Basic intro to vectors and matrices

BILD 62

Vectors contain information about different dimensions of our data

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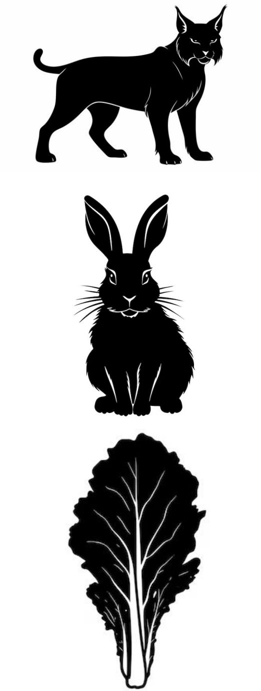
Imagine an environment with **bobcats**, **rabbits** and **lettuce**.

A vector with **3 dimensions** can describe the number of bobcats, rabbits and lettuce in our environment.

$$\vec{x} = \begin{bmatrix} x_{\text{bobcats}} \\ x_{\text{rabbits}} \\ x_{\text{lettuce}} \end{bmatrix}$$



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Then someone takes 2 bobcats, brings 3 rabbits and 4 lettuce leaves.

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For example, initially we have 5 bobcats, 7 rabbits, 22 lettuce leaves.
Then someone takes 2 bobcats, brings 3 rabbits and 4 lettuce leaves.
This is represented by:

$$\vec{x} = \begin{bmatrix} 5 \\ 7 \\ 22 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 26 \end{bmatrix}$$

We also learned from this example that vectors can depend on time

$$\vec{x}(t=0) = \begin{bmatrix} 5 \\ 7 \\ 22 \end{bmatrix} \qquad \vec{x}(t=1) = \begin{bmatrix} 3 \\ 10 \\ 26 \end{bmatrix}$$

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In general:

$$\vec{x}(t) = \begin{bmatrix} x_{\text{bobcats}}(t) \\ x_{\text{rabbits}}(t) \\ x_{\text{lettuce}}(t) \end{bmatrix}$$

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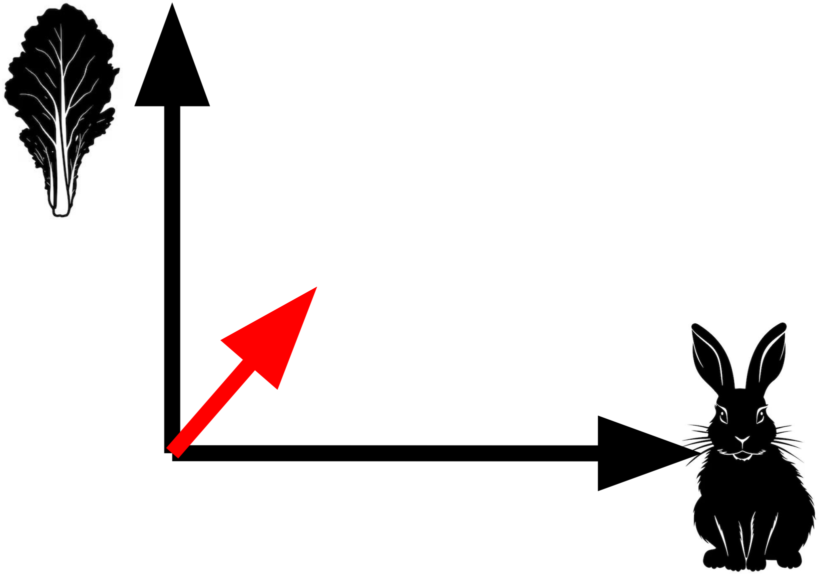

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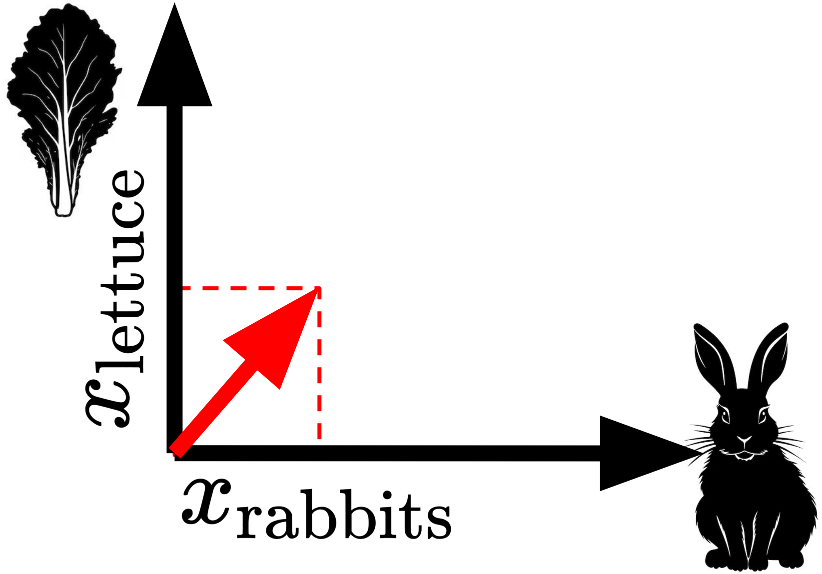

