Ultrasound Elastography: A Dynamic Programming Approach

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***Abstract*—This report focuses on the implementation and study of Dynamic Programming in conjunction with ultrasound elastography. The aim of the project is the use of dynamic programming for the optimization of cost function and creation of strain image to detect the presence of hard tissue (tumor) from pre and post-compression RF data. MATLAB software is used for the entire process. The RF data is used to find the displacement map for both 1D and 2D scenarios. The displacement map is converted into the strain image using different techniques: gradient function and linear regression. The CNR and SNR values for both 1D and 2D strain images are also compiled and represented.**

***Index Terms*—Dynamic programming, strain image, ultrasound elastography.**

I. Introduction

ELASTOGRAPHY is a new emerging modality which helps in detecting the presence of a hard tissue (probably a tumor) on the basis of the elasticity of the tissue. The technique requires application of pressure with mechanical actuators [1], freehand palpations [2] or many other techniques that can be used to apply pressure on the area that needs to be tested. This study uses the data corresponding to static elastography, which applies compression on tissues and provides simultaneous ultrasound images [3]. The ultrasound images have associated radio-frequency (RF) echoes or the RF data for a phantom design. Based on this data from the cases before and after the compression applications, a correlation analysis is done between them.

The correlation analysis puts forth a displacement map which shows how the pixels in pre-compression image have displaced to give the post-compression image. This displacement can be done in both axial and lateral directions. Finally the displacement map is converted into the strain image which gives an idea about the presence of hard and soft tissue by dark and light areas of the image respectively. The focus of the study is to project the function of dynamic programming in a bid to make the cost function more efficient.

Dynamic Programming (DP) is way of solving complicated functions by breaking a bigger part into smaller sub problems, solving them and then storing the values to build back the initial problem recursively. This allows for the computation time to reduce because the compilation process uses the answer to one sub-problem such that there is a relation between the larger problem and its sub-parts. A cost function is involved with the displacement map and DP is used to break the cost function into smaller costs, solving for those smaller costs and then integrating the results into one final function.

The study also showcases two sections for research: the 1D DP and the 2D DP. 2D experiment uses the calculations from the 1D and applies it further for more columns and lateral displacement as well. Finally, CNR (contrast to noise ratio) and SNR (signal to noise ratio) both are calculated for the 1D and 2D strain images.

II. Method

The entire process is initially done for the 1D estimation. At first, two echo signals are considered corresponding to pre-compression and post-compression scenario. One A-line is selected from each signal which gives us g(i) and g'(i) in Fig 1. The requirement is to create a displacement map based on the idea that every pixel in the initial image was displaced after compression and hence a particular displacement range is selected to give a set of responses.

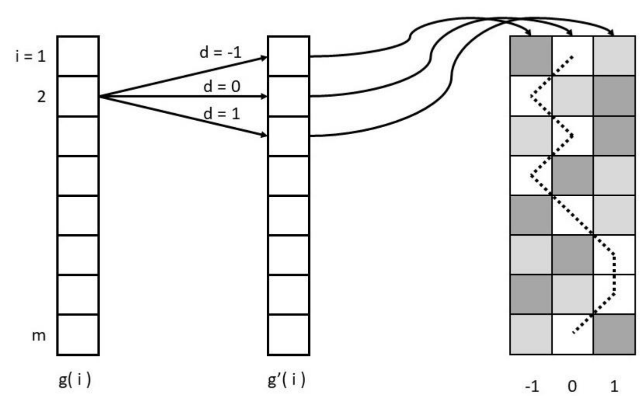

Fig. 1. g(i) and g'(i) are the values for the pre-compression and post-compression RF data. Displacement study gives the cost function C on the right displacement range.

