of the component and $\mathcal{I}_i^{dir}$, $\mathcal{I}_i^{sky}$ and $\mathcal{I}_i^{gnd}$ are the incident solar powers appearing in equations 8-5, 8-7 and 8-8. For an exposed transparent component, $q_i^{sol,ext}$ will have a contribution of the same form, but it will also have a contribution from radiation returning from the zone through the component.

Solar Distribution Inside The Building

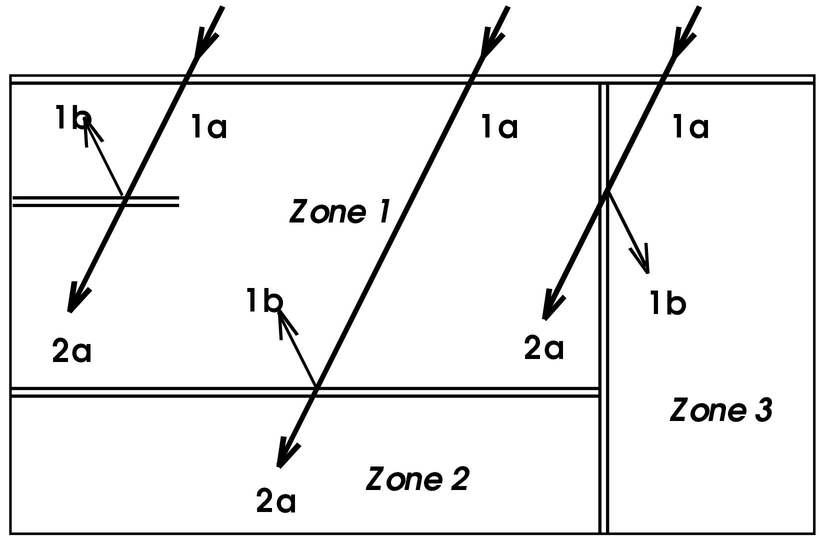
Once it has arrived in a zone, solar radiation is distributed over the zone's internal surfaces. Some of this radiation may be transmitted back out of the building through windows, and some may be transmitted through further transparent constructions to other zones.

Solar Radiation Distribution Algorithm

The final destination of solar radiation entering the building through transparent surfaces is determined in a series of radiation distributions. Each distribution is carried out in turn for each of the building's zones. Its purpose is to distribute the admitted radiation among the zone's surfaces by modelling the repeated bouncing of radiation around the zone. If, in this process, radiation falls on a transparent surface, transmission occurs. Radiation transmitted in this way to other zones is dealt with in the next distribution.

Primary Distribution

Figure 8.2 Primary Distribution of Direct Radiation



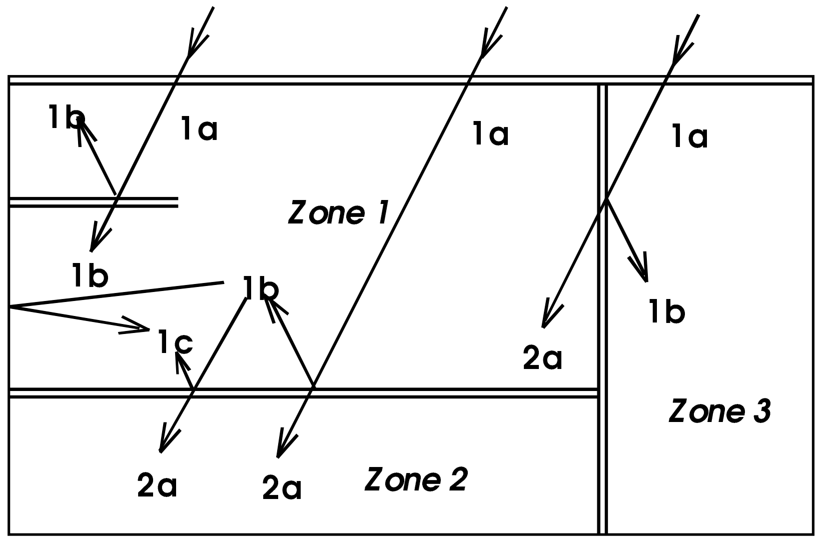
Figures 8.2 and 8.3 illustrate the primary distribution of direct and diffuse radiation entering a building containing three zones. Transparent walls are represented by double lines. Direct and diffuse radiation is represented by thick and thin lines, respectively. Numbers indicate the *order* of the radiation distribution (1=primary distribution, 2=secondary distribution, etc.) with subdivisions as follows:
a = admitted radiation - radiation entering the space through a transparent element.
b = once-scattered radiation - radiation which has undergone one encounter (reflection or transmission) with a zone surface in the current distribution.
c = residual radiation - radiation which has undergone two encounters with zone surface in the current distribution.

A facility exists in 3D-Tas for designating certain external surfaces tracking (indicated by a 'T' in the Building Element table). For such surfaces the direct component of solar radiation is beamed into the building and tracked geometrically onto other surfaces of the zone at each hour. The data describing this tracking process is stored on shading files.

Direct radiation admitted into the building (**1**) is thus in general composed of tracked and non-tracked components. Tracked direct radiation is distributed over the internal zone surfaces according to the data on the shading file. Non-tracked primary direct radiation is distributed amongst surfaces facing the sun in proportion to their projected area in the beam direction.

At each receiving surface the direct radiation is absorbed, reflected and in some cases transmitted, in accordance with the properties of the receiving surface. The reflected radiation becomes *once-scattered primary radiation* (1b). Any transmitted radiation which remains in the building becomes a source of secondary direct radiation (**2**) in the appropriate zone. Any direct radiation transmitted back out of the building, or through *Null Link* surfaces, plays no further part in the analysis.

