ASSIGNMENT 2

PUNDI BINDUSREE
CS21BTECH11048
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outline

1. Question
2. Solution

Question

Question

Show that the four points A,B,C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}, -\hat{j}-\hat{k}, 3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ respectively, are coplanar.

solution

Given four points with position vectors as below

1. $$\mathbf{A} = \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} \tag{1}$$

2. $$\mathbf{B} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \tag{2}$$

3. $$\mathbf{C} = \begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix} \tag{3}$$

4. $$\begin{pmatrix} -4 \end{pmatrix}$$

solution

- Need to prove the above four position vectors are coplanar.

solution

Proof:

- The four position vectors A,B,C and D are coplanar if the scalar triple product of $\mathbf{AB}, \mathbf{AC} and \mathbf{AD}$ is 0.

solution

- In other way scalar triple product of above vectors is

$$\mathbf{AB}.(\mathbf{AC} \times \mathbf{AD}) = \text{determinant of the matrix} \begin{pmatrix} \mathbf{AB} \\ \mathbf{AC} \\ \mathbf{AD} \end{pmatrix} \tag{5}$$

Therefore need to prove determinant of the matrix

$$\begin{pmatrix} \mathbf{AB} \\ \mathbf{AC} \\ \mathbf{AD} \end{pmatrix} = 0 \tag{6}$$

solution

The vectors **AB**, **AC**, **AD** can be found as below

solution

1.

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} \tag{7}$$

$$\mathbf{AB} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0-4 \\ -1-5 \\ -1-1 \end{pmatrix}$$

$$= \begin{pmatrix} -4 \\ -6 \\ -2 \end{pmatrix} \tag{8}$$

soluion

1.

$$\mathbf{AC} = \mathbf{C} - \mathbf{A} \tag{9}$$

$$\begin{aligned}\mathbf{AC} &= \begin{pmatrix}3\\9\\4\end{pmatrix} - \begin{pmatrix}4\\5\\1\end{pmatrix}\\ &= \begin{pmatrix}3-4\\9-5\\4-1\end{pmatrix}\\ &= \begin{pmatrix}-1\\4\\3\end{pmatrix}\end{aligned} \tag{10}$$

solution

1.

$$\mathbf{AD} = \mathbf{D} - \mathbf{A} \tag{11}$$

$$\mathbf{AD} = \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -4-4 \\ 4-5 \\ 4-1 \end{pmatrix}$$

$$= \begin{pmatrix} -8 \\ -1 \\ 3 \end{pmatrix} \tag{12}$$

solution

- Let X be a matrix and

$$X = \begin{pmatrix} \mathbf{AB} \\ \mathbf{AC} \\ \mathbf{AD} \end{pmatrix} \tag{13}$$

solution

- Now the scalar dot product of above three vectors is determinant of matrix X = $|X|$

$$
\begin{aligned}
|X| &= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} \\
&= -4((4 \times 3) - (3 \times -1)) - (-6)((-1 \times 3) - (3 \times -8)) + (-2)((- \\
&= -4(15) + 6(21) - 2(33) \\
&= -60 + 126 - 66 \\
&= 0 \qquad (14)
\end{aligned}
$$

solution

- Determinant of matrix X $= 0$ It says that scalar triple product of the vectors $\mathbf{AB}, \mathbf{AC}$ and $\mathbf{AD}$ is zero.

solution

There the given four position vectors $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ are coplanar.Hence proved.