Dynamic Equations for the 3-DOF Helicopter

URL: http://www.quanser.com

Description

• The Lagrange's method is used to obtain the dynamic model of the system.

The *Quanser_Tools* Package

• The *Quanser_Tools* module defines generic procedures and data in relation to determining the state-space representation of all the Quanser experiments. Specifically, this means deriving and solving the Lagrange's equations of the Quanser systems.

• The *quanser* repository containing the *Quanser_Tools* package is implemented in the 2 following files: *quanser.ind* and *quanser.lib.* If these two files are not readily available, they can be generated by executing the Maple worksheet titled: *quanser_tools.mws*.

• **To install** the *Quanser_Tools* package, copy the two files *quanser.ind* and *quanser.lib* into a directory of your choice, like for example: "C:\Program Files\Quanser\Maple Repository".

• **To use** the *Quanser_Tools* package in a Maple worksheet, add the path to its disk location to the Maple global variable *libname*. For example, this can be achieved by the following Maple command:
libname := *"C:/Program Files/Quanser/Maple Repository", libname:*

Worksheet Initialization

```
> restart: interface( imaginaryunit = j ):
> with( LinearAlgebra ):
> libname := "C:\Maple", ".", libname:
> with( Quanser_Tools );
```

[*HTM*, *deriveA*, *deriveB*, *deriveF*, *kinetic_energy*, *lagrange_equations*, *moment_of_inertia*, *n_norm*, *potential_energy*, *write_ABCD_to_Mfile*] **(2.1)**

environment variable representing the order of series calculations

```
> Order := 2:
```

Notations

Generalized Coordinates: q_i's

The generalized coordinates are also called Lagrangian coordinates.

```
> q := [ epsilon(t), p(t), lambda(t) ];
```

$$q := [\epsilon(t), p(t), \lambda(t)] \quad \textbf{(3.1.1)}$$

Nq = number of Lagrangian coordinates
Nq is also the number of position states.

```
> Nq := nops( q ):
```

qd = first-order time derivative of the generalized coordinates

```
> qd := map( diff, q, t );
```

$$qd := \left[\frac{d}{dt}\,\epsilon(t),\ \frac{d}{dt}\,p(t),\ \frac{d}{dt}\,\lambda(t) \right] \tag{3.1.2}$$

Cartesian Coordinates of the Moving Bodies

```
> HTM_BASE_TO_TRAVEL := HTM( 'rot', 'Z', -lambda(t) );
```

$$HTM\_BASE\_TO\_TRAVEL := \begin{bmatrix} \cos(\lambda(t)) & \sin(\lambda(t)) & 0 & 0 \\ -\sin(\lambda(t)) & \cos(\lambda(t)) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2.1}$$

```
> HTM_TRAVEL_TO_CW := Multiply( HTM( 'rot', 'X', epsilon(t) ),
  HTM( 'trans', 0, -L[w], 0 ) );
```

$$HTM\_TRAVEL\_TO\_CW := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\epsilon(t)) & -\sin(\epsilon(t)) & -\cos(\epsilon(t))\,L_w \\ 0 & \sin(\epsilon(t)) & \cos(\epsilon(t)) & -\sin(\epsilon(t))\,L_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2.2}$$

```
> HTM_TRAVEL_TO_HB := Multiply( HTM( 'rot', 'X', epsilon(t) ),
  HTM( 'trans', 0, L[a], 0 ) );
```

$$HTM\_TRAVEL\_TO\_HB := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\epsilon(t)) & -\sin(\epsilon(t)) & \cos(\epsilon(t))\,L_a \\ 0 & \sin(\epsilon(t)) & \cos(\epsilon(t)) & \sin(\epsilon(t))\,L_a \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2.3}$$

```
> HTM_HB_TO_FM := Multiply( HTM( 'rot', 'Y', -p(t) ), HTM(
  'trans', L[h], 0, 0 ) );
```

$$HTM\_HB\_TO\_FM := \begin{bmatrix} \cos(p(t)) & 0 & -\sin(p(t)) & \cos(p(t))\,L_h \\ 0 & 1 & 0 & 0 \\ \sin(p(t)) & 0 & \cos(p(t)) & \sin(p(t))\,L_h \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2.4}$$

```
> HTM_HB_TO_BM := Multiply( HTM( 'rot', 'Y', -p(t) ), HTM(
  'trans', -L[h], 0, 0 ) );
```

$$HTM\_HB\_TO\_BM := \begin{bmatrix} \cos(p(t)) & 0 & -\sin(p(t)) & -\cos(p(t))\,L_h \\ 0 & 1 & 0 & 0 \\ \sin(p(t)) & 0 & \cos(p(t)) & -\sin(p(t))\,L_h \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2.5}$$

```
> HTM_BASE_TO_CW := Multiply( HTM_BASE_TO_TRAVEL,
  HTM_TRAVEL_TO_CW );
```

$$HTM\_BASE\_TO\_CW := [[\cos(\lambda(t)), \sin(\lambda(t))\cos(\epsilon(t)), -\sin(\lambda(t))\sin(\epsilon(t)), -\sin(\lambda(t))\cos(\epsilon(t))\,L_w], [-\sin(\lambda(t)), \cos(\lambda(t))\cos(\epsilon(t)), -\cos(\lambda(t))\sin(\epsilon(t)), -\cos(\lambda(t))\cos(\epsilon(t))\,L_w], [0, \sin(\epsilon(t)), \cos(\epsilon(t)), -\sin(\epsilon(t))\,L_w], [0, 0, 0, 1]] \tag{3.2.6}$$

```
> HTM_BASE_TO_HB := Multiply( HTM_BASE_TO_TRAVEL,
  HTM_TRAVEL_TO_HB );
```

$$HTM\_BASE\_TO\_HB := [[\cos(\lambda(t)), \sin(\lambda(t))\cos(\epsilon(t)), -\sin(\lambda(t))\sin(\epsilon(t)), \sin(\lambda(t))\cos(\epsilon(t))\,L_a], [-\sin(\lambda(t)), \cos(\lambda(t))\cos(\epsilon(t)), -\cos(\lambda(t))\sin(\epsilon(t)), \cos(\lambda(t))\cos(\epsilon(t))\,L_a], [0, \sin(\epsilon(t)), \cos(\epsilon(t)), \sin(\epsilon(t))\,L_a], [0, 0, 0, 1]] \tag{3.2.7}$$

```
> HTM_BASE_TO_FM := Multiply( HTM_BASE_TO_HB, HTM_HB_TO_FM );
```

$$HTM\_BASE\_TO\_FM := [[\cos(\lambda(t))\cos(p(t)) - \sin(\lambda(t))\sin(\epsilon(t))\sin(p(t)), \sin(\lambda(t))\cos(\epsilon(t)), -\cos(\lambda(t))\sin(p(t)) - \sin(\lambda(t))\sin(\epsilon(t))\cos(p(t)), \cos(\lambda(t))\cos(p(t))\,L_h - \sin(\lambda(t))\sin(\epsilon(t))\sin(p(t))\,L_h + \sin(\lambda(t))\cos(\epsilon(t))\,L_a], [-\sin(\lambda(t))\cos(p(t)) - \cos(\lambda(t))\sin(\epsilon(t))\sin(p(t)), \cos(\lambda(t))\cos(\epsilon(t)), \sin(\lambda(t))\sin(p(t)) - \cos(\lambda(t))\sin(\epsilon(t))\cos(p(t)), -\sin(\lambda(t))\cos(p(t))\,L_h - \cos(\lambda(t))\sin(\epsilon(t))\sin(p(t))\,L_h + \cos(\lambda(t))\cos(\epsilon(t))\,L_a], [\cos(\epsilon(t))\sin(p(t)), \sin(\epsilon(t)), \cos(\epsilon(t))\cos(p(t)), \cos(\epsilon(t))\sin(p(t))\,L_h + \sin(\epsilon(t))\,L_a], \tag{3.2.8}$$

$[0, 0, 0, 1]]$

```
> HTM_BASE_TO_BM := Multiply( HTM_BASE_TO_HB, HTM_HB_TO_BM );
```

$$HTM\_BASE\_TO\_BM := [[\cos(\lambda(t)) \cos(p(t)) - \sin(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)), \sin(\lambda(t)) \cos(\epsilon(t)), -\cos(\lambda(t)) \sin(p(t)) - \sin(\lambda(t)) \sin(\epsilon(t)) \cos(p(t)), -\cos(\lambda(t)) \cos(p(t)) L_h + \sin(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h + \sin(\lambda(t)) \cos(\epsilon(t)) L_a], [-\sin(\lambda(t)) \cos(p(t)) - \cos(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)), \cos(\lambda(t)) \cos(\epsilon(t)), \sin(\lambda(t)) \sin(p(t)) - \cos(\lambda(t)) \sin(\epsilon(t)) \cos(p(t)), \sin(\lambda(t)) \cos(p(t)) L_h + \cos(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h + \cos(\lambda(t)) \cos(\epsilon(t)) L_a], [\cos(\epsilon(t)) \sin(p(t)), \sin(\epsilon(t)), \cos(\epsilon(t)) \cos(p(t)), -\cos(\epsilon(t)) \sin(p(t)) L_h + \sin(\epsilon(t)) L_a], [0, 0, 0, 1]] \tag{3.2.9}$$

```
> x[cw] := HTM_BASE_TO_CW[ 1, 4 ];
  y[cw] := HTM_BASE_TO_CW[ 2, 4 ];
  z[cw] := HTM_BASE_TO_CW[ 3, 4 ];
```

$$x_{cw} := -\sin(\lambda(t)) \cos(\epsilon(t)) L_w$$

$$y_{cw} := -\cos(\lambda(t)) \cos(\epsilon(t)) L_w$$

$$z_{cw} := -\sin(\epsilon(t)) L_w \tag{3.2.10}$$

```
> x[fm] := HTM_BASE_TO_FM[ 1, 4 ];
  y[fm] := HTM_BASE_TO_FM[ 2, 4 ];
  z[fm] := HTM_BASE_TO_FM[ 3, 4 ];
```

$$x_{fm} := \cos(\lambda(t)) \cos(p(t)) L_h - \sin(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h + \sin(\lambda(t)) \cos(\epsilon(t)) L_a$$

$$y_{fm} := -\sin(\lambda(t)) \cos(p(t)) L_h - \cos(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h + \cos(\lambda(t)) \cos(\epsilon(t)) L_a$$

$$z_{fm} := \cos(\epsilon(t)) \sin(p(t)) L_h + \sin(\epsilon(t)) L_a \tag{3.2.11}$$

```
> x[bm] := HTM_BASE_TO_BM[ 1, 4 ];
  y[bm] := HTM_BASE_TO_BM[ 2, 4 ];
  z[bm] := HTM_BASE_TO_BM[ 3, 4 ];
```

$$x_{bm} := -\cos(\lambda(t)) \cos(p(t)) L_h + \sin(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h + \sin(\lambda(t)) \cos(\epsilon(t)) L_a$$

$$y_{bm} := \sin(\lambda(t)) \cos(p(t)) L_h + \cos(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h + \cos(\lambda(t)) \cos(\epsilon(t)) L_a$$

$$z_{bm} := -\cos(\epsilon(t)) \sin(p(t)) L_h + \sin(\epsilon(t)) L_a \tag{3.2.12}$$

```
> xd[cw] := diff( x[cw], t );
  yd[cw] := diff( y[cw], t );
  zd[cw] := diff( z[cw], t );
```

$$xd_{cw} := -\left(\frac{d}{dt} \lambda(t)\right) \cos(\lambda(t)) \cos(\epsilon(t)) L_w + \sin(\lambda(t)) \left(\frac{d}{dt} \epsilon(t)\right) \sin(\epsilon(t)) L_w$$

$$yd_{cw} := \left(\frac{d}{dt} \lambda(t)\right) \sin(\lambda(t)) \cos(\epsilon(t)) L_w + \cos(\lambda(t)) \left(\frac{d}{dt} \epsilon(t)\right) \sin(\epsilon(t)) L_w$$

$$zd_{cw} := -\left(\frac{d}{dt} \epsilon(t)\right) \cos(\epsilon(t)) L_w \tag{3.2.13}$$

```
> xd[fm] := diff( x[fm], t );
  yd[fm] := diff( y[fm], t );
  zd[fm] := diff( z[fm], t );
```

$$xd_{fm} := -\left(\frac{d}{dt} \lambda(t)\right) \sin(\lambda(t)) \cos(p(t)) L_h - \cos(\lambda(t)) \left(\frac{d}{dt} p(t)\right) \sin(p(t)) L_h - \left(\frac{d}{dt} \lambda(t)\right) \cos(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h - \sin(\lambda(t)) \left(\frac{d}{dt} \epsilon(t)\right) \cos(\epsilon(t)) \sin(p(t)) L_h - \sin(\lambda(t)) \sin(\epsilon(t)) \left(\frac{d}{dt} p(t)\right) \cos(p(t)) L_h + \left(\frac{d}{dt} \lambda(t)\right) \cos(\lambda(t)) \cos(\epsilon(t)) L_a - \sin(\lambda(t)) \left(\frac{d}{dt} \epsilon(t)\right) \sin(\epsilon(t)) L_a$$

