

ASSIGNMENT 2

PUNDI BINDUSREE
CS21BTECH11048
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outline

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Question

Question

Show that the four points A,B,C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively, are coplanar.

solution

Given four points with position vectors as below

1

$$\mathbf{A} = \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} \quad (1)$$

2

$$\mathbf{B} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \quad (2)$$

3

$$\mathbf{C} = \begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix} \quad (3)$$

solution

4

$$\mathbf{D} = \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix} \quad (4)$$

solution

- Need to prove the above four position vectors are coplanar.

solution

Proof:

- The four position vectors A, B, C and D are coplanar if the scalar triple product of \mathbf{AB}, \mathbf{AC} and \mathbf{AD} is 0.

solution

- In other way scalar triple product of above vectors is

$$\mathbf{AB} \cdot (\mathbf{AC} \times \mathbf{AD}) = \text{determinant of the matrix } \begin{pmatrix} \mathbf{AB} \\ \mathbf{AC} \\ \mathbf{AD} \end{pmatrix} \quad (5)$$

Therefore need to prove determinant of the matrix

$$\begin{pmatrix} \mathbf{AB} \\ \mathbf{AC} \\ \mathbf{AD} \end{pmatrix} = 0 \quad (6)$$

solution

The vectors **AB**, **AC**, **AD** can be found as below

solution

1

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} \quad (7)$$

$$\mathbf{AB} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 - 4 \\ -1 - 5 \\ -1 - 1 \end{pmatrix}$$

$$= \begin{pmatrix} -4 \\ -6 \\ -2 \end{pmatrix} \quad (8)$$

solution

2

$$\mathbf{AC} = \mathbf{C} - \mathbf{A} \quad (9)$$

$$\begin{aligned} \mathbf{AC} &= \begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 3 - 4 \\ 9 - 5 \\ 4 - 1 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} \quad (10) \end{aligned}$$

solution

3

$$\mathbf{AD} = \mathbf{D} - \mathbf{A} \quad (11)$$

$$\begin{aligned} \mathbf{AD} &= \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} -4 - 4 \\ 4 - 5 \\ 4 - 1 \end{pmatrix} \\ &= \begin{pmatrix} -8 \\ -1 \\ 3 \end{pmatrix} \end{aligned} \quad (12)$$

solution

4 Let X be a matrix and

$$X = \begin{pmatrix} AB \\ AC \\ AD \end{pmatrix} \quad (13)$$

solution

- Now the scalar dot product of above three vectors is determinant of matrix $X = |X|$

$$\begin{aligned}
 |X| &= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} \\
 &= -4((4 \times 3) - (3 \times -1)) - (-6)((-1 \times 3) - (3 \times -8)) + (-2)((-8 \times -1) - (-24)) \\
 &= -4(15) + 6(21) - 2(33) \\
 &= -60 + 126 - 66 \\
 &= 0
 \end{aligned}
 \tag{14}$$

solution

- Determinant of matrix $X = 0$ It says that scalar triple product of the vectors **AB**, **AC** and **AD** is zero.

solution

- There the given four position vectors **A**, **B**, **C** and **D** are coplanar.
Hence proved.