

```

V = {t, r, e, f};
g = Table[0, {4}, {4}];
g[[1, 1]] = -1 + 2 * G / c^2 * M / r;
g[[2, 2]] = +1;
g[[3, 3]] = r^2;
g[[4, 4]] = r^2 * Sin[e]^2;
g[[1, 4]] = 0;
g[[4, 1]] = g[[1, 4]];
invg = Inverse[g]; (*逆ア度ケ*)
dg1 = Outer[D, g, V]; (* $\frac{\partial g_{\alpha\beta}}{\partial x^\gamma}$ *)
dg2 = Transpose[dg1, {1, 3, 2}];
dg3 = Transpose[dg1, {2, 3, 1}];
(*シ算克氏シ罫*)
G0 = Simplify[(1/2) invg. (dg1 + dg2 - dg3)];
(* $\frac{1}{2} g^{\mu\alpha} \left( \frac{\partial g_{\alpha\beta}}{\partial x^\gamma} + \frac{\partial g_{\alpha\gamma}}{\partial x^\beta} - \frac{\partial g_{\beta\gamma}}{\partial x^\alpha} \right)$ *)
ddg1 = Outer[D, G0, V]; (*克氏シ罫做偏才数*)
ddg2 = Transpose[ddg1, {1, 2, 4, 3}];
G1 = Table[Sum[G0[[m, j, k]] G0[[i, l, m]],
  {m, 4}], {i, 4}, {j, 4}, {k, 4}, {l, 4}];
G2 = Transpose[G1, {1, 2, 4, 3}];
R = -Simplify[ddg1 - ddg2 + G1 - G2];
(*尼曼曲率1量*)
Rc =
  Simplify[Table[Sum[R[[i, j, i, n]], {i, 4}],
    {j, 4}, {n, 4}]]; (*里奇1量*)
Rs = Simplify[Sum[Rc[[i, i]], {i, 4}]];
(*曲率ア量*)
R1 = Simplify[
  Table[Sum[R[[m, j, k, l]] g[[i, m]], {m, 4}],
    {i, 4}, {j, 4}, {k, 4}, {l, 4}]];
MatrixForm[g]
Do[Print[{i, j, k}, G0[[i, j, k]]],

```

```

    {i, 4}, {j, 4}, {k, 4}]
Do[Print[{i, j, k, l}, R[[i, j, k, l]]],
    {i, 4}, {j, 4}, {k, 4}, {l, 4}]
Do[Print[{i, j}, Rc[[i, j]]], {i, 4}, {j, 4}]
Rs

```

$$\begin{pmatrix} -1 + \frac{2GM}{c^2 r} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin[e]^2 \end{pmatrix}$$

```
{1, 1, 1}0
```

$$\{1, 1, 2\} \frac{GM}{r (-2GM + c^2 r)}$$

```
{1, 1, 3}0
```

```
{1, 1, 4}0
```

$$\{1, 2, 1\} \frac{GM}{r (-2GM + c^2 r)}$$

```
{1, 2, 2}0
```

```
{1, 2, 3}0
```

```
{1, 2, 4}0
```

```
{1, 3, 1}0
```

```
{1, 3, 2}0
```

```
{1, 3, 3}0
```

```
{1, 3, 4}0
```

```
{1, 4, 1}0
```

```
{1, 4, 2}0
```

```
{1, 4, 3}0
```

```
{1, 4, 4}0
```

$$\{2, 1, 1\} \frac{GM}{c^2 r^2}$$

```
{2, 1, 2}0
```

```
{2, 1, 3}0
```

```
{2, 1, 4}0
```

```
{2, 2, 1}0
```

```
{2, 2, 2}0
```

```
{2, 2, 3}0
```

```
{2, 2, 4}0
```

```
{2, 3, 1}0
```

```
{2, 3, 2}0
```

```
{2, 3, 3}-r
```

```

{2, 3, 4}0
{2, 4, 1}0
{2, 4, 2}0
{2, 4, 3}0
{2, 4, 4}-r Sin[e]^2
{3, 1, 1}0
{3, 1, 2}0
{3, 1, 3}0
{3, 1, 4}0
{3, 2, 1}0
{3, 2, 2}0
{3, 2, 3} $\frac{1}{r}$ 
{3, 2, 4}0
{3, 3, 1}0
{3, 3, 2} $\frac{1}{r}$ 
{3, 3, 3}0
{3, 3, 4}0
{3, 4, 1}0
{3, 4, 2}0
{3, 4, 3}0
{3, 4, 4}-Cos[e] Sin[e]
{4, 1, 1}0
{4, 1, 2}0
{4, 1, 3}0
{4, 1, 4}0
{4, 2, 1}0
{4, 2, 2}0
{4, 2, 3}0
{4, 2, 4} $\frac{1}{r}$ 
{4, 3, 1}0
{4, 3, 2}0
{4, 3, 3}0
{4, 3, 4}Cot[e]
{4, 4, 1}0
{4, 4, 2} $\frac{1}{r}$ 
{4, 4, 3}Cot[e]

