CS390 Computational Game Theory and Mechanism Design July 21, 2013

Problem Set 5

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Problem 1 (No collaborator.)
Proof. The definition of *lower contour* $L(o, v_i)$ ($\forall i \in N, o \subset O, v_i \in \Theta_i$) is

$$L(o, v_i) = \{o' \mid u_i(v_i, o) \geq u_i(v_i, o')\}$$

Since the utility function does not have ties, so in this environment E

$$L(o, v_i) = \{o' \mid u_i(v_i, o) > u_i(v_i, o')\}$$

Conside when $\forall \theta, \theta' \in \Theta$, $\forall o \in f^{CON}(\theta)$, and $\forall i \in N$, $L(o, \theta_i) \subseteq L(o, \theta'_i)$, we have

$$\{i \mid u_i(\theta_i, o) > u_i(\theta_i, o')\} \subseteq \{i \mid u_i(\theta'_i, o) > u_i(\theta'_i, o')\}$$

$$\Big|\{i \mid u_i(\theta_i, o) > u_i(\theta_i, o')\}\Big| \leq \Big|\{i \mid u_i(\theta'_i, o) > u_i(\theta'_i, o')\}\Big|$$

So $o \in f^{CON}(\theta')$ then f^{CON} satisfies monotonicity. $\square$

Problem 2 (No collaborator.)
f^{BC} doesn't satisfy monotonicity.
Proof. Suppose originally player 1 get $(3, 1)$ and player 2 get $(2, 2)$ and player 3 get $(1, 3)$. After the change player 1 get $(3, 1)$ and player 2 get $(1, 2)$ and player 3 get $(2, 3)$, then it satisfies the lower contour property, but player 1 is not the winner any more. So it is not monotone. $\square$

Problem 3 (No collaborator.)
$f(\theta)$ defined in this game is not monotone.
Proof. Let $\theta = (2, 1, 1, ..., 1)$ and $\theta' = (0.5, 1, 1, ..., 1)$ and $o = (1, p)$ while $p = (0.5, 0.5, ..., 0.5)$. Obviously $o \in f(\theta)$ but $o \notin f(\theta')$. Next we will see that for all i, $L(o, \theta_i) \subset L(o, \theta'_i)$ equivalent to

$$\forall o', u_(\theta_i, o) \geq u_i(\theta_i, o') \Rightarrow u'_i(\theta_i, o) \geq u'_i(\theta_i, o')$$

which leads to the statement proved.
For $i = 2, 3, ..., n$ since $\theta_i = \theta'_i$, $u_i(\theta_i, o) = u_i(\theta'_i, o)$ and $u_i(\theta_i, o') = u_i(\theta'_i, o')$, the above statement holds.
For player 1, $u_1(\theta_1, o) = 1.5$ and $u_1(\theta'_1, o) = 0$. Suppose in another o' his price is p_1. If in o', player 1 still win the good, then $u_1(\theta_1, o') = 2 - p_1$ and $u_1(\theta'_1, o') = 0.5 - p_1$. When $p_1 \geq 0.5$, $u_1(\theta_1, o') \leq 1.5$ and $u_1(\theta'_1, o') \leq 0$, so that the above statement also holds.
So the above statement holds for all n, which shows $f(\theta)$ is not monotone. $\square$

References

[1] M. J. Osborne and A. Rubinstein. *A course in game theory.* MIT Press, 1994.

[2] N. Nisan, T. Roughgarden, E. Tardos, and V. Vazirani (eds). *Algorithmic game theory.* Cambridge University Press, 2007. (Available at `http://www.cambridge.org/journals/nisan/downloads/Nisan_Non-printable.pdf`.)