CS1699: Blockchain Technology and Cryptocurrency

7. Proof of Work and Mining

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Recall Our Naive Consensus Protocol

- "Pick a node at random, allow them to add their transactions in their transaction pool to blockchain"
- Problems:
  - How to pick a random node?
  - Assumes honest nodes - easy to subvert
  - Nothing at stake, no reason to be honest

Incentives

- We can’t punish dishonest nodes but…
- What if there was an incentive to be an honest node?
- How about… bitcoins?

Incentives in Bitcoin

- Two incentive mechanisms:
  - Block reward
  - Transaction fees

Block Reward

- The creator of a valid block gets a reward (currently 12.5 bitcoin - halves every 210,000 blocks, will drop to 6.25 around May 2020) - http://www.thehalvening.com/
- Block reward only valid if it's on long-term consensus branch, which means other nodes must accept it
- If longest-valid-branch, will incentivize to work off of longest branch and be honest
- Creating blocks is hard work (as we will see) - don't want to waste it!

The Future of the Block Reward

- No other way to generate bitcoins besides block reward
- Block rewards will end around the year 2140 (they keep halving approximately four years, for much longer than that they will be so small as to be insignificant)
- So… nobody will mine from 2140 on, right?

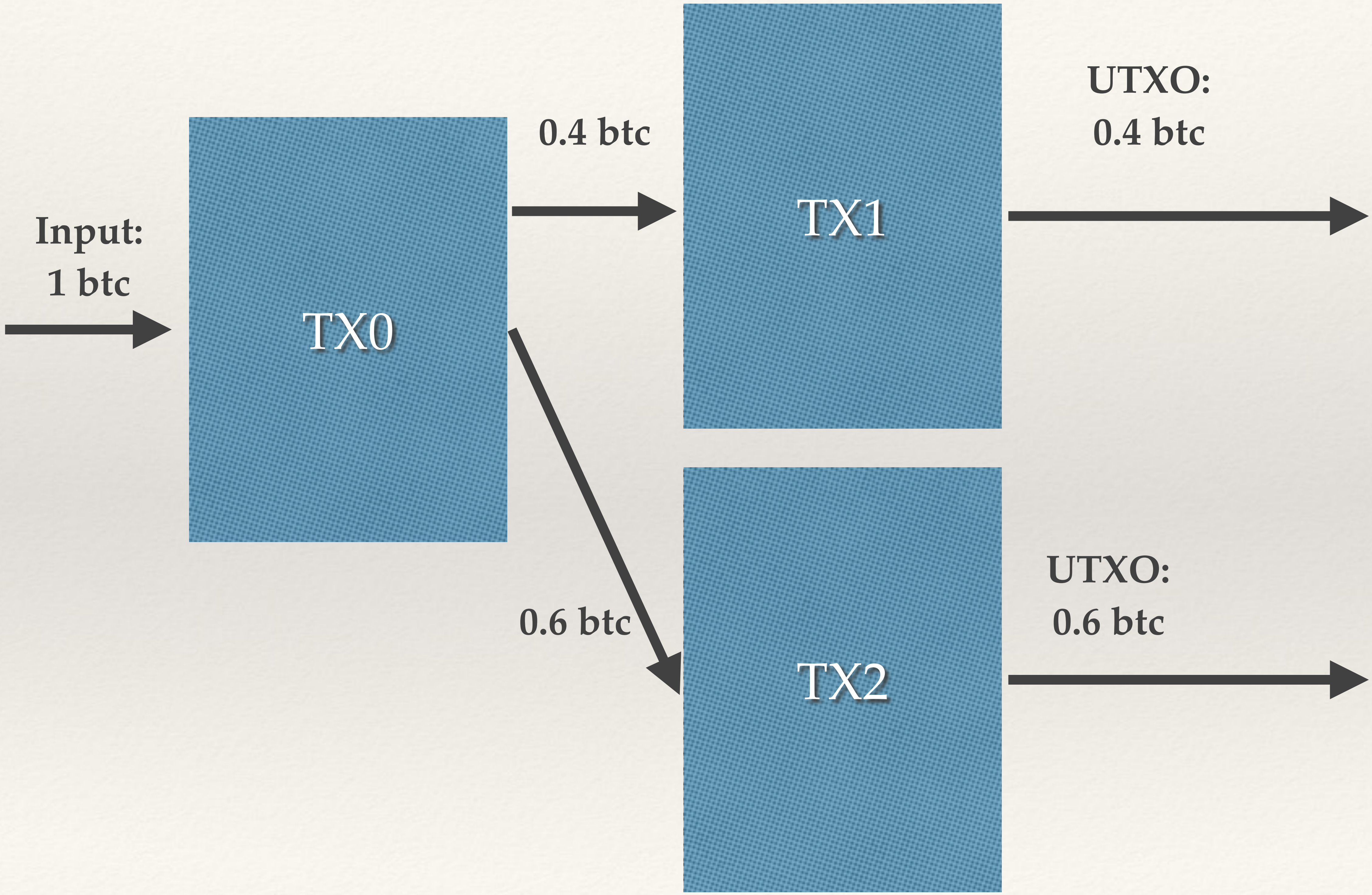
Transaction Fees

- The other incentive mechanism is transaction fees
- The creator of a transaction can decide to give themselves back less in “change” than necessary
- This “missing bitcoin” can be claimed by the miner of a block

