Oriented area of a triangle

Table of Contents

Given three points p_1, p_2 and p_3, calculate an oriented (signed) area of a triangle formed by them. The sign of the area is determined in the following way: imagine you are standing in the plane at point p_1 and are facing p_2. You go to p_2 and if p_3 is to your right (then we say the three vectors turn "clockwise"), the sign of the area is positive, otherwise it is negative. If the three points are collinear, the area is zero.

Using this signed area, we can both get the regular unsigned area (as the absolute value of the signed area) and determine if the points lie clockwise or counterclockwise in their specified order (which is useful, for example, in convex hull algorithms).

Calculation

We can use the fact that a determinant of a 2×2 matrix is equal to the signed area of a parallelogram spanned by column (or row) vectors of the matrix. This is analog to the definition of the cross product in 2D (see Basic Geometry). By dividing this area by two we get the area of a triangle that we are interested in. We will use $\overrightarrow{p_1p_2}$ and $\overrightarrow{p_2p_3}$ as the column vectors and calculate a 2×2 determinant:

$$2S = \begin{vmatrix} x_2 - x_1 & x_3 - x_2 \\ y_2 - y_1 & y_3 - y_2 \end{vmatrix} = (x_2 - x_1)(y_3 - y_2) - (x_3 - x$$

Implementation

```
int signed_area_parallelogram(point2d p1, poin
    return cross(p2 - p1, p3 - p2);
}

double triangle_area(point2d p1, point2d p2, p
    return abs(signed_area_parallelogram(p1, p
}

bool clockwise(point2d p1, point2d p2, point2d
    return signed_area_parallelogram(p1, p2, p
}

bool counter_clockwise(point2d p1, point2d p2,
    return signed_area_parallelogram(p1, p2, p
}
```

Practice Problems

- Codechef - Chef and Polygons