

Structure Coursework

Jiaxuan Tang - 01486551

9th March 2021

1 Tables

ε_y^0/s^{-1}	$\dot{F}_x^0/\text{KN/s}$	$\dot{F}_x^{90}/\text{KN/s}$	$\dot{F}_x^{30}/\text{KN/s}$
-0.00002	0.4823	0.0482	0.1093

Table 1: Input variables

E1/GPa	V12	E2/GPa	V21	G12/GPa	E_{qi}	V_{qi}	A11/ MN/m	A22/ MN/m	A12/ MN/m	V_{xy}^{30}
192.92	0.2	19.28	0.02	10.96	79.64	0.2786	86.34	86.34	24.05	0.3085

Table 2: Engineering constants

2 Graphs

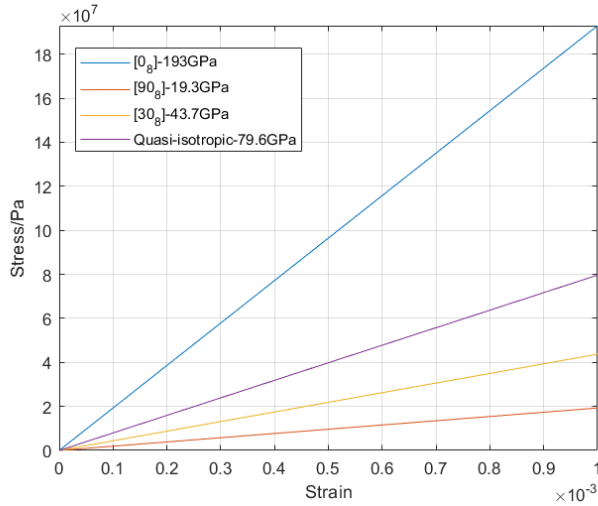


Figure 1: σ_x vs ε_x

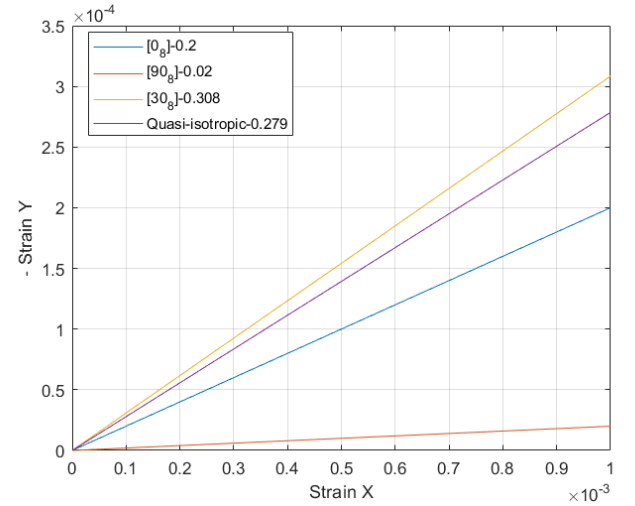


Figure 2: $-\varepsilon_y$ vs ε_x

3 Calculations

Using the constant axial strain rate for the specimens, combined with the force rate, E_1 of this material can be obtained very easily. E_2 of the material is $\sigma_x^{90}/\varepsilon_x^{90}$ which is the traverse young's modulus. v_{12} can be obtained from $\frac{-\varepsilon_y^0}{\varepsilon_x^0}$ which was already given in the lab data. v_{21} is simply $v_{12} * \frac{E_2}{E_1}$. In global calculation of the stiffness matrix, only G_{12} , γ_{xy}^{30} and ϵ_{yy}^{30} are unknown, solving the equation can generate these 3 variables. When solving the equation, turn the compliance matrix S to stiffness matrix Q .

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_6 \end{pmatrix} = [S] \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_6 \end{pmatrix} = \begin{pmatrix} 1/E_1 & -v_{21}/E_2 & 0 \\ -v_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{pmatrix}$$

$$[Q] = [S]^{-1} = \begin{pmatrix} E_1/(1-v_{12}v_{21}) & v_{21}E_1/(1-v_{12}v_{21}) & 0 \\ v_{12}E_2/(1-v_{12}v_{21}) & E_2/(1-v_{12}v_{21}) & 0 \\ 0 & 0 & G_{12} \end{pmatrix}$$

As we need to solve for $\theta = 30$, it needs to be transformed to global coordinates using $[T_\sigma]$ and $[T_\varepsilon]$, and $[\bar{Q}] = [T_\sigma]^{-1} [Q] [T_\varepsilon]$,

where $[T_\sigma] = \begin{pmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{pmatrix}$, $[T_\varepsilon] = \begin{pmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{pmatrix}$ with $c = \cos\theta$ and $s = \sin\theta$

Then engineering constants except A_{11} , A_{12} , A_{22} , E_{qi} and v_{qi} can be obtained. Remained variables can be calculated using laminate in-plane stiffness matrix. The system is 0/45/90/-45/-45/90/45/0 to have the same geometry as the specimens.

$$N_{XX} = \sum_{K=1}^n (\bar{Q}_{11})_K (h_K - h_{K-1}) \varepsilon_{xx} + \sum_{K=1}^n (\bar{Q}_{12})_K (h_K - h_{K-1}) \varepsilon_{yy} + \sum_{K=1}^n (\bar{Q}_{16})_K (h_K - h_{K-1}) \gamma_{xy}$$

$$\begin{pmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{pmatrix} = [A] \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{pmatrix}, \text{ where } A_{ij} = \sum_{K=1}^n (\bar{Q}_{ij})_K (h_K - h_{K-1})$$

A_{11} , A_{12} and A_{22} then can be obtained as $h_K - h_{K-1}$ is always 0.125mm.

$$\begin{pmatrix} N_{XX} \\ 0 \\ 0 \end{pmatrix} = [A] \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 0 \end{pmatrix} \begin{cases} N_{XX} = A_{11}\varepsilon_{xx} + A_{12}\varepsilon_{yy} \\ 0 = A_{12}\varepsilon_{xx} + A_{22}\varepsilon_{yy} \end{cases}$$

Solving the equations can eliminate the N_{xx} and can lead to:

$$\varepsilon_{YY} = -\frac{A_{12}}{A_{22}}\varepsilon_{XX}; \quad N_{XX} = \left(A_{11} - \frac{A_{12}^2}{A_{22}}\right)\varepsilon_{XX}; \quad \sigma_{XX} = E_x\varepsilon_{XX} = \frac{N_{XX}}{h}$$

$$\bar{E}_x = \frac{1}{h} \frac{N_{XX}}{\varepsilon_{XX}} = \frac{1}{h} \left(A_{11} - \frac{A_{12}^2}{A_{22}}\right); \quad \bar{v}_{xy} = -\frac{\varepsilon_{YY}}{\varepsilon_{XX}} = \frac{A_{12}}{A_{22}}$$

All the engineering constants can now be computed.