Imperial College London - Department of Aeronautics

Laboratory Handout for AERO96002 Control Systems

The Twin Rotor Multivariable System: Feedback Control of a Pseudo-Helicopter Pre-lab Tasks

January 14, 2021

- **You are requested to finish the pre-lab tasks before the laboratory session in order to complete the exercise successfully**
- **Note the deadline for Matlab Grader assignment submission**
- **Remember to take the 4 sets of controllers designed (corresponding Q and R values) to the main lab session for testing**

Version 4 (20200109)

Contents

1 Introduction

1.1 Description

The 3 Degrees-Of-Freedom (3 DOF) Helicopter plant is shown in Figure 1. The system consists of a combination of two frames, a long arm, which is free to rotate about its elevation and yaw axes, and a helicopter body frame suspended by an instrumented joint mounted at the end of the long arm, which enables the frame to pitch about its centre. Two DC motors are mounted at each end of the helicopter body and drives two propellers. The thrust vector produced by these is normal to the helicopter body frame. Furthermore, a counterweight is attached to the other end of the long arm such that the effective mass of the helicopter body is reduced and, consequently, allowing the helicopter to be driven by the thrust from the motors. The system resembles a Boeing CH-47 Chinook oriented perpendicular to the support arm.

The system's trajectory and state can be controlled by varying the motors' voltages. An elevation of the body is caused by applying a positive voltage to either motor. A positive voltage applied to the front motor causes a positive pitch while the opposite occurs for a positive voltage applied to the back motor. Such a pitch in the system induces a travel in the body due the rotation of the thrust vector. The presence of a slip ring in the system's pedestal allows electrical signals to be channeled to and from the arm and helicopter, eliminating tangled wires, reducing friction and allowing for unlimited and unhindered travel [1].

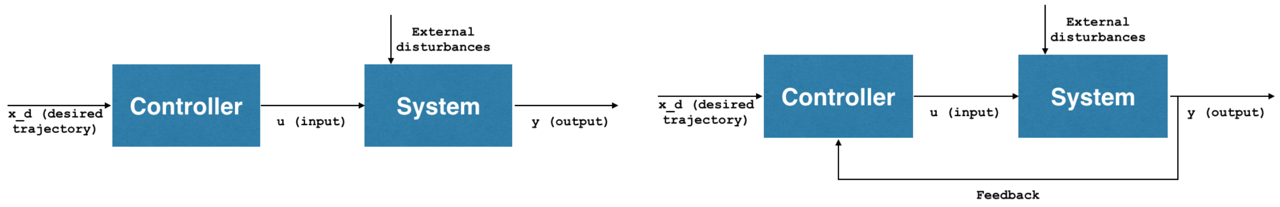
Figure 1: 3 DOF Helicopter when running.

1.2 Objectives

The objective of this laboratory exercise is to introduce some of the ideas and methods that are taught in the AERO96002 Control Systems course.

AERO96002 only deals with relatively low-order linear systems. We shall see that simple linear control theory can be used to control the nonlinear 3 DOF Helicopter system. This experiment will complement the state space based feedback control theory covered in AERO96002, which will allow you to properly analyse and design feedback controllers. You will discover that a coupled linear model can be approximated by linearising the fully nonlinear dynamics of the system. The approximate linear model will then be used to design a controller. The experiment will emphasise that linear controllers designed on a linear approximate model often work reasonably well on the complex nonlinear system.

As one can imagine, our mathematical model may, and indeed will, be approximate and imprecise. Furthermore, unpredicted phenomena, such as disturbances, that influence the behaviour of the system sill also be present. Thus, the necessity of a controller that considers all these. Also, the open-loop unstable nature of the system results in requiring a controller even if it is for stability purposes alone. This necessity of feedback is better represented by the block diagrams shown in Figure 2.



(a) Controller is "blind" to the system outputs (i.e. states).

(b) Output information is fed back into the controller to create a new input.

Figure 2: Block diagrams for the open and closed-loop configurations.

Furthermore, taking into account the fact that we are dealing with a cross-coupled multivariable and multi-degree-of-freedom system, it make sense to design a multivariable controller that considers all these aspects. Hence, the main aim of this experiment will be to design a multivariable PID controller using a Linear-Quadratic Regular scheme to track and regulate the elevation and travel trajectories of the system.