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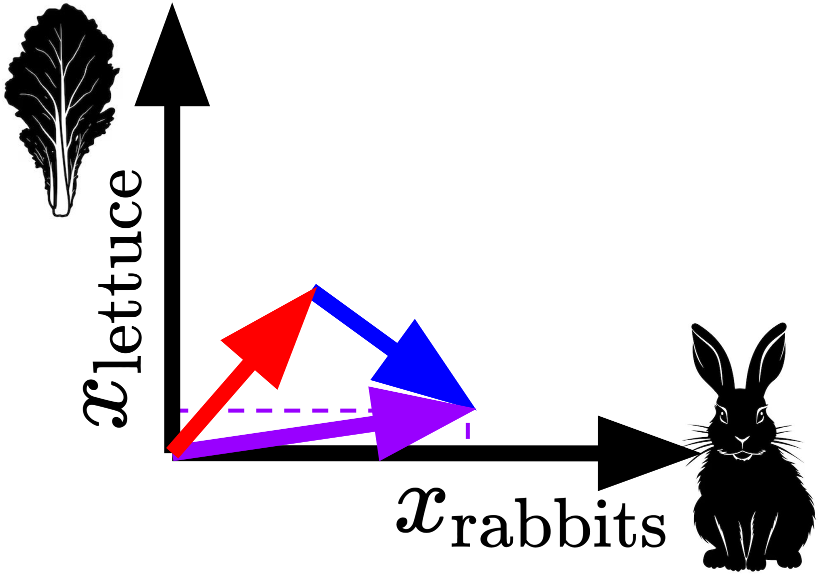

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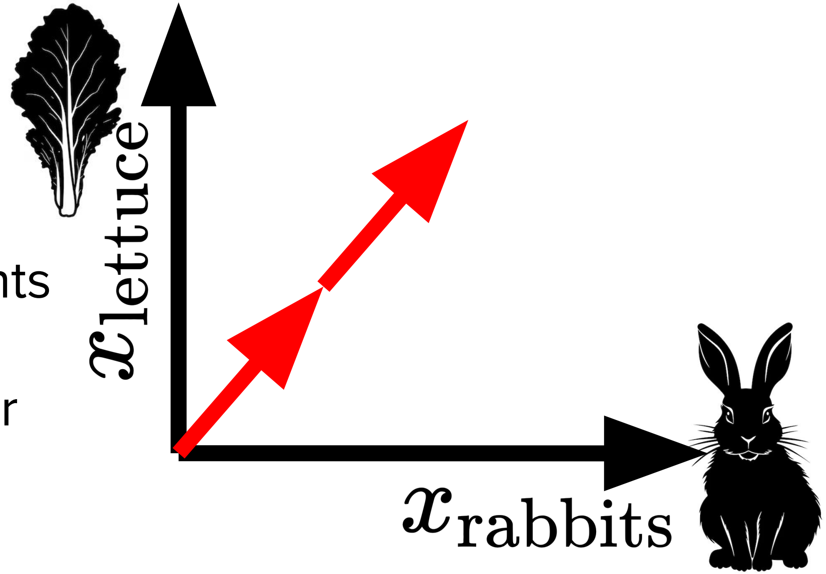
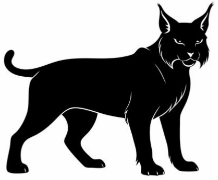
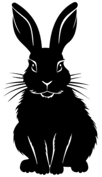
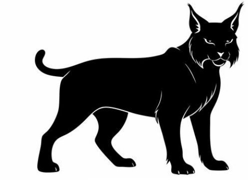
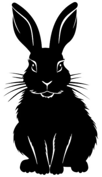
- We can **add vectors** by putting **the start of one vector** at **the end of another**
- We can multiply a vector by a number by multiplying all the elements by the same number.
  This changes the length of the vector but not its direction.



