一、求后牛顿度规的 Christoffel 符号。

$$\Gamma_{ij}^{0} = \frac{1}{2}g^{00}\left(\frac{\partial g_{0i}}{\partial x^{j}} + \frac{\partial g_{0j}}{\partial x^{i}} - \frac{\partial g_{ij}}{c\partial t}\right) + \frac{1}{2}g^{0k}\left(\frac{\partial g_{ki}}{\partial x^{j}} + \frac{\partial g_{kj}}{\partial x^{i}} - \frac{\partial g_{ij}}{\partial x^{k}}\right)$$

$$= \frac{1}{2}(-1 + \mathcal{O}_2)\left(\left(-\frac{4}{c^3}\right)\frac{\partial w_i}{\partial x^j} + \left(-\frac{4}{c^3}\right)\frac{\partial w_j}{\partial x^i} - \frac{\partial}{c\partial t}\left(\delta_{ij}\left(1 + \frac{2w}{c^2}\right) + \mathcal{O}_4\right)\right) + \mathcal{O}_5$$

$$= \frac{2}{c^3}\left(\frac{\partial w_i}{\partial x^j} + \frac{\partial w_j}{\partial x^i}\right) + \delta_{ij}\frac{1}{c^3}\frac{\partial w}{\partial t} + \mathcal{O}_5$$

$$= \frac{1}{c^2}\delta_{ij}w_{,0} + \frac{4}{c^3}w_{(i,j)} + \mathcal{O}_5$$

$$\Gamma_{00}^{i} = \frac{1}{2}g^{i0}\left(2 \times \frac{\partial g_{0i}}{c\partial t} - \frac{\partial g_{00}}{\partial x^{i}}\right) + \frac{1}{2}g^{ik}\left(2 \times \frac{\partial g_{k0}}{c\partial t} - \frac{\partial g_{00}}{\partial x^{k}}\right)$$

$$= \frac{1}{2}\left(-\frac{4w_i}{c^3} + \mathcal{O}_5\right)\left(-\frac{2}{c^2}\frac{\partial w}{\partial x^i} + \mathcal{O}_4\right)$$

$$+ \frac{1}{2}\left(\left(1 - \frac{2w}{c^2}\right) + \mathcal{O}_4\right)\left(-\frac{8}{c^4}\frac{\partial w_i}{\partial t} - \frac{2}{c^2}\frac{\partial w}{\partial x^i} + \frac{4w}{c^4}\frac{\partial w}{\partial x^i} + \mathcal{O}_6\right)$$

$$= \frac{4w_i}{c^5}\frac{\partial w}{\partial x^i} - \frac{4}{c^4}\frac{\partial w_i}{\partial t} - \frac{1}{c^2}\frac{\partial w}{\partial x^i} + \frac{2w}{c^4}\frac{\partial w}{\partial x^i} + \frac{2w}{c^4}\frac{\partial w}{\partial x^i} + \mathcal{O}_6$$

$$= -\frac{1}{c^2}w_{,i} + \frac{4}{c^4}ww_{,i} + -\frac{4}{c^3}w_{i,0} + \frac{4w_i}{c^5}\frac{\partial w}{\partial x^i} + \mathcal{O}_6$$

$$\Gamma_{0j}^{i} = \frac{1}{2}g^{i0}\frac{\partial g_{00}}{\partial x^{j}} + \frac{1}{2}g^{ik}\left(\frac{\partial g_{k0}}{\partial x^{j}} + \frac{\partial g_{kj}}{c\partial t} - \frac{\partial g_{0j}}{\partial x^{k}}\right)$$

$$= \mathcal{O}_5 + \frac{1}{2}\left(\left(1 - \frac{2w}{c^2}\right) + \mathcal{O}_4\right)\left(-\frac{4}{c^3}\frac{\partial w_i}{\partial x^j} + \delta_{ij}\frac{2}{c^3}\frac{\partial w}{\partial t} + \frac{4}{c^3}\frac{\partial w_j}{\partial x^i}\right)$$

$$= \frac{2}{c^3}\frac{\partial w_j}{\partial x^i} - \frac{2}{c^3}\frac{\partial w_i}{\partial x^j} + \delta_{ij}\frac{1}{c^3}\frac{\partial w}{\partial t} + \mathcal{O}_5$$

$$= -\frac{2}{c^3}w_{[i,j]} + \frac{1}{c^2}\delta_{ij}w_{,0} + \mathcal{O}_5$$

$$\Gamma_{jk}^{i} = \frac{1}{2}g^{i0}\left(\frac{\partial g_{0j}}{\partial x^{k}} + \frac{\partial g_{0k}}{\partial x^{j}} - \frac{\partial g_{jk}}{c\partial t}\right) + \frac{1}{2}g^{il}\left(\frac{\partial g_{lj}}{\partial x^{k}} + \frac{\partial g_{lk}}{\partial x^{j}} - \frac{\partial g_{jk}}{\partial x^{l}}\right)$$

$$= \mathcal{O}_6 + \frac{1}{2}\left(\left(1 - \frac{2w}{c^2}\right) + \mathcal{O}_4\right)\left(\delta_{ij}\frac{2}{c^2}\frac{\partial w}{\partial x^k} + \delta_{ik}\frac{2}{c^2}\frac{\partial w}{\partial x^j} - \delta_{jk}\frac{2}{c^2}\frac{\partial w}{\partial x^i}\right)$$

$$= \frac{1}{c^2}\delta_{ij}w_{,k} + \frac{1}{c^2}\delta_{ik}w_{,j} - \frac{1}{c^2}\delta_{jk}w_{,i} + \mathcal{O}_4$$