2 Modelling

2.1 System Dynamics

The system's 3 degrees-of-freedom are shown in Figure 3. As can be seen, the pitch ρ of the helicopter is the rotation of the helicopter about a line perpendicular to the length of the body located at the centre of gravity of the helicopter body. The pitch angle is said to be positive when the front motor is higher than the back motor. The elevation axis is defined as a line parallel to the length of the body, at the base coordinate frame. Therefore, a change in the elevation angles ϵ translates into a change in the altitude of the helicopter as it rotates about the base frame. The elevation angle is positive when the helicopter is at a higher position than the horizontal, which is defined by $\epsilon = 0$. Finally, the travel axis is defined as a vertical line at the base coordinate frame perpendicular to the elevation axis. A change in the travel angle λ translates into forward flight about the travel axis. The travel angle is positive when the helicopter rotates anticlockwise around the base.

The nonlinear equations of motion of the 3 DOF Helicopter in elevation, pitch and travel are obtained through structural dynamics analysis using Euler-Lagrange's method (see [2] for details). The thrust forces acting on the elevation, pitch, and travel axes from the front and back motors are defined and made relative to the quiescent voltage or operating point V_{op} defined as follows

$$V_{op} = \frac{1}{2}\frac{g(L_w m_w - L_a m_f - L_a m_b)}{L_a K_f}, \tag{1}$$

where the symbols are defined in the Appendix.

2.2 System State-Space Model

The nonlinear equations of motion of the 3 DOF helicopter are now linearised about zero, to model the system's dynamics in a linear state-space structure (A, B, C, D). This model describes the voltage-to-angular joint position dynamics of the system. Given the state-space representation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = Ax + Bu \;\text{ and }\; y = Cx + Du, \tag{2}$$

where the state vector for the 3 DOF Helicopter is defined as

$$x^T = \begin{bmatrix} \epsilon \; \rho \; \lambda \; \frac{\mathrm{d}\epsilon}{\mathrm{d}t} \; \frac{\mathrm{d}\rho}{\mathrm{d}t} \; \frac{\mathrm{d}\lambda}{\mathrm{d}t} \end{bmatrix} \tag{3}$$

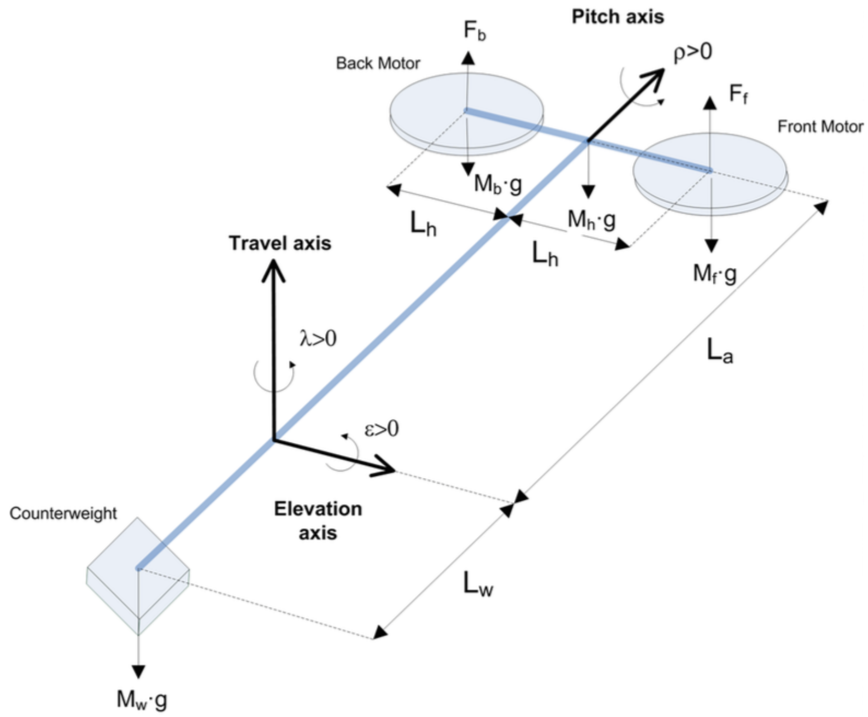


Figure 3: FBD of 3 DOF Helicopter.