Matrices contain information about how different components of our system affect each other. In our example with **bobcats**, **rabbits, lettuce**:

			
	+	−	0
	+	+	−
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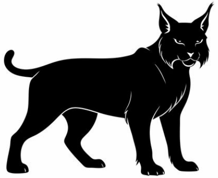
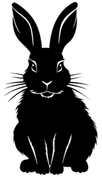
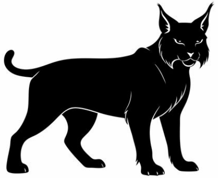
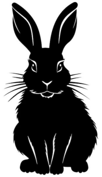
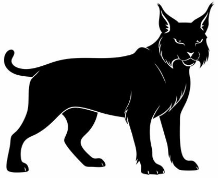
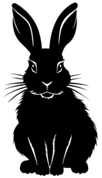
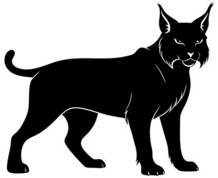
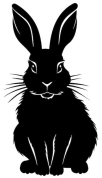
			
	+	−	0
	+	+	−
	0	+	0

Compute the effect of interactions by multiplying the matrix times the vector

Matrices contain information about how different components of our system affect each other

			
	+	−	0
	+	+	−
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Compute the effect of interactions by multiplying the matrix times the vector

Multiplying a matrix by a vector can change the length and direction.

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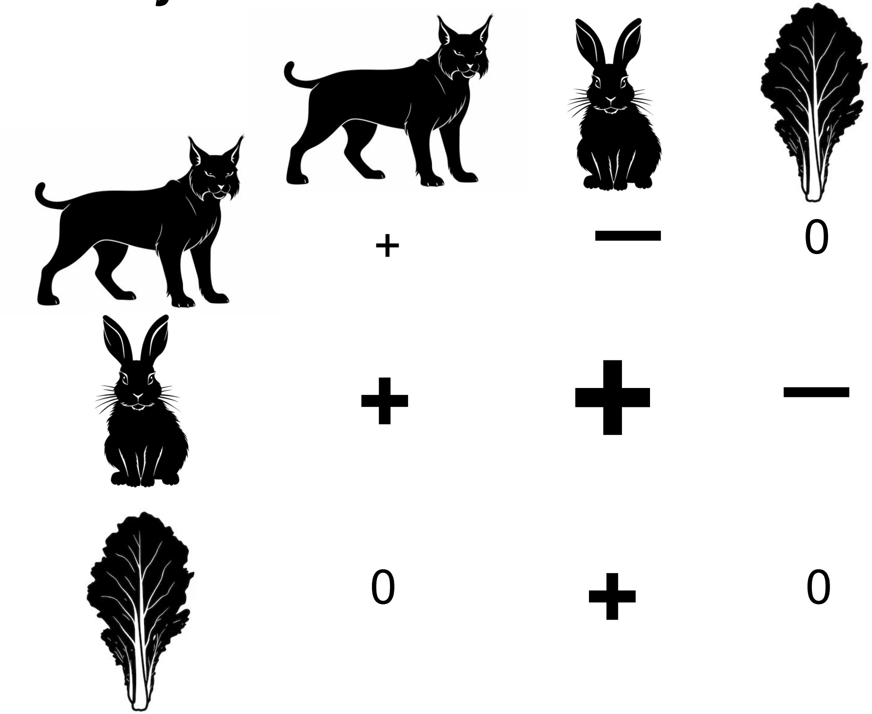

