Low complexity DCT approximation for image compression

1.Introduction

The Discrete Cosine Transform (DCT) is one of the important tools for image/video compression and processing. Due to its remarkable energy compaction properties, the DCT has been adopted in many image and video compression standards, such as JPEG, MPEG-1, MPEG-2 and MPEG-4.

Considering a N-point sequence, which means x(n)=0 when n<0 or n>N-1, the DCT is defined as: $X(k) = e(k)\sum_{n=0}^{N-1} x(n)\cos\left[\frac{(2n+1)k\pi}{2N}\right], k = 0,1,2,\dots,N-1$ (1)

Where $e(k) = \begin{cases} \frac{1}{\sqrt{2}} & if\ k = 0 \\ 1 & otherwise \end{cases}$

Generalizing to the 2-dimension DCT, of order N×N, it is defined as:

$$T_{DCT}(u,v) = \alpha(u)\alpha(v)\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\cos\left[\frac{(2i+1)u\pi}{2N}\right]\cos\left[\frac{(2j+1)v\pi}{2N}\right] \quad (2)$$

Where as $\alpha(u)\alpha(v) = \begin{cases} \frac{1}{\sqrt{N}} & if\ u,v = 0 \\ \frac{2}{\sqrt{N}} & otherwise \end{cases}$

Obviously, the direct implementation of a N × N 2-D DCT requires $O(N^4)$ multiply-add operations, which is hardware expensive. The algorithmic strength reduction can obtain DCT with very few multipliers. For N-points 1-D DCT, Strength reduction can reduce the multiplication complexity of a 8-oint DCT from 56 to 13.[1]

However, multiplications are not expected for real-time image/video processing. Traditional algorithms aim at the computation of exact DCT, which always requires multiplication operations. As a consequence, approximate methods were proposed since it only needs additions. In some applications, a simple DCT approximation can provide meaningful results at low arithmetic complexity [2]. Thus, approximation techniques for the DCT are becoming increasingly popular.

2.Approximate DCT

N-points sequence x(n) and X(k) can be represented as vector: $x(n) = [x(0)\ x(1) \dots x(N-1)]^T$ and $X(k) = [X(0)\ X(1) \dots X(N-1)]^T$. So DCT and 2-D DCT can be represented in matrix form as $\boldsymbol{X} = \boldsymbol{Cx}$ and $\boldsymbol{X} = \boldsymbol{CxC^T}$, where $\boldsymbol{C}$ is a N×

N matrix whose elements are given by (1) and (2).

Generally, a DCT approximation is a transformation $\widehat{\boldsymbol{C}}$ which behaves similarly to exact DCT matrix $\boldsymbol{C}$. In order to ensure orthogonality or quasi-orthogonality, we often have $\widehat{\boldsymbol{C}} = \boldsymbol{S} \cdot \boldsymbol{T}$, where $\boldsymbol{T}$ is a transformation matrix of low computational and $\boldsymbol{S}$ is a scaling diagonal matrix. [3]

The elements of the transformation matrix $\boldsymbol{T}$ often possess null multiplicative complexity. The diagonal matrix $\boldsymbol{S}$ usually contains irrational numbers in the form $1/\sqrt{m}$, where m is a small positive integer. Due to a scaling diagonal matrix can be conveniently merged into the quantization step of compression algorithms, $\boldsymbol{S}$ needs no extra computation. [3]

Haweel introduced a simple approach for designing a DCT approximation. The DCT approximation termed as SDCT was defined as $sign(\boldsymbol{C})$, where $sign(\cdot)$ is the signum function applied to each entry of $\boldsymbol{C}$ and is given by

$$sign(x) = \begin{cases} +1 & if\ x > 0, \\ 0 & if\ x = 0, \\ -1 & if\ x < 0. \end{cases} \tag{3}$$

All the elements in the transform are 0 or ±1, thus avoiding the need of multiplication operation or transcendental expression. The SDCT also maintains the periodicity and spectral structure of its originating DCT and in turn maintains its good decorrelation and energy compaction characteristics. Therefore, SDCT highly preferred for low computation complexity applications. [17]

3.Reported Methods

A typical method requires 14 additions with improved PSNR is proposed in [4]. The proposed transform matrix is obtained by appropriately setting some of the entries of the 8x8 SDCT matrices introduce to zero as following.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 0 & 1 & -1 & 0 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

The transformation matrix $\boldsymbol{C_1} = \boldsymbol{D_1} \cdot \boldsymbol{T_1}$, where $\boldsymbol{D_1} = diag(\frac{1}{\sqrt{8}}, \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{2}}, \frac{1}{8}, \frac{1}{\sqrt{2}})$.The number of additions in the proposed transform can be clearly understood from the butterfly diagram shown in Fig 1. Continuous and dashed line represents +1 and -1respectively. The results of this method are shown in Fig 2 and Fig3.

