MTE321 Formulas

Stresses

Deformation Elongation

$$\delta = \frac{FL}{EA}$$
$$\delta = \frac{\sigma L}{E}$$

Torsional Formulas

Stress

R is the radial distance

$$\tau = \frac{Tr}{J}$$
$$Z_p = \frac{J}{c}$$
$$\tau_{max} = \frac{T}{Z_p}$$

Deformation

θ is the angle of twist across L
For non-circular shafts K is section polar second moment of area and Q the section polar modulus

$$T = \frac{P}{\omega} \quad T_{lb\cdot in} = 63000\frac{P}{\omega}$$
$$\theta = \frac{TL}{GJ}$$
$$Non\text{-}Circular\ \tau = \frac{T}{Q}$$
$$Non\text{-}Circular\ \theta = \frac{TL}{GK}$$

Thin-Walled Closed Tubes

A = median area boundary, U is length of median boundary

$$K = \frac{4A^2t}{U}$$
$$Q = 2tA$$

Shear Stress

V section shear force, Q is the first moment area, and t is the section thickness

$$\tau_{(y)} = \frac{VQ}{It}$$
$$Rectangular\ Beam\ \tau_{max} = \frac{3V}{2A}$$
$$Solid\ Round\ Beam\ \tau_{max} = \frac{4V}{3A}$$
$$Hollow\ Round\ Beam\ \tau_{max} = \frac{2V}{A}$$

Beam Bending

M is the moment at the section, y is the distance from the neutral axis

$$\sigma_y = -\frac{My}{I}$$

Stress Concentrations

Stress Concentration Factor

K_t is material and loading dependent, values greater than 3 are a waste

$$\sigma_{max} = K_t\sigma_{nom}$$

Curved Beam Bending

R = $\frac{A}{ASF}$
r = distance to required stress location
r_c= centroid distance
A = cross-sectional area

$$\sigma_{(r)} = \frac{M(\theta)(R-r)}{Ar(r_c - R)}$$

Thermal Strain

$$\epsilon_x^m = -\alpha\Delta T$$

Principle Stresses

$$tan2\theta_\sigma = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$Max\ \sigma_{norm} = \frac{1}{2}(\sigma_x + \sigma_y) + \sqrt{\left[\frac{1}{2}\sigma_x - \sigma_y\right]^2 + \tau_{xy}^2}$$
$$Min\ \sigma_{norm} = \frac{1}{2}(\sigma_x - \sigma_y) - \sqrt{\left[\frac{1}{2}\sigma_x - \sigma_y\right]^2 + \tau_{xy}^2}$$

Design Factors

A_{95} Equivalence

$$Equivalent\ Diameter:\ D_e = 0.370D$$
$$General:\ 0.0766D_e^2$$

Design: Dynamic Loads

Loading

$$\sigma_m = mean\ stress\ = \frac{\sigma_{max} + \sigma_{min}}{2}$$
$$\sigma_a = stress\ amplitude\ = \frac{\sigma_{max} - \sigma_{min}}{2}$$
$$R = stress\ ratio = \frac{\sigma_{min}}{\sigma_{max}}$$
$$A = stress\ ratio\ = \frac{\sigma_a}{\sigma_m}$$
Loading Cycle: preriod between peaks

Stress

Periodic

Fluctuating $\sigma_m \neq 0$, R = -1
Pulsating $\sigma_m = 0$, R =1

Endurance Limit

s_a =Stress Amplitude Level
N: number of cycles to failure
s_n = fatigue limit
Assume s_n = 0.5s_u if no data

$$s_a = s_nN^b$$
$$s_n^{'} = C_mC_{st}C_RC_Ss_n$$

s_n from table appendix 3
C_m material flaws
C_R Reliability Factor
C_s = size factor (5-12,5-4 circular),(5-13 for other)

4. Apply a material factor, C_m, from the following list.

Wrought steel:	$C_m = 1.00$
Cast steel:	$C_m = 0.80$
Powdered steel:	$C_m = 0.76$
Malleable cast iron:	$C_m = 0.80$
Gray cast iron:	$C_m = 0.70$
Ductile cast iron:	$C_m = 0.66$

5. Apply a type-of-stress factor: $C_{st} = 1.0$ for bending stress; $C_{st} = 0.80$ for axial tension.
6. Apply a reliability factor, C_R, from Table 5–3.
7. Apply a size factor, C_s, using Figure 5–12 and Table 5–4 as guides.

Goodman Method

Dynamic Loads Ductile

$$(\sigma_m < 0)$$

$$\textit{Von Mises: } N_1 = \frac{s_n^{`}}{K_t \sigma_a^{`}}$$

$$\textit{Tresca: } N_1 = \frac{s_n^{`}}{K_t \sigma_a^{`}}$$

Dynamic Loads Tensile

$$(\sigma_m > 0)$$

$$\textit{Von Mises: } \frac{K_t \sigma_a^{`}}{s^{`} n} + \frac{\sigma_m^{`}}{s_u} = \frac{1}{N_1}$$

$$\textit{Tresca: } \frac{2K_t}{s_n^{`}}(\tau_a)_{max} + \frac{4}{3s_u}(\tau_m)_{max} = \frac{1}{N_1}$$

Dynamic Yield Test

$$\textit{for low } \sigma_a \textit{ high } \sigma_m$$

$$\textit{Von Mises: } \frac{K_t \sigma_a^{`}}{s_y} + \frac{K_t \sigma_m^{`}}{s_y} = \frac{1}{N_2}$$

$$\textit{Tresca: } \frac{2K_t}{s_{sy}}(\tau_a)_{max} + \frac{2K_t}{S_{sy}}(\tau_m)_{max} = \frac{1}{N_2}$$

Effective safety factor is < of N_1 and N_2

Gears

Table 8-1

Pitch Line Speed

$$V_T = \frac{\pi D \cdot n_p}{12}$$

Gears

Spur Gears

$$\textit{Speed of Gears: } \frac{n_p}{n_G} = \frac{N_G}{N_P}$$

$$\textit{Common Speed: } v_T = R_1\omega_1 = R_2\omega_2$$

$$\textit{Tangental Acceleration: } a_T = R_1\alpha_1 = R_2\alpha_2$$

$$\textit{Velocity Ratio: } VR = \frac{R_G}{R_P} >= 1$$

$$\textit{Circular Pitch: } p = \frac{\pi D}{N}$$

Contact Ratio:

Helical Gears

$$\textit{Circular/Transverse Pitch: } \frac{\pi}{P_d}$$

$$\textit{Normal Circular: } p_n = p + \cos\psi$$

$$\textit{Axial Pitch: } p_x = \frac{p_t}{\tan\psi}$$

$$\textit{Pitch Diameter: } D_G = \frac{N}{P_d}$$

$$\textit{Normal Pressure Angle: } \phi_n = \tan^{-1}(\tan\phi_t \cdot \cos\psi)$$