CS390 Computational Game Theory and Mechanism Design July 1, 2013

Lecture 1, Part 1:Normal form games

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Game Theory: Game Analysis, Mechanism Design

Definition 1 *Normal-formal game:*

- *A normal-form game(or a strategic game) is a triple(N, S, u).*
- *N is the set of players.* $N = \{1, 2, ..., n\}$.
- $S = S_1 \times S_2 \times ... \times S_n$
- S_i *is the pure strategy set of i. We only discuss finite set here.*
- $(S_1, S_2, ..., S_n) \in S$ *is a 'strategy profile'.*
- $u = (u_1, u_2, ..., u_n), u_i : S \to \mathbb{R}$ *utility of i under S.*

The game is simultaneously played: players don't know other players' option.
Common knowledge of the game: all know the rules and all know other players know them(the rules).

Example 1

1/2	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

Table 1: Matching Pennies

Example 2

1/2	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Table 2: Rock-Paper-Scissors

Example 3

1/2	C	H
C	1,1	0,2
H	2,0	-1,-1

Table 3: Chicken or Hawk

Example 4

1/2	B	S
B	2,1	0,0
S	0,0	1,2

Table 4: BoS(Ballet or Soccer, Bach or Stravinsky, Battle of the sexes)

Example 5

1/2	don't confess	confess
don't confess	3,3	-1,4
confess	4,-1	0,0

Table 5: Prisoners' Dilemma(PD)

Assumption: Players are rational.

Definition 2 *A player i is rational if given $s_{-i} \in S_{-i} = S_1 \times ... \times S_{i-1} \times S_{i+1} \times ... \times S_n$ i choose*

$$s_i \in \underset{s_i^{'} \in S_i}{argmax}\ u_i(s_i^{'}, s_{-i})$$

Definition 3 *A strategy s_i strictly dominates $s_i^{'}$ if*

$$\forall s_{-i} \in S_{-i} \quad u_i(s_i, s_{-i}) > u_i(s_i^{'}, s_{-i})$$

We also can say $s_i^{'}$ is strictly dominated by s_i
s_i is strictly dominated if it is strictly dominated by $s_i^{'}$, $\forall\, s_i^{'} \neq s_i$

Definition 4 *$s \in S$ is a pure Nash equilibrium if*

$$\forall i \quad s_i \in B_i(s_{-i}) \triangleq \{s_i^{'} : u_i(s_i^{'}, s_{-i}) = max_{s_i^{''}}\, u_i(s_i^{''}, s_{-i})\}$$

The NEs are marked by () below.

Example 6

1/2	C	H
C	1,1	(0,2)
H	(2,0)	-1,-1

Table 6: Chicken or Hawk

Example 7

1/2	B	S
B	(2,1)	0,0
S	0,0	(1,2)

Table 7: BoS(Ballet or Soccer, Bach or Stravinsky, Battle of the sexes)

Example 8

1/2	don't confess	confess
don't confess	3,3	-1,4
confess	4,-1	(0,0)

Table 8: Prisoners' Dilemma(PD)

Definition 5 *A mixed strategy: $\sigma_i \in \Delta(S_i)$ (all the probability distribution of choices)*

$$\sum_{s_i} \sigma_i(s_i) = 1 \quad \sigma_i(s_i) \geq 0$$

A mixed strategy profile $\sigma \in \Delta(S_1) \times ... \times \Delta(S_n)$

$$u_i(\sigma_1, ..., \sigma_n) = \sum_s \sigma(s) u_i(s)$$

σ is a mixed NE if

$$\sigma_i \in \underset{\sigma_i' \in \Delta(S_i)}{argmax}\, u_i(\sigma_i', \sigma_{-i})$$

e.g. $\Delta(A, B) = \{(p, 1-p) \,|\, p \in [0, 1]\}$
$S_1 = \{L, R\}$ $S_2 = \{U, D\}$
$\sigma = (\frac{L}{2} + \frac{R}{2}) \times (\frac{U}{2} + \frac{D}{2})$

Theorem 1 *(Nash): Any finite normal-form game has a NE.*

Proof. *Kakutoni's fixed point theorem.*
See reference[1] page 33 proposition 33.1 □

scribers: These notes are wrote by Qizhe Xie and inspected by Yiding Feng.

References

[1] M. J. Osborne and A. Rubinstein. *A course in game theory.* MIT Press, 1994.

[2] N. Nisan, T. Roughgarden, E. Tardos, and V. Vazirani (eds). *Algorithmic game theory.* Cambridge University Press, 2007. (Available at `http://www.cambridge.org/journals/nisan/downloads/Nisan_Non-printable.pdf`.)