Balanced Ternary

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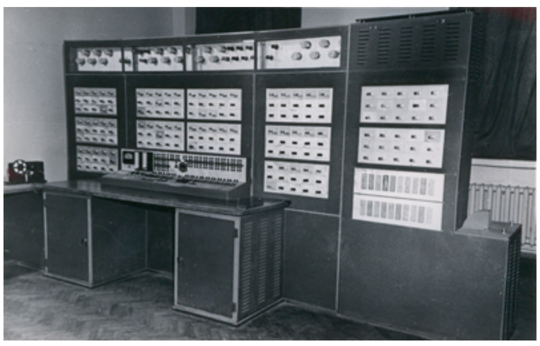

This is a non-standard but still positional **numeral system**. Its feature is that digits can have one of the values `-1`, `0` and `1`. Nevertheless, its base is still `3` (because there are three possible values). Since it is not convenient to write `-1` as a digit, we'll use letter `Z` further for this purpose. If you think it is quite a strange system - look at the picture - here is one of the computers utilizing it.

So here are few first numbers written in balanced ternary:

```
 0    0
 1    1
 2    1Z
 3    10
 4    11
 5    1ZZ
 6    1Z0
 7    1Z1
 8    10Z
 9    100
```

This system allows you to write negative values without leading minus sign: you can simply invert digits in any positive number.

```
-1   Z
-2   Z1
-3   Z0
-4   ZZ
-5   Z11
```

Note that a negative number starts with `Z` and positive with `1`.

Conversion algorithm

It is easy to represent a given number in **balanced ternary** via temporary representing it in normal ternary number system. When value is in standard ternary, its digits are either `0` or `1` or `2`. Iterating from the lowest digit we can safely skip any `0`s and `1`s, however `2` should be turned into `Z` with adding `1` to the next digit. Digits `3` should be turned into `0` on the same terms - such digits are not present in the number initially but they can be encountered after increasing some `2`s.

Example 1: Let us convert `64` to balanced ternary. At first we use normal ternary to rewrite the number:

$$64_{10} = 02101_3$$

Let us process it from the least significant (rightmost) digit:

- `1`,`0` and `1` are skipped as it is.(Because `0` and `1` are allowed in balanced ternary)
- `2` is turned into `Z` increasing the digit to its left, so we get `1Z101`.

The final result is `1Z101`.

Let us convert it back to the decimal system by adding the weighted positional values:

$$1Z101 = 81 \cdot 1 + 27 \cdot (-1) + 9 \cdot 1 + 3 \cdot 0 + 1 \cdot 1 = 64_{10}$$

Example 2: Let us convert `237` to balanced ternary. At first we use normal ternary to rewrite the number:

$$237_{10} = 22210_3$$

Let us process it from the least significant (rightmost) digit:

- `0` and `1` are skipped as it is.(Because `0` and `1` are allowed in balanced ternary)
- `2` is turned into `Z` increasing the digit to its left, so we get `23Z10`.
- `3` is turned into `0` increasing the digit to its left, so we get `30Z10`.
- `3` is turned into `0` increasing the digit to its left(which is by default `0`), and so we get `100Z10`.

The final result is `100Z10`.

Let us convert it back to the decimal system by adding the weighted positional values:

$$100Z10 = 243 \cdot 1 + 81 \cdot 0 + 27 \cdot 0 + 9 \cdot (-1) + 3 \cdot 1 + 1 \cdot 0$$

Practice Problems

- Topcoder SRM 604, Div1-250