Structure Coursework

Jiaxuan Tang

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1 Tables

$\dot{\varepsilon}_y^0/s^{-1}$	$\dot{F}_x^0$/ KN/s	$\dot{F}_x^{90}$/ KN/s	$\dot{F}_x^{30}$/ KN/s
-0.00002	0.4823	0.0482	0.1093

Table 1: Input variables

E1/GPa	V12	E2/GPa	V21	G12/GPa	E_{qi}	V_{qi}	A11/ MN/m	A22/ MN/m	A12/ MN/m	V_{xy}^{30}
192.92	0.2	19.28	0.02	10.96	79.64	0.2786	86.34	86.34	24.05	0.3085

Table 2: Engineering constants

2 Graphs

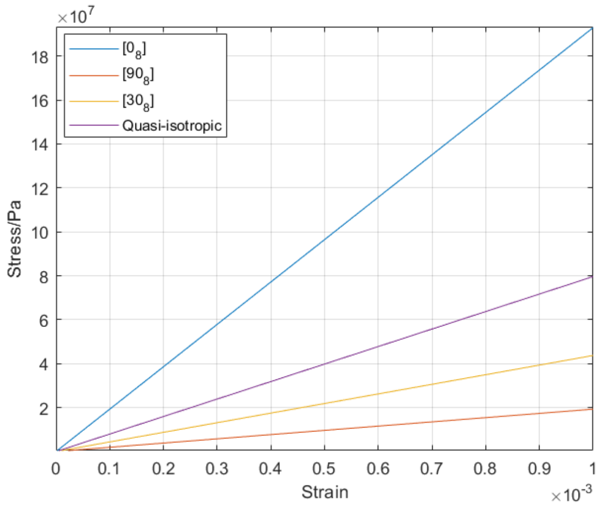

Figure 1: σ_x vs ε_x

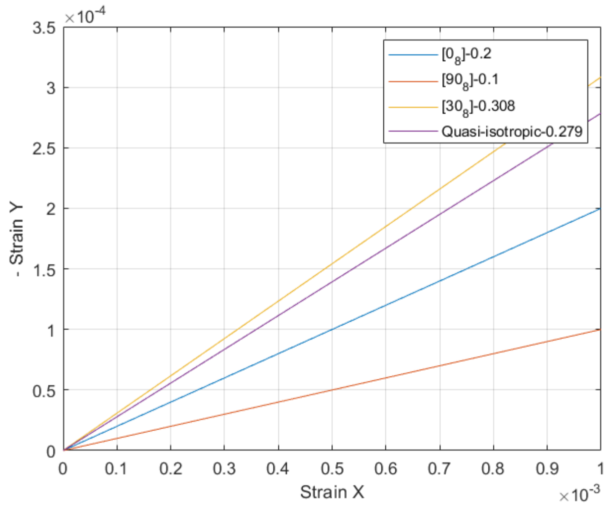

Figure 2: $-\varepsilon_y$ vs ε_x

3 Formula

Using the constant axial strain rate for the specimens, combined with the force rate, E1 of this material can be obtained very easily. E2 of the material is $\sigma_x^{90}/\varepsilon_x^{90}$ which is the traverse young's modulus. v_{12} can be obtained from $\frac{-\varepsilon_y^0}{\varepsilon_x^0}$ which was already given in the lab data. v_{21} is simply $v_{12} * \frac{E2}{E1}$. In global calculation of the stiffness matrix, only G_{12}, γ_{xy}^{30} and ϵ_{yy}^{30} are unknown, solving the equation can generate these 3 variables. When solving the equation, turn the compliance matrix S to stiffness matrix Q.

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_6 \end{pmatrix} = [\mathrm{S}] \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_6 \end{pmatrix} = \begin{pmatrix} 1/\mathrm{E}_1 & -v_{21}/\mathrm{E}_2 & 0 \\ -v_{12}/\mathrm{E}_1 & 1/\mathrm{E}_2 & 0 \\ 0 & 0 & 1/\mathrm{G}_{12} \end{pmatrix}$$

$$[\mathrm{Q}] = [\mathrm{S}]^{-1} = \begin{pmatrix} \mathrm{E}_1/\left(1 - v_{12}v_{21}\right) & v_{21}\mathrm{E}_1/\left(1 - v_{12}v_{21}\right) & 0 \\ v_{12}\mathrm{E}_2/\left(1 - v_{12}v_{21}\right) & \mathrm{E}_2/\left(1 - v_{12}v_{21}\right) & 0 \\ 0 & 0 & \mathrm{G}_{12} \end{pmatrix}$$