The study is done for the displacement range of [-1,0,1]. Three displacements are checked for every pixel in g(i) and compared with g'(i) with a delta function Δ which computes the sum of absolute differences(differences in the signal).

$$delta = abs(pre(current\_row) - post(current\_row+(-(dmin-current\_column)))); \tag{1}$$

Where (1) represents the actual MATLAB equation for delta function. This computes the initial difference between the samples of g(i) and g'(i) ranging from i=1,2….m. The left part of Fig 1. shows the graphical representation of delta. However, the displacement jumps can be abrupt and it might happen that one pixel is slightly displaced while the succeeding pixel is displaced a lot in comparison. To make the displacement changes smooth, a function S called the regularized smoothening function is used.

$$S = (((-(neg\_dmin-k))-(-(neg\_dmin-k-1)))^{\wedge}2) \tag{2}$$

$$Weighted\ S = alpha * S \tag{3}$$

The equations (2) and (3) are used in conjunction with the delta function so that the cost function as seen on the right side of Fig. 1 can be created more efficiently. Alpha regularizes the smoothening and the effect of S can be changed by changing the value of alpha. Once, the delta function and S function are calculated, the C function or the cot function is worked upon. The cost function C stores the costs corresponding to different displacements in our given range for samples in the column g(i) when compared to g'(i). The dark parts in Fig 1 refers to the maximum cost and the least probable displacement. The light parts stand for minimum cost and most probable displacement from the given range.

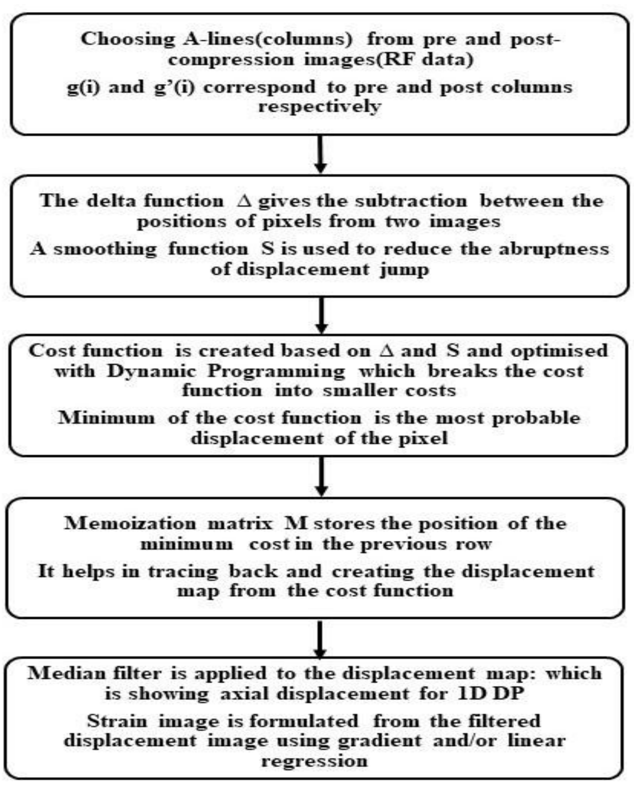

Fig. 2. The steps involved in 1D DP of RF data to obtain the final strain image.

Fig 2 shows the step wise methodology involved in the process. Dynamic programming is involved in the third step of the flow chart wherein the cost function is created. The role of DP is to work on the smallest cost available and then use it recursively to formulate the final cost function. It optimises the function, makes the process more efficient and reduces run time.

Once the cost function is achieved, the goal is to spot the minimum cost values for samples. Matrix M or the memoization function helps in storing the minimum cost position of the last row. This position helps keep track of the sample points for minimum cost. The concept for displacement map comes from back tracing with the help of values stored in M.

$$C = Function(\Delta\ ,\ \alpha S) \tag{4}$$

$$M = Position(min(C)) \tag{5}$$

$$D = Back\text{-}tracing(M) \tag{6}$$

Equations (4), (5) and (6) give the algorithmic understanding of the steps leading to the displacement map D. This displacement map can be showcased as an image in itself. To see the strain image the displacement map has to be theoretically differentiated. In our procedure, we used two methods to procure the strain image. The inbuilt gradient function and the linear regression function were applied.

The 2D DP follows the same methodology except that it is applied over all the A-lines (columns) corresponding to the RF data. The 2D DP also makes a study of lateral displacement in the image along with axial displacement. A combination of the two displacements together provides for strain image which is more accurate and fundamentally covers all bases.