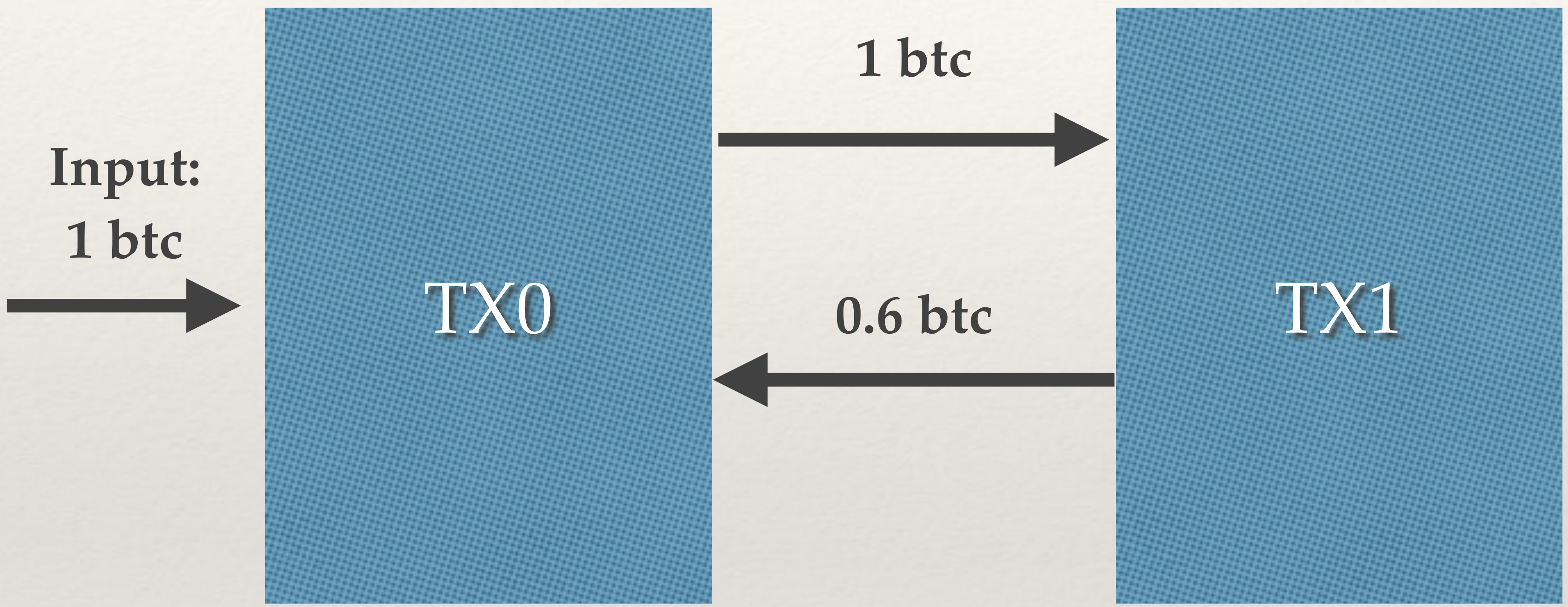
Side Note: UTXOs

- Although we think of Bitcoin as a distributed ledger where address X has 4 bitcoin and address Y has 2.5 bitcoin, really each has a collection of *unspent transaction outputs*, that is, transactions that been input to an address but not yet output
- We can tally up these outputs and determine the full amount of bitcoin that a particular address has, but behind the scenes they are actually a collection of these outputs