$$yd_{fm} := -\left(\frac{d}{dt} \lambda(t)\right) \cos(\lambda(t)) \cos(p(t)) L_h + \sin(\lambda(t)) \left(\frac{d}{dt} p(t)\right) \sin(p(t)) L_h + \left(\frac{d}{dt} \lambda(t)\right) \sin(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h - \cos(\lambda(t)) \left(\frac{d}{dt} \epsilon(t)\right) \cos(\epsilon(t)) \sin(p(t)) L_h - \cos(\lambda(t)) \sin(\epsilon(t)) \left(\frac{d}{dt} p(t)\right) \cos(p(t)) L_h - \left(\frac{d}{dt} \lambda(t)\right) \sin(\lambda(t)) \cos(\epsilon(t)) L_a - \cos(\lambda(t)) \left(\frac{d}{dt} \epsilon(t)\right) \sin(\epsilon(t)) L_a$$

$$zd_{fm} := -\left(\frac{d}{dt} \epsilon(t)\right) \sin(\epsilon(t)) \sin(p(t)) L_h + \cos(\epsilon(t)) \left(\frac{d}{dt} p(t)\right) \cos(p(t)) L_h + \left(\frac{d}{dt} \epsilon(t)\right) \cos(\epsilon(t)) L_a \tag{3.2.14}$$

```
> xd[bm] := diff( x[bm], t );
  yd[bm] := diff( y[bm], t );
  zd[bm] := diff( z[bm], t );
```

$$xd_{bm} := \left(\frac{d}{dt}\lambda(t)\right)\sin(\lambda(t))\cos(p(t))L_h + \cos(\lambda(t))\left(\frac{d}{dt}p(t)\right)\sin(p(t))L_h + \left(\frac{d}{dt}\lambda(t)\right)\cos(\lambda(t))\sin(\epsilon(t))\sin(p(t))L_h + \sin(\lambda(t))\left(\frac{d}{dt}\epsilon(t)\right)\cos(\epsilon(t))\sin(p(t))L_h + \sin(\lambda(t))\sin(\epsilon(t))\left(\frac{d}{dt}p(t)\right)\cos(p(t))L_h + \left(\frac{d}{dt}\lambda(t)\right)\cos(\lambda(t))\cos(\epsilon(t))L_a - \sin(\lambda(t))\left(\frac{d}{dt}\epsilon(t)\right)\sin(\epsilon(t))L_a$$

$$yd_{bm} := \left(\frac{d}{dt}\lambda(t)\right)\cos(\lambda(t))\cos(p(t))L_h - \sin(\lambda(t))\left(\frac{d}{dt}p(t)\right)\sin(p(t))L_h - \left(\frac{d}{dt}\lambda(t)\right)\sin(\lambda(t))\sin(\epsilon(t))\sin(p(t))L_h + \cos(\lambda(t))\left(\frac{d}{dt}\epsilon(t)\right)\cos(\epsilon(t))\sin(p(t))L_h + \cos(\lambda(t))\sin(\epsilon(t))\left(\frac{d}{dt}p(t)\right)\cos(p(t))L_h - \left(\frac{d}{dt}\lambda(t)\right)\sin(\lambda(t))\cos(\epsilon(t))L_a - \cos(\lambda(t))\left(\frac{d}{dt}\epsilon(t)\right)\sin(\epsilon(t))L_a$$

$$zd_{bm} := \left(\frac{d}{dt}\epsilon(t)\right)\sin(\epsilon(t))\sin(p(t))L_h - \cos(\epsilon(t))\left(\frac{d}{dt}p(t)\right)\cos(p(t))L_h + \left(\frac{d}{dt}\epsilon(t)\right)\cos(\epsilon(t))L_a \tag{3.2.15}$$

State-Space Variables

- The chosen states should at least include the generalized coordinates and their first-time derivatives.
- X is the state vector.
- In the state vector X: Lagrangian coordinates are first, followed by their first-time derivatives, and finally any other states, as required.

Substitution sets for the states (to obtain time-independent state equations).

```
> subs_Xq := { seq( q[i] = X[i], i=1..Nq ) };
  subs_Xqd := { seq( qd[i] = X[i+Nq], i=1..Nq ) };
```

$$subs\_Xq := \{\epsilon(t) = X_1, \lambda(t) = X_3, p(t) = X_2\}$$

$$subs\_Xqd := \left\{\frac{d}{dt}\epsilon(t) = X_4, \frac{d}{dt}\lambda(t) = X_6, \frac{d}{dt}p(t) = X_5\right\} \tag{3.3.1}$$

Substitution set for the input(s).

```
> subs_U := { V[f] = U[1], V[b] = U[2] };
```

$$subs\_U := \{V_b = U_2, V_f = U_1\} \quad \textbf{(3.3.2)}$$

Nu = number of inputs; U = input (row) vector

```
> Nu := nops( subs_U ):
```

substitution set for the position states' second time derivatives

```
> subs_Xqdd := { seq( diff( q[i], t$2 ) =  Xd[i+Nq], i=1..Nq )
  };
```

$$subs\_Xqdd := \left\{\frac{d^2}{dt^2}\,\epsilon(t) = Xd_4,\ \frac{d^2}{dt^2}\,\lambda(t) = Xd_6,\ \frac{d^2}{dt^2}\,p(t) = Xd_5\right\} \quad \textbf{(3.3.3)}$$

second time derivatives of the position states (written as time-independent variables).
The set of unknowns is obtained from this list to solve the Lagrange's equations of motion.

```
> Xqdd := [ seq( Xd[i+Nq], i=1..Nq ) ];
```

$$Xqdd := [Xd_4, Xd_5, Xd_6] \quad \textbf{(3.3.4)}$$

substitution set to linearize the state-space matrices (i.e. A and B)
about the quiescent null state vector (small-displacement theory)

```
> subs_XU_op := { seq( X[i] = 0, i=1..2*Nq ), seq( U[i] = 0, i=
  1..Nu ) }:
```

Nx = dim(X) = total number of states (should be greater than or equal to: 2 * Nq)
Ny = chosen number of outputs

```
> Nx := 2 * Nq + 0:
  Ny := Nq:
```

Total Potential and Kinetic Energies of the System

The total potential and kinetic energies are needed to calculate the Lagrangian of the system.

Total Potential Energy: V_T

The total potential energy can be expressed in terms of the generalized coordinates alone.

Ve[T] = Total Elastic Potential Energy of the system

```
> Ve[T] := 0:
```

Vg[T] = Total Gravitational Potential Energy of the system
initialization:

```
> Vg[cw] := potential_energy( 'gravity', m[w], g, z[cw] );
```

$$Vg_{cw} := -m_w\, g \sin(\epsilon(t))\, L_w \quad \textbf{(4.1.1)}$$

```
> Vg[fm] := potential_energy( 'gravity', m[f], g, z[fm] );
```

$$Vg_{fm} := m_f\, g\, \left(\cos(\epsilon(t))\sin(p(t))\, L_h + \sin(\epsilon(t))\, L_a\right) \quad \textbf{(4.1.2)}$$

```
> Vg[bm] := potential_energy( 'gravity', m[b], g, z[bm] );
```

$$Vg_{bm} := m_b\, g\, \left(-\cos(\epsilon(t))\sin(p(t))\, L_h + \sin(\epsilon(t))\, L_a\right) \quad \textbf{(4.1.3)}$$

```
> Vg[T] := Vg[cw] + Vg[fm] + Vg[bm]:
```

V[T] = Total Potential Energy of the system

```
> V[T] := simplify( Ve[T] + Vg[T] );
```

$$V_T := -g\left(\left(\left(-m_b - m_f\right) L_a + L_w m_w\right) \sin(\epsilon(t)) + \sin(p(t)) \cos(\epsilon(t)) L_h \left(m_b - m_f\right)\right) \quad \textbf{(4.1.4)}$$

Total Kinetic Energy: T_T

The total kinetic energy can be expressed in terms of the generalized coordinates and their first-time derivatives.

```
> v[cw] := n_norm( [ xd[cw], yd[cw], zd[cw] ], 2 ):
> Tt[cw] := kinetic_energy( 'translation', m[w], v[cw] );
```

$$\begin{aligned} Tt_{cw} := \frac{1}{2} m_w \Bigg(& \left(-\left(\frac{d}{dt} \lambda(t)\right) \cos(\lambda(t)) \cos(\epsilon(t)) L_w \right. \\ & \left. + \sin(\lambda(t)) \left(\frac{d}{dt} \epsilon(t)\right) \sin(\epsilon(t)) L_w\right)^2 \\ & + \left(\left(\frac{d}{dt} \lambda(t)\right) \sin(\lambda(t)) \cos(\epsilon(t)) L_w + \cos(\lambda(t)) \left(\frac{d}{dt} \epsilon(t)\right) \sin(\epsilon(t)) L_w\right)^2 \\ & + \left(\frac{d}{dt} \epsilon(t)\right)^2 \cos(\epsilon(t))^2 L_w^2 \Bigg) \end{aligned} \quad \textbf{(4.2.1)}$$

```
> v[fm] := n_norm( [ xd[fm], yd[fm], zd[fm] ], 2 ):
> Tt[fm] := kinetic_energy( 'translation', m[f], v[fm] );
```

$$\begin{aligned} Tt_{fm} := \frac{1}{2} m_f \Bigg(& \left(-\left(\frac{d}{dt} \lambda(t)\right) \sin(\lambda(t)) \cos(p(t)) L_h \right. \\ & - \cos(\lambda(t)) \left(\frac{d}{dt} p(t)\right) \sin(p(t)) L_h \\ & - \left(\frac{d}{dt} \lambda(t)\right) \cos(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h \\ & - \sin(\lambda(t)) \left(\frac{d}{dt} \epsilon(t)\right) \cos(\epsilon(t)) \sin(p(t)) L_h \\ & - \sin(\lambda(t)) \sin(\epsilon(t)) \left(\frac{d}{dt} p(t)\right) \cos(p(t)) L_h \\ & \left. + \left(\frac{d}{dt} \lambda(t)\right) \cos(\lambda(t)) \cos(\epsilon(t)) L_a - \sin(\lambda(t)) \left(\frac{d}{dt} \epsilon(t)\right) \sin(\epsilon(t)) L_a\right)^2 \\ & + \left(-\left(\frac{d}{dt} \lambda(t)\right) \cos(\lambda(t)) \cos(p(t)) L_h + \sin(\lambda(t)) \left(\frac{d}{dt} p(t)\right) \sin(p(t)) L_h \right. \\ & + \left(\frac{d}{dt} \lambda(t)\right) \sin(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h \\ & - \cos(\lambda(t)) \left(\frac{d}{dt} \epsilon(t)\right) \cos(\epsilon(t)) \sin(p(t)) L_h \\ & - \cos(\lambda(t)) \sin(\epsilon(t)) \left(\frac{d}{dt} p(t)\right) \cos(p(t)) L_h \end{aligned} \quad \textbf{(4.2.2)}$$

$$- \left(\frac{d}{dt}\lambda(t)\right)\sin(\lambda(t))\cos(\epsilon(t))L_a - \cos(\lambda(t))\left(\frac{d}{dt}\epsilon(t)\right)\sin(\epsilon(t))L_a\Big)^2$$

$$+ \Big(-\left(\frac{d}{dt}\epsilon(t)\right)\sin(\epsilon(t))\sin(p(t))L_h + \cos(\epsilon(t))\left(\frac{d}{dt}p(t)\right)\cos(p(t))L_h$$

$$+ \left(\frac{d}{dt}\epsilon(t)\right)\cos(\epsilon(t))L_a\Big)^2\Big)$$

```
> v[bm] := n_norm( [ xd[bm], yd[bm], zd[bm] ], 2 ):
> Tt[bm] := kinetic_energy( 'translation', m[b], v[bm] );
```

$$Tt_{bm} := \frac{1}{2} m_b \Big(\Big(\Big(\left(\frac{d}{dt}\lambda(t)\right)\sin(\lambda(t))\cos(p(t))L_h \tag{4.2.3}$$

$$+ \cos(\lambda(t))\left(\frac{d}{dt}p(t)\right)\sin(p(t))L_h$$

$$+ \left(\frac{d}{dt}\lambda(t)\right)\cos(\lambda(t))\sin(\epsilon(t))\sin(p(t))L_h$$

$$+ \sin(\lambda(t))\left(\frac{d}{dt}\epsilon(t)\right)\cos(\epsilon(t))\sin(p(t))L_h$$

