Lecture 05:
Language modelling

OVERVIEW

- What is language modeling
- Why do we need language modeling
- Probabilistic Language Models
- Language modeling with n-grams

LANGUAGE MODEL

Formal definition

- Given a finite vocabulary *V* of words (tokens):
- Formal language: let *Ω* be a set of sequences of words from *V,*

$$\forall\, x \in \Omega, x \textit{ is called a sentence}$$

- A **Language Modeling** is a probability distribution over a sequence of words (tokens) from a formal language *Ω*,

$$P : \Omega \rightarrow [0, 1] \qquad \sum_{x \in \Omega} P(x) = 1$$

LANGUAGE MODEL

A language model aims to answer the question which sequences are more likely?

I would like to eat. *I would like eat to.* *I like would eat to.* *I to like would eat.*

P(I would like to eat) *P(I would like eat to)* *P(I like would eat to)* *P(I to like would eat)*

We expect that regular and grammatically correct sentences will occur more often in text and speech than other weird sequences

P(I would like to eat) > P(I like would eat to)

P(I would like to eat) > P(I like would eat to)

P(I would like to eat) > P(I to like would eat)

LANGUAGE MODELS - USAGE

Many NLP tasks require natural language output estimated from a sentence probability

- Machine translation: *"vents violents ce soir"*

$$P(high\ winds\ tonight) > P(large\ winds\ tonight)$$

- Speech recognition: return a transcript of what was spoken

$$P(I\ saw\ a\ van) >> P(eyes\ awe\ of\ an)$$

- Spell-correction: *"the office is about fifteen minuets from my house"*

$$P(about\ fifteen\ minutes\ from) > P(about\ fifteen\ minuets\ from)$$

- Summarization, question answering, etc

LANGUAGE MODELING

- How do we compute the probability of a sequence of tokens $x^1, x^2, \ldots, x^t$ from a vocabulary V?

- The probability of $x^1, x^2, \ldots, x^t$ is the joint probability $P(x^1, x^2, \ldots, x^t)$ and estimate by the chain rule of probability

$$P(x^1, x^2, \ldots, x^{t-1}, x^t) = P(x^1 | x^2, \ldots, x^{t-1}, x^t) P(x^2, \ldots, x^{t-1}, x^t)$$

$$= P(x^1) P(x^2 | x^1) P(x^3 | x^1, x^2) \ldots . P(x^t \, | x^1, \ldots, x^{t-1})$$

$$= P(x^1) \prod_{k=1}^{t} P(x^k | x^1, \ldots, x^{k-1})$$

LANGUAGE MODEL

Given the sentence *"the student open their books"*
How do we compute the probability of *P(the student open their books)*

$$
\begin{aligned}
P(the\ student\ open\ their\ books) = \ & P(books|the\ student\ open\ their) * \\
& P(their|the\ student\ open) * \\
& P(open|the\ student) * \\
& P(student|the) * \\
& P(the)
\end{aligned}
$$

We estimate the probabilities by counting

$$
P(books|the\ student\ open\ their) = \frac{count(the\ student\ open\ their\ books)}{count(the\ student\ open\ their)}
$$

LANGUAGE MODEL – ESTIMATING PROBABILITIES

A language model is a distribution over all word-sequences $x_1 x_2, \dots, x_n$ in a vocabulary V

$$\sum_{\langle x_1 x_2, \dots, x_n \rangle} P(x_1 x_2, \dots, x_n) = 1$$

by derivation is given by $P(x^1, x^2, \dots, x^{t-1}, x^t) = P(x^1) \prod_{k=1}^{t} P(x^k | x^1, \dots, x^{k-1})$

To estimate the probability of each sequence we need to

- first estimate $P(x^1)$
- Estimate probabilities $P(x^k | x^1, \dots, x^{k-1})$ for all $x^1, \dots, x^k$

LANGUAGE MODEL – ESTIMATING PROBABILITIES

- Relative frequency from a corpus

$$P(x_i|x_1,\dots,x_{i-1}) = \frac{count(x_1,\dots,x_{i-1},x_i)}{count(x_1,\dots,x_{i-1})}$$
$$= \frac{count(x_1,\dots,x_{i-1},x_i)}{\sum_{x\in V} count(x_1,\dots,x_{i-1},\ x)}$$

