

ASSIGNMENT 2

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Q.NO:16B SOLUTION:

Given four position vectors A,B,C and D as $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$, $4(-\hat{i} + \hat{j} + \hat{k})$ respectively.

Need to prove given four position vectors are coplanar.

Position vectors A,B,C and D are coplanar if and only if $\vec{AB} \cdot (\vec{AC} \times \vec{AD}) = 0$ i.e., scalar triple product of vectors $\vec{AB}, \vec{AC}, \vec{AD}$ is zero.

$$\begin{aligned}\vec{AB} &= \vec{B} - \vec{A} \\ &= (-\hat{j} - \hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k}) \\ &= -\hat{j} - \hat{k} - 4\hat{i} - 5\hat{j} - \hat{k}\end{aligned}$$

$$\vec{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k} \quad (1)$$

$$\begin{aligned}\vec{AC} &= \vec{C} - \vec{A} \\ &= (3\hat{i} + 9\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k}) \\ &= 3\hat{i} + 9\hat{j} + 4\hat{k} - 4\hat{i} - 5\hat{j} - \hat{k}\end{aligned}$$

$$\vec{AC} = -1\hat{i} + 4\hat{j} + 3\hat{k} \quad (2)$$

$$\begin{aligned}\vec{AD} &= \vec{D} - \vec{A} \\ &= (-4\hat{i} + 4\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k}) \\ &= -4\hat{i} + 4\hat{j} + 4\hat{k} - 4\hat{i} - 5\hat{j} - \hat{k}\end{aligned}$$

$$\vec{AD} = -8\hat{i} - 1\hat{j} + 3\hat{k} \quad (3)$$

$$\begin{aligned}\text{Scalar triple product of vectors } \vec{AB}, \vec{AC}, \vec{AD} &= \vec{AB} \cdot (\vec{AC} \times \vec{AD}) \\ &= (-4\hat{i} - 6\hat{j} - 2\hat{k}) \cdot ((-1\hat{i} + 4\hat{j} + 3\hat{k}) \times (-8\hat{i} - 1\hat{j} + 3\hat{k})) \\ &= (-4\hat{i} - 6\hat{j} - 2\hat{k}) \cdot (15\hat{i} + 21\hat{j} + 33\hat{k}) \\ &= 60 - 126 - 66 \\ &= 0\end{aligned}$$

As the scalar triple product value is 0 the condition for coplanar vectors is satisfied therefore the given four positional vectors are coplanar.

Hence it is proved that the given positional vectors are coplanar vectors.