As we need to solve for $\theta = 30$, it needs to be transformed to global coordinates using $[T_\sigma]$ and $[T_\varepsilon]$, and $[\overline{\mathrm{Q}}] = [\mathrm{T}_\sigma]^{-1}[\mathrm{Q}][\mathrm{T}_\varepsilon]$,

where $[\mathrm{T}_\sigma] = \begin{pmatrix} \mathrm{c}^2 & \mathrm{s}^2 & 2\mathrm{cs} \\ \mathrm{s}^2 & \mathrm{c}^2 & -2\mathrm{cs} \\ -\mathrm{cs} & \mathrm{cs} & \mathrm{c}^2 - \mathrm{s}^2 \end{pmatrix}$, $[\mathrm{T}_\varepsilon] = \begin{pmatrix} \mathrm{c}^2 & \mathrm{s}^2 & \mathrm{cs} \\ \mathrm{s}^2 & \mathrm{c}^2 & -\mathrm{cs} \\ -2\mathrm{cs} & 2\mathrm{cs} & \mathrm{c}^2 - \mathrm{s}^2 \end{pmatrix}$ with $c = cos\theta$ and$s = sin\theta$

Then engineering constants except A_{11}, A_{12}, A_{22}, E_{qi} and v_{qi} can be obtained. Remained variables can calculated using laminate in-plane stiffness matrix. The system is 0/45/90/-45/-45/90/45/0 to have the same geometry as the specimens.

$$\mathrm{N_{XX}} = \sum_{\mathrm{K}=1}^{\mathrm{n}} \left(\overline{\mathrm{Q}}_{11}\right)_{\mathrm{k}} (\mathrm{h_k} - \mathrm{h_{k-1}})\,\varepsilon_{\mathrm{xx}} + \sum_{\mathrm{K}=1}^{\mathrm{n}} \left(\overline{\mathrm{Q}}_{12}\right)_{\mathrm{k}} (\mathrm{h_k} - \mathrm{h_{k-1}})\,\varepsilon_{\mathrm{yy}} + \sum_{\mathrm{K}=1}^{\mathrm{n}} \left(\overline{\mathrm{Q}}_{16}\right)_{\mathrm{k}} (\mathrm{h_k} - \mathrm{h_{k-1}})\,\gamma_{\mathrm{xy}}$$

$$\begin{pmatrix} \mathrm{N_{xx}} \\ \mathrm{N_{yy}} \\ \mathrm{N_{xy}} \end{pmatrix} = [\mathrm{A}] \begin{pmatrix} \varepsilon_{\mathrm{xx}} \\ \varepsilon_{\mathrm{yy}} \\ \gamma_{\mathrm{xy}} \end{pmatrix}, \text{ where } \mathrm{A_{ij}} = \sum_{\mathrm{K}=1}^{\mathrm{n}} \left(\overline{\mathrm{Q}}_{\mathrm{ij}}\right)_{\mathrm{k}} (\mathrm{h_k} - \mathrm{h_{k-1}})$$

A_{11}, A_{12} and A_{22} then can be obatined as $h_k - h_{k-1}$ is always 0.125mm.

$$\begin{pmatrix} \mathrm{N_{XX}} \\ 0 \\ 0 \end{pmatrix} = [\mathrm{A}] \begin{pmatrix} \varepsilon_{\mathrm{xx}} \\ \varepsilon_{\mathrm{yy}} \\ 0 \end{pmatrix} \begin{cases} \mathrm{N_{XX}} = \mathrm{A}_{11}\varepsilon_{\mathrm{xx}} + \mathrm{A}_{12}\varepsilon_{\mathrm{yy}} \\ 0 = \mathrm{A}_{12}\varepsilon_{\mathrm{xx}} + \mathrm{A}_{22}\varepsilon_{\mathrm{yy}} \end{cases}$$

Solving the equations can eliminate the N_{xx} and can lead to:

$$\varepsilon_{YY} = -\frac{A_{12}}{A_{22}}\varepsilon_{XX}; \quad N_{XX} = \left(A_{11} - \frac{A_{12}^2}{A_{22}}\right)\varepsilon_{XX}; \quad \sigma_{XX} = E_x\varepsilon_{XX} = \frac{N_{XX}}{h}$$

$$\bar{E}_x = \frac{1}{h}\frac{N_{XX}}{\varepsilon_{XX}} = \frac{1}{h}\left(A_{11} - \frac{A_{12}^2}{A_{22}}\right); \bar{v}_{xy} = -\frac{\varepsilon_{YY}}{\varepsilon_{XX}} = \frac{A_{12}}{A_{22}}$$

All the engineering constants can now be computed.