```

$$\{4, 4, 4\}0$$

$$\{1, 1, 1, 1\}0$$

$$\{1, 1, 1, 2\}0$$

$$\{1, 1, 1, 3\}0$$

$$\{1, 1, 1, 4\}0$$

$$\{1, 1, 2, 1\}0$$

$$\{1, 1, 2, 2\}0$$

$$\{1, 1, 2, 3\}0$$

$$\{1, 1, 2, 4\}0$$

$$\{1, 1, 3, 1\}0$$

$$\{1, 1, 3, 2\}0$$

$$\{1, 1, 3, 3\}0$$

$$\{1, 1, 3, 4\}0$$

$$\{1, 1, 4, 1\}0$$

$$\{1, 1, 4, 2\}0$$

$$\{1, 1, 4, 3\}0$$

$$\{1, 1, 4, 4\}0$$

$$\{1, 2, 1, 1\}0$$

$$\{1, 2, 1, 2\} - \frac{GM (3GM - 2c^2 r)}{r^2 (-2GM + c^2 r)^2}$$

$$\{1, 2, 1, 3\}0$$

$$\{1, 2, 1, 4\}0$$

$$\{1, 2, 2, 1\} - \frac{GM (-3GM + 2c^2 r)}{r^2 (-2GM + c^2 r)^2}$$

$$\{1, 2, 2, 2\}0$$

$$\{1, 2, 2, 3\}0$$

$$\{1, 2, 2, 4\}0$$

$$\{1, 2, 3, 1\}0$$

$$\{1, 2, 3, 2\}0$$

$$\{1, 2, 3, 3\}0$$

$$\{1, 2, 3, 4\}0$$

$$\{1, 2, 4, 1\}0$$

$$\{1, 2, 4, 2\}0$$

$$\{1, 2, 4, 3\}0$$

$$\{1, 2, 4, 4\}0$$

$$\{1, 3, 1, 1\}0$$

$$\{1, 3, 1, 2\}0$$

$$\{1, 3, 1, 3\} - \frac{GM}{-2GM + c^2 r}$$

$$\{1, 3, 1, 4\}0$$

$$\{1, 3, 2, 1\}0$$

$$\{1, 3, 2, 2\}0$$

$$\{1, 3, 2, 3\}0$$

$$\{1, 3, 2, 4\}0$$

$$\{1, 3, 3, 1\} - \frac{GM}{2GM - c^2 r}$$

$$\{1, 3, 3, 2\}0$$

$$\{1, 3, 3, 3\}0$$

$$\{1, 3, 3, 4\}0$$

$$\{1, 3, 4, 1\}0$$

$$\{1, 3, 4, 2\}0$$

$$\{1, 3, 4, 3\}0$$

$$\{1, 3, 4, 4\}0$$

$$\{1, 4, 1, 1\}0$$

$$\{1, 4, 1, 2\}0$$

$$\{1, 4, 1, 3\}0$$

$$\{1, 4, 1, 4\} - \frac{GM \sin[e]^2}{-2GM + c^2 r}$$

$$\{1, 4, 2, 1\}0$$

$$\{1, 4, 2, 2\}0$$

$$\{1, 4, 2, 3\}0$$

$$\{1, 4, 2, 4\}0$$

$$\{1, 4, 3, 1\}0$$

$$\{1, 4, 3, 2\}0$$

$$\{1, 4, 3, 3\}0$$

$$\{1, 4, 3, 4\}0$$

$$\{1, 4, 4, 1\} - \frac{GM \sin[e]^2}{2GM - c^2 r}$$

$$\{1, 4, 4, 2\}0$$

$$\{1, 4, 4, 3\}0$$

$$\{1, 4, 4, 4\}0$$

$$\{2, 1, 1, 1\}0$$

$$\{2, 1, 1, 2\} - \frac{GM (3GM - 2c^2 r)}{c^2 r^3 (-2GM + c^2 r)}$$

$$\{2, 1, 1, 3\}0$$

$$\{2, 1, 1, 4\}0$$

$$\{2, 1, 2, 1\} - \frac{GM (-3GM + 2c^2 r)}{c^2 r^3 (-2GM + c^2 r)}$$

$$\{2, 1, 2, 2\}0$$

{2, 1, 2, 3}0  
{2, 1, 2, 4}0  
{2, 1, 3, 1}0  
{2, 1, 3, 2}0  
{2, 1, 3, 3}0  
{2, 1, 3, 4}0  
{2, 1, 4, 1}0  
{2, 1, 4, 2}0  
