SOFTWARE DIFFERENTIAL FORMULAS

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Introduction

During the implementation of my R/C Car I decided to design the formulas for the software differential. The R/C Car will have 4 motors, so the software differential is the only way. Apart from that, I believe that mechanical differentials:

- Cannot be improved further
- Cannot be smaller in size with the stiffness which a car needs
- Cannot be lighter (probably the can but they could be much more expensive)
- Cannot be eco-friendly (need fluids, power loss due to their moving parts and extra axles)
- Cannot take less space in the cabin (sometimes the place where you rest your hand is a camouflage for the axle that goes to the rear).

On the other side, electric motors just started to exist in automotive industry and probably they have more secrets to reveal! How about changing your car's differential from open to locked in a single update? Now that I have your attention, we can continue!

Formulas

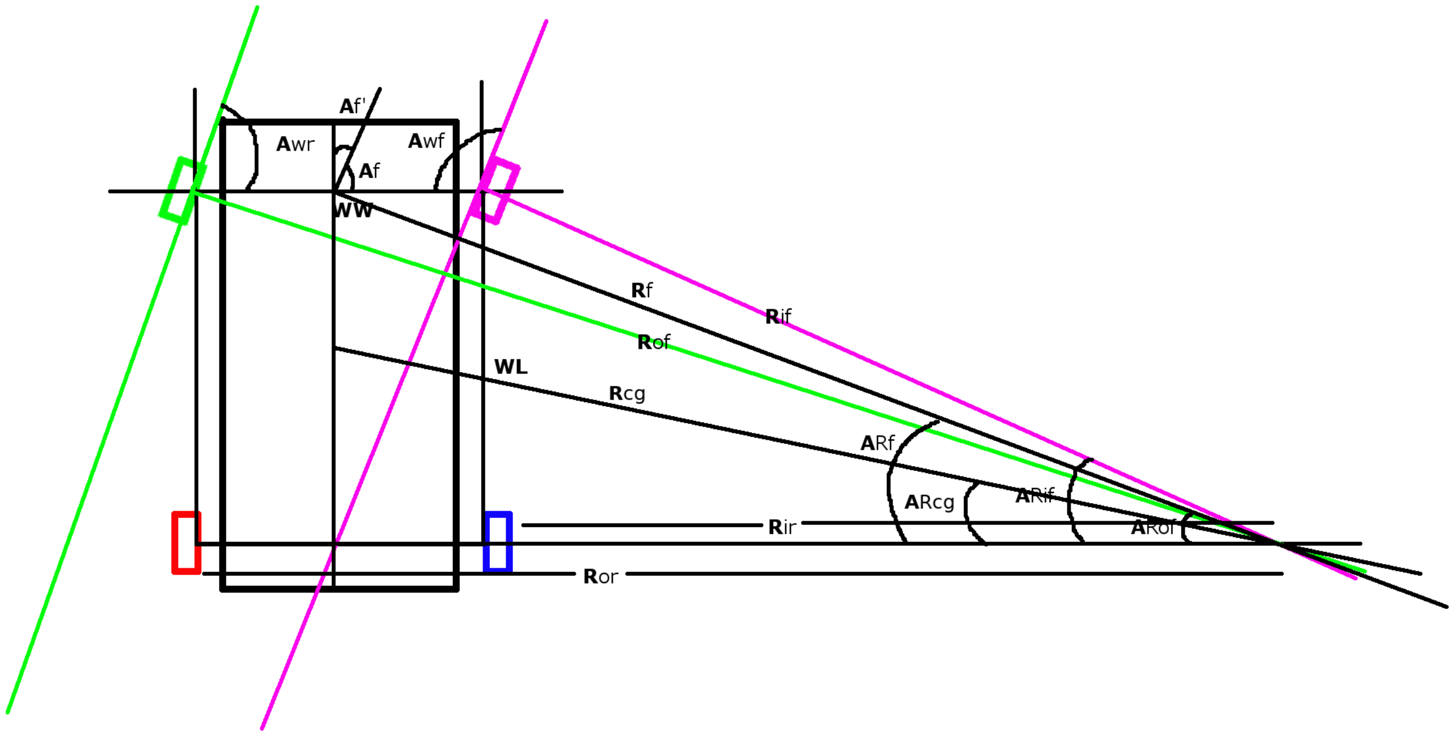
Before starting the calculation we have to say that we know the following:

- **WL** (*wheel length distance, kingpin to kingpin*),
- **WW** (*wheel width distance, kingpin to kingpin*),
- **WR** (*wheel radius*),
- **A_{wr}** (*outer front wheel turning angle*),
- **A_{wf}** (*inner front wheel turning angle*),
- **Af** (*car turning angle*).

The inner wheel formula should be equal to the real turning angle in order to achieve concentric circles.

