

# lab-7-22mcb1002

April 23, 2023

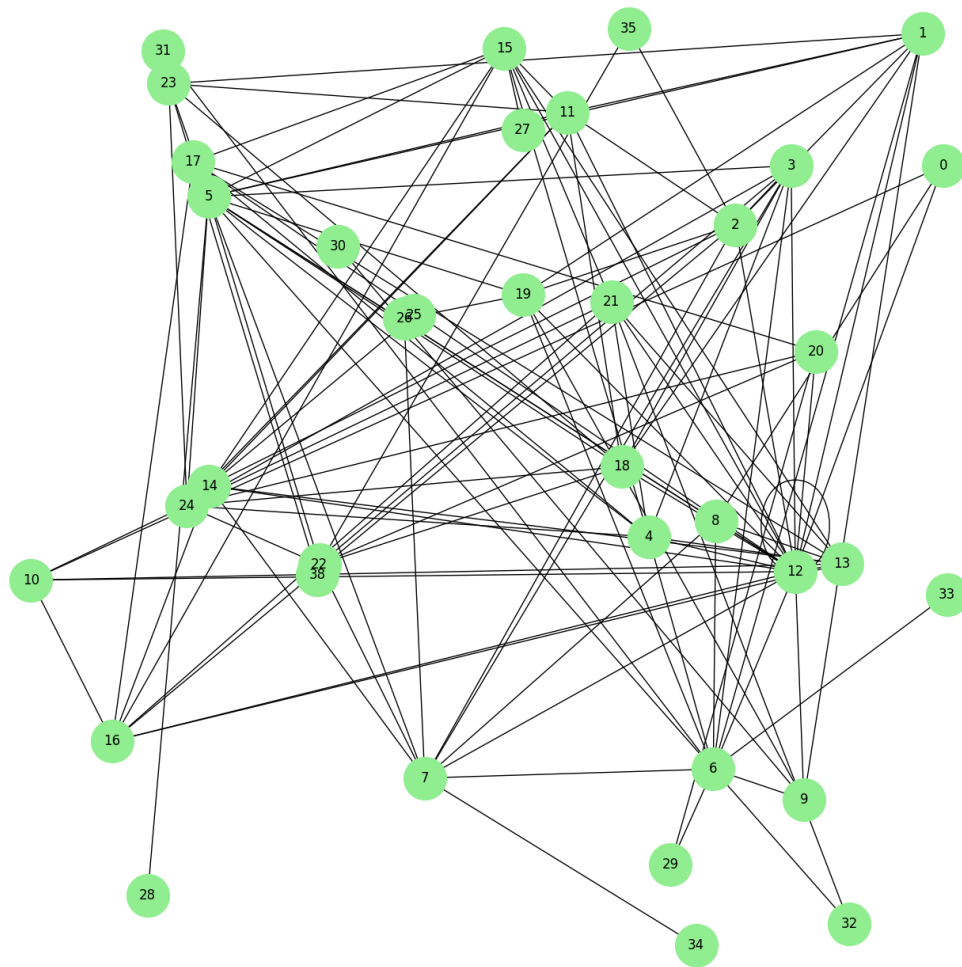
## 1 Visualization of friendship dataset

```
[1]: import networkx as nx
import pandas as pd
import matplotlib.pyplot as plt
from networkx.algorithms.community import girvan_newman
```

```
[7]: G = nx.Graph()
df = pd.read_csv("edgelist.txt", header = None).apply(lambda edgeData:
    ↪(edgeData[0], edgeData[1]), axis=1)
G.add_edges_from(df)
```

### i. Network Visualization

```
[10]: plt.figure(figsize = (14,14))
nx.draw(G,nx.random_layout(G), with_labels = True, node_size=1500, node_color =
    ↪'#90ee90')
plt.show()
```



