Data Preprocessing for

Pose Estimation & Prediction with Geodesic Regression on SE(3)

Wu Shuang

May 2018

Table of Contents

Modelling Articulate Objects with Kinematic Chains

- The problem of 3D pose estimation / prediction in articulate objects assumes an anatomically motivated skeletal structure.
- The predefined set of keypoints correspond to joint locations of the skeletal structure.
- A kinematic chain is a model for the skeletal structure (an assembly of bones connected and constrained by the joints).
- Since bones are rigid, any pose configuration can be represented as an ordered set of rigid transformations.

Illustration of Skeletal Models

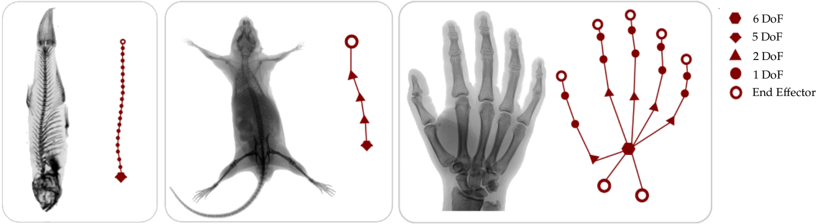

Figure: An illustration of the skeletal models adopted for three different articulate objects: (from left to right) fish, mouse, and human hand.

The rigid transformation connecting two joints is an element of the Special Euclidean SE(3) Lie group. Moving along a root joint to an end effector constitutes a kinematic chain and the ordered set of SE(3) transformations characterizes the pose.

Rigid transformations and SE(3) group

- A rigid transformation in 3D consists of a rotation and a translation.
- Any rigid transformation can be represented as matrices of the form

$$\begin{pmatrix} R & T \\ 0 & 1 \end{pmatrix}$$

  where R is a 3×3 rotation matrix and T is a 3D translation vector.
- The set of all rigid transformations, together with the composition operation is a group structure known as the SE(3) group.

Remark

In the above representation, composition of rigid transformations correspond to matrix multiplication. Check that SE(3) matrices indeed form a group.

$\mathfrak{se}(3)$ Lie algebra

- In addition to its group structure, SE(3) also has a manifold structure (locally resembles [1] $\mathbb{R}^n$).
- Technically, SE(3) is known as a Lie group and the tangent space at its identity is known as its Lie algebra $\mathfrak{se}(3)$.
- The dimension of SE(3) and $\mathfrak{se}(3)$ is 6; every rigid transformation can be fully parametrized by 6 Lie algebra parameters.
- The goal is to obtain the Lie algebra parametrization of a pose given the 3D joint locations (inverse kinematics) and vice versa (forward kinematics).

Remark

Lie groups are geometric objects, i.e. manifolds while Lie algebras are linear objects, i.e. vector spaces. For this reason, it is advantageous to use Lie algebra parameters over the SE(3) matrix representation.

[1] The technical term is diffeomorphism. An intuitive example of a manifold is the Earth which, as a spherical manifold, locally resembles a plane.

Connecting the SE(3) matrix representation and $\mathfrak{se}(3)$ parameters

- The matrix exponential is a surjective map from $\mathfrak{se}(3)$ to SE(3) and the inverse is given by the matrix logarithm map.
- Given $\begin{pmatrix} R & T \\ 0 & 1 \end{pmatrix}$, we obtain the $\mathfrak{se}(3)$ parametrization $\xi = \begin{pmatrix} \omega \\ t \end{pmatrix}$ where ω corresponds to the axis-angle representation of the rotation R while t can be identified with T [2].
- For the axis-angle representation $\omega = \theta \hat{n}$, the angle of rotation is given by $\theta = \arccos\left(\frac{\mathrm{Tr}(R)-1}{2}\right)$ while the axis is given by

$$\hat{n} = \frac{1}{2\sin\theta} \begin{pmatrix} R(3,2) - R(2,3) \\ R(1,3) - R(3,1) \\ R(2,1) - R(1,2) \end{pmatrix}.$$

[2] Technically, $t \neq T$ but the correct expression for t will not be useful for our project. For all practical purpose, it is sufficient to assign $t = T$.

Connecting the SE(3) matrix representation and $\mathfrak{se}(3)$ parameters

- Given the axis-angle representation $\omega = \theta\hat{n}$, the rotation matrix is obtained as

$$R = I + \sin\theta\,\hat{n}_\times + (1-\cos\theta)\hat{n}_\times^2$$

where

$$\hat{n}_\times = \begin{pmatrix} 0 & -\hat{n}_3 & \hat{n}_2 \\ \hat{n}_3 & 0 & -\hat{n}_1 \\ -\hat{n}_2 & \hat{n}_1 & 0 \end{pmatrix}.$$

Table of Contents

Parametrizing a Kinematic Chain with $\mathfrak{se}(3)$

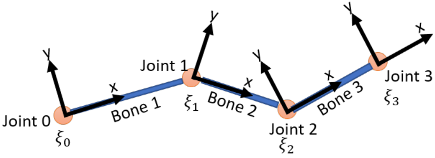

Figure: An illustration of a kinematic chain with n+1 joints and n bones; n=3.

- Joint 0: Base joint, Joint 1-2: Internal joints, Joint 3: End effector
- A local coordinate system is defined at each joint such that the x-axis of joint i aligns with bone $i+1$.
- The coordinate transformation is effectuated by ξ_i:
  ξ_0 transforms the world reference frame to the local coordinates system of at joint 0; ξ_1 transforms the local coordinates system at joint 0 to the local coordinates system at joint 1 and so on $\cdots$

Now you are ready to implement the inverse kinematics in inverse.m.
If you need hints, click here.

Table of Contents

Computing Joint Locations given $\mathfrak{se}(3)$ parameters

Once you have obtained the $\mathfrak{se}(3)$ parameters, implement the forward kinematics in forward.m.

If you need hints, click here.