{2, 1, 4, 3}0  
{2, 1, 4, 4}0  
{2, 2, 1, 1}0  
{2, 2, 1, 2}0  
{2, 2, 1, 3}0  
{2, 2, 1, 4}0  
{2, 2, 2, 1}0  
{2, 2, 2, 2}0  
{2, 2, 2, 3}0  
{2, 2, 2, 4}0  
{2, 2, 3, 1}0  
{2, 2, 3, 2}0  
{2, 2, 3, 3}0  
{2, 2, 3, 4}0  
{2, 2, 4, 1}0  
{2, 2, 4, 2}0  
{2, 2, 4, 3}0  
{2, 2, 4, 4}0  
{2, 3, 1, 1}0  
{2, 3, 1, 2}0  
{2, 3, 1, 3}0  
{2, 3, 1, 4}0  
{2, 3, 2, 1}0  
{2, 3, 2, 2}0  
{2, 3, 2, 3}0  
{2, 3, 2, 4}0  
{2, 3, 3, 1}0  
{2, 3, 3, 2}0  
{2, 3, 3, 3}0  
{2, 3, 3, 4}0  
{2, 3, 4, 1}0

$$\{2, 3, 4, 2\}0$$

$$\{2, 3, 4, 3\}0$$

$$\{2, 3, 4, 4\}0$$

$$\{2, 4, 1, 1\}0$$

$$\{2, 4, 1, 2\}0$$

$$\{2, 4, 1, 3\}0$$

$$\{2, 4, 1, 4\}0$$

$$\{2, 4, 2, 1\}0$$

$$\{2, 4, 2, 2\}0$$

$$\{2, 4, 2, 3\}0$$

$$\{2, 4, 2, 4\}0$$

$$\{2, 4, 3, 1\}0$$

$$\{2, 4, 3, 2\}0$$

$$\{2, 4, 3, 3\}0$$

$$\{2, 4, 3, 4\}0$$

$$\{2, 4, 4, 1\}0$$

$$\{2, 4, 4, 2\}0$$

$$\{2, 4, 4, 3\}0$$

$$\{2, 4, 4, 4\}0$$

$$\{3, 1, 1, 1\}0$$

$$\{3, 1, 1, 2\}0$$

$$\{3, 1, 1, 3\} - \frac{GM}{c^2 r^3}$$

$$\{3, 1, 1, 4\}0$$

$$\{3, 1, 2, 1\}0$$

$$\{3, 1, 2, 2\}0$$

$$\{3, 1, 2, 3\}0$$

$$\{3, 1, 2, 4\}0$$

$$\{3, 1, 3, 1\} - \frac{GM}{c^2 r^3}$$

$$\{3, 1, 3, 2\}0$$

$$\{3, 1, 3, 3\}0$$

$$\{3, 1, 3, 4\}0$$

$$\{3, 1, 4, 1\}0$$

$$\{3, 1, 4, 2\}0$$

$$\{3, 1, 4, 3\}0$$

$$\{3, 1, 4, 4\}0$$

$$\{3, 2, 1, 1\}0$$

$$\{3, 2, 1, 2\}0$$

{3, 2, 1, 3}0  
{3, 2, 1, 4}0  
{3, 2, 2, 1}0  
{3, 2, 2, 2}0  
{3, 2, 2, 3}0  
{3, 2, 2, 4}0  
{3, 2, 3, 1}0  
{3, 2, 3, 2}0  
{3, 2, 3, 3}0  
{3, 2, 3, 4}0  
{3, 2, 4, 1}0  
{3, 2, 4, 2}0  
{3, 2, 4, 3}0  
{3, 2, 4, 4}0  
{3, 3, 1, 1}0  
{3, 3, 1, 2}0  
{3, 3, 1, 3}0  
{3, 3, 1, 4}0  
{3, 3, 2, 1}0  
{3, 3, 2, 2}0  
{3, 3, 2, 3}0  
{3, 3, 2, 4}0  
{3, 3, 3, 1}0  
{3, 3, 3, 2}0  
{3, 3, 3, 3}0  
{3, 3, 3, 4}0  
{3, 3, 4, 1}0  
{3, 3, 4, 2}0  
{3, 3, 4, 3}0  
{3, 3, 4, 4}0  
{3, 4, 1, 1}0  
{3, 4, 1, 2}0  
{3, 4, 1, 3}0  
{3, 4, 1, 4}0  
{3, 4, 2, 1}0  
{3, 4, 2, 2}0  
{3, 4, 2, 3}0  
{3, 4, 2, 4}0  
{3, 4, 3, 1}0