$$+ \sin(\lambda(t))\sin(\epsilon(t))\left(\frac{d}{dt}p(t)\right)\cos(p(t))L_h$$

$$+ \left(\frac{d}{dt}\lambda(t)\right)\cos(\lambda(t))\cos(\epsilon(t))L_a - \sin(\lambda(t))\left(\frac{d}{dt}\epsilon(t)\right)\sin(\epsilon(t))L_a\Big)^2$$

$$+ \Big(\Big(\left(\frac{d}{dt}\lambda(t)\right)\cos(\lambda(t))\cos(p(t))L_h - \sin(\lambda(t))\left(\frac{d}{dt}p(t)\right)\sin(p(t))L_h$$

$$- \left(\frac{d}{dt}\lambda(t)\right)\sin(\lambda(t))\sin(\epsilon(t))\sin(p(t))L_h$$

$$+ \cos(\lambda(t))\left(\frac{d}{dt}\epsilon(t)\right)\cos(\epsilon(t))\sin(p(t))L_h$$

$$+ \cos(\lambda(t))\sin(\epsilon(t))\left(\frac{d}{dt}p(t)\right)\cos(p(t))L_h$$

$$- \left(\frac{d}{dt}\lambda(t)\right)\sin(\lambda(t))\cos(\epsilon(t))L_a - \cos(\lambda(t))\left(\frac{d}{dt}\epsilon(t)\right)\sin(\epsilon(t))L_a\Big)^2$$

$$+ \Big(\Big(\left(\frac{d}{dt}\epsilon(t)\right)\sin(\epsilon(t))\sin(p(t))L_h - \cos(\epsilon(t))\left(\frac{d}{dt}p(t)\right)\cos(p(t))L_h$$

$$+ \left(\frac{d}{dt}\epsilon(t)\right)\cos(\epsilon(t))L_a\Big)^2\Big)$$

T_T = Total Kinetic Energy of the system

```
> T[T] := Tt[cw] + Tt[fm] + Tt[bm]:
  T[T] := collect( T[T], diff );
```

$$T_T := \left(\frac{1}{2}\, m_w\left(\cos(\lambda(t))^2 \sin(\epsilon(t))^2 L_w^2 + \sin(\lambda(t))^2 \sin(\epsilon(t))^2 L_w^2 + \cos(\epsilon(t))^2 L_w^2\right) \quad \textbf{(4.2.4)}$$

$$+ \frac{1}{2}\, m_f\left(\left(-\sin(\lambda(t)) \cos(\epsilon(t)) \sin(p(t)) L_h - \sin(\lambda(t)) \sin(\epsilon(t)) L_a\right)^2 + \left(\right.\right.$$

$$-\cos(\lambda(t)) \cos(\epsilon(t)) \sin(p(t)) L_h - \cos(\lambda(t)) \sin(\epsilon(t)) L_a\Big)^2 + \Big($$

$$-\sin(\epsilon(t)) \sin(p(t)) L_h + \cos(\epsilon(t)) L_a\Big)^2\Big)$$

$$+ \frac{1}{2}\, m_b\left(\left(\sin(\lambda(t)) \cos(\epsilon(t)) \sin(p(t)) L_h - \sin(\lambda(t)) \sin(\epsilon(t)) L_a\right)^2\right.$$

$$+ \left(\cos(\lambda(t)) \cos(\epsilon(t)) \sin(p(t)) L_h - \cos(\lambda(t)) \sin(\epsilon(t)) L_a\right)^2$$

$$\left.\left. + \left(\sin(\epsilon(t)) \sin(p(t)) L_h + \cos(\epsilon(t)) L_a\right)^2\right)\right) \left(\frac{\mathrm{d}}{\mathrm{d}t}\, \epsilon(t)\right)^2 + \Big(\Big(\frac{1}{2}\, m_f\Big(2\,\Big($$

$$-\sin(\lambda(t)) \cos(p(t)) L_h - \cos(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h$$

$$+ \cos(\lambda(t)) \cos(\epsilon(t)) L_a\Big) \Big(-\sin(\lambda(t)) \cos(\epsilon(t)) \sin(p(t)) L_h$$

$$- \sin(\lambda(t)) \sin(\epsilon(t)) L_a\Big) + 2\, \Big(\sin(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h$$

$$- \cos(\lambda(t)) \cos(p(t)) L_h - \sin(\lambda(t)) \cos(\epsilon(t)) L_a\Big) \Big($$

$$-\cos(\lambda(t)) \cos(\epsilon(t)) \sin(p(t)) L_h - \cos(\lambda(t)) \sin(\epsilon(t)) L_a\Big)\Big)$$

$$+ \frac{1}{2}\, m_b\Big(2\, \Big(\sin(\lambda(t)) \cos(p(t)) L_h + \cos(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h$$

$$+ \cos(\lambda(t)) \cos(\epsilon(t)) L_a\Big) \Big(\sin(\lambda(t)) \cos(\epsilon(t)) \sin(p(t)) L_h$$

$$- \sin(\lambda(t)) \sin(\epsilon(t)) L_a\Big) + 2\, \Big(-\sin(\lambda(t)) \sin(\epsilon(t)) \sin(p(t)) L_h$$

$$+ \cos(\lambda(t)) \cos(p(t)) L_h - \sin(\lambda(t)) \cos(\epsilon(t)) L_a\Big)$$

$$\Big(\cos(\lambda(t)) \cos(\epsilon(t)) \sin(p(t)) L_h - \cos(\lambda(t)) \sin(\epsilon(t)) L_a\Big)\Big)\Big) \left(\frac{\mathrm{d}}{\mathrm{d}t}\, \lambda(t)\right)$$

$$+ \Big(\frac{1}{2}\, m_f\Big(2\, \Big(-\sin(\lambda(t)) \cos(p(t)) \sin(\epsilon(t)) L_h - \cos(\lambda(t)) \sin(p(t)) L_h\Big) \Big($$

$$-\sin(\lambda(t)) \cos(\epsilon(t)) \sin(p(t)) L_h - \sin(\lambda(t)) \sin(\epsilon(t)) L_a\Big) + 2\, \Big($$

$$-\cos(\lambda(t)) \cos(p(t)) \sin(\epsilon(t)) L_h + \sin(\lambda(t)) \sin(p(t)) L_h\Big) \Big($$

$$-\cos(\lambda(t))\cos(\epsilon(t))\sin(p(t))L_h-\cos(\lambda(t))\sin(\epsilon(t))L_a)$$

$$+2\cos(\epsilon(t))\cos(p(t))L_h\left(-\sin(\epsilon(t))\sin(p(t))L_h+\cos(\epsilon(t))L_a\right))$$

$$+\frac{1}{2}m_b\left(2\left(\sin(\lambda(t))\cos(p(t))\sin(\epsilon(t))L_h\right.\right.$$

$$+\cos(\lambda(t))\sin(p(t))L_h)\left(\sin(\lambda(t))\cos(\epsilon(t))\sin(p(t))L_h\right.$$

$$-\sin(\lambda(t))\sin(\epsilon(t))L_a)+2\left(\cos(\lambda(t))\cos(p(t))\sin(\epsilon(t))L_h\right.$$

$$-\sin(\lambda(t))\sin(p(t))L_h)\left(\cos(\lambda(t))\cos(\epsilon(t))\sin(p(t))L_h\right.$$

$$-\cos(\lambda(t))\sin(\epsilon(t))L_a)-2\cos(\epsilon(t))\cos(p(t))L_h\left(\sin(\epsilon(t))\sin(p(t))L_h\right.$$

$$+\cos(\epsilon(t))L_a)))\left(\frac{d}{dt}p(t)\right))\left(\frac{d}{dt}\epsilon(t)\right)+\left(\frac{1}{2}m_w\left(\cos(\lambda(t))^2\cos(\epsilon(t))^2L_w^2\right.\right.$$

$$+\sin(\lambda(t))^2\cos(\epsilon(t))^2L_w^2)+\frac{1}{2}m_f\left(\left(-\sin(\lambda(t))\cos(p(t))L_h\right.\right.$$

$$-\cos(\lambda(t))\sin(\epsilon(t))\sin(p(t))L_h+\cos(\lambda(t))\cos(\epsilon(t))L_a)^2$$

$$+\left(\sin(\lambda(t))\sin(\epsilon(t))\sin(p(t))L_h-\cos(\lambda(t))\cos(p(t))L_h\right.$$

$$-\sin(\lambda(t))\cos(\epsilon(t))L_a)^2)+\frac{1}{2}m_b\left(\left(\sin(\lambda(t))\cos(p(t))L_h\right.\right.$$

$$+\cos(\lambda(t))\sin(\epsilon(t))\sin(p(t))L_h+\cos(\lambda(t))\cos(\epsilon(t))L_a)^2+($$

$$-\sin(\lambda(t))\sin(\epsilon(t))\sin(p(t))L_h+\cos(\lambda(t))\cos(p(t))L_h$$

$$-\sin(\lambda(t))\cos(\epsilon(t))L_a)^2))\left(\frac{d}{dt}\lambda(t)\right)^2+\left(\frac{1}{2}m_f\left(2\left(\right.\right.\right.$$

$$-\sin(\lambda(t))\cos(p(t))\sin(\epsilon(t))L_h-\cos(\lambda(t))\sin(p(t))L_h)($$

$$-\sin(\lambda(t))\cos(p(t))L_h-\cos(\lambda(t))\sin(\epsilon(t))\sin(p(t))L_h$$

$$+\cos(\lambda(t))\cos(\epsilon(t))L_a)+2\left(-\cos(\lambda(t))\cos(p(t))\sin(\epsilon(t))L_h\right.$$

$$+\sin(\lambda(t))\sin(p(t))L_h)\left(\sin(\lambda(t))\sin(\epsilon(t))\sin(p(t))L_h\right.$$

$$-\cos(\lambda(t))\cos(p(t))L_h-\sin(\lambda(t))\cos(\epsilon(t))L_a))$$

$$+\frac{1}{2}\,m_b\Big(2\,\big(\sin(\lambda(t))\cos(p(t))\sin(\epsilon(t))\,L_h+\cos(\lambda(t))\sin(p(t))\,L_h\big)\,\big(\sin(\lambda(t))\cos(p(t))\,L_h+\cos(\lambda(t))\sin(\epsilon(t))\sin(p(t))\,L_h+\cos(\lambda(t))\cos(\epsilon(t))\,L_a\big)+2\,\big(\cos(\lambda(t))\cos(p(t))\sin(\epsilon(t))\,L_h-\sin(\lambda(t))\sin(p(t))\,L_h\big)\,\big(-\sin(\lambda(t))\sin(\epsilon(t))\sin(p(t))\,L_h+\cos(\lambda(t))\cos(p(t))\,L_h-\sin(\lambda(t))\cos(\epsilon(t))\,L_a\big)\Big)\left(\frac{d}{dt}\,p(t)\right)\left(\frac{d}{dt}\,\lambda(t)\right)+\Big(\frac{1}{2}\,m_f\Big(\big(-\sin(\lambda(t))\cos(p(t))\sin(\epsilon(t))\,L_h-\cos(\lambda(t))\sin(p(t))\,L_h\big)^2+\big(-\cos(\lambda(t))\cos(p(t))\sin(\epsilon(t))\,L_h+\sin(\lambda(t))\sin(p(t))\,L_h\big)^2+\cos(\epsilon(t))^2\cos(p(t))^2\,L_h^2\Big)+\frac{1}{2}\,m_b\Big(\big(\sin(\lambda(t))\cos(p(t))\sin(\epsilon(t))\,L_h+\cos(\lambda(t))\sin(p(t))\,L_h\big)^2+\big(\cos(\lambda(t))\cos(p(t))\sin(\epsilon(t))\,L_h-\sin(\lambda(t))\sin(p(t))\,L_h\big)^2+\cos(\epsilon(t))^2\cos(p(t))^2\,L_h^2\Big)\Big)\left(\frac{d}{dt}\,p(t)\right)^2$$

Generalized Forces: Q_i's

Qe: on epsilon: elevation

```
> Q[1] := L[a] * K[f] * ( V[f] + V[b] );
```

$$Q_1 := L_a\,K_f\,(V_f+V_b) \tag{5.1}$$

Qp: on pitch, p

```
> Q[2] := K[f] * ( V[f] - V[b] ) * L[h];
```

$$Q_2 := K_f\,(V_f-V_b)\,L_h \tag{5.2}$$

Qt: on travel, lambda

```
> Q[3] := L[a] * K[f] * ( V[f] + V[b] ) * sin( p(t) );
```

$$Q_3 := L_a\,K_f\,(V_f+V_b)\,\sin(p(t)) \tag{5.3}$$

```
> Q := [ seq( Q[i], i=1..Nq ) ];
```

$$Q := [L_a\,K_f\,(V_f+V_b),\,K_f\,(V_f-V_b)\,L_h,\,L_a\,K_f\,(V_f+V_b)\,\sin(p(t))] \tag{5.4}$$

quiescent voltage: operating point

```
> EOM_OP_PT := L[w] * m[w] * g = L[a] * g * ( m[f] + m[b] ) + L
  [a] * ( V[op] + V[op] ) * K[f];
```

$$EOM\_OP\_PT := L_w\,m_w\,g = L_a\,g\,(m_f+m_b)+2\,L_a\,V_{op}\,K_f \tag{5.5}$$

```
> V[op] := solve( EOM_OP_PT, V[op] );
```

(5.6)

$$V_{op} := -\frac{1}{2}\,\frac{g\,(L_a m_b + L_a m_f - L_w m_w)}{L_a K_f} \tag{5.6}$$

```
> subs_U_op := { V[f] = V[f] - V[op], V[b] = V[b] - V[op] }:
> Q := subs( subs_U_op, Q );
```

$$Q := \left[L_a K_f \left(V_f + \frac{g\,(L_a m_b + L_a m_f - L_w m_w)}{L_a K_f} + V_b\right), K_f\,(V_f - V_b)\,L_h, L_a K_f \left(V_f + \frac{g\,(L_a m_b + L_a m_f - L_w m_w)}{L_a K_f} + V_b\right)\sin(p(t))\right] \tag{5.7}$$

Euler-Lagrange's Equations

For a N-DOF system, the Lagrange's equations can be written:

$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial qdot_i} L\right) - \left(\frac{\partial}{\partial q_i} L\right) = Q_i$ for $i = 1$ through N

where:

Q_i's are special combinations of external forces and called the *generalized forces,*

q_1, ..., q_N, are N independent coordinates chosen to describe the system and called the *generalized coordinates*,

and L is the *Lagrangian* of the system.