## ii. Centrality measures

```
[11]: nx.closeness centrality(G)
```

```
[11]: {0: 0.4235294117647059,
      8: 0.5070422535211268,
      10: 0.43902439024390244,
      1: 0.5454545454545454,
      2: 0.5070422535211268,
      5: 0.6101694915254238,
      6: 0.5714285714285714,
      9: 0.48,
      11: 0.5142857142857142,
```

```

18: 0.4931506849315068,
19: 0.5217391304347826,
23: 0.43902439024390244,
7: 0.5538461538461539,
3: 0.5373134328358209,
4: 0.5217391304347826,
12: 0.6923076923076923,
14: 0.5070422535211268,
16: 0.47368421052631576,
22: 0.45569620253164556,
24: 0.4675324675324675,
13: 0.48,
15: 0.5142857142857142,
17: 0.46153846153846156,
21: 0.46153846153846156,
20: 0.4675324675324675,
26: 0.37894736842105264,
29: 0.391304347826087,
32: 0.3956043956043956,
35: 0.3673469387755102,
38: 0.3076923076923077,
25: 0.33962264150943394,
27: 0.34285714285714286,
28: 0.3829787234042553,
30: 0.32727272727272727,
31: 0.3673469387755102,
33: 0.3673469387755102,
34: 0.36}

```

```
[12]: nx.betweenness_centrality(G)
```

```

[12]: {0: 0.000529100529100529,
      8: 0.013569974998546426,
      10: 0.0025132275132275133,
      1: 0.03941317941317941,
      2: 0.048016674683341344,
      5: 0.14578312780693728,
      6: 0.19515529444100874,
      9: 0.05555555555555555,
      11: 0.03349773242630386,
      18: 0.01187188252664443,
      19: 0.02978784443070158,
      23: 0.05617913832199546,
      7: 0.10986720510530033,
      3: 0.020551926028116504,
      4: 0.02967248580343818,
      12: 0.2755905734477164,

```

```

14: 0.066578128661462,
16: 0.01241118669690098,
22: 0.026445578231292514,
24: 0.01675979223598271,
13: 0.012667233560090701,
15: 0.07248389837675552,
17: 0.005967458467458468,
21: 0.018154761904761906,
20: 0.02175866461580747,
26: 0.0006349206349206349,
29: 0.00291005291005291,
32: 0.001984126984126984,
35: 0.0006734006734006733,
38: 0.0,
25: 0.0,
27: 0.0,
28: 0.0,
30: 0.0,
31: 0.0,
33: 0.0,
34: 0.0}

```

```
[13]: nx.edge_betweenness centrality(G)
```

```

[13]: {(0, 8): 0.010378235378235378,
(0, 10): 0.004379379379379379,
(0, 12): 0.040297440297440285,
(8, 7): 0.011841511841511844,
(8, 5): 0.016585541585541583,
(8, 6): 0.01569069069069069,
(8, 13): 0.011592361592361593,
(8, 12): 0.01363863863863864,
(10, 12): 0.037412412412412416,
(10, 13): 0.005442942942942942,
(10, 16): 0.006006006006006005,
(10, 21): 0.0055680680680680675,
(1, 2): 0.01319831736498403,
(1, 5): 0.00967168467168467,
(1, 6): 0.02238845988845989,
(1, 9): 0.01691951691951692,
(1, 11): 0.008260045760045761,
(1, 18): 0.007648232648232647,
(1, 19): 0.006783569283569284,
(1, 23): 0.024989274989274987,
(1, 12): 0.01876042709376042,
(2, 11): 0.01068777110443777,
(2, 7): 0.013689338689338686,