The formulas for the Ackerman steering according to this (based on simple trigonometry of course, interior-exterior corresponding angles and triangle angles equality to 180 ^{o}), are:

- $(A_{wf} - 90^o) = \tan^{-1}(WL / R_{ir})$ (a)
- $(90^o - A_{wr}) = \tan^{-1}(WL / R_{or})$ (b)



Rear Inner Wheel

According to the Ackerman formula (a) we have:

$$(A_{wf} - 90^o) = \tan^{-1}(WL / R_{ir}) \Rightarrow$$

$$(A_{wf} - 90^o) = atan(WL / R_{ir}) \Rightarrow$$

$$\tan(A_{wf} - 90^o) = WL / R_{ir} \Rightarrow$$

$$R_{ir} = WL / (\tan(A_{wf} - 90^o))$$

Therefore, the rear inner wheel is moving into a circle with radius:

$$\boxed{R_{ir} = WL / (\tan(A_{wf} - 90^o)) \quad (c)}$$

Rear Outer Wheel

Without further ado, from (c) we have:

$$R_{or} = WW + R_{ir}$$

Therefore, the rear outer wheel is moving into a circle with radius:

$$\boxed{R_{or} = WW + R_{ir} \quad (d)}$$

Front Outer Wheel

Using the steering angles we have:

$$\sin(A_{Rof}) = WL / R_{of} \Rightarrow$$

$$R_{of} = (WL / \sin(A_{Rof})) \Rightarrow$$

$$R_{of} = (WL / \sin(90^o - A_{wr}))$$

Therefore, the front outer wheel is moving into a circle with radius:

$$\boxed{R_{of} = (WL / \sin(90^o - A_{wr})) \quad (e)}$$

Front Inner Wheel

Using the steering angles we have:

$$\sin(A_{Rif}) = WL / R_{if} \Rightarrow$$

$$R_{if} = (WL / \sin(A_{Rif})) \Rightarrow$$

$$R_{if} = (WL / \sin(A_{wf} - 90^o))$$

Therefore, the front inner wheel is moving into a circle with radius:

$$R_{if} = (WL / \sin(A_{wf} - 90^o)) \quad (f)$$

Center of the car

We are going to use also data about the car as an object. We have different options for the steering geometry (ackerman, parallel, reverse) so we cannot be based directly on the pythagorean theorem for radius at the center of the car because possibly the car will not turn in a single concentric circle. By having a device measuring the angle of the car (A_f) we could calculate the A_f angle.

The formula to calculate R_{ir}' (in ackerman $R_{ir} = R_{ir}'$, but not at the other two options) is:

$$\tan(A_f') = WL / (R_{ir}' + WW/2) \Rightarrow$$

$$R_{ir}' = (WL / \tan(A_f')) - WW/2$$

The straight lines R_f, R_{cg} are starting from the car chassis, so the whole car chassis is moving in concentric circles (that is personal opinion, might be wrong and needs checking). So, the R_f and R_{cg} belong into the same triangle. Here, from pythagorean theorem we have:

$$(R_{cg})^2 = (R_{ir}' + WW/2)^2 + (WL / 2)^2 \Rightarrow$$

$$R_{cg} = SQRT (((WL / \tan(A_f')) - WW/2 + WW/2)^2 + (WL / 2)^2) \Rightarrow$$

$$R_{cg} = SQRT (((WL / \tan(A_f')))^2 + (WL / 2)^2)$$

The formulas should be:

$$R_{cg} = SQRT (((WL / \tan(A_f')))^2 + (WL / 2)^2) \quad (g)$$

True Ackerman only formula:

$$R_{cg} = SQRT ((WL / 2)^2 + (R_{ir} + WW/2)^2) \quad (g')$$

Wheel Rotations

The front outer wheel will travel the longest distance of all four wheels. Our calculations will be based on this wheel because if the driver is in 100% throttle, we cannot make a motor to work at more than 100%. On the other side, if the outer wheel is not moving we could apply 0 power, or negative (in a future implementation…probably!).

From paper "Electronic Differential System for an Electric Vehicle with In-Wheel Motor" we have the following formulas for Estimated Rotations (ER):

$ER_{of} = W_{of} = V \times R_{of} / (R_{cg} \times WR)$ (h)
$ER_{if} = W_{if} = V \times R_{if} / (R_{cg} \times WR)$ (i)
$ER_{or} = W_{or} = V \times R_{or} / (R_{cg} \times WR)$ (j)
$ER_{ir} = W_{ir} = V \times R_{ir} / (R_{cg} \times WR)$ (k)

for open differential. For locked differential just (?!) apply the same rotation (**not PWM value**) value to each wheel. For in-between values just pick some values in-between!

For example:

- Open Differential -> Outer front wheel will have max rotations and the others according to the previous calculations
- Semi-open Differential Front -> Outer front wheel will have max rotations and the opposing wheel will have an in-between (from open to locked) value
- Semi-open Differential Rear -> Outer rear wheel will have max rotations (as max as possible in relation with the outer front wheel) and the opposing wheel will have an in-between (from open to locked) value
- Locked Differential -> Apply everywhere the same rotations

Predefined Table Example

This example is at the ideal scenario where all motors have exactly the same specs.