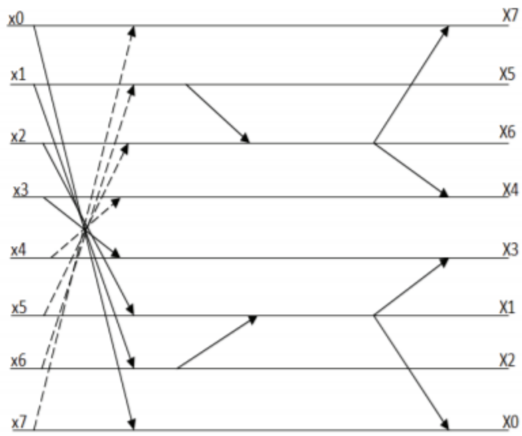

Fig. 1. Butterfly structure for the poposed transform of order N = 8

The parametric transform proposed in 2011 by Bouguezel–Ahmad–Swamy [23] is an 8-point orthogonal transform containing a single parameter "a." Its mathematical structure is $\boldsymbol{C_1} = \boldsymbol{D_1} \cdot \boldsymbol{T_1}$, where $\boldsymbol{D_1} = diag(\frac{1}{\sqrt{8}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{4+4a^2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{4+4a^2}})$. It requires only 16 additions for its computation.

In [5], a DCT approximation tailored for a particular radio-frequency (RF) application was obtained in accordance with an exhaustive computational search. The transformation matrix $\boldsymbol{C_1} = \boldsymbol{D_1} \cdot \boldsymbol{T_1}$, where $\boldsymbol{D_1} = \frac{1}{2} diag(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{3}})$. It requires only 24 additions and six shifts for its computation.

In [6], a low-complexity approximate DCT defines the cost of a transformation matrix as the number of arithmetic operations required for its computation. Elements of matrix intend to be in $\{0, \pm 1, \pm 2\}$ to insure that resulting multiplicative complexity is null. The transformation matrix $\boldsymbol{C_1} = \boldsymbol{D_1} \cdot \boldsymbol{T_1}$, where $\boldsymbol{D_1} = diag(\frac{1}{\sqrt{8}}, \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{2}}, \frac{1}{8}, \frac{1}{\sqrt{2}})$. It requires only 14 additions for its computation