```

(2, 3): 0.010467366717366717,  
 (2, 12): 0.02186293436293436,  
 (2, 14): 0.011861861861861861,  
 (2, 16): 0.015468011301344632,  
 (2, 19): 0.012927212927212923,  
 (2, 35): 0.03473359723359723,  
 (5, 3): 0.006856856856856856,  
 (5, 6): 0.03594974844974845,  
 (5, 4): 0.012656612656612658,  
 (5, 11): 0.010535535535535535,  
 (5, 15): 0.03353736895403561,  
 (5, 18): 0.008451308451308452,  
 (5, 19): 0.018446530946530947,  
 (5, 22): 0.022964210464210457,  
 (5, 23): 0.039330997664331,  
 (5, 24): 0.013924638924638922,  
 (5, 7): 0.024715787215787214,  
 (5, 12): 0.022179095095761766,  
 (5, 28): 0.05405405405405406,  
 (6, 4): 0.022654797654797654,  
 (6, 3): 0.01885944385944386,  
 (6, 9): 0.02548652548652548,  
 (6, 19): 0.01553160303160303,  
 (6, 7): 0.03516146016146016,  
 (6, 29): 0.04062276562276562,  
 (6, 31): 0.05405405405405406,  
 (6, 32): 0.03908908908908909,  
 (6, 33): 0.05405405405405406,  
 (6, 12): 0.043724081224081224,  
 (9, 19): 0.009997009997009997,  
 (9, 30): 0.05405405405405406,  
 (9, 12): 0.0527020527020527,  
 (11, 15): 0.014850266933600264,  
 (11, 18): 0.005180180180180179,  
 (11, 23): 0.01986986986986987,  
 (11, 24): 0.010315672815672815,  
 (11, 14): 0.01855814147480814,  
 (11, 12): 0.019170658753992084,  
 (18, 3): 0.00646003146003146,  
 (18, 22): 0.009316042649375982,  
 (18, 23): 0.009050717384050716,  
 (18, 24): 0.004142535392535392,  
 (18, 12): 0.02626532418199085,  
 (19, 26): 0.023709423709423708,  
 (19, 12): 0.02301408551408551,  
 (23, 24): 0.013043996377329713,  
 (23, 38): 0.05405405405405406,

(7, 3): 0.009187759187759187,  
 (7, 14): 0.027992240492240493,  
 (7, 22): 0.020693416526749855,  
 (7, 26): 0.03154583154583154,  
 (7, 34): 0.05405405405405406,  
 (7, 12): 0.033029528862862194,  
 (3, 4): 0.008229566562899896,  
 (3, 22): 0.008968038134704802,  
 (3, 24): 0.008576433576433578,  
 (3, 12): 0.015330579913913248,  
 (4, 13): 0.009712115962115962,  
 (4, 15): 0.010379129129129128,  
 (4, 17): 0.008547535630868965,  
 (4, 21): 0.010606141856141857,  
 (4, 24): 0.015279771529771526,  
 (4, 12): 0.012125518375518375,  
 (12, 13): 0.01889581889581889,  
 (12, 14): 0.033533533533533534,  
 (12, 15): 0.028528528528528524,  
 (12, 16): 0.022375026541693203,  
 (12, 21): 0.0294663711330378,  
 (12, 12): 0.0,  
 (12, 17): 0.025086991753658416,  
 (12, 20): 0.03804257762591096,  
 (14, 13): 0.011301686301686301,  
 (14, 16): 0.008246693663360331,  
 (14, 15): 0.014464464464464467,  
 (14, 25): 0.05405405405405406,  
 (16, 13): 0.0036473973973973972,  
 (16, 15): 0.008367647950981284,  
 (16, 21): 0.0070293507793507785,  
 (16, 17): 0.006394543894543894,  
 (22, 20): 0.01398544731878065,  
 (22, 24): 0.007564615897949231,  
 (22, 35): 0.020594458094458097,  
 (24, 20): 0.012914104580771248,  
 (13, 15): 0.006722106722106722,  
 (13, 17): 0.005184855184855185,  
 (13, 21): 0.00551980551980552,  
 (15, 17): 0.008789742123075456,  
 (15, 21): 0.011492444825778159,  
 (15, 27): 0.05405405405405406,  
 (17, 20): 0.011340171756838423,  
 (21, 32): 0.018718718718718715,  
 (20, 29): 0.01893679393679393}