NB: this is the estimate for all sequences of length *l*

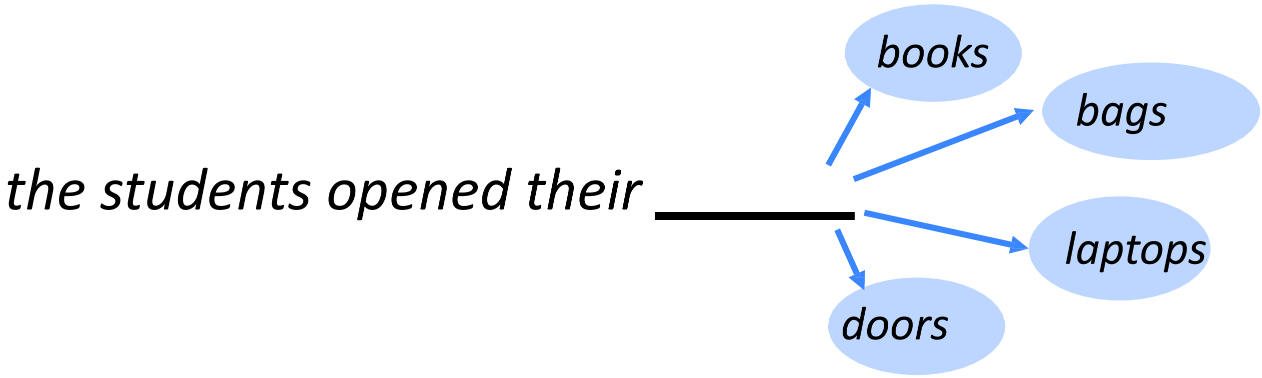
- Suppose /V/ = 1000, all sentences are approximately 10 word long, then we need to estimate 1000^{10} probabilities.

- no corpus is large enough to obtain an unbiased estimate of the probabilities

LANGUAGE MODELING

Language Modeling can be reduced to the task of predicting the next

$$P(x^1, x^2, \ldots, x^{t-1}, x^t) = P(x^1) \prod_{n=2}^{t} \boldsymbol{P(x^n | x^1, \ldots, x^{n-1})}$$



Given a sequence of words $x^{(1)}, x^{(2)}, \ldots, x^{(t)}$ compute the probability distribution of the next word $x^{(t+1)}$:

$$P(\boldsymbol{x}^{(t+1)} | \ \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})$$

LANGUAGE MODEL

- Probabilistic language models are based on the grouping of words into chunks called n-grams in order to estimate

$$P(\boldsymbol{x}^{(t+1)} | \, \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})$$

- The main idea:
  - collect statistics about the frequency of different *n-grams*
    - n-gram probabilities are estimated by counting the frequency of occurrence in the vocabulary
  - use this to estimate the next word given a history of observed words

LANGUAGE MODEL – N-GRAMS

- N-gram language models are based on probabilities of chunks of word.

The student open their

- An n-gram is a chunk of n consecutive words.
  - *unigram: unit of single word – "the", "student", "opened", "their"*
  - *bigrams: unit of double words – "the student", "student opened", "opened their"*
  - *trigram: unit of triple words – "the student opened", "student opened their"*
  - *4-gram: unit of 4 words – "the student opened their"*

- The main idea behind *n-gram* models:
  - collect statistics about the frequency of different *n-grams*
  - use this to predict the next word.

LANGUAGE MODELING – MARKOV ASSUMPTION

Markov assumption

- Independent and identical trials
- There is a fixed and finite *k* such that all word depends only on the preceding *k-1* words

$$P(x_{i+1}|x_1,\dots,x_i) \approx P(x_{i+1}|x_{i-k},\dots,x_i)\ \forall k \geq 0$$

- Model: an k^{th}- order Markov model
- n-gram: statistics of an k-order Markov model is k $+1 - gram$ model

$$P(x_i|\boxed{x_{i-k}},\dots,x_{i-1}) = \frac{count(\boxed{x_{i-k}}\dots,x_{i-1},x_i)}{\sum_{x\in V} count(\boxed{x_{i-k}}\dots,x_{i-1},\ x)}$$