L is defined by:

$L = T - U$

where T is the total kinetic energy of the system and U the total potential energy of the system.

```
> EOM_orig := lagrange_equations( T[T], V[T], q, Q ):
```

this is to display the EOM's

```
> EOM_orig := collect( EOM_orig, { seq( diff( q[i], t$2 ), i=1..
  Nq ), seq( diff( q[i], t ), i=1..Nq ), seq( q[i], i=1..Nq ) } )
  ;
```

$$EOM\_orig := \Big[\left(-\cos(p(t))^2 L_h^2 m_b - \cos(p(t))^2 L_h^2 m_f + L_a^2 m_b + L_a^2 m_f + L_h^2 m_b + L_h^2 m_f + L_w^2 m_w\right)\left(\frac{d^2}{dt^2}\epsilon(t)\right) + \big(\cos(p(t))\cos(\epsilon(t))\sin(p(t))\,L_h^2 m_b + \cos(p(t))\cos(\epsilon(t))\sin(p(t))\,L_h^2 m_f - \cos(p(t))\sin(\epsilon(t))\,L_a L_h m_b + \cos(p(t))\sin(\epsilon(t))\,L_a L_h m_f\big)\left(\frac{d^2}{dt^2}\lambda(t)\right) + \left(-\cos(p(t))\,L_a L_h m_b + \cos(p(t))\,L_a L_h m_f\right)\left(\frac{d^2}{dt^2}p(t)\right) + \left(2\cos(p(t))\sin(p(t))\,L_h^2 m_b + 2\cos(p(t))\sin(p(t))\,L_h^2 m_f\right)\left(\frac{d}{dt}p(t)\right)\left(\frac{d}{dt}\epsilon(t)\right) \tag{6.1}$$

$$+\left(\cos(p(t))^2\sin(\epsilon(t))\cos(\epsilon(t))L_h^2 m_b+\cos(p(t))^2\sin(\epsilon(t))\cos(\epsilon(t))L_h^2 m_f\right.$$

$$-2\cos(\epsilon(t))^2\sin(p(t))L_a L_h m_b+2\cos(\epsilon(t))^2\sin(p(t))L_a L_h m_f$$

$$+\sin(\epsilon(t))\cos(\epsilon(t))L_a^2 m_b+\sin(\epsilon(t))\cos(\epsilon(t))L_a^2 m_f-\sin(\epsilon(t))\cos(\epsilon(t))L_h^2 m_b$$

$$-\sin(\epsilon(t))\cos(\epsilon(t))L_h^2 m_f+\sin(\epsilon(t))\cos(\epsilon(t))L_w^2 m_w+\sin(p(t))L_a L_h m_b$$

$$\left.-\sin(p(t))L_a L_h m_f\right)\left(\frac{d}{dt}\lambda(t)\right)^2+\left(2\cos(p(t))^2\cos(\epsilon(t))L_h^2 m_b\right.$$

$$+2\cos(p(t))^2\cos(\epsilon(t))L_h^2 m_f+2\sin(\epsilon(t))\sin(p(t))L_a L_h m_b$$

$$\left.-2\sin(\epsilon(t))\sin(p(t))L_a L_h m_f-2\cos(\epsilon(t))L_h^2 m_b-2\cos(\epsilon(t))L_h^2 m_f\right)$$

$$\left(\frac{d}{dt}p(t)\right)\left(\frac{d}{dt}\lambda(t)\right)+\left(\sin(p(t))L_a L_h m_b-\sin(p(t))L_a L_h m_f\right)\left(\frac{d}{dt}p(t)\right)^2$$

$$+\sin(\epsilon(t))\sin(p(t))g L_h m_b-\sin(\epsilon(t))\sin(p(t))g L_h m_f+\cos(\epsilon(t))g L_a m_b$$

$$+\cos(\epsilon(t))g L_a m_f-\cos(\epsilon(t))g L_w m_w=L_a K_f\left(V_f+\frac{g\left(L_a m_b+L_a m_f-L_w m_w\right)}{L_a K_f}\right.$$

$$\left.+V_b\right), -L_h\left(\cos(p(t))L_a m_b-\cos(p(t))L_a m_f\right)\left(\frac{d^2}{dt^2}\epsilon(t)\right)-L_h\Big($$

$$-\cos(\epsilon(t))\sin(p(t))L_a m_b+\cos(\epsilon(t))\sin(p(t))L_a m_f-\sin(\epsilon(t))L_h m_b$$

$$-\sin(\epsilon(t))L_h m_f\Big)\left(\frac{d^2}{dt^2}\lambda(t)\right)-L_h\left(-L_h m_b-L_h m_f\right)\left(\frac{d^2}{dt^2}p(t)\right)$$

$$-L_h\left(\cos(p(t))\sin(p(t))L_h m_b+\cos(p(t))\sin(p(t))L_h m_f\right)\left(\frac{d}{dt}\epsilon(t)\right)^2$$

$$-L_h\left(2\cos(p(t))^2\cos(\epsilon(t))L_h m_b+2\cos(p(t))^2\cos(\epsilon(t))L_h m_f\right.$$

$$+2\sin(\epsilon(t))\sin(p(t))L_a m_b-2\sin(\epsilon(t))\sin(p(t))L_a m_f-2\cos(\epsilon(t))L_h m_b$$

$$- 2 \cos(\epsilon(t))\, L_h\, m_f\Big) \left(\frac{d}{dt}\, \lambda(t)\right) \left(\frac{d}{dt}\, \epsilon(t)\right) - L_h \Big($$

$$-\cos(p(t))\, \cos(\epsilon(t))^2 \sin(p(t))\, L_h\, m_b - \cos(p(t))\, \cos(\epsilon(t))^2 \sin(p(t))\, L_h\, m_f$$

$$+ \cos(p(t))\, \sin(\epsilon(t))\, \cos(\epsilon(t))\, L_a\, m_b - \cos(p(t))\, \sin(\epsilon(t))\, \cos(\epsilon(t))\, L_a\, m_f\Big)$$

$$\left(\frac{d}{dt}\, \lambda(t)\right)^2 - L_h \left(\cos(p(t))\, \cos(\epsilon(t))\, g\, m_b - \cos(p(t))\, \cos(\epsilon(t))\, g\, m_f\right) = K_f \Big(V_f$$

$$- V_b\Big)\, L_h, \Big(\cos(p(t))\, \cos(\epsilon(t))\, \sin(p(t))\, L_h^2\, m_b + \cos(p(t))\, \cos(\epsilon(t))\, \sin(p(t))\, L_h^2\, m_f$$

$$- \cos(p(t))\, \sin(\epsilon(t))\, L_a\, L_h\, m_b + \cos(p(t))\, \sin(\epsilon(t))\, L_a\, L_h\, m_f\Big) \left(\frac{d^2}{dt^2}\, \epsilon(t)\right)$$

$$+ \Big(\cos(p(t))^2 \cos(\epsilon(t))^2 L_h^2\, m_b + \cos(p(t))^2 \cos(\epsilon(t))^2 L_h^2\, m_f$$

$$+ 2 \sin(\epsilon(t))\, \cos(\epsilon(t))\, \sin(p(t))\, L_a\, L_h\, m_b - 2 \sin(\epsilon(t))\, \cos(\epsilon(t))\, \sin(p(t))\, L_a\, L_h\, m_f$$

$$+ \cos(\epsilon(t))^2 L_a^2\, m_b + \cos(\epsilon(t))^2 L_a^2\, m_f - \cos(\epsilon(t))^2 L_h^2\, m_b - \cos(\epsilon(t))^2 L_h^2\, m_f$$

$$+ \cos(\epsilon(t))^2 L_w^2\, m_w + L_h^2\, m_b + L_h^2\, m_f\Big) \left(\frac{d^2}{dt^2}\, \lambda(t)\right) + \Big(\cos(\epsilon(t))\, \sin(p(t))\, L_a\, L_h\, m_b$$

$$- \cos(\epsilon(t))\, \sin(p(t))\, L_a\, L_h\, m_f + \sin(\epsilon(t))\, L_h^2\, m_b + \sin(\epsilon(t))\, L_h^2\, m_f\Big) \left(\frac{d^2}{dt^2}\, p(t)\right) + \Big($$

$$-\cos(p(t))\, \sin(\epsilon(t))\, \sin(p(t))\, L_h^2\, m_b - \cos(p(t))\, \sin(\epsilon(t))\, \sin(p(t))\, L_h^2\, m_f$$

$$- \cos(p(t))\, \cos(\epsilon(t))\, L_a\, L_h\, m_b + \cos(p(t))\, \cos(\epsilon(t))\, L_a\, L_h\, m_f\Big) \left(\frac{d}{dt}\, \epsilon(t)\right)^2 + \Big(\Big($$

$$-2 \cos(p(t))^2 \sin(\epsilon(t))\, \cos(\epsilon(t))\, L_h^2\, m_b - 2 \cos(p(t))^2 \sin(\epsilon(t))\, \cos(\epsilon(t))\, L_h^2\, m_f$$

$$+ 4 \cos(\epsilon(t))^2 \sin(p(t))\, L_a\, L_h\, m_b - 4 \cos(\epsilon(t))^2 \sin(p(t))\, L_a\, L_h\, m_f$$

$$- 2 \sin(\epsilon(t))\, \cos(\epsilon(t))\, L_a^2\, m_b - 2 \sin(\epsilon(t))\, \cos(\epsilon(t))\, L_a^2\, m_f + 2 \sin(\epsilon(t))\, \cos(\epsilon(t))$$

$$L_h^2 m_b + 2\sin(\epsilon(t))\cos(\epsilon(t)) L_h^2 m_f - 2\sin(\epsilon(t))\cos(\epsilon(t)) L_w^2 m_w$$

$$-2\sin(p(t)) L_a L_h m_b + 2\sin(p(t)) L_a L_h m_f\Big)\left(\frac{d}{dt}\lambda(t)\right) + \Big(2\cos(p(t))^2\cos(\epsilon(t))$$

$$L_h^2 m_b + 2\cos(p(t))^2\cos(\epsilon(t)) L_h^2 m_f\Big)\left(\frac{d}{dt}p(t)\right)\Bigg)\left(\frac{d}{dt}\epsilon(t)\right) + \Big($$

$$-2\cos(p(t))\cos(\epsilon(t))^2\sin(p(t)) L_h^2 m_b - 2\cos(p(t))\cos(\epsilon(t))^2\sin(p(t)) L_h^2 m_f$$

$$+2\cos(p(t))\sin(\epsilon(t))\cos(\epsilon(t)) L_a L_h m_b$$

$$-2\cos(p(t))\sin(\epsilon(t))\cos(\epsilon(t)) L_a L_h m_f\Big)\left(\frac{d}{dt}p(t)\right)\left(\frac{d}{dt}\lambda(t)\right)$$

$$+\Big(\cos(p(t))\cos(\epsilon(t)) L_a L_h m_b - \cos(p(t))\cos(\epsilon(t)) L_a L_h m_f\Big)\left(\frac{d}{dt}p(t)\right)^2$$

$$= L_a K_f\left(V_f + \frac{g\,(L_a m_b + L_a m_f - L_w m_w)}{L_a K_f} + V_b\right)\sin(p(t))\Bigg]$$