Transactions Behind the Scenes

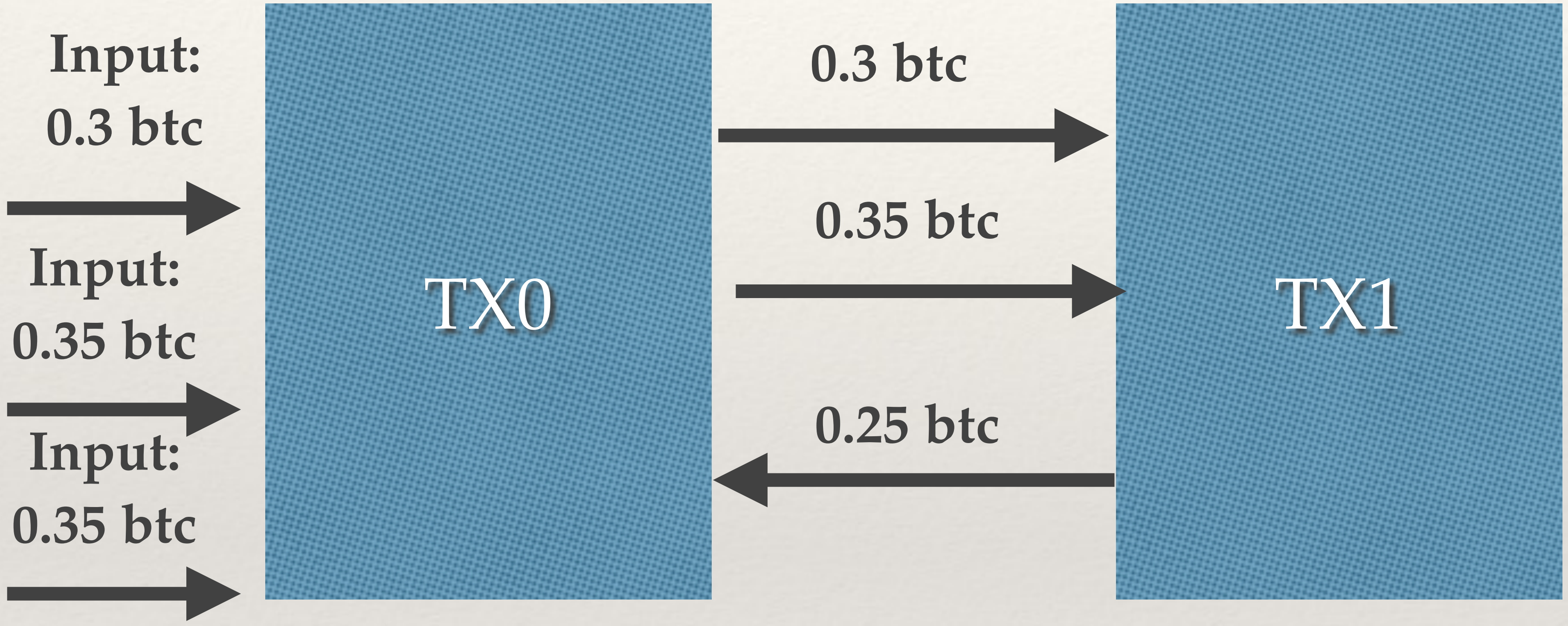


Splitting TXs - Sending 0.4 btc



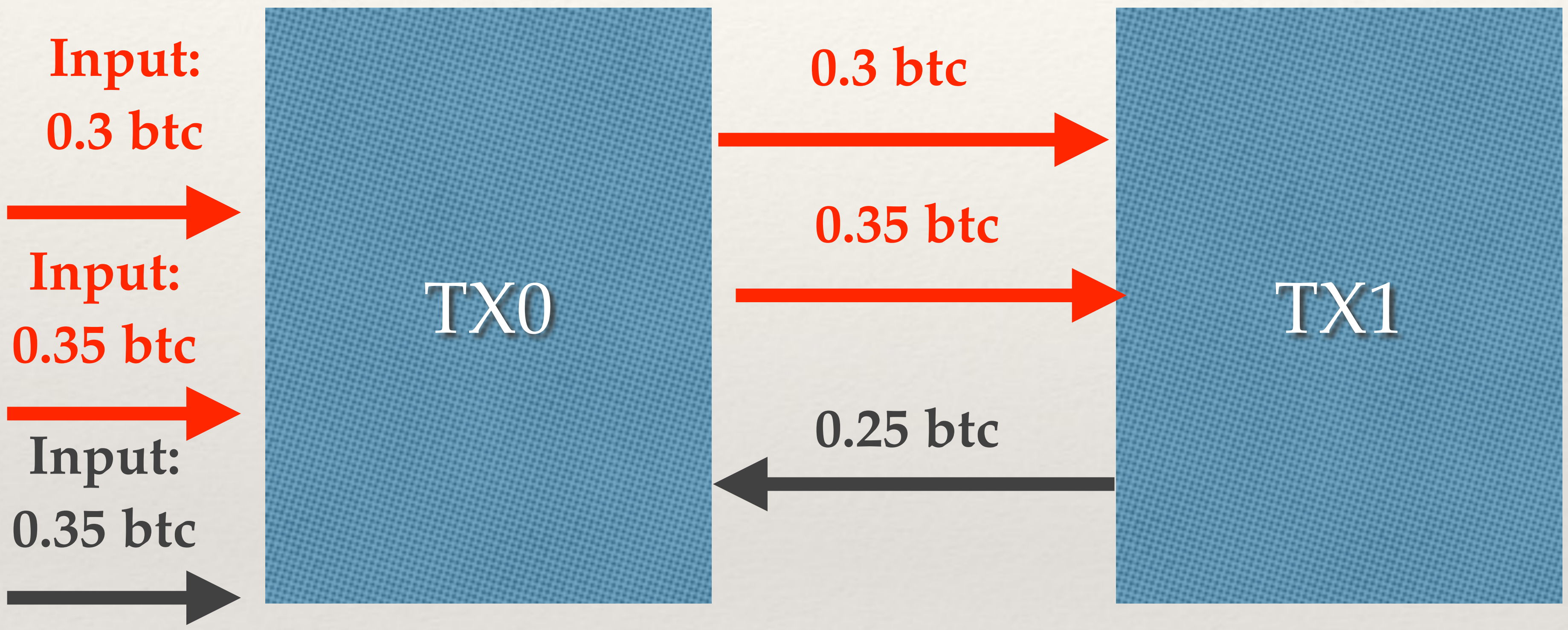
Multiple TX Input/Output

Sending 0.4 btc



Spent vs Unspent Tx's

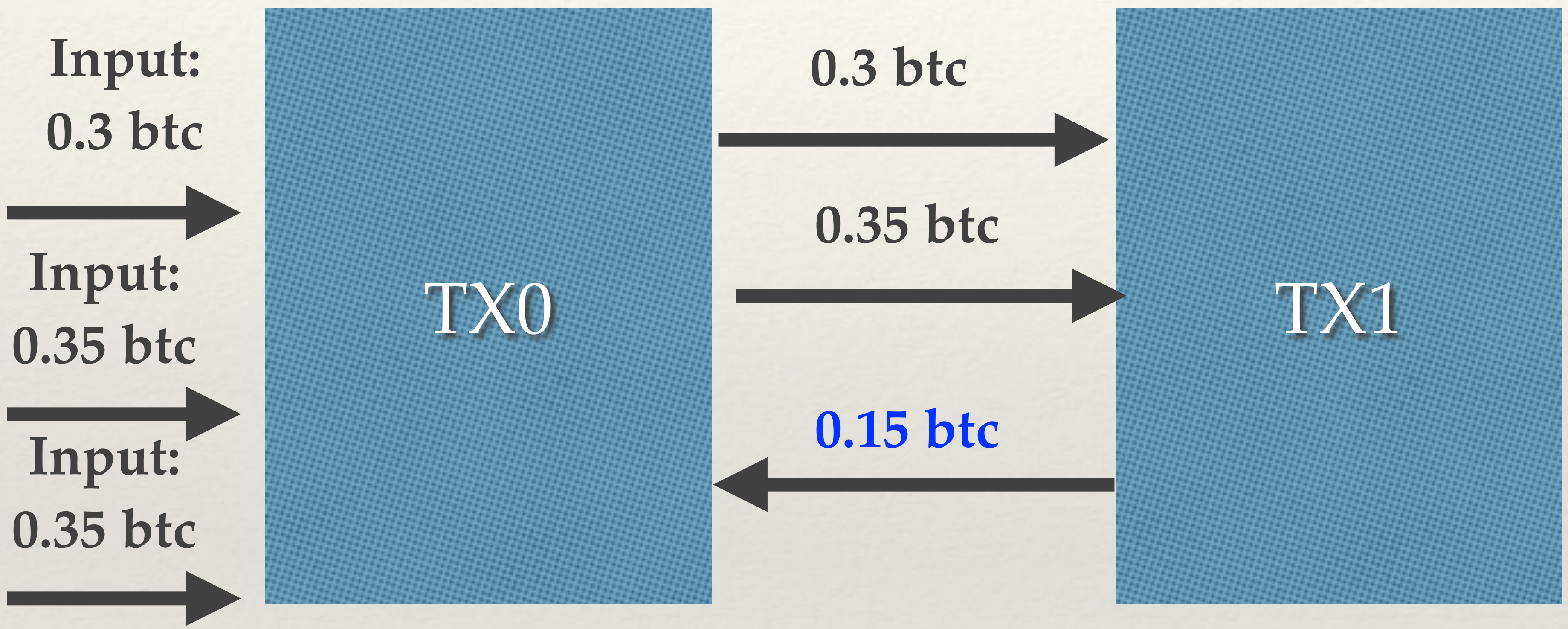
Sending 0.4 btc



Generator of TX0 now has two UTXOs: 0.35 btc and 0.25 btc
Most bitcoin wallets will just show you 0.35 + 0.25 = 0.6 btc
1 btc - 0.4 btc = 0.6 btc

Multiple TX Input/Output

Sending 0.4 btc



What if we only asked for 0.15 btc back instead of 0.25?
Extra money is the *transaction fee* - whoever mines a block with this transaction can claim it!

Transaction Fees