LANGUAGE MODELING – MARKOV ASSUMPTION

The order of a Markov model is defined by the length of its history or *n-gram* *(n = k+1)*

$0^{th} - order$: $P(x_1, \ldots, x_n) \approx P(x_1) \prod_{i=1}^{n-1} P(x_{i+1}) \approx P(x_1)P(x_2) \ldots P(x_n)$

|history| = 0

$1^{st} - order$: $P(x_1, \ldots, x_n) \approx P(x_1) \prod_{i=1}^{n-1} P(x_{i+1}|x_i)$

|history| = 1

$2^{nd} - order$: $P(x_1, \ldots, x_n) \approx P(x_1) \prod_{i=1}^{n-1} P(x_{i+1}|x_i, x_{i-1})$

|history| = 2

$k^{th} - order$: $P(x_1, \ldots, x_n) \approx P(x_1) \prod_{i=1}^{n-1} P(x_{i+1}|x_i, \ldots, x_{i-k})$

|history| = k

LANGUAGE MODEL - FROM N-GRAM PROBABILITIES

In a trigram model

$$P(x^1x^2x^3) = P(x^1)P(x^2|x^1)P(x^3|x^1x^2)$$

- The only trigram is $P(x^3|x^1x^2)$
  - $P(x^1)$ and $P(x^2|x^1)$ are not trigrams thus are from a different probability distribution

- Solution:
  - add *n-1* beginning of sentence (**<s>**) symbols

    <s><s>$x^1x^2x^3$……

  - similarly add *n-1* end of sentence symbols

    $\ldots\ldots x^1x^2x^3$ </s></s>

FROM N-GRAM PROBABILITIES TO LANGUAGE MODEL

- With the start or end of sentence token(s) we define a new vocabulary

$$V^* = V \, U \, \{</s>\}$$
$$or$$
$$V^* = V \, U \, \{<s>\}$$

- With the new vocabulary we can get a single distribution over strings of any length

- **Why?**
  - because *P(</s>|…)* will be high enough that we are always guaranteed to stop after generating a finite number of words.

LANGUAGE MODELING WITH N-GRAM

Maximum likelihood estimate

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

Example

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$P(\texttt{I} \mid \texttt{<s>}) = \frac{2}{3} = .67$ $\quad P(\texttt{Sam} \mid \texttt{<s>}) = \frac{1}{3} = .33$ $\quad P(\texttt{am} \mid \texttt{I}) = \frac{2}{3} = .67$

ESTIMATING PROBABILITIES - PRACTICAL ISSUES

- We do everything in log space
  - Avoid underflow
  - Computationally adding is faster than multiplying

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

NUMBER OF POSSIBLE PARAMETERS

- Estimating the number of parameter per n-gram language model.
- Given a vocabulary V of $|V|$ unique tokens, where $|V| = 10^4$

  - **Unigram** model: $|V|$ parameters ⇔ 10^4 parameters
  - **Bigram** model: $|V|^2$ parameters ⇔ 10^8 parameters
  - **Trigram** model: $|V|$ parameters ó 10^{12} parameters

SHAKESPEARE AS CORPUS

- Number of words (symbols) = 884,647
- Tokens, V=29,066
- Shakespeare produced 300,000 bigrams
- bigram types out of V^2= 844 million possible bigrams
- So, 99.96% of the possible bigrams were never seen (*have zero entries in the table*)

    844 million – 300,000 unused bigrams
- Quadrigrams worse:
    - What's coming out looks like Shakespeare because it ***is*** Shakespeare

EVALUATION: HOW GOOD IS OUR MODEL?

- Does our language model prefer good sentences to bad ones?
- Assign higher probability to "real" or "frequently observed" sentences
  - than "ungrammatical" or "rarely observed" sentences?
- We train parameters of our model on a **training set**.
- We test the model's performance on data we haven't seen.
  - A **test set** is an unseen dataset that is different from our training set, totally unused.
  - An **evaluation metric** tells us how well our model does on the test set.