Figure 8.3 Primary Distribution of Diffuse Radiation



Primary diffuse radiation (1) is distributed in three stages:

1a - Admitted primary diffuse radiation. Sky and ground-scattered portions of the admitted primary diffuse radiation are treated separately. For each admitting surface, the sky radiation is distributed over the zone's surfaces in accordance with estimated shape factors from the surface to the sky as seen through the window. The ground radiation is distributed in an analogous way.

The shape factors are generated in a three-stage process. A set of *area-based shape factors* are first obtained for each pair of zone surfaces. These are calculated using only the areas of the surfaces, as described under Internal Long-wave Radiation. Adjustments are then made to the area-based shape factors in the case of pairs of surfaces with similar orientation, to yield a set of *orientation-adjusted shape factors*. The adjustment ensures that the shape factor for no pair of surfaces exceeds the theoretical maximum shape factor for two surfaces in the given mutual orientation, and in particular, that the shape factor between two coplanar surfaces is zero. Any corrections for close orientation are distributed amongst other surfaces so that the sum of the shape factors for a given surface remains unity. Finally, shape factors from each surface to a) the sky and b) the ground, viewed through each window, are estimated using the orientation-adjusted shape factor between the receiving surface and the window, together with the orientations of these surfaces.

1b - Once-scattered primary radiation. This radiation, which consists of reflected primary direct and diffuse radiation, plus primary diffuse radiation which has been transmitted through internal zone partitions, is distributed over the zone's surfaces on the basis of the area-based shape factors. These are such that radiation emanating from a particular surface is distributed over all the *other* surfaces in the zone - the source surface being excluded from the distribution. Absorption, reflection and transmission occurs at each receiving surface, with the reflected portions being collected in the *residual primary radiation* pool (1c). *Null Link* surfaces receive special treatment in the distribution of once-scattered diffuse radiation. These are surfaces for which conditions on the far side are assumed to be the same as those in the zone under consideration. In accordance with this principle, radiation transmitted through such surfaces is assumed to be matched by an identical amount of radiation transmitted from the other side. This radiation is added to the residual radiation pool.

1c - Residual primary radiation. The residual pool of radiation is distributed according to an (area x *acceptance*) weighting, where
acceptance = 1 - reflectance
(with the exception of *Null Link* surfaces, where, in accordance with the assumed boundary condition for such surfaces, *acceptance* = absorptance). This produces

the same distribution as an infinite number of distributions on a simple area-weighted basis.

Secondary Distribution

If any radiation has been transmitted through inter-zone partitions in the primary distribution, a secondary distribution is carried out. This follows exactly the same pattern as the primary distribution, with the exception that there is no tracked radiation.

Tertiary (etc.) Distribution

The number of distributions is currently set to 8. These all follow the same pattern except the last, where, to ensure energy conservation, inter-zone transmission is returned to the source zone. This is achieved as follows:

1. Direct radiation transmitted through inter-zone partitions is returned to the source zone as once-scattered diffuse radiation.
2. Admitted diffuse radiation transmitted through inter-zone partitions is returned to the source zone as once-scattered radiation.
3. Once-scattered radiation transmitted through inter-zone partitions is returned to the source zone as residual radiation.
4. Residual radiation is distributed according to a modified (area x acceptance) weighting from which transmission to other zones is excluded.

The number of distributions carried out by the software puts a limit on the number of zone boundary surfaces through which radiation may be transmitted. With 8 distributions, radiation will be transmitted through a maximum of 8 such surfaces.

Surface Solar Gain and Zone Solar Gain

The *surface solar gain* ($q_i^{sol,\mathrm{int}}$, $q_i^{sol,ext}$) calculated by Tas for an internal or external zone surface is the total solar radiation (in kW) absorbed by the surface in the course of the solar distributions described above.

In the case of a transparent construction, radiation is actually absorbed within the construction, rather than at its surfaces, and in this case the *surface solar gains* are calculated on the basis of the equivalent surface absorptances discussed in Section **Solar Transmission and Absorption**.

Zone solar gain is the sum of the *surface solar gains* for all the surfaces facing into the zone:

$$Q^{sol} = \sum_i q_i^{sol,\mathrm{int}} \qquad (8-34)$$

References

1. Joseph J. Michalsky. The Astronomical Almanac's algorithm for approximate solar position (1950-2050). *Solar Energy* Vol. 40, No. 3. pp.227-235, 1988.
2. Eugene Hecht and Alfred Zajac. Optics. Addison-Wesley, 1974.

Casual Gains

Specification of Gains

The Tas *Internal Conditions* facility allows the user to specify the following casual gains as time-varying quantities in each zone:

W^{light} *: Lighting Gains* (W/m^2)

W^{occS} *: Occupancy Sensible Gains* (W/m^2)

W^{occL} *: Occupancy Latent Gains* (W/m^2)

W^{equS} *: Equipment Sensible Gains* (W/m^2)

W^{equL} *: Equipment Latent Gains* (W/m^2)

The gains are expressed in terms of Watts per square metre of floor area. The entered figures are multiplied by the zone floor area displayed in the Tas *Surface Geometry* facility, A^{floor}, which is the internal area of the floor. The sensible gains in the zone (Watts) are thus

$$Q^{light} = A^{floor} W^{light} \qquad (9-1)$$

$$Q^{occS} = A^{floor} W^{occS} \qquad (9-2)$$

$$Q^{equS} = A^{floor} W^{equS} \qquad (9-3)$$

These sensible gains may be viewed after simulation in the *Sensible Load Breakdown* table for any zone or combination of zones.

