ASSIGNMENT 2

PUNDI BINDUSREE
CS21BTECH11048
ICSE 2018 12TH 16(b)

May 20, 2022

outline

Question

Question

Show that the four points A,B,C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}, -\hat{j}-\hat{k}, 3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ respectively, are coplanar.

solution

Given four points with position vectors as below

1. $$\mathbf{A} = \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} \tag{1}$$

2. $$\mathbf{B} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \tag{2}$$

3. $$\mathbf{C} = \begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix} \tag{3}$$

solution

4.
$$\mathbf{D} = \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix} \tag{4}$$

solution

- Need to prove the above four position vectors are coplanar.

solution

Proof:

- The four position vectors A,B,C and D are coplanar if the scalar triple product of $\mathbf{AB}, \mathbf{AC} and \mathbf{AD}$ is 0.

solution

- In other way scalar triple product of above vectors is

$$\mathbf{AB}.(\mathbf{AC} \times \mathbf{AD}) = \text{determinant of the matrix} \begin{pmatrix} \mathbf{AB} \\ \mathbf{AC} \\ \mathbf{AD} \end{pmatrix} \tag{5}$$

Therefore need to prove determinant of the matrix

$$\begin{pmatrix} \mathbf{AB} \\ \mathbf{AC} \\ \mathbf{AD} \end{pmatrix} = 0 \tag{6}$$

solution

The vectors $\mathbf{AB}, \mathbf{AC}, \mathbf{AD}$ can be found as below

solution

1.

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} \tag{7}$$

$$\mathbf{AB} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0-4 \\ -1-5 \\ -1-1 \end{pmatrix}$$

$$= \begin{pmatrix} -4 \\ -6 \\ -2 \end{pmatrix} \tag{8}$$

soluion

2

$$\mathbf{AC} = \mathbf{C} - \mathbf{A} \tag{9}$$

$$\begin{aligned}\mathbf{AC} &= \begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 3-4 \\ 9-5 \\ 4-1 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}\end{aligned} \tag{10}$$

solution

3.

$$\mathbf{AD} = \mathbf{D} - \mathbf{A} \tag{11}$$

$$\begin{aligned}\mathbf{AD} &= \begin{pmatrix}-4\\4\\4\end{pmatrix} - \begin{pmatrix}4\\5\\1\end{pmatrix}\\ &= \begin{pmatrix}-4-4\\4-5\\4-1\end{pmatrix}\\ &= \begin{pmatrix}-8\\-1\\3\end{pmatrix}\end{aligned} \tag{12}$$

solution

- Let X be a matrix and

$$X = \begin{pmatrix} \mathbf{AB} \\ \mathbf{AC} \\ \mathbf{AD} \end{pmatrix} \tag{13}$$

solution

- Now the scalar dot product of above three vectors is determinant of matrix X $= |X|$

$$|X| = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

solution

- Applying row reduction method for above determinant $|X|$.
- Let C_1, C_2, C_3 be first,second,third columns respectively and R_1, R_2, R_3 be first,second,third columns respectively.

solution

1. Multiplying the determinant with 2 and dividing R_1 in the determinant with 2 gives

$$|X| = 2\begin{vmatrix} -2 & -3 & -1 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} \tag{14}$$

solution

2. taking common factor -1 from the R_1 in the determinant gives

$$|X| = -2\begin{vmatrix} 2 & 3 & 1 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} \tag{15}$$

solution

3. on interchanging C_1 and C_2 gives

$$|X| = 2\begin{vmatrix} 3 & 2 & 1 \\ 4 & -1 & 3 \\ -1 & -8 & 3 \end{vmatrix} \tag{16}$$

solution

4. $C_1 \longrightarrow C_1 - C_2$ gives

$$|X| = 2\begin{vmatrix} 1 & 2 & 1 \\ 5 & -1 & 3 \\ 7 & -8 & 3 \end{vmatrix} \tag{17}$$

solution

5. $C_2 \longrightarrow C_2 - C3$ gives

$$|X| = 2\begin{vmatrix} 1 & 1 & 1 \\ 5 & -4 & 3 \\ 7 & -11 & 3 \end{vmatrix} \tag{18}$$

solution

6. $C_1 \longrightarrow C_1 - C_2$ gives

$$= 2\begin{vmatrix} 0 & 1 & 1 \\ 9 & -4 & 3 \\ 18 & -11 & 3 \end{vmatrix} \tag{19}$$

solution

7. $C_2 \longrightarrow C_2 - C_3$ gives

$$|X| = 2\begin{vmatrix} 0 & 0 & 1 \\ 9 & -7 & 3 \\ 18 & -14 & 3 \end{vmatrix} \tag{20}$$

solution

8. taking common factor 9 from C_1 and -7 from C_2 gives

$$|X| = -156 \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 2 & 3 \end{vmatrix} \tag{21}$$

finally two columns are obtained with the same entries

solution

- If a determinant have two rows or two columns with same entries, then the value of the determinant is zero.
  As the above determinant have two columns with same entries the value of the determinant of matrix X i.e.,—X—= 0.

solution

- As determinant of matrix $X = 0$ it says that scalar triple product of the vectors $\mathbf{AB}, \mathbf{AC}$ and $\mathbf{AD}$ is zero.

solution

- Therefore the given four position vectors $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ are coplanar. Hence proved.