Express the Euler-Lagrange equations of motion as functions of the states:
1) substitute (i.e. name) the acceleration states first!
2) then substitute the velocity states!
3) and only after, the position states, and the inputs!

```
> EOM_states := subs( subs_Xqd, subs( subs_Xqdd, EOM_orig ) ):
> EOM_states := subs( subs_Xq, subs_U, EOM_states );
```

$$EOM\_states := \Big[\Big(-\cos(X_2)^2 L_h^2 m_b - \cos(X_2)^2 L_h^2 m_f + L_a^2 m_b + L_a^2 m_f + L_h^2 m_b + L_h^2 m_f + \qquad (6.2)$$

$$L_w^2 m_w\Big) Xd_4 + \Big(\cos(X_2)\cos(X_1)\sin(X_2) L_h^2 m_b + \cos(X_2)\cos(X_1)\sin(X_2) L_h^2 m_f$$

$$-\cos(X_2)\sin(X_1) L_a L_h m_b + \cos(X_2)\sin(X_1) L_a L_h m_f\Big) Xd_6 + \Big(-\cos(X_2) L_a L_h m_b$$

$$+\cos(X_2) L_a L_h m_f\Big) Xd_5 + \Big(2\cos(X_2)\sin(X_2) L_h^2 m_b + 2\cos(X_2)\sin(X_2) L_h^2 m_f\Big) X_5 X_4$$

$$+\Big(\cos(X_2)^2\sin(X_1)\cos(X_1) L_h^2 m_b + \cos(X_2)^2\sin(X_1)\cos(X_1) L_h^2 m_f$$

$$-2\cos(X_1)^2\sin(X_2) L_a L_h m_b + 2\cos(X_1)^2\sin(X_2) L_a L_h m_f + \sin(X_1)\cos(X_1) L_a^2 m_b$$

$$+\sin(X_1)\cos(X_1)L_a^2 m_f-\sin(X_1)\cos(X_1)L_h^2 m_b-\sin(X_1)\cos(X_1)L_h^2 m_f$$

$$+\sin(X_1)\cos(X_1)L_w^2 m_w+\sin(X_2)L_a L_h m_b-\sin(X_2)L_a L_h m_f\big)X_6^2$$

$$+\big(2\cos(X_2)^2\cos(X_1)L_h^2 m_b+2\cos(X_2)^2\cos(X_1)L_h^2 m_f+2\sin(X_1)\sin(X_2)L_a L_h m_b$$

$$-2\sin(X_1)\sin(X_2)L_a L_h m_f-2\cos(X_1)L_h^2 m_b-2\cos(X_1)L_h^2 m_f\big)X_5 X_6$$

$$+\left(\sin(X_2)L_a L_h m_b-\sin(X_2)L_a L_h m_f\right)X_5^2+\sin(X_1)\sin(X_2)g L_h m_b$$

$$-\sin(X_1)\sin(X_2)g L_h m_f+\cos(X_1)g L_a m_b+\cos(X_1)g L_a m_f-\cos(X_1)g L_w m_w$$

$$=L_a K_f\left(U_1+\frac{g\left(L_a m_b+L_a m_f-L_w m_w\right)}{L_a K_f}+U_2\right),\ -L_h\left(\cos(X_2)L_a m_b\right.$$

$$-\cos(X_2)L_a m_f\big)Xd_4-L_h\left(-\cos(X_1)\sin(X_2)L_a m_b+\cos(X_1)\sin(X_2)L_a m_f\right.$$

$$-\sin(X_1)L_h m_b-\sin(X_1)L_h m_f\big)Xd_6-L_h\left(-L_h m_b-L_h m_f\right)Xd_5$$

$$-L_h\left(\cos(X_2)\sin(X_2)L_h m_b+\cos(X_2)\sin(X_2)L_h m_f\right)X_4^2$$

$$-L_h\big(2\cos(X_2)^2\cos(X_1)L_h m_b+2\cos(X_2)^2\cos(X_1)L_h m_f+2\sin(X_1)\sin(X_2)L_a m_b$$

$$-2\sin(X_1)\sin(X_2)L_a m_f-2\cos(X_1)L_h m_b-2\cos(X_1)L_h m_f\big)X_6 X_4-L_h\big($$

$$-\cos(X_2)\cos(X_1)^2\sin(X_2)L_h m_b-\cos(X_2)\cos(X_1)^2\sin(X_2)L_h m_f$$

$$+\cos(X_2)\sin(X_1)\cos(X_1)L_a m_b-\cos(X_2)\sin(X_1)\cos(X_1)L_a m_f\big)X_6^2$$

$$-L_h\left(\cos(X_2)\cos(X_1)g m_b-\cos(X_2)\cos(X_1)g m_f\right)=K_f\left(U_1-U_2\right)L_h,$$

$$\big(\cos(X_2)\cos(X_1)\sin(X_2)L_h^2 m_b+\cos(X_2)\cos(X_1)\sin(X_2)L_h^2 m_f$$

$$- \cos(X_2) \sin(X_1) L_a L_h m_b + \cos(X_2) \sin(X_1) L_a L_h m_f) Xd_4 + (\cos(X_2)^2 \cos(X_1)^2$$

$$L_h^2 m_b + \cos(X_2)^2 \cos(X_1)^2 L_h^2 m_f + 2 \sin(X_1) \cos(X_1) \sin(X_2) L_a L_h m_b$$

$$- 2 \sin(X_1) \cos(X_1) \sin(X_2) L_a L_h m_f + \cos(X_1)^2 L_a^2 m_b + \cos(X_1)^2 L_a^2 m_f - \cos(X_1)^2$$

$$L_h^2 m_b - \cos(X_1)^2 L_h^2 m_f + \cos(X_1)^2 L_w^2 m_w + L_h^2 m_b + L_h^2 m_f) Xd_6$$

$$+ (\cos(X_1) \sin(X_2) L_a L_h m_b - \cos(X_1) \sin(X_2) L_a L_h m_f + \sin(X_1) L_h^2 m_b + \sin(X_1)$$

$$L_h^2 m_f) Xd_5 + (-\cos(X_2) \sin(X_1) \sin(X_2) L_h^2 m_b - \cos(X_2) \sin(X_1) \sin(X_2) L_h^2 m_f$$

$$- \cos(X_2) \cos(X_1) L_a L_h m_b + \cos(X_2) \cos(X_1) L_a L_h m_f) X_4^2 + (($$

$$-2 \cos(X_2)^2 \sin(X_1) \cos(X_1) L_h^2 m_b - 2 \cos(X_2)^2 \sin(X_1) \cos(X_1) L_h^2 m_f$$

$$+ 4 \cos(X_1)^2 \sin(X_2) L_a L_h m_b - 4 \cos(X_1)^2 \sin(X_2) L_a L_h m_f - 2 \sin(X_1) \cos(X_1)$$

$$L_a^2 m_b - 2 \sin(X_1) \cos(X_1) L_a^2 m_f + 2 \sin(X_1) \cos(X_1) L_h^2 m_b + 2 \sin(X_1) \cos(X_1) L_h^2 m_f$$

$$- 2 \sin(X_1) \cos(X_1) L_w^2 m_w - 2 \sin(X_2) L_a L_h m_b + 2 \sin(X_2) L_a L_h m_f) X_6$$

$$+ (2 \cos(X_2)^2 \cos(X_1) L_h^2 m_b + 2 \cos(X_2)^2 \cos(X_1) L_h^2 m_f) X_5) X_4 + ($$

$$-2 \cos(X_2) \cos(X_1)^2 \sin(X_2) L_h^2 m_b - 2 \cos(X_2) \cos(X_1)^2 \sin(X_2) L_h^2 m_f$$

$$+ 2 \cos(X_2) \sin(X_1) \cos(X_1) L_a L_h m_b - 2 \cos(X_2) \sin(X_1) \cos(X_1) L_a L_h m_f) X_5 X_6$$

$$+ (\cos(X_2) \cos(X_1) L_a L_h m_b - \cos(X_2) \cos(X_1) L_a L_h m_f) X_5^2 = L_a K_f \Bigg(U_1$$

$$+ \frac{g (L_a m_b + L_a m_f - L_w m_w)}{L_a K_f} + U_2 \Bigg) \sin(X_2) \Bigg]$$

Linearization in the EOM's of the Trigonometric Functions

Linearization of the equations of motion around the quiescent point of operation (in order to solve them).
Here, linearization around the zero angles, i.e. for small-amplitude oscillations.
Linearization around: alpha0 = 0, and alpha_dot0 = 0

```
> EOM_ser := EOM_states:
```

Generalized series expansions of the trigonometric functions is used (for small angles).

```
> for i from 1 to Nq do
    EOM_ser[i] := subsop( 1 = convert( series( op( 1, EOM_ser
  [i] ), X[1] ), polynom ), EOM_ser[i] );
    EOM_ser[i] := subsop( 1 = convert( series( op( 1, EOM_ser
  [i] ), X[2] ), polynom ), EOM_ser[i] );
    EOM_ser[i] := subsop( 1 = convert( series( op( 1, EOM_ser
  [i] ), X[3] ), polynom ), EOM_ser[i] );
    EOM_ser[i] := simplify( EOM_ser[i] );
  end do:
```

```
> EOM_ser;
```

$$\Big[\left(X_1 X_6^2+Xd_4\right)\left(m_f+m_b\right)L_a^2+\Big(-\left(m_b-m_f\right)\Big(\left(-2\,X_1 X_5 X_6-X_5^2+X_6^2\right)X_2+Xd_6 X_1+Xd_5\Big)L_h+g\left(m_f+m_b\right)\Big)L_a+X_2\left(2\,X_4 X_5+Xd_6\right)\left(m_f+m_b\right)L_h^2+g\,X_1 X_2\left(m_b-m_f\right)L_h-m_w L_w\left(-L_w X_1 X_6^2-L_w Xd_4+g\right)=\Big(g\left(m_f+m_b\right)+K_f\left(U_1+U_2\right)\Big)L_a-L_w m_w g,\ -\Big(-\left(m_f+m_b\right)\Big(\left(-X_4^2+X_6^2\right)X_2+Xd_6 X_1+Xd_5\Big)L_h+\Big(\Big(\left(2\,X_1 X_4 X_6-Xd_6\right)X_2+X_1 X_6^2+Xd_4\Big)L_a+g\Big)\left(m_b-m_f\right)\Big)L_h=K_f\left(U_1-U_2\right)L_h,\ \left(m_f+m_b\right)\Big(\left(-X_2 X_4^2+Xd_5\right)X_1+\left(-2\,X_5 X_6+Xd_4\right)X_2+2\,X_5 X_4+Xd_6\Big)L_h^2+\left(m_f-m_b\right)\Big(\left(-2\,X_2 Xd_6-2\,X_5 X_6+Xd_4\right)X_1+\left(-2\,X_4 X_6-Xd_5\right)X_2+X_4^2-X_5^2\Big)L_a L_h+\left(-2\,X_1 X_4 X_6+Xd_6\right)\Big(\left(m_f+m_b\right)L_a^2+L_w^2 m_w\Big)=-\sin\left(X_2\right)\Big(\Big(\left(-m_b-m_f\right)g-K_f\left(U_1+U_2\right)\Big)L_a+L_w m_w g\Big)\Big] \quad (6.1.1)$$