EXTRINSIC EVALUATION OF N-GRAM MODELS

- Best evaluation for comparing models A and B
- Embed each model in a task
  - spelling corrector,
  - speech recognizer,
  - Machine Translation system
- Run the task, get an accuracy for A and for B
  - How many misspelled words corrected properly
  - How many words translated correctly
- Compare accuracy for A and B

Time-consuming; can take days, weeks or even months

INTRINSIC EVALUATION OF N-GRAM MODELS

- Sometimes use **intrinsic** evaluation: **perplexity**
- Bad approximation
  - unless the test data looks **just** like the training data
  - So **generally, only useful in pilot experiments**
- But is helpful to think about.

PERPLEXITY

- Perplexity is the inverse probability of the test set, normalized by the number of words *N*:

$$perplexity(W) = P(x_1 x_2, \dots, x_N)^{-1/N} = \sqrt[N]{\frac{1}{P(x_1 x_2, \dots, x_N)}} = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(x_i | x_1, \dots, x_{i-1})}}$$

- For unigram $perplexity(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(x_i)}}$

- For bigram $perplexity(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(x_i | x_{i-1})}}$

NB: Minimizing perplexity is the same as maximizing probability

INTUITION OF PERPLEXITY

- The **Shannon Game**:
  - How well can we predict the next word?

I always order pizza with cheese and ____

The 33rd President of the US was ____

I saw a ____

laptop	0.09
mushrooms	0.02
pepperoni	0.1
anchovies	0.0001
fried rice	0.005
and	1e-10
cook	2e-15

  - Unigrams are terrible at this game. (Why?)
- A better language model is the one that assigns a higher probability to the most appropriate word

LIMITATIONS – STORAGE PROBLEMS

Storage Problem:
The need to store count for all n-grams you saw in the corpus.

$$P(\boldsymbol{w}|\ I\,want\,to\,eat) = \frac{\boxed{count(I\,want\,to\,eat\,\boldsymbol{w})}}{count(I\,want\,to\,eat)}$$

Increasing *n* or increasing corpus size <=> increases model size

LIMITATIONS - SPARSITY PROBLEM

Sparsity Problem 1:

- What if "*I want to w*" never occurred in the corpus?
- Then *w* has probability 0

Solution - smoothing: Add small δ to the count for every $w \in V$.

$$P(\boldsymbol{w}|waiting\ for\ the) = \frac{\boxed{count(I\ want\ to\ \boldsymbol{w})}}{\boxed{count(I\ want\ to)}}$$

Sparsity Problem 2:

- What if "*I want to*" never occurred in the corpus?
- Then we cannot calculate probabiliity of any *w*.

Solution - backoff: condition on the (n-1)-gram, i.e. "*I want*"

Larger *n* makes sparsity problem worse. Typically *n* should be less than or equal to 5

THE PERILS OF OVERFITTING - SPARSITY

N-grams only work well for word prediction if the test corpus looks like the training corpus.

- In real life, it often doesn’t
  - We need to train robust models that generalize!

- One kind of generalization: Zeros!
  - Things that don’t ever occur in the training set but occur in the test set

SMOOTHING METHODS

Smoothing methods

- Additive smoothing
- Good-Turing estimate
- Jelinek-Mercer smoothing (interpolation)
- Katz smoothing (backoff)
- Witten-Bell smoothing
- Absolute discounting
- Kneser-Ney smoothing

ADDITIVE SMOOTHING

- Idea: pretend we've seen each n-gram δ times more than we have.
  - Typically, $0 < \delta \leq 1$.
  - Lidstone and Jeffreys advocate $\delta = 1$.
  - Gale & Church (1994) argue that this method performs poorly.

$$p_{add}(w_i|w_{i-n+1}^{i-1}) = \frac{\delta + c(w_{i-n+1}^{i})}{\delta|V| + \sum_{w_i} c(w_{i-n+1}^{i})}$$

ADD-ONE ESTIMATION

- Also called Laplace smoothing
- Pretend we saw each word one more time than we did
- Just add one to all the counts!
- MLE estimate:

$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-1 estimate:

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

BACKOFF AND INTERPOLATION

- Sometimes it helps to use **less** context
  - Condition on less context for contexts you haven't learned much about
- **Backoff:**
  - use trigram if you have good evidence,
  - otherwise, bigram, otherwise unigram
- **Interpolation:**
  - mix unigram, bigram, trigram
- NB: Interpolation works better

REFERENCES