Latent gains from occupants and equipment represent sources of moisture in the zone. The rate of addition of moisture in the form of water vapour (kg/s) is related to the latent gain by

$$w^{occL} = \frac{A^{floor} W^{occL}}{L} \qquad (9-4)$$

$$w^{equL} = \frac{A^{floor} W^{equL}}{L} \qquad (9-5)$$

where

L is the latent heat of evaporation of water, for which Tas assumes the value 2450 kJ/kg[1] (a value appropriate to a temperature of about 20C and standard atmospheric pressure).

The total moisture addition rate in the zone from casual gains is

$$w^{gains} = w^{occL} + w^{equL} \qquad (9-6)$$

Radiant Proportion

For each type of sensible gain, and for each zone and day-type, a *radiant proportion* and a *view coefficient* must be specified.

Radiant proportion is a number between 0 and 1 indicating the proportion of the sensible heat gain which is in the form of radiant heat. If the radiant proportions for lights, occupants and equipment are p^{light}, p^{occ} and p^{equ}, respectively, then the total radiant gain will be

$$Q^{gains,rad} = p^{light}Q^{light} + p^{occ}Q^{occS} + p^{equ}Q^{equs} \quad (9-7)$$

During simulation, this part of the gain is distributed over the zone's interior surfaces on the basis of an (area x emissivity) weighting. Surface i thus receives a radiant heat input

$$q_i^{gains,\text{int}} = \frac{Q^{gains,rad}\varepsilon_i^{\text{int}}A_i}{\sum \varepsilon_k^{\text{int}}A_k} \quad (9-8)$$

where

$\varepsilon_i^{\text{int}}$ is the emissivity of internal surface i;

A_i is the *component area* of surface i.

This (area x emissivity) weighting is consistent with the assumption that both direct and scattered radiant gains are incident on surfaces in proportion to their area. Under the same assumption we find that the direct and scattered portions of the radiant flux from casual gains incident on surface i are given by

$$W_i^{dir\ gains,\text{int}} = \frac{Q^{gains,rad}}{\sum A_K} \quad (9-9)$$

$$\begin{aligned} W_i^{scat\ gains,\text{int}} &= \frac{q_i^{gains,\text{int}}}{A_i} - W_i^{dir\ gains,\text{int}} \\ &= Q^{gains,rad}\left(\frac{1}{\sum \varepsilon_k^{\text{int}}A_k} - \frac{1}{\sum A_k}\right) \quad (9-10) \end{aligned}$$

Equation 9-10 is used in the calculation of the Tas output *Mean Radiant Temperature* (see Section **Mean Radiant Temperature**).

The convective portion of the gain, which we denote by $Q^{gains,conv}$, is input to the zone air:

$$Q^{gains,conv} = (1-p^{light})Q^{light} + (1-p^{occ})Q^{oocS} + (1-p^{equ})Q^{equS} \quad (9-11)$$

View Coefficient

View coefficient is a parameter specifying the degree to which radiation from the emitter contributes to the occupants' perception of the radiant environment. This parameter is used solely in the calculation of the Tas outputs *Mean Radiant Temperature* and *Resultant Temperature*.

The A-Tas User Manual provides formulae and tables to aid the selection of suitable values of *radiant proportion* and *view coefficient* for gains of various types.

References

1. CIBSE Guide, Part B3, page B3-11. The Chartered Institution of Building Services Engineers. Delta House, 222 Balham High Road, London SW12 9BS.

Plant

Plant Parameters in Internal Conditions

Heating; Cooling; Humidification & Dehumidification

The following parameters may be set in the Tas *Internal Conditions* facility to specify the operation of heating, cooling, humidification and dehumidification plant for each zone and for each day-type:

- Temperature Upper Limit (C)
- Temperature Lower Limit (C)
- Proportional Control Band (K)
- On/Off Control Band (K)
- Humidity Upper Limit (%)
- Humidity Lower Limit (%)
- Plant Maximum Outside Temperature (C)
- Plant Off Outside Temperature (C)

and for up to 4 periods in the day:

- Time Plant On (hours)
- Time Plant Off (hours)
- Maximum Heating (kW)
- Maximum Cooling (kW)

The meanings of these parameters are explained in the user manual.

Radiant Proportion and View Coefficient

A *radiant proportion* and a *view coefficient* must be specified for heating and for cooling, for each zone and day-type. These concepts were introduced in Section . **Casual Gains**, and the same principles apply in the case of plant inputs.

The A-Tas User Manual provides formulae and tables to aid the selection of suitable values of *radiant proportion* and *view coefficient* for various heating and cooling devices.

Plant Operation: Sensible Load

We denote the sensible power input (Watts) to a zone from the plant by Q^{plantS}. This quantity, which is also referred to as the *Sensible Load*, is positive for heating and negative for cooling.

The heating and cooling controls adjust the power input as a function of zone air temperature, as described in the A-Tas User Manual, Section **Internal Conditions**. Outside temperature is also taken into account if the *Plant Maximum Outside Temperature* and *Plant Off Outside Temperature* are set or if External Frost Protection applies. The functional relationship between power input and zone air temperature, which we term the *plant control characteristic*, is assumed to consist of a series of straight-line segments. (See for example the Proportional Control diagram in the User Manual, Section **Internal Conditions**.) Within one such segment the relationship can thus be represented by

$$P_Q Q^{plantS} + P_T T^{air} = P_{QT} \quad (10-1)$$

where

P_Q , P_T and P_{TQ} are constants (with P_Q P_T non-negative in every case).