Additional Insight: Inertia (or mass) Matrix: Fi

The nonlinear system of equations resulting from the Lagrangian mechanics can be written in the following matrix form:
F(q) . qdd + G(q, qd) . qd + H(q) . q = L(q, qd, u)
F, G, and H are called, respectively, the mass, damping, and stiffness matrices.
They are symmetric in form.
The inertia (a.k.a. mass) matrix, F, gives indications regarding the coupling existing in the system.

```
> Fi := Matrix( Nq, Nq ):
```

```
> for i from 1 to Nq do
    for k from 1 to Nq do
      Fi[ i, k ] := simplify( diff( op( 1, EOM_states[i] ), Xd
```

```
[k+Nq] ) );
    Fi[ i, k ] := collect( combine( Fi[ i, k ], trig ), cos )
;
  end do;
end do:
> 'F[i]' = Fi;
```

$$F_i = \left[\left[\left(-\frac{1}{2}L_h^2 m_b - \frac{1}{2}L_h^2 m_f\right)\cos(2X_2) + \frac{1}{2}L_h^2 m_b + \frac{1}{2}L_h^2 m_f + L_a^2 m_b + L_a^2 m_f + L_w^2 m_w, \left(-L_a L_h m_b + L_a L_h m_f\right)\cos(X_2), \frac{1}{4}L_h^2 m_b \sin(X_1 + 2X_2) + \frac{1}{4}L_h^2 m_f \sin(X_1 + 2X_2) - \frac{1}{4}L_h^2 m_b \sin(X_1 - 2X_2) - \frac{1}{4}L_h^2 m_f \sin(X_1 - 2X_2) - \frac{1}{2}L_a L_h m_b \sin(X_1 + X_2) - \frac{1}{2}L_a L_h m_b \sin(X_1 - X_2) + \frac{1}{2}L_a L_h m_f \sin(X_1 + X_2) + \frac{1}{2}L_a L_h m_f \sin(X_1 - X_2)\right],\right.$$

$$\left[\left(-L_a L_h m_b + L_a L_h m_f\right)\cos(X_2), L_h^2 m_b + L_h^2 m_f, \frac{1}{2}L_a L_h m_b \sin(X_1 + X_2) - \frac{1}{2}L_a L_h m_f \sin(X_1 + X_2) - \frac{1}{2}L_a L_h m_b \sin(X_1 - X_2) + \frac{1}{2}L_a L_h m_f \sin(X_1 - X_2) + \sin(X_1) L_h^2 m_b + \sin(X_1) L_h^2 m_f\right],$$

$$\left[\frac{1}{4}L_h^2 m_b \sin(X_1 + 2X_2) + \frac{1}{4}L_h^2 m_f \sin(X_1 + 2X_2) - \frac{1}{4}L_h^2 m_b \sin(X_1 - 2X_2) - \frac{1}{4}L_h^2 m_f \sin(X_1 - 2X_2) - \frac{1}{2}L_a L_h m_b \sin(X_1 + X_2) - \frac{1}{2}L_a L_h m_b \sin(X_1 - X_2) + \frac{1}{2}L_a L_h m_f \sin(X_1 + X_2) + \frac{1}{2}L_a L_h m_f \sin(X_1 - X_2), \frac{1}{2}L_a L_h m_b \sin(X_1 + X_2) - \frac{1}{2}L_a L_h m_f \sin(X_1 + X_2) - \frac{1}{2}L_a L_h m_b \sin(X_1 - X_2) + \frac{1}{2}L_a L_h m_f \sin(X_1 - X_2) + \sin(X_1) L_h^2 m_b + \sin(X_1) L_h^2 m_f, \left(\frac{1}{2}L_w^2 m_w + \frac{1}{2}L_a^2 m_b + \frac{1}{2}L_a^2 m_f - \frac{1}{4}L_h^2 m_b - \frac{1}{4}L_h^2 m_f\right)\cos(2X_1) + \left(\frac{1}{4}L_h^2 m_f + \frac{1}{4}L_h^2 m_b\right)\cos(2X_2) + \left(\frac{1}{8}L_h^2 m_b + \frac{1}{8}L_h^2 m_f\right)\cos(2X_1 - 2X_2) + \left(-\frac{1}{2}L_a L_h m_f + \frac{1}{2}L_a L_h m_b\right)\cos(2X_1 - X_2) + \left(\frac{1}{2}L_a L_h m_f - \frac{1}{2}L_a L_h m_b\right)\cos(2X_1 + X_2) + \left(\frac{1}{8}L_h^2 m_b + \frac{1}{8}L_h^2 m_f\right)\cos(2X_1 + 2X_2) + \frac{1}{2}L_a^2 m_b + \frac{1}{2}L_a^2 m_f + \frac{3}{4}L_h^2 m_b + \frac{3}{4}L_h^2 m_f + \frac{1}{2}L_w^2 m_w\right]\right] \quad \textbf{(6.2.1)}$$

Linearization of the inertia matrix for small-displacements

```
> Fi_lin := Matrix( Nq, Nq ):
```

```
> for i from 1 to Nq do
    for k from 1 to Nq do
      Fi_lin[ i, k ] := Fi[ i, k ];
      Fi_lin[ i, k ] := convert( series( Fi_lin[ i, k ], X[ 2 ]
  ), polynom );
      Fi_lin[ i, k ] := subs( subs_XU_op, Fi_lin[ i, k ] );
    end do;
  end do:
```

```
> 'F_lin' = Fi_lin;
```

$$F\_lin = \left[\left[L_a^2 m_b + L_a^2 m_f + L_w^2 m_w, -L_a L_h m_b + L_a L_h m_f, 0\right], \left[-L_a L_h m_b + L_a L_h m_f, L_h^2 m_b + L_h^2 m_f, 0\right], \left[0, 0, L_a^2 m_b + L_a^2 m_f + L_h^2 m_b + L_h^2 m_f + L_w^2 m_w\right]\right] \tag{6.2.2}$$

Solving the Euler-Lagrange's Equations

To solve the Euler-Lagrange's equations, they need to be linear.

Reverse State Substitution for Pretty Display of the Solved EOM's

only for pretty print

```
> subs_Xq_rev := { seq( X[i] = q[i], i=1..Nq ) }:
  subs_Xqd_rev := { seq( X[i+Nq] = qd[i], i=1..Nq ) }:
```

```
> subs_U_rev := { U[1] = V[f], U[2] = V[b] };
```

$$subs\_U\_rev := \left\{U_1 = V_f, U_2 = V_b\right\} \tag{7.1.1}$$

```
> eom_collect_list := { seq( diff( q[i], t ), i=1..Nq ), seq( q
  [i], i=1..Nq ) };
```

$$eom\_collect\_list := \left\{\frac{\mathrm{d}}{\mathrm{d}t}\,\epsilon(t), \frac{\mathrm{d}}{\mathrm{d}t}\,\lambda(t), \frac{\mathrm{d}}{\mathrm{d}t}\,p(t), \epsilon(t), \lambda(t), p(t)\right\} \tag{7.1.2}$$

Solution to the Non-Linear Equations of Motion

Solve the non-linear form of the equations of motion for the states' second time derivatives

```
> Xqdd_solset_nl := solve( convert( EOM_states, set ), convert(
  Xqdd, set ) ):
```

```
> assign( Xqdd_solset_nl );
```

```
> Xd_nl[Nq+1] := simplify( Xd[Nq+1] ):
  Xd_nl[Nq+2] := simplify( Xd[Nq+2] ):
  Xd_nl[Nq+3] := simplify( Xd[Nq+3] ):
  unassign( 'Xd[Nq+1]', 'Xd[Nq+2]', 'Xd[Nq+3]' ):
```

pretty display w.r.t. the named system states

```
> Xd_nl[Nq+1] := simplify( subs( subs_U_rev, subs_Xq_rev,
  subs_Xqd_rev, Xd_nl[Nq+1] ) ):
  diff( qd[1], t ) = collect( Xd_nl[Nq+1], eom_collect_list );
```

$$\frac{d^2}{dt^2}\,\epsilon(t) = -\frac{\sin(p(t))\,L_a\,(m_f - m_b)\,\cos(p(t))^2\,L_h\left(\frac{d}{dt}\,\epsilon(t)\right)^2}{(m_f+m_b)\,L_a^2 + L_h^2\,m_b + L_h^2\,m_f + L_w^2\,m_w} + \Bigg($$

$$-\frac{1}{(m_f+m_b)\,L_a^2 + L_h^2\,m_b + L_h^2\,m_f + L_w^2\,m_w}\Big(2\cos(p(t))\,\big(\cos(\epsilon(t))\,L_a\,(m_f$$

$$- m_b)\cos(p(t))^2 - \cos(\epsilon(t))\,L_a\,(m_f - m_b) + \sin(p(t))\,\sin(\epsilon(t))\,L_h\,(m_f+m_b)\big)$$

$$L_h\left(\frac{d}{dt}\,\lambda(t)\right)\Big) - \frac{2\sin(p(t))\,(m_f+m_b)\cos(p(t))\,L_h^2\left(\frac{d}{dt}\,p(t)\right)}{(m_f+m_b)\,L_a^2 + L_h^2\,m_b + L_h^2\,m_f + L_w^2\,m_w}\Bigg)\left(\frac{d}{dt}\,\epsilon(t)\right)$$

$$+\frac{1}{(m_f+m_b)\,L_a^2 + L_h^2\,m_b + L_h^2\,m_f + L_w^2\,m_w}\Big(\big(\cos(\epsilon(t))\,\big(\sin(p(t))\,L_a\,(m_f$$

$$- m_b)\cos(\epsilon(t)) - 2\sin(\epsilon(t))\,L_h\,(m_f+m_b)\big)\,L_h\cos(p(t))^2$$

$$-2\sin(p(t))\,L_a\,L_h\,(m_f-m_b)\cos(\epsilon(t))^2 - \big((m_f+m_b)\,L_a^2 - L_h^2\,m_b - L_h^2\,m_f +$$

$$L_w^2\,m_w\big)\sin(\epsilon(t))\cos(\epsilon(t)) + \sin(p(t))\,L_a\,L_h\,(m_f-m_b)\big)\left(\frac{d}{dt}\,\lambda(t)\right)^2\Big)$$

$$-\frac{1}{(m_f+m_b)\,L_a^2 + L_h^2\,m_b + L_h^2\,m_f + L_w^2\,m_w}\Big(2\,\big(\cos(\epsilon(t))\,L_h\,(m_f+m_b)\cos(p(t))^2$$

$$-\cos(\epsilon(t))\,L_h\,(m_f+m_b) - \sin(p(t))\,\sin(\epsilon(t))\,L_a\,(m_f-m_b)\big)$$

$$\left(\frac{d}{dt}\,p(t)\right)L_h\left(\frac{d}{dt}\,\lambda(t)\right)\Big) + \frac{\sin(p(t))\,L_a\,(m_f-m_b)\,L_h\left(\frac{d}{dt}\,p(t)\right)^2}{(m_f+m_b)\,L_a^2 + L_h^2\,m_b + L_h^2\,m_f + L_w^2\,m_w}$$

$$+\Big(\big(\big((m_f-m_b)^2\,L_a^2 + L_h^2\,(m_f+m_b)^2\big)\,\big(\big(g\,m_b + g\,m_f + K_f\,(V_f+V_b)\big)\,L_a - L_w\,m_w\,g\big)\cos(p(t))^3 + \mathrm{co}$$

$$L_h^2\,L_w\,(m_f+m_b)^2\big)\,g\cos(\epsilon(t)) + \big((m_f-m_b)^2\,L_a^2 + L_h^2\,(m_f+m_b)^2\big)\,\big(\big(g\,m_b + g\,m_f$$

$$+K_f\,(V_f+V_b)\big)\,L_a - L_w\,m_w\,g\big)\cos(p(t))^2 + \big((-V_f+V_b)\,L_a\,\big((m_f+m_b)\,L_a^2 +$$

$$L_h^2\,m_b + L_h^2\,m_f + L_w^2\,m_w\big)\,(m_f-m_b)\,K_f\cos(\epsilon(t)) - \big((m_f-m_b)^2\,L_a^2 + L_h^2\,(m_f$$

$$+m_b)^2\big)\ \big(\sin(\epsilon(t))\ K_f L_h\ (-V_f+V_b)\ \sin(p(t)) + \big(g\,m_b + g\,m_f + K_f\,(V_f+V_b)\big)\ L_a$$

$$-L_w\,m_w\,g\big)\big)\ \cos(p(t)) + \cos(\epsilon(t))\ \big(-\big((m_f+m_b)\ L_a - L_w\,m_w\big)\ g\cos(\epsilon(t))$$

$$+g\sin(\epsilon(t))\ L_h\ (m_f-m_b)\ \sin(p(t)) + \big(g\,m_b + g\,m_f + K_f\,(V_f+V_b)\big)\ L_a$$

$$-L_w\,m_w\,g\big)\ \big(4\,L_a^2\,m_b\,m_f + m_w\,L_w^2\ (m_f+m_b)\big)\big)\Big/\Big(\cos(\epsilon(t))\ \big((m_f+m_b)\ L_a^2 + L_h^2\,m_b$$

$$+L_h^2\,m_f + L_w^2\,m_w\big)\ \big(4\,L_a^2\,m_b\,m_f + m_w\,L_w^2\ (m_f+m_b)\big)\Big)$$

```
> Xd_nl[Nq+2] := simplify( subs( subs_U_rev, subs_Xq_rev,
  subs_Xqd_rev, Xd_nl[Nq+2] ) ):
  diff( qd[2], t ) = collect( Xd_nl[Nq+2], eom_collect_list );
```

$$\frac{d^2}{dt^2}\,p(t) = \Bigg(\Big(\sin(p(t))\ \big((m_f+m_b)\ L_h^2 + (m_f+m_b)\ L_a^2 + L_w^2\,m_w\big)\ \cos(\epsilon(t))$$

$$+\cos(p(t))^2\sin(\epsilon(t))\ L_a\,L_h\ (m_f-m_b)\Big)\ \cos(p(t))\ \left(\frac{d}{dt}\,\epsilon(t)\right)^2\Bigg)\Bigg/$$