- *Ceteris paribus*, miners prefer to include transactions with higher fees
    - There are reasons not to do so, however, which we will discuss later…
- Think of it as bidding for space or express mail - if you want your transaction in a block quickly, you gotta pay
- https://bitcoinfees.info/

Proof of Work

- So what's to stop you from going out there and making a block yourself and gathering some transaction fees?

- **NOTHING!***

… As Long You Can Provide a Valid Nonce

- To create a block is going to require work (a tax on identity - avoids Sybil attacks, or at least cheap ones)
- Bitcoin uses SHA-256 hash puzzles, i.e., find a nonce (**n**umber used **once**) such that a hash of concatenation of the nonce, the hash of the previous block, and all transactions in the block is less than some target
- $H(nonce \mid\mid prev\_hash \mid\mid tx_1 \mid\mid tx_2 \mid\mid \ldots tx_n) < target$

Recall Puzzle-Friendliness of SHA-256 Hash

- No solving strategy for $H(id \mid\mid x) \in Y$ better than trying possible values of *id*
- In other words, assuming a non-trivial target, we are going to have to try some very large number of nonces (with no shortcuts) before we find a nonce such that $H(nonce \mid\mid rest\_of\_block) < target$
- And trying different values is the ONLY way to do so - no shortcut, one-way function