The *Radiant Proportion* parameter for heating or cooling is used to divide the heating or cooling power into radiant and convective portions. The radiant portion of plant input is

$$q_i^{plant,rad} = pQ^{plantS}$$

where p is the radiant proportion:

$$p = \begin{cases} p^{htg} & (Q^{plantS} > 0) \\ p^{c\lg} & (Q^{plantS} < 0) \end{cases} \quad (10-3)$$

where

p^{htg} and $p^{c\lg}$ are the radiant proportions for heating and cooling, respectively.

The radiant plant input is distributed over the zone's interior surfaces on the basis of an (area x emissivity) weighting. Surface i thus receives a radiant heat input

$$\begin{aligned} q_i^{plant,\text{int}} &= \frac{Q^{plant,rad}\varepsilon_i^{\text{int}} A_i}{\sum \varepsilon_k^{\text{int}} A_k} \quad (10-4) \\ &= \frac{pQ^{plantS}\varepsilon_i^{\text{int}} A_i}{\sum \varepsilon_k^{\text{int}} A_k} \quad (10-5) \end{aligned}$$

The scattered portion of the plant radiant flux incident on surface *i*, which is used in the calculation of *Mean Radiant Temperature* (see Section **Mean Radiant Temperature**), is (by analogy with equation 9-10)

$$W_i^{scat\ gains,\text{int}} = Q^{plant,rad}\left(\frac{1}{\sum \varepsilon_k^{\text{int}} A_k} - \frac{1}{\sum A_k}\right) \quad (10-6)$$

The convective portion of the gain, which we denote by $Q^{plant,conv}$, is input to the zone air:

$$Q^{plant,conv} = (1-p)Q^{plantS} \quad (10-7)$$

Plant Operation: Latent Load

The humidity controls add or remove water vapour from each zone in order to maintain the relative humidity between the *Humidity Upper* and *Lower Limits*, as described in Section *11. Zone Heat Balance*. From the required moisture addition rate, w^{plant} (kg/s), the *Latent Load,* Q^{plantL} (Watts), is calculated:

$$Q^{plantL} = Lw^{plant} \qquad (10-8)$$

where

L is the latent heat of evaporation of water, for which Tas assumes the value 2450 kJ/kg[1] (a value appropriate to a temperature of about 20C and standard atmospheric pressure).

Frost Protection; Optimum Start; Master Zone Control; Boilers & Heat Pumps; B-Tas

Various facilities for modifying the plant and control characteristics specified in Internal Conditions are available under the Tas *Plant and Controls* menu. For a description of these facilities the reader is referred to User Manual. A much wider class of systems and controls is available in the B-Tas program.

References

1. CIBSE Guide, Part B3, page B3-11. The Chartered Institution of Building Services Engineers. Delta House, 222 Balham High Road, London SW12 9BS.

Zone Heat Balance

Sensible Balance

Air Heat Balance

The zone air heat balance takes the following form, equating the rate at which heat is added to the air to the total heat gain from infiltration, ventilation, air movement, casual gains, plant and surface convection:

$$Q^{air} = Q^{\inf} + Q^{vent} + Q^{am} + Q^{gains,conv} + Q^{plant,conv} - \sum q_i^{conv,\text{int}} \qquad (11-1)$$

where

$$Q^{air} = \frac{\left(p^{air} V c_p\right)\left(T^{air} - \left\langle T^{air} \right\rangle\right)}{\Delta} \qquad (6-2\ bis)$$

$$Q^{\inf} = m^{\inf} c_p (T_0^{air} - T^{air}) \qquad (5-2\ bis)$$

$$Q^{vent} = m^{vent} c_p (T_0^{air} - T^{air}) \qquad (5-4\ bis)$$

$$Q^{am} = \sum_{s=0}^{Z} m_{sz} c_p (T_s^{air} - T^{air}) \qquad (5-6\ bis)$$

$$Q^{plant,conv} = (1-p) Q^{plantS} \qquad (10-7\ bis)$$

$$P_Q Q^{plantS} + P_T T^{air} = P_{QT} \qquad (10-1\ bis)$$

$$q_i^{conv,\text{int}} = A h_i^{\text{int}} (T^{air} - T_i^{\text{int}}) \qquad (4-14\ bis)$$

and $Q^{gains,conv}$ is calculated by the method described in Section **Casual Gains**.

Expanding equation 11-1 using the equations which follow it, we obtain

$$\frac{\left(p^{air}Vc_p\right)\left(T^{air}-\left\langle T^{air}\right\rangle\right)}{\Delta}=(m^{\inf}+m^{vent})c_p(T_0^{air}-T^{air})$$
$$+\sum_{s=0}^{Z}m_{sz}c_p(T_s^{air}-T^{air})+Q^{gains,conv}+(1-p)Q^{plantS}-\qquad(11-2)$$
$$\sum A_i h_i^{\text{int}}(T^{air}-T_i^{air})$$

in which all quantities are known except T^{air}, T_s^{air} and Q^{plantS}.

Equation 11-2 expresses a linear relationship between zone air temperature, zone surface temperatures and plant input power.