$$\Big(\cos(\epsilon(t))\ \big((m_f+m_b)\ L_h^2 + (m_f+m_b)\ L_a^2 + L_w^2\,m_w\big)\Big) + \Bigg(-\Big(2\ \Big(-\big((m_f+m_b)$$

$$L_a^2 + L_w^2\,m_w\big)\ \cos(p(t))^2\cos(\epsilon(t))^2 + \cos(p(t))^2\sin(p(t))\ \sin(\epsilon(t))\ L_a\,L_h\ (m_f$$

$$-m_b)\ \cos(\epsilon(t)) - L_h^2\ (m_f+m_b)\ \cos(p(t))^2 + (m_f+m_b)\ L_h^2 + (m_f+m_b)\ L_a^2 +$$

$$L_w^2\,m_w\Big)\ \left(\frac{d}{dt}\,\lambda(t)\right)\Big)\Big/\Big(\cos(\epsilon(t))\ \big((m_f+m_b)\ L_h^2 + (m_f+m_b)\ L_a^2 + L_w^2\,m_w\big)\Big)$$

$$+\frac{2\ (m_f+m_b)\ \sin(\epsilon(t))\ \left(\frac{d}{dt}\,p(t)\right)\ \cos(p(t))^2\ L_h^2}{\cos(\epsilon(t))\ \big((m_f+m_b)\ L_h^2 + (m_f+m_b)\ L_a^2 + L_w^2\,m_w\big)}\Bigg)\ \left(\frac{d}{dt}\,\epsilon(t)\right)$$

$$-\Big(\cos(p(t))\ \Big(\Big(\big((-m_b-m_f)\ L_h^2 + (m_f+m_b)\ L_a^2 + L_w^2\,m_w\big)\ \sin(p(t))\ \cos(\epsilon(t))^3 + \sin(\epsilon(t))\ L_a\,L_h\ (c$$

$$\Big(\cos(\epsilon(t))\ \big((m_f+m_b)\ L_h^2 + (m_f+m_b)\ L_a^2 + L_w^2\,m_w\big)\Big) - \Big(2\ \left(\frac{d}{dt}\,p(t)\right)\ \Big($$

$$-L_a\ (m_f-m_b)\ \cos(\epsilon(t))^2 + \sin(p(t))\ \sin(\epsilon(t))\ L_h\ (m_f+m_b)\ \cos(\epsilon(t)) + L_a\ (m_f$$

$$-m_b)\Big)\ \cos(p(t))\ L_h\ \left(\frac{d}{dt}\,\lambda(t)\right)\Big)\Big/\Big(\cos(\epsilon(t))\ \big((m_f+m_b)\ L_h^2 + (m_f+m_b)\ L_a^2 +$$

$$L_w^2\, m_w)) - \frac{\sin(\epsilon(t))\, L_a\, (m_f - m_b)\, \cos(p(t))\, L_h \left(\frac{d}{dt}\, p(t)\right)^2}{\cos(\epsilon(t))\, ((m_f + m_b)\, L_h^2 + (m_f + m_b)\, L_a^2 + L_w^2\, m_w)} + (-(L_a\, ($$

$$-4\, L_a\, m_b\, m_f + m_w\, L_w\, (m_f + m_b))\, L_h^2 + m_w\, (L_a + L_w)\, L_w\, ((m_f + m_b)\, L_a^2 + L_w^2\, m_w))$$

$$(m_f - m_b)\, g\, \cos(p(t))\, \cos(\epsilon(t))^3 + (-(-V_f + V_b)\, ((m_f - m_b)^2\, L_a^2 + L_h^2\, (m_f$$

$$+ m_b)^2)\, K_f L_h^2 \cos(p(t))^2 + (\sin(\epsilon(t))\, ((m_f + m_b)\, (-4\, L_a\, m_b\, m_f + m_w\, L_w\, (m_f$$

$$+ m_b))\, L_h^2 + L_w\, L_a\, m_w\, (m_f - m_b)^2\, (L_a + L_w))\, g\, L_h \sin(p(t)) - L_a\, ((m_f + m_b)\, L_h^2$$

$$+ (m_f + m_b)\, L_a^2 + L_w^2\, m_w)\, (m_f - m_b)\, ((g\, m_b + g\, m_f + K_f\, (V_f + V_b))\, L_a$$

$$- L_w\, m_w\, g))\, \cos(p(t)) - (-V_f + V_b)\, ((-m_b - m_f)\, L_h^2 + (m_f + m_b)\, L_a^2 +$$

$$L_w^2\, m_w)\, ((m_f + m_b)\, L_h^2 + (m_f + m_b)\, L_a^2 + L_w^2\, m_w)\, K_f)\, \cos(\epsilon(t))^2 + (-L_a\, ((m_f$$

$$+ m_b)\, L_h^2 + (m_f + m_b)\, L_a^2 + L_w^2\, m_w)\, (m_f - m_b)\, ((g\, m_b + g\, m_f + K_f\, (V_f + V_b))\, L_a$$

$$- L_w\, m_w\, g)\, \cos(p(t))^2 + (\sin(\epsilon(t))\, ((m_f - m_b)^2\, L_a^2 + L_h^2\, (m_f + m_b)^2)\, ((g\, m_b$$

$$+ g\, m_f + K_f\, (V_f + V_b))\, L_a - L_w\, m_w\, g)\, \sin(p(t)) - (4\, L_a^2\, m_b\, m_f + m_w\, L_w^2\, (m_f$$

$$+ m_b))\, (m_f - m_b)\, g\, L_h)\, L_h \cos(p(t)) + L_a\, ((m_f + m_b)\, L_h^2 + (m_f + m_b)\, L_a^2 +$$

$$L_w^2\, m_w)\, (m_f - m_b)\, (2\, \sin(\epsilon(t))\, K_f L_h\, (-V_f + V_b)\, \sin(p(t)) + (g\, m_b + g\, m_f + K_f\, (V_f$$

$$+ V_b))\, L_a - L_w\, m_w\, g))\, \cos(\epsilon(t)) + (((m_f - m_b)^2\, L_a^2 + L_h^2\, (m_f + m_b)^2)\, \cos(p(t))^2$$

$$- ((m_f + m_b)\, L_h^2 + (m_f + m_b)\, L_a^2 + L_w^2\, m_w)\, (m_f + m_b))\, (\sin(\epsilon(t))\, ((g\, m_b + g\, m_f$$

$$+ K_f\, (V_f + V_b))\, L_a - L_w\, m_w\, g)\, \sin(p(t)) + K_f L_h\, (-V_f + V_b))\, L_h) \Big/$$

$$(\cos(\epsilon(t))^2\, ((m_f + m_b)\, L_h^2 + (m_f + m_b)\, L_a^2 + L_w^2\, m_w)\, (4\, L_a^2\, m_b\, m_f + m_w\, L_w^2\, (m_f$$

$$+ m_b))\, L_h)$$

```
> Xd_nl[Nq+3] := simplify( subs( subs_U_rev, subs_Xq_rev,
  subs_Xqd_rev, Xd_nl[Nq+3] ) ):
  diff( qd[3], t ) = collect( Xd_nl[Nq+3], eom_collect_list );
```

$$\frac{d^2}{dt^2}\, \lambda(t) = -\frac{L_a\, (m_f - m_b)\, \cos(p(t))^3\, L_h \left(\frac{d}{dt}\, \epsilon(t)\right)^2}{\cos(\epsilon(t))\, ((m_f + m_b)\, L_a^2 + L_h^2\, m_b + L_h^2\, m_f + L_w^2\, m_w)}$$

$$+ \Bigg(\Big(2\, ((\sin(p(t))\, L_a\, (m_f - m_b)\, \cos(\epsilon(t)) - \sin(\epsilon(t))\, L_h\, (m_f$$

$$+ m_b)) L_h \cos(p(t))^2 + \sin(\epsilon(t)) ((m_f + m_b) L_a^2 + L_h^2 m_b + L_h^2 m_f + L_w^2 m_w))$$

$$\left(\frac{d}{dt} \lambda(t)\right)) \Big/ (\cos(\epsilon(t)) ((m_f + m_b) L_a^2 + L_h^2 m_b + L_h^2 m_f + L_w^2 m_w))$$

$$- \frac{2 (m_f + m_b) \left(\frac{d}{dt} p(t)\right) \cos(p(t))^2 L_h^2}{\cos(\epsilon(t)) ((m_f + m_b) L_a^2 + L_h^2 m_b + L_h^2 m_f + L_w^2 m_w)}) \left(\frac{d}{dt} \epsilon(t)\right)$$

$$+ ((L_a (m_f - m_b) \cos(\epsilon(t))^2 \cos(p(t))^2 + 2 \sin(p(t)) \sin(\epsilon(t)) L_h (m_f + m_b) \cos(\epsilon(t)) - 2 (\cos($$

$$L_w^2 m_w)) + (2 (\cos(\epsilon(t)) L_h (m_f + m_b) \sin(p(t)) + \sin(\epsilon(t)) L_a (m_f$$

$$- m_b)) \left(\frac{d}{dt} p(t)\right) \cos(p(t)) L_h \left(\frac{d}{dt} \lambda(t)\right)) \Big/ (\cos(\epsilon(t)) ((m_f + m_b) L_a^2 +$$

$$L_h^2 m_b + L_h^2 m_f + L_w^2 m_w)) + \frac{L_a (m_f - m_b) \cos(p(t)) L_h \left(\frac{d}{dt} p(t)\right)^2}{\cos(\epsilon(t)) ((m_f + m_b) L_a^2 + L_h^2 m_b + L_h^2 m_f + L_w^2 m_w)}$$

$$+ (-((m_f - m_b)^2 L_a^2 + L_h^2 (m_f + m_b)^2) (((g m_b + g m_f + K_f (V_f + V_b)) L_a$$

$$- L_w m_w g) \sin(p(t)) + \sin(\epsilon(t)) K_f L_h (-V_f + V_b)) \cos(p(t))^2$$

$$- \cos(\epsilon(t)) (((m_w L_w (m_f - m_b)^2 L_a^2 + ((-4 L_h^2 m_f + L_w^2 m_w) m_b^2 + (-4 L_h^2 m_f^2 - 2$$

$$L_w^2 m_f m_w) m_b + m_f^2 m_w L_w^2) L_a + m_w L_h^2 L_w (m_f + m_b)^2) g \cos(\epsilon(t)) + ((m_f - m_b)^2 L_a^2$$

$$+ L_h^2 (m_f + m_b)^2) ((g m_b + g m_f + K_f (V_f + V_b)) L_a - L_w m_w g)) \sin(p(t))$$

$$- \sin(\epsilon(t)) (4 L_a^2 m_b m_f + m_w L_w^2 (m_f + m_b)) (m_f - m_b) g L_h) \cos(p(t)) + ((m_f$$

$$+ m_b) L_a^2 + L_h^2 m_b + L_h^2 m_f + L_w^2 m_w) ((-K_f L_a (-V_f + V_b) (m_f - m_b) \cos(\epsilon(t))$$

$$+ ((g m_b + g m_f + K_f (V_f + V_b)) L_a - L_w m_w g) (m_f + m_b)) \sin(p(t))$$

$$+ \sin(\epsilon(t)) K_f L_h (-V_f + V_b) (m_f + m_b))) \Big/ (\cos(\epsilon(t))^2 ((m_f + m_b) L_a^2 + L_h^2 m_b +$$

$$L_h^2 m_f + L_w^2 m_w) (4 L_a^2 m_b m_f + m_w L_w^2 (m_f + m_b)))$$

Solution to the Linearized EOM's

Solve the linear form of the equations of motion for the states' second time derivatives

```
> Xqdd_solset_ser := solve( convert( EOM_ser, set ), convert(
  Xqdd, set ) ):
> assign( Xqdd_solset_ser );
```