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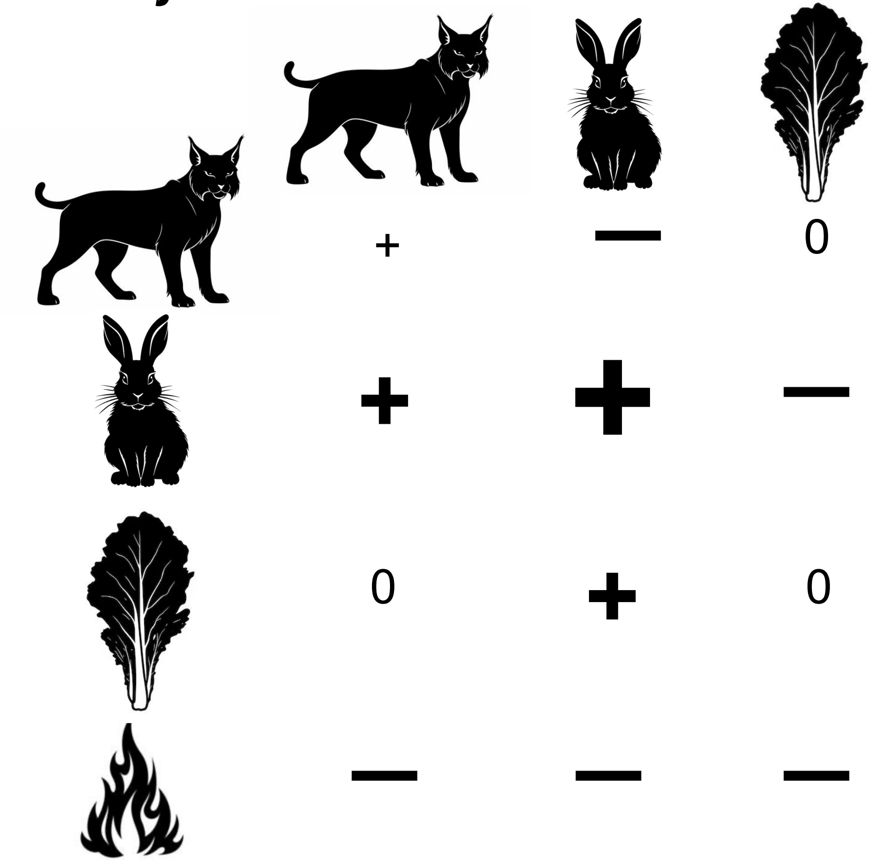
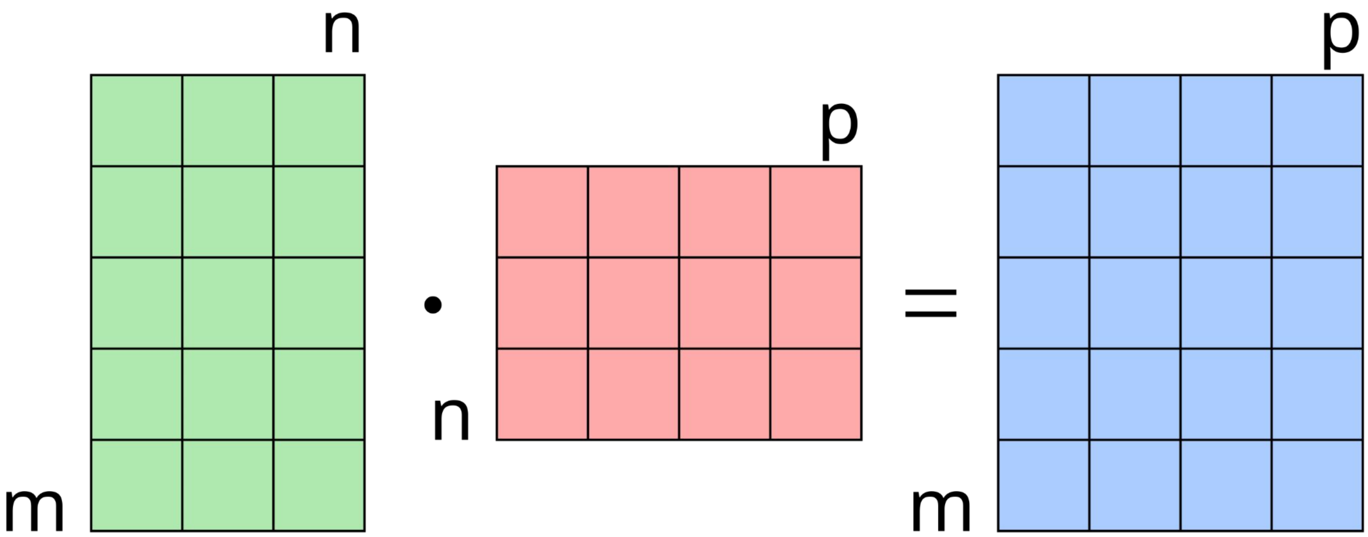


Compute the effect of interactions by multiplying the matrix times the vector
Multiplying a matrix by a vector can change the length and direction.
The number of columns of the matrix has to be equal to the number of rows of the vector.
The number of rows of the matrix can be different from the number of columns.

We can multiply matrices, if the numbers of rows and columns match



$$A \cdot B = C$$

We can multiply matrices, if the numbers of rows and columns match

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} \end{bmatrix}$$

In our example,
vectors represented the state of an **ecological system**,
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Vectors and matrices are useful in all fields of biology and medicine

- Activity of neurons and interactions between neurons
- Gene expression and its regulation
- Population ecology (abundance of species), and interactions
- Efficacy of drugs and combinations of drugs. Side effects of drugs
- Other ..

In BILD 62 we will only learn to do very basic computations with vectors and matrices.

Learning about more powerful methods to

- Gain insight on how a biological system works
- How to analyze complex data

requires more knowledge in linear algebra.