After the strain images are created, two important parameters i.e. CNR (Contrast-noise Ratio) and SNR (Signal-Noise Ratio) are calculated for both 1D and 2D simulations. The values are calculated on the basis of equation (7) given in [3].

$$\mathrm{CNR} = \frac{C}{N} = \sqrt{\frac{2(\bar{s}_b - \bar{s}_t)^2}{\sigma_b^2 + \sigma_t^2}},\ \ \mathrm{SNR} = \frac{\bar{s}}{\sigma} \tag{7}$$

For CNR, the image is divided into different background and target frames according to which the value is calculated. The yellow box in the middle of the image is taken as the target window and the rest are taken as backgrounds to calculate different contrast to noise ratio. In (7), $\bar{s}_b$ and $\bar{s}_t$ are strain averages or the mean of the background and target respectively. The σ_b and σ_t are strain variance of the background and target window respectively. In case of SNR, the $\bar{s}$ stands for the mean while σ stands for standard deviation.

III. Distribution

The work for the entire project was handled by two students in a group. The responsibilities were divided as illustrated in Table I. The code was written by both together but the code in itself had many parts and equations which were divided between the two students to find faults and correct errors.

TABLE I
WORK DISTRIBUTION

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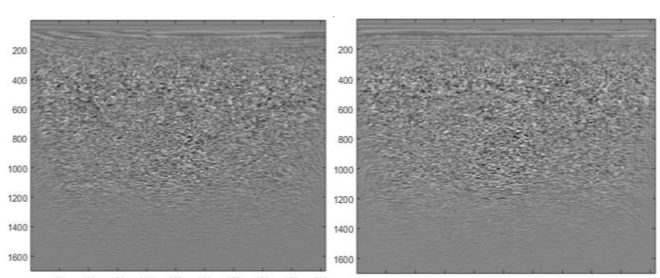
1. Find faults in the delta function Δ for the 1 D and work on error correction	1. Find faults in the S function (smoothening) for the 1 D and work on error correction with weighted S.
2. Work on getting the matrix D and corresponding displacement image from the memoization function M.	2. Error detection and enhancement of cost function C for 1 D DP from delta and S.
3. Final implementation of functions to procure the strain image for different sample windows (5, 25 and 43) and methods.	3. The formation of memoization function M and errors were handled. Many difficulties were discovered and acted upon.
4. 2D implementation of the delta function was done with the incorporation of different columns.	4. The corresponding 2D S function was worked upon for smooth displacement jumps.
5. Lateral and axial strain images were procured with lateral and axial displacement images as well.	5. The cost function C and M function for 2D was thoroughly vetted for errors and optimised.

IV. Results

The radio-frequency data for this particular set of results was taken from [4]. The data is from the compression experiment of a phantom designed with a hard object in the middle. Fig 3 shows the pre-compression and post-compression images corresponding to the RF data. These are the input images Im1 and Im2 which are used in the code.



(a) (b)

Fig. 3. (a) Pre-compression image for phantom. (b) Post-compression image for phantom.

The data for these images is used as an input and first the 1D DP is calculated. The data chosen is of the form which has the effect of lateral displacement also. The 1D DP only focuses on axial displacement. The MATLAB code is run and used to obtain the axial displacement image as a result. A median filter of window size 3x3 is used to filter the noise from the image received. Fig 4 (b) shows how the filter removes some noise from the middle of the image. Since the 1D DP works with axial displacement, the output images have slight edge distortions.