Moreover, for small angles

```
> subs_small_angles_list := { X[1]^2 = 0, X[2]^2 = 0, X[3]^2 =
  0, X[Nq+1]^2 = 0, X[Nq+2]^2 = 0, X[Nq+3]^2 = 0, m[b] = m[f] }
  :
> Xd[Nq+1] := algsubs( sin(X[2]) = X[2], Xd[Nq+1] ):
> Xd[Nq+1] := subs( subs_small_angles_list, Xd[Nq+1] ):
> Xd[Nq+1] := algsubs( X[Nq+1] * X[Nq+2] = 0, Xd[Nq+1] ):
> Xd[Nq+1] := algsubs( X[Nq+1] * X[Nq+3] = 0, Xd[Nq+1] ):
> Xd[Nq+1] := algsubs( X[Nq+2] * X[Nq+3] = 0, Xd[Nq+1] ):
> Xd[Nq+1] := algsubs( X[1] * X[2] = 0, Xd[Nq+1] ):
> Xd[Nq+1] := algsubs( X[1] * X[3] = 0, Xd[Nq+1] ):
> Xd[Nq+1] := algsubs( X[2] * X[3] = 0, Xd[Nq+1] );
```

$$Xd_4 := \left(-4 K_f L_a^3 U_1 m_f^2 - 4 K_f L_a^3 U_2 m_f^2 - 4 K_f L_a L_h^2 U_1 m_f^2 - 4 K_f L_a L_h^2 U_2 m_f^2 - 2 K_f L_a L_w^2 U_1 m_f m_w - 2 K_f L_a L_w^2 U_2 m_f m_w\right) \Big/ \left(-8 L_a^4 m_f^3 - 8 L_a^2 L_h^2 m_f^3 - 8 L_a^2 L_w^2 m_f^2 m_w - 4 L_h^2 L_w^2 m_f^2 m_w - 2 L_w^4 m_f m_w^2\right) \tag{7.3.1}$$

```
> Xd[Nq+2] := algsubs( sin(X[2]) = X[2], Xd[Nq+2] ):
> Xd[Nq+2] := subs( subs_small_angles_list, Xd[Nq+2] ):
> Xd[Nq+2] := algsubs( X[Nq+1] * X[Nq+2] = 0, Xd[Nq+2] ):
> Xd[Nq+2] := algsubs( X[Nq+1] * X[Nq+3] = 0, Xd[Nq+2] ):
> Xd[Nq+2] := algsubs( X[Nq+2] * X[Nq+3] = 0, Xd[Nq+2] ):
> Xd[Nq+2] := algsubs( X[1] * X[2] = 0, Xd[Nq+2] ):
> Xd[Nq+2] := algsubs( X[1] * X[3] = 0, Xd[Nq+2] ):
> Xd[Nq+2] := algsubs( X[2] * X[3] = 0, Xd[Nq+2] );
```

$$Xd_5 := \left(-4 K_f L_a^4 U_1 m_f^2 + 4 K_f L_a^4 U_2 m_f^2 - 4 K_f L_a^2 L_h^2 U_1 m_f^2 + 4 K_f L_a^2 L_h^2 U_2 m_f^2 - 4 K_f L_a^2 L_w^2 U_1 m_f m_w + 4 K_f L_a^2 L_w^2 U_2 m_f m_w - 2 K_f L_h^2 L_w^2 U_1 m_f m_w + 2 K_f L_h^2 L_w^2 U_2 m_f m_w - K_f L_w^4 U_1 m_w^2 + K_f L_w^4 U_2 m_w^2\right) \Big/ \left(\left(-8 L_a^4 m_f^3 - 8 L_a^2 L_h^2 m_f^3 - 8 L_a^2 L_w^2 m_f^2 m_w - 4 L_h^2 L_w^2 m_f^2 m_w - 2 L_w^4 m_f m_w^2\right) L_h\right) \tag{7.3.2}$$

```
> Xd[Nq+3] := algsubs( sin(X[2]) = X[2], Xd[Nq+3] ):
> Xd[Nq+3] := subs( subs_small_angles_list, Xd[Nq+3] ):
> Xd[Nq+3] := algsubs( X[Nq+1] * X[Nq+2] = 0, Xd[Nq+3] ):
> Xd[Nq+3] := algsubs( X[Nq+1] * X[Nq+3] = 0, Xd[Nq+3] ):
> Xd[Nq+3] := algsubs( X[Nq+2] * X[Nq+3] = 0, Xd[Nq+3] ):
> Xd[Nq+3] := algsubs( X[1] * X[2] = 0, Xd[Nq+3] ):
> Xd[Nq+3] := algsubs( X[1] * X[3] = 0, Xd[Nq+3] ):
> Xd[Nq+3] := algsubs( X[2] * X[3] = 0, Xd[Nq+3] );
```

(7.3.3)

$$Xd_6 := -\left(-2\, m_f \left(-8\, L_a^4 X_4 X_6\, m_f^2 - 8\, L_a^2 L_w^2 X_4 X_6\, m_f m_w - 2\, L_w^4 X_4 X_6\, m_w^2 + 2\, K_f L_a^2 L_h U_1\, m_f - 2\, K_f L_a^2 L_h U_2\, m_f + K_f L_h L_w^2 U_1\, m_w - K_f L_h L_w^2 U_2\, m_w\right) X_1 + 2\, m_f \left(4\, g\, L_a^3 m_f^2 - 2\, g\, L_a^2 L_w\, m_f m_w + 2\, g\, L_a L_w^2\, m_f m_w - g\, L_w^3\, m_w^2 + 2\, K_f L_a^3 U_1\, m_f + 2\, K_f L_a^3 U_2\, m_f - 2\, K_f L_a L_h^2 U_1\, m_f - 2\, K_f L_a L_h^2 U_2\, m_f + K_f L_a L_w^2 U_1\, m_w + K_f L_a L_w^2 U_2\, m_w\right) X_2\right) \Big/ \left(-8\, L_a^4 m_f^3 - 8\, L_a^2 L_h^2 m_f^3 - 8\, L_a^2 L_w^2 m_f^2 m_w - 4\, L_h^2 L_w^2 m_f^2 m_w - 2\, L_w^4 m_f m_w^2\right) \qquad \textbf{(7.3.3)}$$

pretty display w.r.t. the named system states
1st Linearized Equation Of Motion in the time domain

```
> L_EOM_1_DT := diff( qd[1], t ) = collect( subs( subs_U_rev,
  subs_Xq_rev, subs_Xqd_rev, Xd[Nq+1] ), eom_collect_list ):
  L_EOM_1_DT;
```

$$\frac{d^2}{dt^2}\,\epsilon(t) = \left(-4\, K_f L_a^3 V_b\, m_f^2 - 4\, K_f L_a^3 V_f m_f^2 - 4\, K_f L_a L_h^2 V_b\, m_f^2 - 4\, K_f L_a L_h^2 V_f m_f^2 - 2\, K_f L_a L_w^2 V_b\, m_f m_w - 2\, K_f L_a L_w^2 V_f m_f m_w\right) \Big/ \left(-8\, L_a^4 m_f^3 - 8\, L_a^2 L_h^2 m_f^3 - 8\, L_a^2 L_w^2 m_f^2 m_w - 4\, L_h^2 L_w^2 m_f^2 m_w - 2\, L_w^4 m_f m_w^2\right) \qquad \textbf{(7.3.4)}$$

2nd Linearized Equation Of Motion in the time domain

```
> L_EOM_2_DT := diff( qd[2], t ) = collect( subs( subs_U_rev,
  subs_Xq_rev, subs_Xqd_rev, Xd[Nq+2] ), eom_collect_list ):
  L_EOM_2_DT;
```

$$\frac{d^2}{dt^2}\,p(t) = \left(4\, K_f L_a^4 V_b\, m_f^2 - 4\, K_f L_a^4 V_f m_f^2 + 4\, K_f L_a^2 L_h^2 V_b\, m_f^2 - 4\, K_f L_a^2 L_h^2 V_f m_f^2 + 4\, K_f L_a^2 L_w^2 V_b\, m_f m_w - 4\, K_f L_a^2 L_w^2 V_f m_f m_w + 2\, K_f L_h^2 L_w^2 V_b\, m_f m_w - 2\, K_f L_h^2 L_w^2 V_f m_f m_w + K_f L_w^4 V_b\, m_w^2 - K_f L_w^4 V_f m_w^2\right) \Big/ \left(\left(-8\, L_a^4 m_f^3 - 8\, L_a^2 L_h^2 m_f^3 - 8\, L_a^2 L_w^2 m_f^2 m_w - 4\, L_h^2 L_w^2 m_f^2 m_w - 2\, L_w^4 m_f m_w^2\right) L_h\right) \qquad \textbf{(7.3.5)}$$

2nd Linearized Equation Of Motion (i.e., re. alpha) in the time domain

```
> L_EOM_3_DT := diff( qd[3], t ) = collect( subs( subs_U_rev,
  subs_Xq_rev, subs_Xqd_rev, Xd[Nq+3] ), eom_collect_list ):
  L_EOM_3_DT;
```

$$\frac{d^2}{dt^2}\,\lambda(t) = \frac{2\, m_f \left(-8\, L_a^4 m_f^2 - 8\, L_a^2 L_w^2 m_f m_w - 2\, L_w^4 m_w^2\right) \epsilon(t) \left(\frac{d}{dt}\,\lambda(t)\right)\left(\frac{d}{dt}\,\epsilon(t)\right)}{-8\, L_a^4 m_f^3 - 8\, L_a^2 L_h^2 m_f^3 - 8\, L_a^2 L_w^2 m_f^2 m_w - 4\, L_h^2 L_w^2 m_f^2 m_w - 2\, L_w^4 m_f m_w^2} + \frac{2\, m_f \left(-2\, K_f L_a^2 L_h V_b\, m_f + 2\, K_f L_a^2 L_h V_f m_f - K_f L_h L_w^2 V_b\, m_w + K_f L_h L_w^2 V_f m_w\right) \epsilon(t)}{-8\, L_a^4 m_f^3 - 8\, L_a^2 L_h^2 m_f^3 - 8\, L_a^2 L_w^2 m_f^2 m_w - 4\, L_h^2 L_w^2 m_f^2 m_w - 2\, L_w^4 m_f m_w^2} - \left(2\, m_f \left(4\, g\, L_a^3 m_f^2 - 2\, g\, L_a^2 L_w\, m_f m_w + 2\, g\, L_a L_w^2\, m_f m_w - g\, L_w^3\, m_w^2 + 2\, K_f \right.\right. \qquad \textbf{(7.3.6)}$$

$L_a^3\, V_b\, m_f + 2\, K_f L_a^3\, V_f m_f - 2\, K_f L_a\, L_h^2\, V_b\, m_f - 2\, K_f L_a\, L_h^2\, V_f m_f + K_f L_a\, L_w^2\, V_b\, m_w + K_f L_a$
$L_w^2\, V_f m_w\big)\, p(t)\Big) \Big/ \Big(-8\, L_a^4\, m_f^3 - 8\, L_a^2\, L_h^2\, m_f^3 - 8\, L_a^2\, L_w^2\, m_f^2\, m_w - 4\, L_h^2\, L_w^2\, m_f^2\, m_w - 2\, L_w^4\, m_f$
$m_w^2\Big)$

Determine the System State-Space Matrices: A, B, C, and D

```
> A_ss := Matrix( Nx, Nx ):
> A_ss := deriveA( Xqdd, A_ss, Nq, subs_XU_op ):
  'A' = A_ss;
```

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{(2\, L_a\, m_f - L_w\, m_w)\, g}{2\, L_a^2\, m_f + 2\, L_h^2\, m_f + L_w^2\, m_w} & 0 & 0 & 0 & 0 \end{bmatrix} \tag{8.1}$$

```
> B_ss := Matrix( Nx, Nu ):
> B_ss := deriveB( Xqdd, B_ss, Nq, subs_XU_op ):
  'B' = B_ss;
```

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_f L_a}{2\, L_a^2\, m_f + L_w^2\, m_w} & \frac{K_f L_a}{2\, L_a^2\, m_f + L_w^2\, m_w} \\ \frac{1}{2}\, \frac{K_f}{L_h\, m_f} & -\frac{1}{2}\, \frac{K_f}{L_h\, m_f} \\ 0 & 0 \end{bmatrix} \tag{8.2}$$

```
> C_ss := IdentityMatrix( Nq, Nx ):
  C = C_ss;
```

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{8.3}$$

```
> #lprint( C_ss );
```

```
> D_ss := Matrix( Nq, Nu, 0 ):
  D = D_ss;
```

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{8.4}$$

```
> #lprint( D_ss );
```

Write A, B, C, and D to a Matlab file

Save the state-space matrices A, B, C and D to a MATLAB file.

```
> Matlab_File_Name := "HELI3D":
> Matlab_File_Name := cat( Matlab_File_Name, "_ABCD_eqns.m" );
```

$$\mathit{Matlab\_File\_Name} := \text{"HELI3D\_ABCD\_eqns.m"} \tag{9.1}$$

substitution set containing a notation consistent with that used in the MATLAB design script(s)

```
> Matlab_Notations := { L[w] = Lw, L[h] = Lh, L[a] = La, m[b] =
  m_b, m[w] = m_w, m[f] = m_f, K[f] = Kf, g = g }:
> Experiment_Name := "3-DOF Helicopter":
> write_ABCD_to_Mfile( Matlab_File_Name, Experiment_Name,
  Matlab_Notations, A_ss, B_ss, C_ss, D_ss );
```

Procedure Printing

default:

```
> #interface( verboseproc = 1 );
> #eval( lagrange_equations );
> #eval( deriveB );
```

Click here to go back to top: Description Section.