MTE321 Formulas

Stresses

Deformation Elongation

$$\delta = \frac{FL}{EA}$$
$$\delta = \frac{\sigma L}{E}$$

Torsional Formulas

Stress

R is the radial distance

$$\tau = \frac{Tr}{J}$$
$$Z_p = \frac{J}{c}$$
$$\tau_{max} = \frac{T}{Z_p}$$
$$Hollow\!: \; J = \frac{\pi}{2}(C^4 - C_i^4)$$
$$Solid\!: \; J = \frac{\pi}{2}C^4$$

Deformation

θ is the angle of twist across L
For non-circular shafts K is section polar second moment of area and Q the section polar modulus

$$T = \frac{P_W}{\omega} \quad T_{lb\cdot in} = 63000\frac{P_{hp}}{\omega}$$
$$\theta = \frac{TL}{GJ}$$
$$Non\text{-}Circular \; \tau = \frac{T}{Q}$$
$$Non\text{-}Circular \; \theta = \frac{TL}{GK}$$

Thin-Walled Closed Tubes

A = median area boundary, U is length of median boundary

$$K = \frac{4A^2t}{U}$$
$$Q = 2tA$$

Shear Stress

V section shear force, Q is the first moment area, and t is the section thickness

$$\tau_{(y)} = \frac{VQ}{It}$$
$$Rectangular \; Beam \; \tau_{max} = \frac{3V}{2A}$$
$$Solid \; Round \; Beam \; \tau_{max} = \frac{4V}{3A}$$
$$Hollow \; Round \; Beam \; \tau_{max} = \frac{2V}{A}$$
$$Q = A_p\bar{y}$$
$$\bar{y} = Distance \; to \; central \; axis$$
$$A_p = \frac{1}{12}t \cdot h \; rectangle$$

Beam Bending

M is the moment at the section, y is the distance from the neutral axis

$$\sigma_y = -\frac{My}{I}$$
$$\sigma_{max} = \frac{M}{S}$$

Stress Concentrations

Stress Concentration Factor

K_t is material and loading dependent, values greater than 3 are a waste

$$\sigma_{max} = K_t\sigma_{nom}$$

Curved Beam Bending

R = $\frac{A}{ASF}$
r = distance to required stress location
r_c= centroid distance
A = cross-sectional area

$$\sigma_{(r)} = \frac{M(R-r)}{Ar(r-R)}$$

Thermal Strain

$$Fixed \; between \; two \; walls \; \epsilon_x^m = -\alpha\Delta T$$
$$\epsilon_x^t = \alpha\Delta T$$

Principle Stresses

$$tan2\theta_\sigma = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$Max \; \sigma_{norm} = \frac{1}{2}(\sigma_x + \sigma_y) + \sqrt{\left[\frac{1}{2}\sigma_x - \sigma_y\right]^2 + \tau_{xy}^2}$$
$$Min \; \sigma_{norm} = \frac{1}{2}(\sigma_x - \sigma_y) - \sqrt{\left[\frac{1}{2}\sigma_x - \sigma_y\right]^2 + \tau_{xy}^2}$$
$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Static Loads

Effective Stress

$$Tresca\!: \; \sigma^{`} = \frac{\sigma_1 - \sigma_3}{2}$$
$$Von \; Mises\!: \; \sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$

Static Loading

$$N = \frac{s_y}{2\tau_{max}}$$
$$N = \frac{s_y}{\sigma_e}$$
$$If \; brittle$$
$$N = \frac{S_{ut}}{K_t\sigma_1}$$
$$N = \frac{S_{uc}}{K_t\sigma_3}$$
$$\sigma_1 > \sigma_2 > \sigma_3$$

Design Factors

A_{95} Equivalence

$$Equivalent \; Diameter\!: \; D_e = 0.370D$$
$$General\!: \; 0.0766D_e^2$$

Design: Dynamic Loads

Loading

$$\sigma_m = mean\ stress \ = \frac{\sigma_{max}+\sigma_{min}}{2}$$
$$\sigma_a = stress\ amplitude \ = \frac{\sigma_{max}-\sigma_{min}}{2}$$
$$R = stress\ ratio = \frac{\sigma_{min}}{\sigma_{max}}$$
$$A = stress\ ratio \ = \frac{\sigma_a}{\sigma_m}$$
$$Loading\ Cycle:\ preriod\ between\ peaks$$