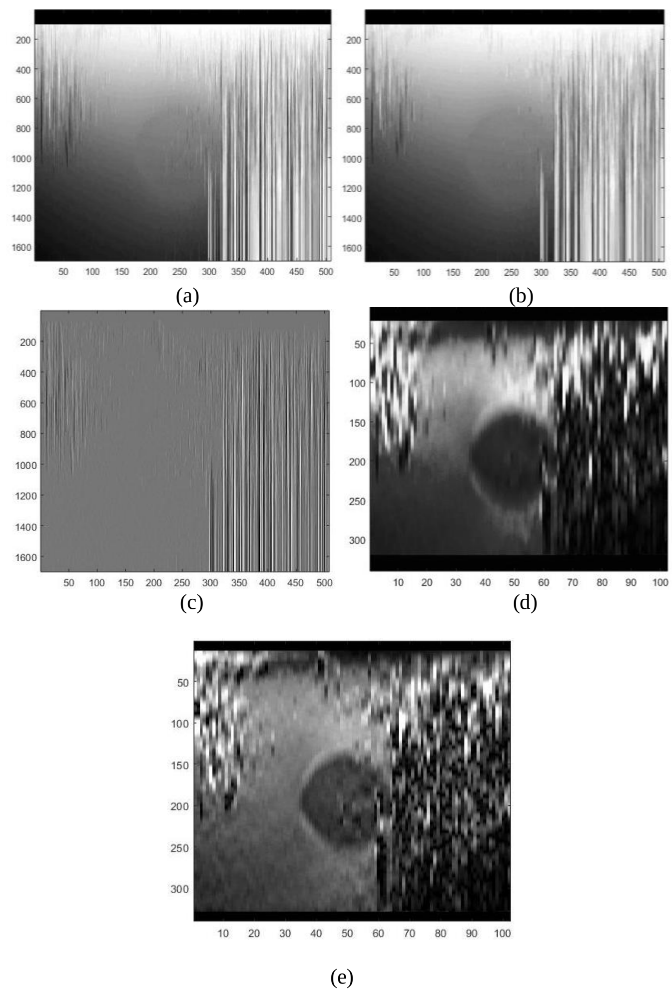


Fig. 4. (a) The axial displacement image for 1D DP. (b) The axial displacement image for 1D DP after applying median filter. (c) Strain image achieved from gradient function. (d) Strain image achieved from linear regression (sample window size 43). (e) Strain image with sampling window size of 25.

The edge distortions that can be seen here may exist because of lateral displacements which were not calculated for the first scenario. However, while the edge distortion might exist, the image is able to detect the presence of hard part in the middle of the tissue very accurately. The displacement image is converted into the strain image through differentiation. Our initial approach was the use of inbuilt gradient function to find the strain image. The result can be seen as image (c) in Fig 4. The source research [3] uses linear regression method to find the strain image which was used as the final attempt. The contrast between the two methods can be easily seen by comparing the results achieved by them.

The gradient function was unable to retain the hardened section in the middle of the tissue which was achieved by the displacement image.

The linear regression method on the other hand, makes it more prominent. The sampling window size of linear regression method also affects the kind of image that is obtained. By reducing the window size, the image is sampled more number of times hence a more compact image is obtained but there is more noise as compared to an image sampled with a larger window. Fig 4, (d) shows the strain image sampled with a window of size 43 while (e) shows the same image sampled at window size 25.

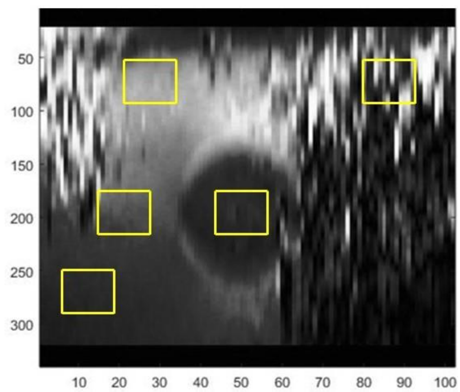


Fig. 5. 1D DP strain image with CNR windows of the target and backgrounds.

Table II showcases the CNR values for various frame windows in the 1D DP strain image. The average CNR value for different background windows and single target window comes out to be around 1.75 for the 1D DP. The value can be made more accurate by taking different sized windows all over the image and taking more values to improve the average. Fig 5 shows the different background and target windows used for measurements.

The SNR value is calculated for a frame window in the middle of the image where the hard tissue is detected while another window is designed just above it in the top center of the image. Fig 6 shows the window frames for which the SNR is calculated. The SNR values comes out to be 2.2158 and 1.5582 for the top and middle frame respectively.