The corresponding state-space matrices are given by [3] as

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{(L_w m_w - 2L_a m_f)g}{m_w L_w^2 + 2m_f L_h^2 + 2m_f L_a^2} & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{L_a K_f}{2m_f L_a^2 + m_w L_w^2} & \frac{L_a K_f}{2m_f L_a^2 + m_w L_w^2} \\ \frac{1}{2}\frac{K_f}{m_f L_h} & -\frac{1}{2}\frac{K_f}{m_f L_h} \\ 0 & 0 \end{bmatrix}, \tag{4}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \text{ and } D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

where the model parameters are defined in Appendix.

3 Control Design

3.1 State-Feedback

As mentioned at the beginning, the aim of the laboratory exercise is to design a controller to stabilise the dynamic response of the system and regulate the elevation and travel angles of the 3 DOF Helicopter at set positions. In our case a PID controller is to be designed, whose gains are to be determined from the solution to a Linear-Quadratic Regulator problem. The state-feedback controller voltage entering the front motor V_f and the back motor V_b is defined as

$$\begin{bmatrix} V_f \\ V_b \end{bmatrix} = K_{PD}(x_d - x) + V_i + \begin{bmatrix} V_{op} \\ V_{op} \end{bmatrix}, \tag{5}$$

where

$$K_{PD} = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} & k_{1,5} & k_{1,6} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & k_{2,5} & k_{2,6} \end{bmatrix} \tag{6}$$

is the proportional-derivative control gain matrix,

$$x_d^T = \begin{bmatrix} \epsilon_d \; \rho_d \; \lambda_d \; 0 \; 0 \; 0 \end{bmatrix} \tag{7}$$

is the desired state vector,

$$V_i = \begin{bmatrix} \int k_{1,7}(x_{d,1} - X_1)\mathrm{d}t + \int k_{1,8}(x_{d,3} - X_3)\mathrm{d}t \\ \int k_{2,7}(x_{d,1} - X_1)\mathrm{d}t + \int k_{2,8}(x_{d,3} - X_3)\mathrm{d}t \end{bmatrix} \tag{8}$$

is the integral control, and V_{op} is the operating point voltage defined in (4). Note the first three columns of the K_{PD} matrix are the proportional gains and the last three columns are the derivative gains.

3.2 Linear Quadratic Regulator (LQR)

The PID control gains are computed from the solution to a Linear-Quadratic Regular problem. However, in order to apply integral gains, the system state first needs to be augmented to include the integrals of the elevation and travel states, i.e. we redefine the states and system matrices respectively as follows

$$\tilde{x}^T = \begin{bmatrix} \epsilon \; \rho \; \lambda \; \frac{\mathrm{d}\epsilon}{\mathrm{d}t} \; \frac{\mathrm{d}\rho}{\mathrm{d}t} \; \frac{\mathrm{d}\lambda}{\mathrm{d}t} \; \int \epsilon \, \mathrm{d}t \; \int \lambda \, \mathrm{d}t \end{bmatrix}, \tag{9}$$

$$\tilde{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{(L_w m_w - 2L_a m_f)g}{m_w L_w^2 + 2m_f L_h^2 + 2m_f L_a^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{L_a K_f}{2m_f L_a^2 + m_w L_w^2} & \frac{L_a K_f}{2m_f L_a^2 + m_w L_w^2} \\ \frac{1}{2}\frac{K_f}{m_f L_h} & -\frac{1}{2}\frac{K_f}{m_f L_h} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{10}$$

Using the feedback law

$$u = -K(\tilde{x} - \tilde{x}_d) + \begin{bmatrix} V_{op} \\ V_{op} \end{bmatrix}, \qquad \tilde{x}_d^T = \begin{bmatrix} \epsilon_d \; \rho_d \; \lambda_d \; 0 \; 0 \; 0 \; 0 \; 0 \end{bmatrix} \tag{11}$$

the weighting matrices

$$Q = \mathrm{diag}\left(\begin{bmatrix} q_\epsilon \; q_\rho \; q_\lambda \; q_{\dot\epsilon} \; q_{\dot\rho} \; q_{\dot\lambda} \; q_{\int\epsilon} \; q_{\int\lambda} \end{bmatrix} \right) \quad \text{and} \quad R = \mathrm{diag}\left(\begin{bmatrix} r_f \; r_b \end{bmatrix} \right) \tag{12}$$

and the state-space matrices $\tilde{A}$ and $\tilde{B}$, the control gain K is calculated by minimising the cost function

$$J = \int_0^\infty (\tilde{x} - \tilde{x}_d)^T Q (\tilde{x} - \tilde{x}_d) + u^T R u \, \mathrm{d}t. \tag{13}$$

Thus, the full PID control gain is expressed as

$$K = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} & k_{1,5} & k_{1,6} & k_{1,7} & k_{1,8} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & k_{2,5} & k_{2,6} & k_{2,7} & k_{2,8} \end{bmatrix}. \tag{14}$$

For further information on PID and LQR control you can read Chapter 10 and 7.3 respectively in [4]. Also note that the Linear-Quadratic Regular scheme, used in this experiment, is one way of calculating the gains for a PID controller. Indeed, there are many other approaches that can be used to determine these, such as “Pole Placement”, which you will encounter during the lectures.