PWM value (% throttle value)	Rotations @ wheel (per second)			
	Front Left	Front Right	Rear Left	Rear Right
50 (19.61%)	5	6	5	4
100 (39.21%)	12	14	16	11
150 (58.82%)	22	22	18	25
190 (74.51%)	44	43	45	48
220 (86.27%)	50	49	47	53
255 (100%)	56	57	57	55

Vehicle Dynamics

Later… with a VBOX?

Use case (Ackerman scenario)

For a car during a single right turn we have the input:

$$WL = 4m,\ WW = 1.5m,\ A_{wr} = 71^o,\ A_{wf} = 111.5^o,\ A_f' = 20.15^o,\ 195/50R15,$$

$$\text{throttle} = 50\%$$

and according to the previous theory we have:

$$R_{ir} = WL / (\tan(A_{wf} - 90^o)) = 4 / (\tan(111.5^o - 90^o)) = 10.152m$$

$$R_{or} = WW + R_{ir} = 1.5 + 10.152 = 11.652m$$

$$R_{of} = (WL / \sin(90^o - A_{wr})) = 4 / \sin(90^o - 71^o) = 12.270m$$

$$R_{if} = (WL / \sin(A_{wf} - 90^o)) = 4 / \sin(111.5^o - 90^o) = 10.899m$$

And the rim calculations are:

$$((15 / 39.37) / 2) + (0.195 * 0.5) = 0.190 + 0.0975 = 0.2875m$$

Calculating the wheel rotations we have:

For throttle = 50%, we have 127.5 of 255 PWM value

so from table almost 17 rotations

Ackerman, parallel, reverse	Ackerman only
$R_{cg} = SQRT(((WL / \tan(A_f')))^2 + (WL / 2)^2) =>$ $SQRT((4 / 0.367)^2 + 4 =>$ $SQRT(10.899^2 + 4) =>$ $11.081m$	$R_{cg} = SQRT((WL / 2)^2 + (R_{ir} + WW/2)^2) =>$ $R_{cg} = 9.612m$
$W_{of} = V \times R_{of} / R_{cg} \times WR =>$ $17 = V \times 12.270 / 11.081 \times 0.2875 =>$ $V = 4.412\ ^m/_s$	$W_{of} = V \times R_{of} / R_{cg} \times WR =>$ $17 = V \times 12.270 / 9.612 \times 0.2875 =>$ $V = 3.829\ ^m/_s$
$W_{if} = V \times R_{if} / (R_{cg} \times WR) =>$ $W_{if} = 4.412 \times 10.899 / 11.081 \times 0.2875 =>$ $W_{if} = 15.09$ rps	$W_{if} = V \times R_{if} / (R_{cg} \times WR) =>$ $W_{if} = 3.829 \times 10.899 / 9.612 \times 0.2875 =>$ $W_{if} = 15.10$ rps
$W_{or} = V \times R_{or} / (R_{cg} \times WR) =>$ $W_{or} = 4.412 \times 11.652 / 3.186 =>$ $W_{or} = 16.14$ rps	$W_{or} = V \times R_{or} / (R_{cg} \times WR) =>$ $W_{or} = 3.829 \times 11.652 / 2.763 =>$ $W_{or} = 16.14$ rps
$W_{ir} = V \times R_{ir} / (R_{cg} \times WR) =>$	$W_{ir} = V \times R_{ir} / (R_{cg} \times WR) =>$

$W_{ir} = 4.412 \times 10.152 / 3.186 \Rightarrow$ $W_{ir} = 14.06rps$	$W_{ir} = 3.829 \times 10.152 / 2.763 \Rightarrow$ $W_{ir} = 14.06rps$

$W_{FrontLeft}$ = 17rps = 127.5 PWM ≈ 50% throttle
$W_{FrontRight}$ = 15.09 OR 15.10 rps ≈ 107 PWM ≈ 41.96% throttle
$W_{rearLeft}$ = 16.14 rps ≈ 100 PWM ≈ 39.21% throttle
$W_{rearRight}$ = 14.06 rps ≈ 125 PWM ≈ 49.01% throttle

Flowchart

This flowchart is in early stage; so:

- It is not describing an algorithm in actual code,
- It needs an algorithm to calculate the PWM value for specified rotations,
- It has some restrictions (such as the differential settings could be applied when car is not moving).
- It cannot calculate ranges 29.5° in a turning wheel is different from 29.6°. This should not happen, at least in small speeds!
- It lacks hardware features (such us interruption) because I still don't have a lot of experience on these. Programmed I/O is a good start, though!
- I have not decided if this software will work on an a controller or on a OS to work with threads.