Hash Puzzle Properties

1. Difficult to compute
2. Parameterizable (i.e. adjustable) cost
3. Trivial to verify

1. Difficult to Compute

- We want people to prove that they have worked for this (to avoid Sybil attacks / nothing-at-stake problem)
- As of the creation of this slide, miners around the world are trying ~ 50,419,619,703,000,000,000 (> 50 quintillion) possible nonces every second, and they generally don't find a solution until after about ten minutes of this!
- This number of attempts per second is called the *hash rate* or *hashpower* of the Bitcoin network

2. Paramaterizable Cost

- January 12, 2009 - hash rate was (best estimate) ~ 4,500,000 (orders of magnitude lower than today)
- But blocks still were generated at a rate of about one every ten minutes
- Target re-adjusts itself every 2016 blocks based on (self-reported!) block times
  - If blocks are taking longer than 10 minutes, difficulty adjusts down (target increases, making it easier to find nonce)
  - If blocks are taking less than 10 minutes, difficulty adjusts up (target decreases, making it harder to find nonce)

Impact of Puzzle-Friendliness

- Solving hash values is probabilistic - some blocks are produced within seconds (or even “negative” time - remember dealing with time in a distributed system!), others can take hours
- Average is ~ 10 minutes
- https://data.bitcoinity.org/bitcoin/block_time/all?f=m10&t=l