Stress

Periodic

Fluctuating $\sigma_m \neq 0$, R = -1
Pulsating $\sigma_{min} = 0$, R =1

Endurance Limit

s_a =Stress Amplitude Level
N: number of cycles to failure
s_n = fatigue limit
Assume $s_n = 0.5 s_u$ if no data

$$s_a = s_n N^b$$
$$s_n^{`} = C_m C_{st} C_R C_S s_n$$
$$C_S\ only\ in\ bending$$

s_n from table appendix 3
C_m material flaws
C_R Reliability Factor Assume 0.99 reliability
C_s = size factor (5-12,5-4 circular),(5-13 for other)

4. Apply a material factor, C_m, from the following list.

Wrought steel:	$C_m = 1.00$
Cast steel:	$C_m = 0.80$
Powdered steel:	$C_m = 0.76$
Malleable cast iron:	$C_m = 0.80$
Gray cast iron:	$C_m = 0.70$
Ductile cast iron:	$C_m = 0.66$

5. Apply a type-of-stress factor: $C_{st} = 1.0$ for bending stress; $C_{st} = 0.80$ for axial tension.
6. Apply a reliability factor, C_R, from Table 5–3.
7. Apply a size factor, C_s, using Figure 5–12 and Table 5–4 as guides.

Goodman Method

Dynamic Loads Compressive

$$(\sigma_m \leq 0)$$
$$Von\ Mises:\ N_1 = \frac{s_n^{`}}{K_t \sigma_a^{`}}$$
$$Tresca:\ N_1 = \frac{s_n^{`}}{K_t \sigma_a^{`}}$$

Dynamic Loads Tensile

$$(\sigma_m > 0)$$
$$Von\ Mises:\ \frac{K_t \sigma_a^{`}}{s^{`}n} + \frac{\sigma_m^{`}}{s_u} = \frac{1}{N_1}$$
$$Tresca:\ \frac{2K_t}{s_n^{`}}(\tau_a)_{max} + \frac{4}{3s_u}(\tau_m)_{max} = \frac{1}{N_1}$$

Dynamic Yield Test

$$for\ low\ \sigma_a\ high\ \sigma_m$$
$$Von\ Mises:\ \frac{K_t \sigma_a^{`}}{s_y} + \frac{K_t \sigma_m^{`}}{s_y} = \frac{1}{N_2}$$
$$Tresca:\ \frac{2K_t}{s_{sy}}(\tau_a)_{max} + \frac{2K_t}{S_{sy}}(\tau_m)_{max} = \frac{1}{N_2}$$

Effective safety factor is < of N_1 and N_2

Gears

Table 8-1

Pitch Line Speed

$$V_T = \frac{\pi D \cdot n_p}{12}$$

Gears

Spur Gears

$$Center\ Distance\ C = R_P + R_G$$
$$Speed\ of\ Gears:\ \frac{n_p}{n_G} = \frac{N_G}{N_P}$$
$$Common\ Speed:\ v_T = R_1\omega_1 = R_2\omega_2$$
$$Tangental\ Acceleration:\ a_T = R_1\alpha_1 = R_2\alpha_2$$
$$Velocity\ Ratio:\ VR = \frac{R_G}{R_P} \geq 1 = \frac{N_G}{N_P} = \frac{n_p}{n_G} = \frac{\omega_P}{\omega_G}$$
$$Circular\ Pitch:\ p = \frac{\pi D}{N}$$
$$Contact\ Ratio:\ m_f = \frac{\sqrt{R_{oP}^2 - R_{bP}} + \sqrt{R_{oG}^2 - R_{bG}} - c\sin\phi}{p\cos\phi}$$
$$P = T\omega$$
$$backlash: = w - t$$
$$w \rightarrow Tooth\ Space,\ distance\ pitch\ circle\ travels\ between\ teeth$$

Helical Gears

$$Circular/Transverse\ Pitch:\ p = \frac{\pi}{P_d}$$
$$Normal\ Circular:\ p_n = p\cos\psi$$
$$Axial\ Pitch:\ p_x = \frac{p_t}{\tan\psi}$$
$$Pitch\ Diameter:\ D_G = \frac{N}{P_d}$$
$$Normal\ Pressure\ Angle:\ \phi_n = \tan^{-1}(\tan\phi_t \cdot \cos\psi)$$
$$Diametral\ Pitch:\ P_d = \frac{N}{D}$$
$$Axial\ Pitches\ in\ Face:\ \frac{F_w}{P_x}$$
$$Normal\ Diametral\ Pitch:\ P_{nd} = \frac{P_d}{\cos\psi}$$