Mean Radiant Temperature Node Heat Balance

The heat balance at the Mean Radiant Temperature node is given by equation 7-25.

$$T^{MRT(C)}=\frac{\sum\left(G_k T_k^{\text{int}}\right)}{\sum G_k}\quad(7-25\,bis)$$

Internal Surface Heat Balance

The internal surface heat balance is expressed by the following relation which equates conduction heat flow into the internal surface of a component to the total heat gain from convection, long-wave radiant exchange, solar gain, casual gains and the radiant portion of plant input:

$$q_i^{cond,\text{int}}=q_i^{conv,\text{int}}+q_i^{rad,\text{int}}+q_i^{sol,\text{int}}+q_i^{gains,\text{int}}+q_i^{plant,\text{int}}\quad(11-3)$$

where

$$q_i^{cond,\text{int}}=A_i X_i^0 T_i^{\text{int}}-A_i Y_i^0 T_i^{ext}+q_i^{his\_cond,\text{int}}\quad(3-10\,bis)$$

$$q_i^{conv,\text{int}}=A_i h_i^{\text{int}}(T^{air}-T_i^{\text{int}})\quad(4-14\,bis)$$

$$q_i^{rad,\text{int}}=G_i(T^{MRT(C)}-T_i^{\text{int}})\quad(7-26\,bis)$$

$$p_i^{plant,\text{int}}=\frac{pQ^{plantS}\varepsilon_i^{\text{int}}A_i}{\sum\varepsilon_k^{\text{int}}A_k}\quad(10-5\,bis)$$

and $q_i^{sol,\text{int}}$ and $q_i^{gains,\text{int}}$ are calculated by methods described in Sections **Solar Radiation** and **Casual Gains**.

Expanding equation 11-2,

$$A_i X_i^0 T_i^{\text{int}}-A_i Y_i^0 T_i^{ext}+q_i^{his\_cond,\text{int}}=A_i h_i^{\text{int}}(T^{air}-T_i^{\text{int}})+$$
$$G_i(T^{MRT(C)}-T_i^{\text{int}})+q_i^{sol,\text{int}}+q_i^{gains,\text{int}}+q_i^{plant,\text{int}}\qquad(11-4)$$

which can be written

$$A_i X_i^0 T_i^{\text{int}}-A_i Y_i^0 T_i^{ext}+q_i^{hist\_cond,\text{int}}=A_i h_i^{conv/rad,\text{int}}(T_i^{env,\text{int}}-T_i^{\text{int}})\quad(11-5)$$

where $A_i h_i^{conv/rad,\text{int}}$ is a combined convective/radiant coupling coefficient:

$$A_i h_i^{conv/rad,\text{int}}=A_i h_i^{\text{int}}+G_i\quad(11-6)$$

and $T_i^{env,\mathrm{int}}$ is an effective internal environmental temperature (as seen by surface *i*):

$$T_i^{env,\mathrm{int}} = \frac{A_i h_i^{\mathrm{int}} T^{air} + G_i T^{MRT(C)} + q_i^{sol,\mathrm{int}} + q_i^{gains,\mathrm{int}} + q_i^{plant,\mathrm{int}}}{h_i^{conv/rad,\mathrm{int}}} \quad (11-7)$$

External Surface Heat Balance

The form taken by the heat balance at the external surface of a building component is dependent on the type of boundary condition at this surface, which in turn is determined by the Building Feature Type.

1) Building Feature Types *external* and *Transparent*.
   These Building Feature Types are exposed to outside environment on the external side. The surface heat balance therefore takes the form

   $$q_i^{cond,ext} = q_i^{cond,ext} + q_i^{rad,ext} + q_i^{sol,ext} \quad (11-8)$$

   where

   $$q_i^{cond,ext} = A_i Z_i^0 T_i^{ext} - A_i Y_i^0 T_i^{\mathrm{int}} + q_i^{hist\_cond,ext} \quad (3-12\ bis)$$

   $$q_i^{conv,ext} = A_i h^{ext} (T_0^{air} - T_i^{ext}) \quad (4-4\ bis)$$

   $$q_i^{rad,ext} = \varepsilon_i^{ext} A_i R_i^{env} - A_i (R_i^{(ref)} + h_i^{rad,ext} T_i^{ext}) \quad (7-22\ bis)$$

   and $q_i^{sol,ext}$ is calculated by the method described in Section **Solar Radiation**.

   Expanding equation 11-8,

   $$\begin{aligned} A_i Z_i^0 T_i^{ext} - A_i Y_i^0 T_i^{\mathrm{int}} + q_i^{hist\_cond,ext} &= A_i h^{ext} (T_0^{air} - T_i^{ext}) + \\ \varepsilon_i^{ext} A_i R_i^{env} - A_i (R_i^{(ref)} + h_i^{rad,ext} T_i^{ext}) &+ q_i^{sol,ext} \end{aligned} \quad (11-9)$$

   which can be written

   $$A_i Z_i^0 T_i^{ext} - A_i Y_i^0 T_i^{\mathrm{int}} + q_i^{hist\_cond,ext} = A_i h_i^{conv/rad,ext} (T_i^{env,ext} - T_i^{ext}) \quad (11-10)$$

   where $A_i h_i^{conv/rad,ext}$ is a combined convective/radiant coupling coefficient:

   $$A_i h_i^{conv/rad,ext} = A_i (h^{ext} + h_i^{rad,ext}) \quad (11-11)$$

   and $T_i^{env,ext}$ is an effective external environmental (or *sol-air*) temperature:

   $$T_i^{env,ext} = \frac{h^{ext} T_0^{air} + \varepsilon_i^{ext} R_i^{env} - R_i^{(ref)} + \dfrac{q_i^{sol,ext}}{A_i}}{h_i^{conv/rad,ext}} \quad (11-12)$$

   Equation 11-10 constitutes a linear relationship between T_i^{ext} and T_i^{int}. Such a relationship will be established for each Building Feature Type.

2) Building Feature Type *Ground Floor*:
   Here the external surface temperature is assumed to be equal to the

groundwater temperature, T^{gnd}, so the external boundary condition is represented by the simple equation

$$T_i^{ext} = T^{gnd} \quad (11-13)$$

3) Building Feature Type *internal*:
In this case the *external* surface is another surface within the same zone, so the external surface heat balance is taken care of by equation 11-3 applied to the other surface.