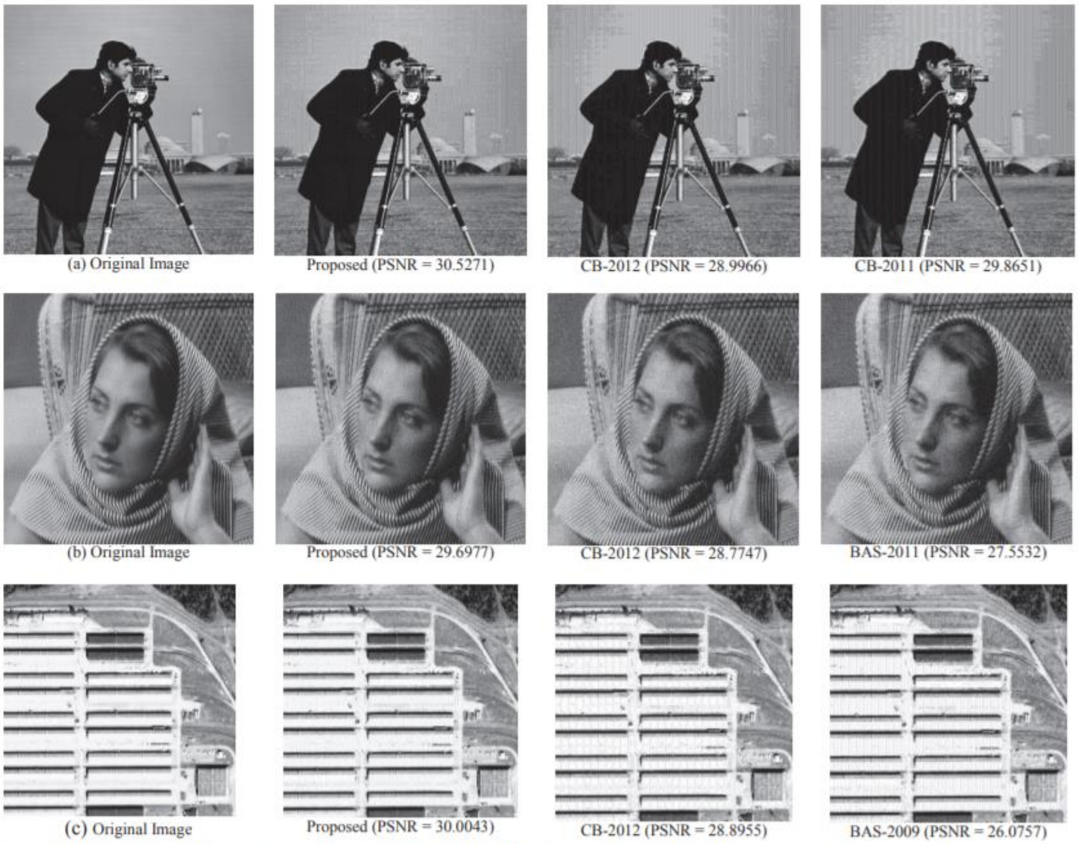

Fig. 2. Reconstructed from 1.3 Compression ratio using proposed transform, CB-2012 and BAS-2011 for (a) Cameraman (b) Woman (c) Satellite

Transform	Cameraman	Lena	Pepper	Woman	Parrots	Mandrill	Satellite
Proposed Transform	41.5088	40.9173	40.4654	41.6039	41.5357	41.5804	41.4661
CB-2012 [15]	40.6501	40.2659	39.8267	40.7240	40.8240	40.3848	40.7233
CB-2011 [14]	41.0770	40.2734	39.7537	40.5738	41.0896	40.1573	40.7923
BAS2011(a=1) [13]	39.3991	38.9538	38.4759	39.5487	39.6602	39.1116	39.6460
BAS2010 [12]	51.0451	50.5509	50.0564	51.0304	51.0823	50.6601	51.0959
BAS2009 [11]	38.5415	38.1404	37.7655	38.8138	38.8580	38.3438	38.9184
BAS2008 [9]	46.4782	45.9307	45.4787	46.2487	46.5429	46.2642	46.4464
SDCT [8]	48.0072	47.5725	47.0718	47.9427	48.0677	47.7219	48.0371

Fig. 3 PSNR OBTAINED BY DIFFERENT 8X8

In [7], the proposed energy-efficient 8-point DCT approximation requires only 12 additions. In this paper, a new DCT approximation scheme is developed by identifying and sharing the common computations to remove the redundancy. The forward transform matrices are obtained as follows.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

We have introduced several existing methods of DCT approximations. The number of additions, multiplications, and bit-shift operations required for these transforms and the original DCT is presented in Fig 5.

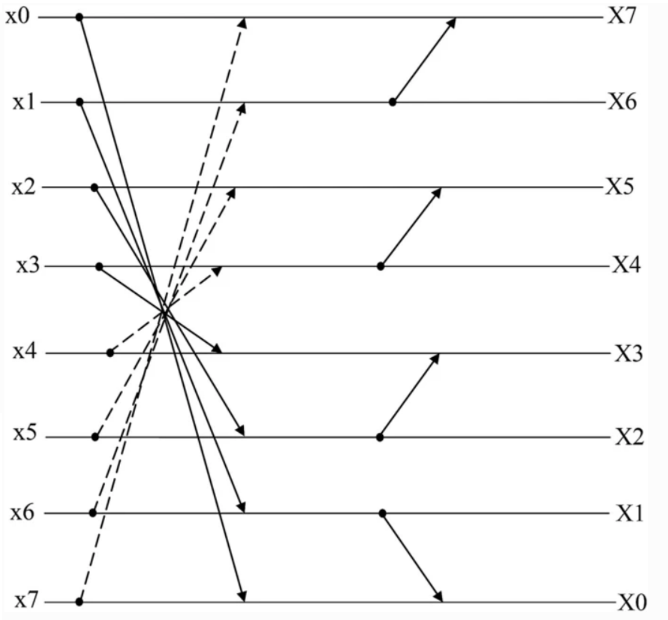

Fig. 4 The Butterfly Structure for the 12 Additions transform

Transform	Addition	Multiplication	Shifts
DCT (by definition)	56	64	0
Arai et al.[38]	29	5	0
SDCT[17]	24	0	0
Level 1 approximation[18]	24	0	2
Bouguezel et al.[20]	21	0	0
Bouguezel et al.[19]	18	0	2
Bouguezel et al.[21]	18	0	0
Bouguezel et al.[22]	24	0	4
Bouguezel et al.[23] (a = 0)	16	0	0
Bouguezel et al.[23] (a = 1)	18	0	0
Bouguezel et al.[23] (a = 2)	18	2	0
Senapati et al.[26]	14	0	2
Cintra and Bayer[24]	22	0	0
Bayer and Cintra[27]	14	0	0
Transform in[29]	16	0	0
Transform in[30]	14	0	0
Proposed transform	12	0	0

Fig 5 Numbers of additions, multiplications and bit-shift

4.VLSI implementation

The digital architecture of the proposed approximate DCT in [7]is shown in Figure 6. The hardware cost is measured by the number of adders, multipliers, and shifters used in the architecture, and the computing time is normalized as clock cycles.