Gear Train

$$TV_{nom} = \frac{n_{in}}{n_{out}}$$

Racks

$$Velocity\ of\ Rack:\ V_R = V_T = R_p\omega_p = \left(\frac{D_p}{2}\right)\omega_p$$
$$Displacement\ of\ Rack:\ s = \frac{D_p}{2}\theta_p$$

Gear Stress

Bending Stress

$$\sigma_t = \frac{W_t P_d}{FJ}$$
$$J = \frac{Y}{K_t}$$
$$Y = \textit{Lewis Form Factor}$$
$$s_t = \sigma_t K_0 K_S K_m K_B K_v$$

- The overload factor, Table 9-1: K_o
- The size factor, Table 9-2: K_s
- The load distribution factor: $K_m = 1.0 + C_{pf} + C_{ma}$
  C_{pf}: Figure 9-12, C_{ma}: Figure 9-13.
- The rim thickness factor, Figure 9-14: K_B
- The dynamic factor, Table 9-5, Figure 9-16, & Table 9-6: K_v

Contact Stress

$$\sigma_c = C_P \sqrt{\frac{W_t}{FD_P I}}$$
$$s_c = C_p \sqrt{\frac{W_t K_o K_S K_m K_v}{FD_P I}}$$

- C_p: The elastic coefficient for the gear set materials (see Eq. (9-20) and Table 9-7).
- I: The geometry factor depends on the gear ratio, the pressure angle, and the number of teeth in the pinion (see Figure 9-17)
- D_P Pinion's pitch diameter

Power Capacity

Bending

$$P_{CAP} = \frac{s_{at} Y_N F J n_p D_P}{(126000) P_d (SF) K_R K_O K_S K_m K_B K_v}$$

Contact

$$P_{CAP} = \frac{n_p F I}{126000 K_o K_s K_m K_v}\left[\frac{s_{ac} D_P Z_N}{(SF) K_R C_P}\right]^2$$

Driven Forces

Spur Gears

$$\textit{Tangental Forces: } W_t = \frac{T}{\frac{D}{2}}$$
$$\textit{Radial Force: } W_r = W_t \tan\phi$$

Chains/Belts

Chain/Sproket:

$$F_{cx} = F_c \cos\phi \qquad F_{cy} = F_c \sin\phi$$

Belt Sheave

$$\textit{V-Belt: } F_B = 1.5\frac{T}{\frac{D}{2}}$$
$$\textit{Flat Belt: } F_B = 2\frac{T}{\frac{D}{2}}$$
$$VR = \frac{\omega_{driving}}{\omega_{driven}}$$

Shoulder Fillets

$$\textit{Sharp Fillet (Bearing): } K_t = 2.5$$
$$\textit{Rounded Fillet: } K_t = 1.5$$
$$\textit{Retaing Ring: } K_t = 3.0$$
$$\textit{Profile Keyseat: } K_t = 2.0$$
$$\textit{Sled Runner Keyseat: } K_t = 1.6$$

Design

Shear Stress

$$\tau_d = K_t\left(\frac{4V}{3A}\right) \qquad N = 0.577\frac{s'_n}{\tau_d} \qquad A = \frac{\pi D^2}{A}$$
$$D = \sqrt{\frac{2.94 K_t V N}{s'_n}}$$

Bending\Torsion

$$M = \sqrt{M_x^2 + M_y^2}$$
$$\left(\frac{\sigma}{s'_n}\right)^2 + \left(\frac{\tau}{s_{ys}}\right)^2 = 1$$
$$s_{ys} = \frac{s_y}{\sqrt{3}} \qquad s_{ys} = \frac{s_y}{\sqrt{3}}$$
$$\left(\frac{K_T N \sigma}{s'_n}\right)^2 + \left(\frac{N_T}{s_{ys}}\right)^2 = 1$$
$$D^3 = \frac{32N}{\pi}\sqrt{\left(\frac{K_t M}{s'_n}\right)^2 + \frac{3}{4}\left(\frac{\tau}{s_y}\right)^2}$$
$$\textit{Steel Shafts: } s'_n = s_n C_R C_s$$