4) Building Feature Type *Link*:
Here the building component links to an adjacent zone. The heat balance in this case is expressed by

$$q_i^{cond,ext} = q_i^{conv,ext} + q_i^{rad,ext} + q_i^{sol,ext} + q_i^{gains,ext} + q_i^{plant,ext} \quad (11-14)$$

where the quantities on the right hand side, with superscript 'ext', are identified with the same quantities with superscript 'int' in the adjacent zone. Suppose the external surface is surface *j* of zone *s*. Expanding equation 11-14 then yields

$$A_i Z_i^0 T_0^{ext} - A_i Y_i^0 T_i^{\text{int}} + q_i^{his\_cond,ext} = A_{s,j} h_{s,j}^{\text{int}} (T_s^{air} - T_{s,j}^{\text{int}}) + G_{s,j}(T_s^{MRT(C)} - T_{s,j}^{\text{int}}) + q_{s,j}^{sol,\text{int}} + q_{s,j}^{gains,\text{int}} + q_{s,j}^{plant,\text{int}} \quad (11-15)$$

where the zone number subscript, *s*, is required for the right-hand-side quantities to indicate that these apply in the adjacent zone.

Collecting terms on the right hand side, and noting that $A_{s,j} = A_i$ and $T_{s,j}^{\text{int}} = T_i^{ext}$, this equation can be written

$$A_i Z_i^0 T_i^{ext} - A_i Y_i^0 T_i^{\text{int}} + q_i^{hist\_cond,ext} = A_i h_i^{conv/rad,ext} (T_i^{env,ext} - T_i^{ext}) \quad (11-16)$$

where

$$T_i^{env,ext} = \frac{A_i h_{s,j}^{\text{int}} T_s^{air} + G_{s,j} T_s^{MRT(C)} + q_{s,j}^{sol,\text{int}} + q_{s,j}^{gains,\text{int}} + q_{s,j}^{plant,\text{int}}}{A_i h_i^{conv/rad,ext}} \quad (11-17)$$

is an effective *environmental temperature* for the adjacent zone (as seen by surface *j*) to which the surface is coupled with a combined convective/radiant surface coupling coefficient

$$A_i h_i^{conv/rad,ext} = A_i h_{s,j}^{\text{int}} + G_{s,j} \quad (11-18)$$

Equation 11-16 represents the linear relationship which we require between T_i^{ext} and T_i^{int}.

5) Building Feature Type *Null Link*:
For a *Null Link* the boundary condition on the external surface of the wall is assumed to be the same as that on the internal surface. The internal surface boundary condition is given by equation 11-5.

$$A_i X_i^0 T_i^{\text{int}} - A_i Y_i^0 T_i^{ext} + q_i^{hist\_cond,\text{int}} = A_i h_i^{conv/rad,\text{int}} (T_i^{env,\text{int}} - T_i^{\text{int}}) \quad (11-5\ bis)$$

Assuming the same boundary condition for the external surface we have

$$A_i Z_i^0 T_i^{ext} - A_i Y_i^0 T_i^{\text{int}} + q_i^{hist\_cond,ext} = A_i h_i^{conv/rad,\text{int}} (T_i^{env,\text{int}} - T_i^{ext}) \quad (11-19)$$

Subtracting equation 11-19 from equation 11-4 gives the required linear relationship between T_i^{ext} and T_i^{int}:

$$\begin{aligned}&(A_i X_i^0 + A_i Y_i^0 + A_i h_i^{conv/rad,\text{int}}) T_i^{\text{int}} \\ &-(A_i Z_i^0 + A_i Y_i^0 + A_i h_i^{conv/rad,\text{int}}) T_i^{ext} \quad (11-20) \\ &+ q_i^{hist\_cond,\text{int}} - q_i^{hist\_cond,ext} = 0\end{aligned}$$

Solution of The Heat Balance Equations

Matrix Solution

If we assume that the plant sensible heat input, Q^{plantS}, is known, we now have 2*M*+2 linear equations for the 2*M*+2 unknown temperatures, where *M* is the number of zone surfaces:

Equations		**Unknowns**	
Air heat balance:	1	Air temperature:	1
MRT node heat balance:	1	Mean radiant temperature:	1
Internal surface heat balance:	M	Internal surface temperatures:	M
External surface heat balance:	M	External surface temperatures:	M
Total:	**2M+2**	**Total:**	**2M+2**

The 2*M*+2 linear equations are solved, using an efficient matrix technique, to establish:

1) The zone air temperature, T^0, with no plant input;
2) The rise in zone air temperature, $\Theta^{htg10000}$, which results from an increase of 10kW in the plant input, with a radiant proportion appropriate to heating ($p = p^{htg}$);
3) The rise in zone air temperature, $\Theta^{c\lg 10000}$, which results from an increase of 10kW in the plant input, with a radiant proportion appropriate to cooling ($p = p^{c\lg}$).

Under the linearity assumptions, this establishes the following *room response characteristic* providing a relationship between zone air temperature, T^{air}, and plant input, Q^{plantS} at each time-step:

$$T^{air} = T^0 + \frac{\Theta^{htg10000} Q^{plantS}}{10000} \quad (Q^{plantS} > 0) \quad (11-21)$$

$$T^{air} = T^0 + \frac{\Theta^{c\lg 10000} Q^{plantS}}{10000} \quad (Q^{plantS} < 0) \quad (11-22)$$

The plant operating point is found as the intersection between this *room response characteristic* and the *plant control characteristic*. As explained in *Section 10. Plant*, the plant control characteristic is represented as a series of straight line segments, each of which may be represented by the equation

$$P_Q Q^{plantS} + P_T T^{air} = P_{QT} \quad (10-1\,bis)$$

The intersection of the two characteristics is found by examining in turn each segment of the *plant control characteristic*, finding its intersection with the *room response characteristic* and testing for consistency. The opposing gradients of the two characteristics ensure that the intersection always exists.