TABLE II
CNR VALUES FOR 1D DP

S_b	S_t	σ_b	σ_t	CNR
0.2817	0.1987	0.0279	0.0172	3.5792
0.1869	0.1987	0.0153	0.0172	0.7267
0.2397	0.1987	0.0376	0.0172	1.4026
0.2224	0.1987	0.0189	0.0172	1.3120
			Mean =	1.75

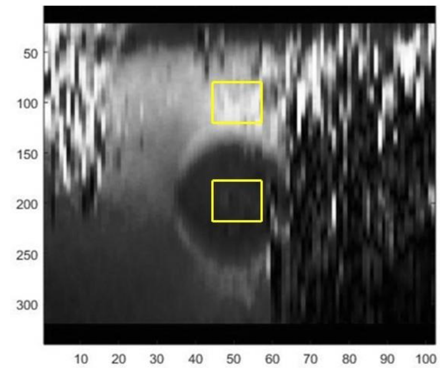
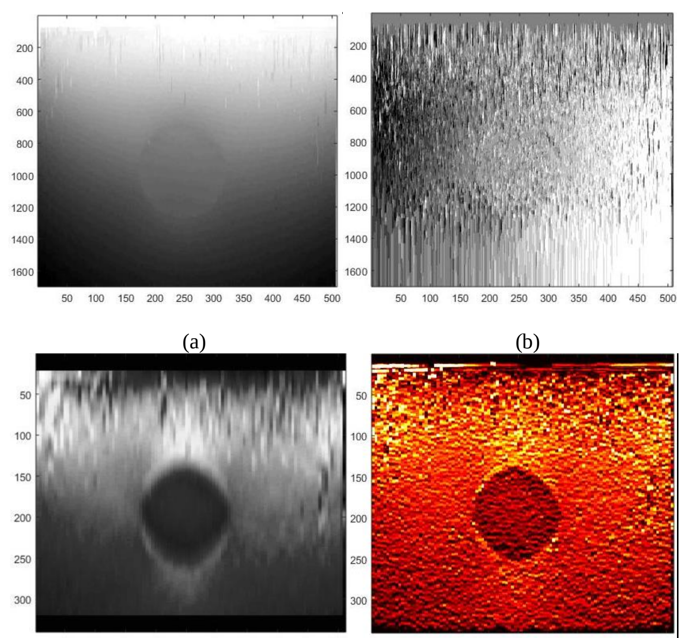


Fig. 6. The 1D DP strain image with SNR frame windows.

The results for the 2D DP are showcased in Fig 7 images (a), (b), (c) and (d). Image (a) shows the axial displacement of the 2D DP. When compared with the initial displacement achieved from the 1D DP we can see that the edge distortion has subsided due to inclusion of more columns into the study. Along with that lateral displacement is also done such that all functions: delta, S and C use axial displacement ‘a’ and lateral displacement ‘l’ along with rows (i) and columns (j). The succeeding matrices M and D are able to incorporate the 2D cost function in the same way.

$$D(j,i) = M(j+1,D(j+1,i)) \tag{8}$$



(a) (b) (c) (d)

Fig. 7. (a) The axial displacement image for 2D DP. (b) The lateral displacement image for 2D DP. (c) Strain image achieved from linear regression. (d) Axial strain image for sample window size 5.

The equation (8) shows the 1D representation of the relation between matrices D and M, which is incorporated in the 2D format for all rows and columns and lateral displacement as well. The images (c) and (d) show the axial strain images for the 2D DP. We can clearly observe the hard, darker tissue detected in the middle. The images differ with the sample window size of the linear regression model used. Image (d) uses the window of size 5 to show the comparison.