### iii. CLIQUES Analysis

```
[14]: cliques = list(nx.find_cliques(G))
```

```
one_cliques = [c for c in cliques if len(c) == 1]
print("1-clique(s):", one_cliques)
```

```
1-clique(s): []
```

```
[15]: two_cliques = [c for c in cliques if len(c) == 2]
print("2-clique(s):", two_cliques)
```

```
2-clique(s): [[32, 21], [32, 6], [33, 6], [34, 7], [35, 2], [35, 22], [38, 23],
[25, 14], [26, 19], [26, 7], [27, 15], [28, 5], [29, 20], [29, 6], [30, 9], [31,
6]]
```

```
[16]: three_cliques = [c for c in cliques if len(c) == 3]
print("3-clique(s):", three_cliques)
```

```
3-clique(s): [[12, 0, 8], [12, 0, 10], [12, 13, 8], [12, 17, 20], [22, 20, 24]]
```

```
[18]: # Maximal Cliques
maximal_cliques = [c for c in cliques if len(c) == len(max(cliques, key=len))]
print("Maximal clique(s):", maximal_cliques)
```

```
Maximal clique(s): [[12, 5, 18, 11, 1], [12, 5, 6, 8, 7], [12, 5, 6, 1, 19],
[12, 5, 6, 3, 4], [12, 5, 6, 3, 7], [12, 9, 1, 19, 6], [12, 10, 16, 13, 21],
[12, 13, 15, 16, 17], [12, 13, 15, 16, 21], [12, 13, 15, 16, 14], [12, 13, 15,
4, 17], [12, 13, 15, 4, 21], [22, 3, 5, 24, 18], [23, 5, 18, 11, 24], [23, 5,
18, 11, 1]]
```

```
[19]: # Cohesive subgroups
cohesive_subgroups = []
for c in cliques:
    subgraph = G.subgraph(c)
    subgraph_density = nx.density(subgraph)
    if subgraph_density >= 0.8:
        cohesive_subgroups.append(subgraph)
print("Cohesive subgroups formed from cliques:")
for sg in cohesive_subgroups:
    print(sg.nodes())
```

```
Cohesive subgroups formed from cliques:
```

```
[32, 21]
[32, 6]
[33, 6]
[34, 7]
[2, 35]
[35, 22]
[38, 23]
```

```

[0, 8, 12]
[0, 10, 12]
[1, 2, 19, 12]
[1, 2, 11, 12]
[2, 3, 12, 7]
[16, 2, 12, 14]
[2, 11, 12, 14]
[2, 12, 14, 7]
[18, 3, 12, 5]
[1, 5, 11, 12, 18]
[11, 12, 5, 15]
[5, 6, 7, 8, 12]
[1, 5, 6, 12, 19]
[3, 4, 5, 6, 12]
[3, 5, 6, 7, 12]
[4, 12, 5, 15]
[1, 6, 9, 12, 19]
[10, 12, 13, 16, 21]
[8, 12, 13]
[12, 13, 15, 16, 17]
[12, 13, 15, 16, 21]
[12, 13, 14, 15, 16]
[4, 12, 13, 15, 17]
[4, 12, 13, 15, 21]
[11, 12, 14, 15]
[17, 12, 20]
[3, 5, 18, 22, 24]
[3, 5, 22, 7]
[24, 20, 22]
[5, 11, 18, 23, 24]
[1, 5, 11, 18, 23]
[24, 3, 4, 5]
[25, 14]
[26, 19]
[26, 7]
[27, 15]
[28, 5]
[20, 29]
[29, 6]
[9, 30]
[6, 31]

```

```

[20]: # overlapping cliques
communities = girvan_newman(G)

overlapping_subgroups = []
for c in communities:

```



```
print(overlapping_subgroups)
```

$$\{33\}, \{34\}), (\{0\}, \{8\}, \{10, 13, 15, 16, 17, 21\}, \{1, 3, 4, 5, 6, 7, 11, 12, 18,$$

22, 24}, {2}, {9}, {19}, {23}, {14}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10, 13, 15, 16, 17, 21}, {1, 3, 4, 5, 6, 7, 11, 12, 18, 24}, {2}, {9}, {19}, {23}, {14}, {22}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10, 13, 15, 16, 17, 21}, {1, 3, 4, 5, 6, 11, 12, 18, 24}, {2}, {9}, {19}, {23}, {7}, {14}, {22}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10, 13, 15, 16, 17, 21}, {1}, {2}, {3, 4, 5, 6, 11, 12, 18, 24}, {9}, {19}, {23}, {7}, {14}, {22}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10, 13, 15, 16, 17, 21}, {1}, {2}, {3, 4, 5, 6, 12, 18, 24}, {9}, {11}, {19}, {23}, {7}, {14}, {22}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10}, {1}, {2}, {3, 4, 5, 6, 12, 18, 24}, {9}, {11}, {19}, {23}, {7}, {14}, {13, 15, 16, 17, 21}, {22}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10}, {1}, {2}, {3, 4, 5, 6, 12, 24}, {9}, {11}, {18}, {19}, {23}, {7}, {14}, {13, 15, 16, 17, 21}, {22}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10}, {1}, {2}, {3, 4, 5, 6, 12}, {9}, {11}, {18}, {19}, {23}, {7}, {14}, {13, 15, 16, 17, 21}, {22}, {24}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10}, {1}, {2}, {3, 4, 5, 6, 12}, {9}, {11}, {18}, {19}, {23}, {7}, {14}, {16, 17, 13, 15}, {22}, {24}, {21}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10}, {1}, {2}, {5}, {3, 4, 12, 6}, {9}, {11}, {18}, {19}, {23}, {7}, {14}, {16, 17, 13, 15}, {22}, {24}, {21}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10}, {1}, {2}, {5}, {6}, {9}, {11}, {18}, {19}, {23}, {7}, {3}, {4, 12}, {14}, {16, 17, 13, 15}, {22}, {24}, {21}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10}, {1}, {2}, {5}, {6}, {9}, {11}, {18}, {19}, {23}, {7}, {3}, {4}, {12}, {14}, {16, 17, 13, 15}, {22}, {24}, {21}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10}, {1}, {2}, {5}, {6}, {9}, {11}, {18}, {19}, {23}, {7}, {3}, {4}, {12}, {14}, {16, 17, 13, 15}, {22}, {24}, {21}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10}, {1}, {2}, {5}, {6}, {9}, {11}, {18}, {19}, {23}, {7}, {3}, {4}, {12}, {14}, {16}, {22}, {24}, {17, 13, 15}, {21}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10}, {1}, {2}, {5}, {6}, {9}, {11}, {18}, {19}, {23}, {7}, {3}, {4}, {12}, {14}, {16}, {22}, {24}, {13}, {17, 15}, {21}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34}), ({0}, {8}, {10}, {1}, {2}, {5}, {6}, {9}, {11}, {18}, {19}, {23}, {7}, {3}, {4}, {12}, {14}, {16}, {22}, {24}, {13}, {15}, {17}, {21}, {20}, {26}, {29}, {32}, {35}, {38}, {25}, {27}, {28}, {30}, {31}, {33}, {34})]

```
[22]: # Density of maximal cliques
mcn = nx.Graph()
for clique in cliques:
    if len(clique) > 1:
        mcn.add_edges_from([(u, v) for u in clique for v in clique if u < v])
```

```
density = nx.density(mcn)

print("Density of maximal clique network:", density)
```