The intersection immediately yields T^{air} and Q^{plantS}, and the remaining temperature variables (surface temperatures and mean radiant temperature) are obtained from the matrix equations.

Iteration

The matrix solution procedure assumes linearity. In reality, due to the dependence of convection coefficients on temperature difference, the system of equations is non-linear. For this reason, the matrix solution is repeated iteratively until the assumed convection coefficients are consistent with the calculated temperature differences. Convergence is deemed to have been achieved when no surface temperature varies by more than 0.2K from one iteration to the next.

After the heat balance has been obtained for each zone, the whole procedure is repeated in an iterative process which ensures that inter-zone connections (via conduction and advection) are correctly accounted for.

Initialisation & Preconditioning

At the start of a simulation, the thermal state of all building components is initialised to a steady state condition consistent with the outside air temperature read from the weather file for the first hour of the simulation, and an inside air temperature of 18C.

Every simulation should include a preconditioning period of 10 days or more to ensure that the effect of the initial condition has become negligible at the start of the period of interest. The required preconditioning period will vary with building type and internal conditions.

Latent Balance

Air Moisture Balance

The zone air moisture balance is expressed by the equation:

$$w^{air} = w^{\inf} + w^{vent} + w^{am} + w^{gains} + w^{plant} \quad (11-23)$$

where

$$w^{air} = \frac{\left(\rho^{air}V\right)\left(x - \langle x \rangle\right)}{\Delta} \quad (6-4\,bis)$$

$$w^{\inf} = m^{\inf}(x_0 - x) \quad (5-3\,bis)$$

$$w^{vent} = m^{vent}(x_0 - x) \quad (5-5\,bis)$$

$$w^{am} = \sum_{s=0}^{Z} m_{sz}(x_s - x) \quad (5-7\,bis)$$

$$w^{gains} = w^{occL} + w^{equL} \quad (9-6\,bis)$$

Equation 11-23 states that the rate of moisture addition to the air is equal to the total moisture gain from infiltration, ventilation, air movement, casual gains and plant.

Expanding equation 11-23 we find

$$\frac{\left(\rho^{air}V\right)\left(x-\langle x\rangle\right)}{\Delta} = (m^{\inf} + m^{vent})(x_0 - x) + \sum_{s=0}^{Z} m_{sz}(x_s - x) + w^{gains} + w^{plant} \qquad (11-24)$$

or

$$\left(\frac{\rho^{air}V}{\Delta} + m^{\inf} + m^{vent} + \sum_{s=0}^{Z} m_{sz}\right)x = \left(\frac{\rho^{air}V}{\Delta}\langle x\rangle + (m^{\inf} + m^{vent})x_0 + \sum_{s=0}^{Z} m_{sz}x_s + w^{gains} + w^{plant}\right) \qquad (11-25)$$

in which all quantities are known except x, x_s and w^{plant}. To minimise iteration, the previous time step's values are used for x_s, the humidity ratios in air movement source zones.

Equation 11-25 is first solved for x on the assumption of no latent addition or removal by the plant:

$$w^{plant} = 0 \qquad (11-26)$$

The humidity is then checked against the humidity control limits. Tas first calculates the humidity ratios corresponding to the *Humidity Upper* and *Lower Limits* set in Internal Conditions at the computed zone air temperature (see Section . **Psychrometrics**). If x lies between these limits, the latent load Q^{plantL} is set to zero and no further analysis is done. If x is greater than the upper limit or less than the lower limit, x is set to the limit in question and the plant moisture input, w^{plant}, is calculated from equation 11-3. Finally the latent load is calculated from equation 10-7:

$$Q^{plantL} = Lw^{plant} \qquad (10-8\ bis)$$

Initialisation & Preconditioning

At the start of a simulation, the humidity ratio in all zones is initialised to the value 0.005 kg/kg.

The time constant of a zone for humidity dynamics is the inverse of the advection rate (infiltration plus ventilation plus air movement) expressed in air changes per hour. Unless the advection rate is very low, this time constant is seldom more than a few hours, so the initial humidity condition soon becomes negligible during preconditioning.

Zone Temperature Outputs

Air Temperature

A-Tas assumes that the air in a zone is well mixed, and therefore representable by a single zone temperature. This zone temperature, T^{air}, is calculated by balancing heat gains and losses at the *air point* as described in **Zone Heat Balance.**

Mean Radiant Temperature

The Tas output *Mean Radiant Temperature*, T^{MRT}, is an estimate of an occupant's perception of the radiant temperature in the zone. (Note that this mean radiant temperature is defined in a different way from the mean radiant temperature $T^{MRT(C)}$ used in Carroll's method for calculating radiant exchange between surfaces - see Section **Internal Long-wave Radiation**). T^{MRT} is calculated as a weighted average of the zone's surface temperatures, modified by the effects of radiant gains (plant, incidental gains and the diffuse component of solar gain). The procedure for the calculation of T^{MRT} is as follows:

1. The *mean effective irradiance*, R^{int} (W/m^2), at the occupant position is calculated as

$$R^{\text{int}} = \underbrace{\frac{\sum_i A_i \varepsilon_i^{\text{int}} \sigma (T_i + 273.15)^4}{\sum_i \varepsilon_i^{\text{int}} A_i}}_{\text{surface radiation}} + \underbrace{\sum_j \frac{1}{2} C_j \frac{Q_j^{rad}}{A^{floor}}}_{\text{direct radiant plant \& casual gain inputs}} +$$

$$\underbrace{\left\{ \frac{1}{\sum_{\text{i}} \varepsilon_{\text{i}}^{\text{int}} A_i} - \frac{1}{\sum_i A_i} \right\} Q^{rad}}_{\text{Scattered radiant plant \& casual gain inputs}} + \underbrace{\frac{\alpha^{skin}}{\varepsilon^{skin}} I^{dif}}_{\text{Solar gain}} \qquad (12-2)$$

