Assignment 5

PUNDI BINDUSREE
CS21BTECH11048
Papoulis Chapter6 Ex-6.25

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Outline

1. Question

2. Solution

Question

Question

Let **x** be the lifetime of a certain electric bulb, and y that of its replacement after the failure of the first bulb. Suppose **x** and **y** are independent with common exponential density function with parameter λ.Find the probability that the combined lifetime exceeds 2λ. What is the probability that the replacement outlasts the original component by λ?

solution

Given

- the lifetime of a certain electric bulb = **x**
- and lifetime of its replacement after the failure of the first bulb = **y**
- and also given that **x** and **y** are independent with common exponential density function with parameter λ .

solution

- density function of $\mathbf{x} = f_{\mathbf{x}}(x) = \frac{1}{\lambda}e^{-\frac{x}{\lambda}}U(x)$
- density function of $\mathbf{y} = f_{\mathbf{y}}(y) = \frac{1}{\lambda}e^{-\frac{y}{\lambda}}U(y)$

solution

As it is given that **x** and **y** are independent

1. $f_{\mathbf{xy}}(xy) = f_{\mathbf{x}}(x)f_{\mathbf{y}}(y)$
2. $f_{\mathbf{xy}}(xy) = \frac{1}{\lambda^2}e^{-\frac{x+y}{\lambda}}U(x)U(y)$

solution

3. let **z**=**x**+**y**

$$\phi_{\mathbf{z}}(w) = \phi_{\mathbf{x}}(w)\phi_{\mathbf{y}}(w)\frac{1}{(1-jw\lambda)^2} \tag{1}$$

$$\mathbf{z}\ Gamma(2, \lambda) \tag{2}$$

Therefore it gives

$$f_{\mathbf{z}}(z) = \frac{z}{\lambda^2}e^{-\frac{z}{\lambda}}U(z) \tag{3}$$

solution

4.

$$P(Z > 2) = \int_{2\lambda}^{\infty} \frac{z}{\lambda^2} e^{\frac{z}{\lambda}} dz \tag{4}$$

$$= \int_{2}^{\infty} xe^{-x} dx$$

$$= 3e^{-2}$$

$$= 0.406 \tag{5}$$

solution

Now need to find the probability of lifetime of replacement exceeds the original by λ

1. This can be found by calculating $P((\mathbf{y}-\mathbf{x})>\lambda)$

solution

2. let $\mathbf{w} = \mathbf{y}\text{-}\mathbf{x}$

$$P((\mathbf{y} - \mathbf{x}) > \lambda) = P(\mathbf{w} > \lambda) \tag{6}$$

$$= \int_{\lambda}^{\infty} f_{\mathbf{w}}(w)dw \tag{7}$$

solution

3.

$$\text{For w} > 0\text{, this gives} \tag{8}$$

$$f_{\mathbf{w}}(w) = \int_0^{infty} \frac{1}{\lambda^2} e^{-\frac{w+2y}{\lambda}} dy \tag{9}$$

$$= \frac{1}{\lambda^2} e^{-\frac{w}{\lambda}} \int_0^{\infty} e^{-\frac{2y}{\lambda}} dy \tag{10}$$

$$= \frac{1}{2\lambda} e^{-\frac{w}{\lambda}}, \text{w} > 0 \tag{11}$$

solution

4. Hence $P(\mathbf{y}-\mathbf{x}>\lambda)=P(\mathbf{w}>\lambda)=\int_{\lambda}^{\infty}\frac{1}{2\lambda}e^{-\frac{w}{\lambda}}dw=\frac{1}{2e}$