TABLE III
CNR VALUES FOR 2D DP

S_b	S_t	σ_b	σ_t	CNR
0.3780	0.1763	0.0176	0.0055	15.4674
0.1735	0.1763	0.0078	0.0055	0.4178
0.2280	0.1763	0.0288	0.0055	2.4955
0.2570	0.1763	0.0083	0.0055	10.5881
			Mean =	7.2422

Table III shows the CNR values for 2D DP simulation according to the background and target windows chosen similar to the 1D DP setting. The mean value of CNR comes out to be 7.2422 which is comparable to the CNR values mentioned in [3]. More background windows can be taken at different positions to enhance the result and make it more accurate. The SNR values for the same simulation come out to be 2.3829 for the window in the middle of the hard tissue and 3.1708 for the window frame just above the circular tissue at the top center of the image. Table IV shows the comparison of SNR values for 1D and 2D setup.

TABLE IV
SNR COMPARISON FOR 1D AND 2D

Position	SNR-1D	SNR-2D
Top frame	2.2158	3.1708
Middle frame	1.5582	2.3829

V. CONCLUSION

The study focused on the introduction of ultrasound elastography as a sophisticated and non-invasive technique to be used for detection of hard tissue or tumor by the means of application of pressure. The corresponding radio-frequency data of the area before and after application of pressure is used to find the displacement of the pixels. This displacement map helps in plotting the strain image which is able to provide with the required information.

There can be error due to decorrelation as the compression maybe due to freehand palpations too. A novel idea of Dynamic Programming is introduced which helps in cutting losses by making the cost function efficient. This cost function is responsible for the main function of the elastography technique. The use of DP to make the cost function efficient involves the breaking down of the main function into smaller sub-costs and then building the function back up through block matching algorithm.

The entire process is initially done for 1 dimension i.e. the rows or samples from i=1 to m. In this case, single columns or A-lines are selected for the experiment. The 1 D results show only axial displacement and hence the strain image gives some sort of edge distortions in the strain image. The strain image is calculated using both the gradient function and linear regression out of which only linear regression shows good results. CNR and SNR values are calculated for the setup. The mean CNR comes out to be 1.75 while the SNR inside the hard tissue structure comes out to be 1.5582 as compared to the 2.2158 outside the hard part.

For 2D simulation, the cost function C, memoization function M and displacement map D are calculated over the entire range of columns i.e 1 to j and rows i=1 to m. The study includes axial as well as lateral displacement. This is the reason that the strain image for the 2D study does not show any edge distortions and accurately detects the hard tissue. The strain image is calculated for different frame sizes and the result section shows a comparison between the two. The CNR value for the 2D DP comes out to be 7.2422 while the SNR values at the top and middle of the image are 3.1708 and 2. 3829 respectively.

Elastography is a clear method of detection but it can have errors due to application of uneven pressure or error due to freehand palpations. The DP method is also not very efficient if compared with the AM (Analytic minimization) as if offers integral displacements. However, the system is able to detect accurately, handle small errors, combat decorrelation and relate tangible results. DP can be used in conjunction with other techniques as well which can work on enhancing the outputs received from this method.

REFERENCES

[1] B. Garra, E. Céspedes, J. Ophir, S. Spratt, R. Zuurbier, C.Magnant, and
M. Pennanen, “Elastography of breast lesions: Initial clinical results,”
Radiology, vol. 202, pp. 79–86, 1997.
[2]K. Hiltawsky, M. Kruger, C. Starke, L. Heuser, H. Ermert, and A.
Jensen, “Freehand ultrasound elastography of breast lesions: Clinical
results,” *Ultrasound Med. Biol.*, vol. 27, pp. 1461–1469, 2001.
[3] H. Rivaz, E. Boctor, P. Foroughi, G. Fichtinger, and G. Hager, “Ultrasound elastography: a dynamic programming approach,” IEEE Trans
Med Imaging, vol. 27, no. 10, pp. 1373–1377, Oct. 2008.
[4]"Hassan Rivaz", *Users.encs.concordia.ca*, 2018. [Online]. Available:
https://users.encs.concordia.ca/~hrivaz/Ultrasound_Elastography/. [Accessed: 04- Dec- 2018].
[5]H. Rivaz, E. Boctor, M. Choti and G. Hager, "Real-Time Regularized Ultrasound Elastography", *IEEE Transactions on Medical Imaging*, vol. 30, no. 4, pp. 928-945, 2011.