Where

T_i = Temperature of surface i (degrees C)

A_i = area of surface i (m^2)

$\varepsilon_i^{\text{int}}$ = emissivity of internal surface I

σ = Stefan-Bolzmann constant (W/m^{2}K^4)

C_j = view coefficient for radiant heat input Q_j^{rad} (set in Internal Conditions)

$Q^{rad} = Q^{plant,rad} + Q^{gains,rad} = \sum_j Q_j^{rad}$ = total radiant gain (W) from plant and casual gains.

A^{floor} = zone floor area (m^2)

I^{dif} = mean surface diffuse solar irradiance (including scattered) (W/m^2)

$\alpha^{skin} / \varepsilon^{skin}$ = ratio of solar absorptance to emissivity for skin = 0.65/0.98

The contribution to R^{int} from scattered radiant plant and casual gain inputs is taken from equations 9-10 and 10-6.

2. The mean radiant temperature T^{MRT} (degrees C) is calculated from

$$R^{\text{int}} = \sigma(T^{MRT} + 273.15)^4 \quad (12-2)$$

Resultant Temperature

Resultant temperature is calculated as the mean of air temperature and mean radiant temperature:

$$T^{res} = \frac{T^{air} + T^{MRT}}{2} \quad (12-3)$$

This is a measure of the temperature perceived by occupants at low air speeds (of the order of 0.1m/s)[1].

Surface Temperatures

The Tas output dataset includes calculated surface temperatures for internal and external zone surfaces. These are calculated as described in Section **Zone Heat Balance**.

References

1. CIBSE Guide, Part A1, Equation A1.2. The Chartered Institution of Building Services Engineers. Delta House, 222 Balham High Road, London SW12 9BS.

Sensible Load Breakdown

General

The Sensible Load Breakdown facility provides an hourly breakdown of sensible heat flows into and out of a zone or group of zones. The components of the load breakdown are also available through the Tas Report Generator.

The energy flows are broken down into the following categories: plant input, solar gain, lighting gain, occupancy gain, equipment gain, infiltration and ventilation, building heat transfer, external conduction (opaque components), external conduction (transparent components). Positive values indicate a heat gain and negative values a heat loss.

These figures represent a balanced account of energy entering and leaving the zone in question. For this purpose the boundaries of the zone are the inner surfaces of its walls, windows, floors and ceilings.

Plant, Q^{plantS} , is the total power input from the plant (the sum of radiant and convective portions).

Solar gain, Q^{sol} , is the sum of the *surface solar gains* for all the surfaces facing into the zone.

Lighting gain, Q^{light} is the power input from lights (sum of radiant and convective portions).

Occupancy gain, Q^{occS} is the sensible power input from occupants (sum of radiant and convective portions).

Equipment gain, Q^{equS} is the sensible power input from equipment (sum of radiant and convective portions).

Infiltration/ventilation heat gain, $Q^{\mathrm{inf}/vent}$, represents the heat gained (or if negative lost) by the zone due to the exchange of air between the zone and the external environment:

$$Q^{\mathrm{int}/vent} = Q^{\mathrm{int}} + Q^{vent} + m_{0z} c_p (T_0^{air} - T^{air}) \quad (13-1)$$

Building heat transfer represents the sum of heat gains from 2 sources: 1) heat entering the zone from *Link, Null Link* or *internal* building components, and 2) heat released into the zone which had been temporarily stored in the air (this quantity is positive when the air temperature is falling and negative when it is rising):

$$Q^{BHT} = -\sum_{i} q_i^{cond,\text{int}} - Q^{air} \quad (13-2)$$

(Links Null Links & Internals)

Air movement represents heat gained via specified inter-zone air movement flows and via air movement through apertures linking to other zones:

$$Q^{AM} = \sum_{s=1}^{Z} m_{sz} c_p (T_s^{air} - T^{air}) \quad (13-3)$$

E*xternal conduction (opaque)* is the heat gained by conduction through the inside surfaces of exposed opaque components and ground floors:

$$Q^{cond(op)} = -\sum_{i} q_i^{cond,\text{int}} \quad (13-4)$$

(exposed opaque components and Ground Floors)

External conduction (glazing) is the heat gained by conduction through the inside surfaces of exposed transparent components:

$$Q^{cond(transap)} = -\sum_{i} q_i^{cond,\text{int}} \quad (13-5)$$

(exposed Transparent components)

If a transparent component is replaced by an *opaque shutter* component, the heat gain is still assigned to the *external conduction (glazing)* column. Similarly, if an opaque component is replaced by a transparent 'shutter' component, the heat gain is still appears as *external conduction (opaque).*

Conservation of energy (as represented by the energy balance equations set out in Section **Zone Heat Balance**), requires that the sum of the gains and losses within a zone for each hour should be zero:

$$\begin{aligned} &Q^{plantS} + Q^{sol} + Q^{light} + Q^{occS} + Q^{\text{inf}/vent} \\ &+ Q^{BHT} + Q^{AM} + Q^{cond(op)} + Q^{cond(transp)} = 0 \end{aligned} \quad (13-6)$$

The sum of the gains and losses for the zone in question over the whole 24 hour period is displayed as *Net Imbalance*. This figure is a measure of the accuracy of the numerical algorithms employed within Tas**.** Typically this figure is less than 0.002 kWh.

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