$$\{3, 4, 3, 2\}0$$

$$\{3, 4, 3, 3\}0$$

$$\{3, 4, 3, 4\}0$$

$$\{3, 4, 4, 1\}0$$

$$\{3, 4, 4, 2\}0$$

$$\{3, 4, 4, 3\}0$$

$$\{3, 4, 4, 4\}0$$

$$\{4, 1, 1, 1\}0$$

$$\{4, 1, 1, 2\}0$$

$$\{4, 1, 1, 3\}0$$

$$\{4, 1, 1, 4\} - \frac{GM}{c^2 r^3}$$

$$\{4, 1, 2, 1\}0$$

$$\{4, 1, 2, 2\}0$$

$$\{4, 1, 2, 3\}0$$

$$\{4, 1, 2, 4\}0$$

$$\{4, 1, 3, 1\}0$$

$$\{4, 1, 3, 2\}0$$

$$\{4, 1, 3, 3\}0$$

$$\{4, 1, 3, 4\}0$$

$$\{4, 1, 4, 1\} - \frac{GM}{c^2 r^3}$$

$$\{4, 1, 4, 2\}0$$

$$\{4, 1, 4, 3\}0$$

$$\{4, 1, 4, 4\}0$$

$$\{4, 2, 1, 1\}0$$

$$\{4, 2, 1, 2\}0$$

$$\{4, 2, 1, 3\}0$$

$$\{4, 2, 1, 4\}0$$

$$\{4, 2, 2, 1\}0$$

$$\{4, 2, 2, 2\}0$$

$$\{4, 2, 2, 3\}0$$

$$\{4, 2, 2, 4\}0$$

$$\{4, 2, 3, 1\}0$$

$$\{4, 2, 3, 2\}0$$

$$\{4, 2, 3, 3\}0$$

$$\{4, 2, 3, 4\}0$$

$$\{4, 2, 4, 1\}0$$

$$\{4, 2, 4, 2\}0$$

$$\{4, 2, 4, 3\}0$$

$$\{4, 2, 4, 4\}0$$

$$\{4, 3, 1, 1\}0$$

$$\{4, 3, 1, 2\}0$$

$$\{4, 3, 1, 3\}0$$

$$\{4, 3, 1, 4\}0$$

$$\{4, 3, 2, 1\}0$$

$$\{4, 3, 2, 2\}0$$

$$\{4, 3, 2, 3\}0$$

$$\{4, 3, 2, 4\}0$$

$$\{4, 3, 3, 1\}0$$

$$\{4, 3, 3, 2\}0$$

$$\{4, 3, 3, 3\}0$$

$$\{4, 3, 3, 4\}0$$

$$\{4, 3, 4, 1\}0$$

$$\{4, 3, 4, 2\}0$$

$$\{4, 3, 4, 3\}0$$

$$\{4, 3, 4, 4\}0$$

$$\{4, 4, 1, 1\}0$$

$$\{4, 4, 1, 2\}0$$

$$\{4, 4, 1, 3\}0$$

$$\{4, 4, 1, 4\}0$$

$$\{4, 4, 2, 1\}0$$

$$\{4, 4, 2, 2\}0$$

$$\{4, 4, 2, 3\}0$$

$$\{4, 4, 2, 4\}0$$

$$\{4, 4, 3, 1\}0$$

$$\{4, 4, 3, 2\}0$$

$$\{4, 4, 3, 3\}0$$

$$\{4, 4, 3, 4\}0$$

$$\{4, 4, 4, 1\}0$$

$$\{4, 4, 4, 2\}0$$

$$\{4, 4, 4, 3\}0$$

$$\{4, 4, 4, 4\}0$$

$$\{1, 1\} \frac{G^2 M^2}{c^2 r^3 (2 G M - c^2 r)}$$

$$\{1, 2\}0$$

$$\{1, 3\}0$$

$$\{1, 4\}0$$

$$\{2, 1\}0$$

$$\{2, 2\}-\frac{GM\left(3GM-2c^2r\right)}{r^2\left(-2GM+c^2r\right)^2}$$

$$\{2, 3\}0$$

$$\{2, 4\}0$$

$$\{3, 1\}0$$

$$\{3, 2\}0$$

$$\{3, 3\}\frac{GM}{2GM-c^2r}$$

$$\{3, 4\}0$$

$$\{4, 1\}0$$

$$\{4, 2\}0$$

$$\{4, 3\}0$$

$$\{4, 4\}\frac{GM\sin[e]^2}{2GM-c^2r}$$

$$\left(GM\left(2G^2M^2+2c^2GMr\left(-2+r^2\right)-c^4r^2\left(-2+r^2\right)-c^2r^3\left(-2GM+c^2r\right)\sin[e]^2\right)\right)/\left(r^3\left(-2cGM+c^3r\right)^2\right)$$