4 Files for Pre-lab Tasks

The following Matlab and Simulink files can be found in Blackboard in the appropriate folder. You are requested to use these files to investigate how the controllers' estimated performance in the nonlinear simulations change as the controller design parameters are modified; in this case mainly the Q weights.

The files and their descriptor are the following:

- `setup_lab_heli_3d.m` - Computes the controller gains using a linearised model. It also computes the parameters used in the linear and nonlinear simulations. For the file to run properly, you will need to reproduce the controller design from Matlab Grader assignments in the *'Controller design'* section and provide the gain matrix. **RUN THIS FILE BEFORE SIMULATION!**
- `NonlinearModel.slx` - The nonlinear model, to be used for the linearisation exercise.
- `ClosedLoopNonLinearModel.slx` - Closed-loop simulation of the nonlinear model.

  For most of the Simulink models, you may visualize relevant outputs via Simscape animations (Figure 4), as well as with the included scopes. The data will be transferred back to the Matlab workspace using `To Workspace` blocks, with the name of the workspace variables corresponding to the variable name shown on the block. The simulation time will be stored by default in the `tout` variable.

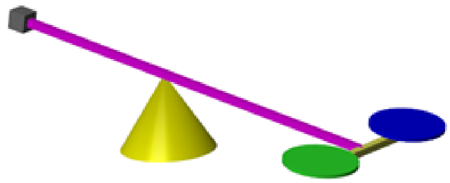


Figure 4: Simscape animation of the lab helicopter setup.

The `lib` folder contains the following supplementary files:

- `setup_heli_3d_configuration.m` - Matlab function used to set the values of the system's parameters.
- `calc_conversion_constants.m` - Calculates useful conversion factors w.r.t. units.
- `Nonlinear_Helicopter_SFunc.m` - The S-Function for the nonlinear model of the helicopter derived using Euler-Lagrange's method.

5 Pre-lab Task

Complete Matlab Grader Assignment 1. All relevant knowledge in this assignment may be examined during the oral assessment for the lab exercise.

6 Appendix

Table 1: System parameters. [3]

Symbol	Description	Value	Units
K_f	Propeller force-thrust constant	0.119	NV^{-1}
m_f, m_b	Mass of front and back fans	0.654	kg
m_w	Mass of counterweight	1.924	kg
m_h	Mass of helicopter body		
L_a	Distance between travel axis to helicopter body	0.66	m
L_h	Distance between pitch axis to each motor	0.178	m
L_w	Distance between travel axis to the counterweight	0.47	m
g	Gravitational constant	9.81	ms^{-2}
F_f	Front motor thrust		
F_b	Back motor thrust		
V_f	Front motor voltage		
V_b	Back motor voltage		
ϵ, $\dot{\epsilon}$	Elevation angle and angular rates		
ρ, $\dot{\rho}$	Pitch angle and angular rate		
λ, $\dot{\lambda}$	Travel angle and angular rate		
J_ϵ	Moment of inertia of system about elevation axis		
J_ρ	Moment of inertia of helicopter body about pitch axis		
J_λ	Moment of inertia of system about travel axis		
K,K_{PD}	PID/PD gain		

References

[1] "User Manual - 3 DOF Helicopter Experiment: Set Up and Configuration," *Quanser Inc.*, 2012.

[2] "Dynamic Equations for the 3-DOF Helicopter," *Quanser Inc.*, 2004. available from BlackBoard.

[3] J. Apkarian, M. Levis, and C. Fulford, "Laboratory Guide - 3 DOF Helicopter Experiment for LabVIEW Users," *Quanser Inc.*, 2012.

[4] K. J. Åström and R. M. Murray, "Feedback Systems: An introduction for scientists and engineers." Second Edition, `http://fbsbook.org`, Princeton University Press, 2020.