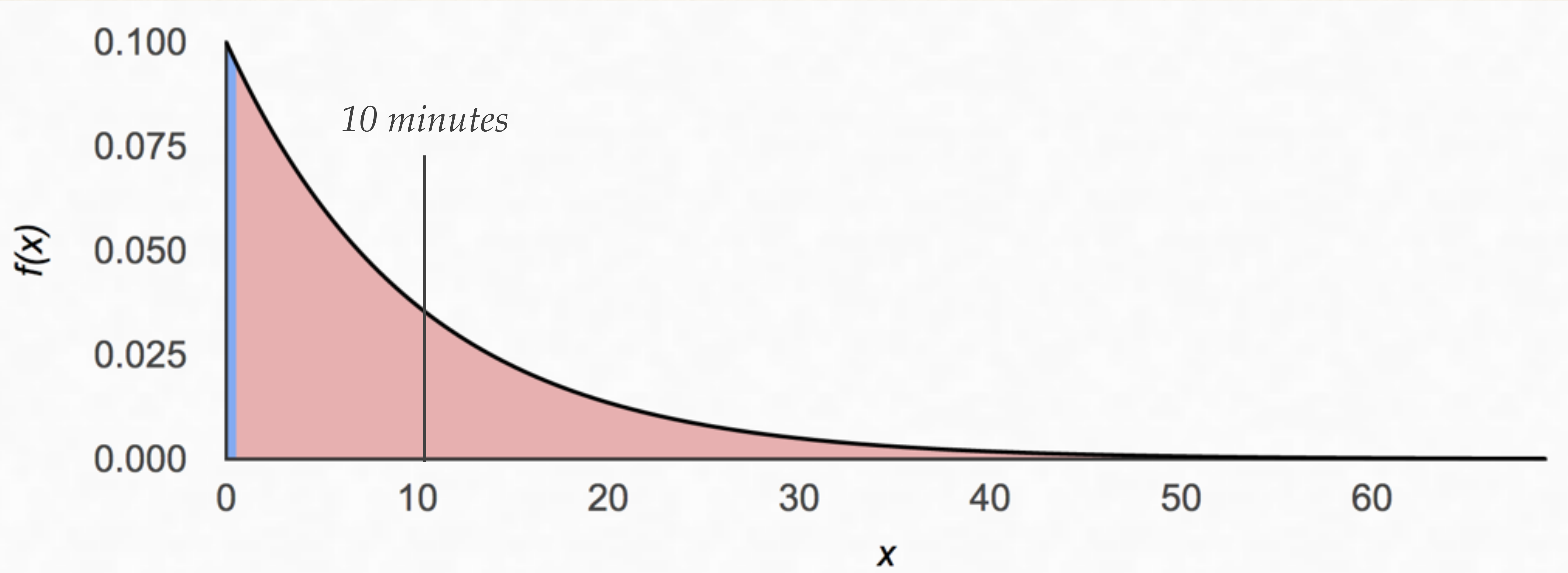
Bernoulli Trials

- A Bernoulli trial is a process with two possible outcomes (success and failure), and the probability of being successful (and by implication, failure) is fixed between trials
- Examples: coin flip, dice, roulette
- Mining: Success - *H(nonce || rest_of_block)* $<$ *target*
  Failure - *H(nonce || rest_of_block)* $>=$ *target*

Poisson Process

- Iterated Bernoulli trials (at large enough numbers so that we can basically act as if the result is continuous instead of discrete) represent a *Poisson process*
  - A continous probabilistic process where events occur independently at a constant *average* rate
  - Likelihood of a block occurring follows an exponential distribution

Exponential Distribution



Note graph is positively skewed - median < mean.

Generated with Matt Bognar's Exponential Distribution Applet, https://homepage.divms.uiowa.edu/~mbognar/applets/exp-like.html
To reproduce: $\lambda = 0.1$, $x = 0.1$

Poisson Process - Mining Bitcoin

- No nonces have been tried - what is the mean time until *H(nonce || rest_of_block)* < *target*?
- 500 quintillion nonces have been tried - what is the mean time until *H(nonce || rest_of_block)* < *target?*

Poisson Process - Dice

- I just rolled 1. How many rolls on average until I roll a 6?
- I just rolled 4, 3, 1, 5, 2, 3, 3, 3, 2, 5, 4. How many rolls on average until I roll a 6?

Poisson Process - Next Successful Event?

- For dice: on average (mean), 6 rolls
- For Bitcoin blocks: on average (mean), 10 minutes
- ALWAYS, at any point in time - Bernoulli trials have no "memory", thus they are always — "on average", i.e. mean — 10 minutes away no matter how much time has already elapsed since the previous block.
- It's been 45 minutes since the last block? On average, you *still* have 10 more minutes to wait! Poor you.

How Long for a Given Miner To Find a Block?

- I don’t care about the network as a whole, I care about me!

$$\textit{mean time to my next block} = \frac{\textit{10 minutes}}{\textit{fraction of hashpower I control}}$$

3. Trivial to Verify

- It should be very difficult to MINE a block, but easy for others to verify that it is valid
- Computationally simple
    - Run SHA-256 hash function against block with miner-calculated nonce
    - If $H(nonce \mid\mid rest\_of\_block) < target$, block is valid

Mining Overview

```
// Should I mine Bitcoin?
mining_reward = block_reward + tx_fees
mining_cost = hardware_cost + operating_costs
if (mining_reward > mining_cost)
  return true
else
  return false
```

51% Attack

- Recall that a key tenet of Bitcoin is decentralization
- Mining allows us to perform distributed consensus and avoid a ScroogeCoin-like (or a Scrooge hiding behind an army of Sybils) centralization
- But what if more than half of the hashpower is owned by a single entity? Is this a new Scrooge?

What Can Our Pseudo-Scrooge Do?

- *Steal coins?* No. The ONLY way to move coins is knowing the secret key of the owner (public key).
- *Suppress transactions?* Yes. Just like Scrooge, they can ignore transactions and refuse to put them in blocks - but users will see transactions in mempool and realize this attack is occurring.
- *Change the block reward?* No. They cannot change the rules of consensus - valid nodes will ignore the blocks.
- *Hurt confidence in Bitcoin?* Yes. A key benefit of Bitcoin lies in its decentralized nature; destroying this may start an exodus to alternatives.

A Pale Yet Dangerous Shadow

- Scrooge could change the block reward, suppress transactions without others realizing, or even change the rules of consensus - none of which our Pseudo-Scrooge can do.
- Decentralization has benefits even when it's been centralized!