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Pundi Bindusree
cs21btech11048

QUESTION 5@ SOLUTION:

Given a matrix $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$ and a matrix X such that $X = B^2 - 4B$

$$\begin{aligned} B^2 &= \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1\times1+1\times8 & 1\times1+1\times3 \\ 8\times1+3\times8 & 8\times1+3\times3 \end{bmatrix} \\ &= \begin{bmatrix} 1+8 & 1+3 \\ 8+24 & 8+9 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} \end{aligned} \tag{1}$$

$$\begin{aligned} 4B &= \begin{bmatrix} 4\times1 & 4\times1 \\ 4\times8 & 4\times3 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix} \end{aligned} \tag{2}$$

substituting (1) and (2) in $X = B^2 - 4B$ gives

$$\begin{aligned} X &= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix} \\ &= \begin{bmatrix} 9-4 & 4-4 \\ 32-32 & 17-12 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \end{aligned}$$

Therefore, $X = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

Given that $X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$$\begin{aligned} X \begin{bmatrix} a \\ b \end{bmatrix} &= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \\ &= \begin{bmatrix} 5\times a+0\times b \\ 0\times a+5\times b \end{bmatrix} \\ &= \begin{bmatrix} 5a+0 \\ 0+5b \end{bmatrix} \\ &= \begin{bmatrix} 5a \\ 5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix} \end{aligned} \tag{3}$$

as

$$\begin{bmatrix} 5a \\ 5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

comparing L.H.S values in left side matrix with R.H.S values in right side matrix in (3) gives

$$5a = 5$$
$$5b = 50$$

from $5a = 5$, $a = 1$ and $5b = 50$, $b = 10$.
Therefore from above the values of a,b are 1,10 respectively.