Density of maximal clique network: 0.17117117117117117

```
[25]: max_clique = max(cliques, key=len)
print("Maximum clique:", max_clique)
print("Maximal clique(s):", maximal_cliques)
```

Maximum clique: [12, 5, 18, 11, 1]  
 Maximal clique(s): [[12, 5, 18, 11, 1], [12, 5, 6, 8, 7], [12, 5, 6, 1, 19],  
 [12, 5, 6, 3, 4], [12, 5, 6, 3, 7], [12, 9, 1, 19, 6], [12, 10, 16, 13, 21],  
 [12, 13, 15, 16, 17], [12, 13, 15, 16, 21], [12, 13, 15, 16, 14], [12, 13, 15,  
 4, 17], [12, 13, 15, 4, 21], [22, 3, 5, 24, 18], [23, 5, 18, 11, 24], [23, 5,  
 18, 11, 1]]

#### iv. Hubs and Authority using HITS algorithm

```
[27]: hub, authority = nx.hits(G)
print("Hubs:", sorted(hub, key = hub.get, reverse = True))
print("Authorities:", sorted(authority, key = authority.get, reverse = True))
```

Hubs: [12, 5, 6, 3, 4, 15, 11, 1, 7, 2, 18, 13, 14, 16, 19, 24, 8, 21, 17, 22,  
 23, 9, 10, 20, 0, 32, 26, 29, 35, 28, 31, 33, 27, 34, 25, 38, 30]  
 Authorities: [12, 5, 6, 3, 4, 15, 11, 1, 7, 2, 18, 13, 14, 16, 19, 24, 8, 21,  
 17, 22, 23, 9, 10, 20, 0, 32, 26, 29, 35, 28, 31, 33, 27, 34, 25, 38, 30]

#### v. Cliques, Clans, Plexes, Cores

```
[28]: n = 3
cliques = list(nx.find_cliques(G))
n_cliques = [c for c in cliques if len(c) == n]

print("n-cliques:", n_cliques)
```

n-cliques: [[12, 0, 8], [12, 0, 10], [12, 13, 8], [12, 17, 20], [22, 20, 24]]

```
[30]: k = 3
clique_communities = list(nx.find_cliques(G))
k_plexes = [c for c in clique_communities if len(c) >= k and
             all(len(set(G.neighbors(n)).intersection(set(c))) >= k-1 for n in c)]

print("k-plexes:", k_plexes)
```

k-plexes: [[12, 0, 8], [12, 0, 10], [12, 2, 19, 1], [12, 2, 1, 11], [12, 2, 3,  
 7], [12, 2, 14, 16], [12, 2, 14, 11], [12, 2, 14, 7], [12, 5, 18, 3], [12, 5,  
 18, 11, 1], [12, 5, 11, 15], [12, 5, 6, 8, 7], [12, 5, 6, 1, 19], [12, 5, 6, 3,

```
4], [12, 5, 6, 3, 7], [12, 5, 15, 4], [12, 9, 1, 19, 6], [12, 10, 16, 13, 21],  
[12, 13, 8], [12, 13, 15, 16, 17], [12, 13, 15, 16, 21], [12, 13, 15, 16, 14],  
[12, 13, 15, 4, 17], [12, 13, 15, 4, 21], [12, 14, 15, 11], [12, 17, 20], [22,  
3, 5, 24, 18], [22, 3, 5, 7], [22, 20, 24], [23, 5, 18, 11, 24], [23, 5, 18, 11,  
1], [24, 5, 3, 4]]
```

```
[31]: G.remove_edges_from(nx.selfloop_edges(G))
```

```
[32]: k = 3  
k_core = nx.k_core(G, k)  
print("Nodes in the 3-core:", k_core.nodes())
```

```
Nodes in the 3-core: [0, 8, 10, 1, 2, 5, 6, 9, 11, 18, 19, 23, 7, 3, 4, 12, 14,  
16, 22, 24, 13, 15, 17, 21, 20]
```