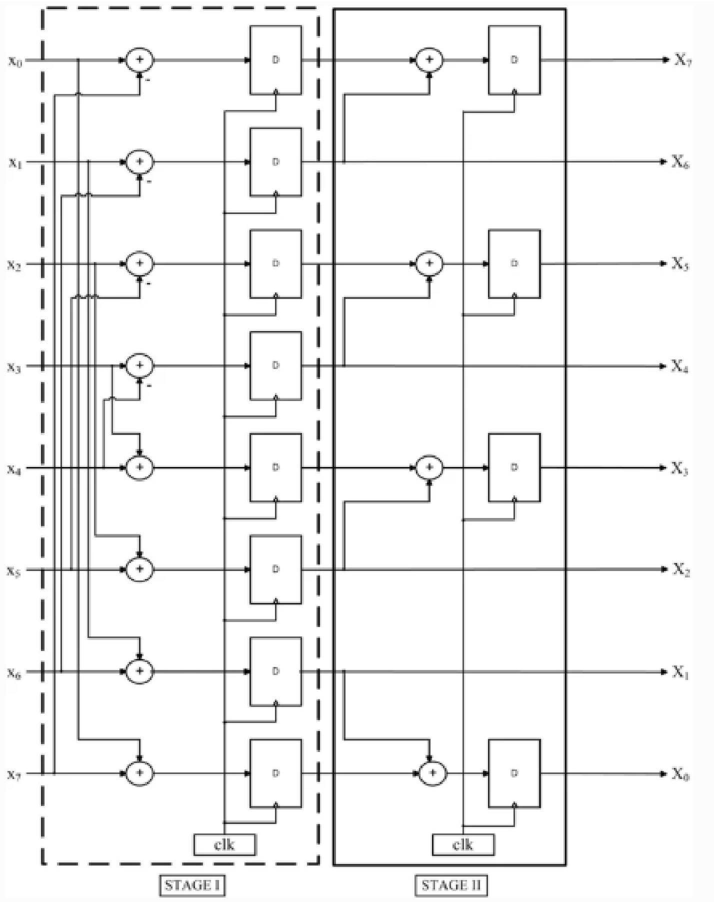

Fig 6 Hardware architecture for DCT approximation in [7]

The proposed approximation DCT matrix in [7] and other matrices were physically implemented on a Xilinx Virtex 7 XC7V585T-2LFFG1761C device. The inputs were assumed at an 8-bit resolution and are realized with pipelining in order to increase the throughput. To get the accurate timing result, post-place and route (PAR) is done for each run of the design flow. Since the hardware resource requirements become low for the proposed method, it gains greater flexibility in placement and routing to get the optimized delay. The resource utilization (area) is measured as the numbers of the cell

Transform	LUTs	Cell usage	Delay (ns)
SDCT[17]	272	274	5.113
Bouguezel et al.[19]	267	269	4.149
Bouguezel et al.[21]	204	206	5.716
Bouguezel et al.[22]	271	273	5.153
Bouguezel et al.[23] (*a* = 1)	204	205	5.593
Senapati et al.[26]	186	189	5.914
Cintra and Bayer[24]	226	228	5.171
Bayer and Cintra[27]	153	155	4.580
Transform in[29]	167	168	6.738
Transform in[30]	156	157	5.924
Proposed transform	132	134	3.247

Fig 7 Comparison of hardware resource consumption with the reported architectures on Xilinx Virtex-7

usage (input/output buffers and global clock buffers) and lookup tables (LUTs). The resources used by the implementation are listed in Fig 7.

5.Conclusions

In this literature survey, I introduced the mathematical concepts of exact DCT and approximations, reported approximate methods, and their hardware implementations. Low power and area minimization are the two indispensable requirements for portable multimedia devices, which employs various signal and image processing algorithms. The proposed approximations DCT in references for image compression is a simple, efficient architecture having lower computational complexity with improvement in the peak signal-to-noise ratio. The hardware implementation that has been carried out clearly shows that these architectures is best suited for real-time low power and high speed applications of image and video.

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