

ASSIGNMENT 2

ICSE 12 2018 PAPER

PUNDI BINDUSREE

CS21BTECH11048

Q.No:16b solution:

Given four points with position vectors A,B,C and D as $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively.

Need to prove given four position vectors are coplanar.

Position vectors A,B,C and D are coplanar if and only if $\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = 0$ i.e., scalar triple product of \overrightarrow{AB} , \overrightarrow{AC} and \overrightarrow{AD} is zero.

$$\begin{aligned}\overrightarrow{AB} &= \vec{B} - \vec{A} \\ &= (-\hat{j} - \hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k}) \\ &= -\hat{j} - \hat{k} - 4\hat{i} - 5\hat{j} - \hat{k}\end{aligned}$$

$$\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k} \quad (1)$$

$$\begin{aligned}\overrightarrow{AC} &= \vec{C} - \vec{A} \\ &= (3\hat{i} + 9\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k}) \\ &= 3\hat{i} + 9\hat{j} + 4\hat{k} - 4\hat{i} - 5\hat{j} - \hat{k}\end{aligned}$$

$$\overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k} \quad (2)$$

$$\begin{aligned}\overrightarrow{AD} &= \vec{D} - \vec{A} \\ &= (4(-\hat{i} + \hat{j} + \hat{k})) - (4\hat{i} + 5\hat{j} + \hat{k}) \\ &= -4\hat{i} + 4\hat{j} + 4\hat{k} - 4\hat{i} - 5\hat{j} - \hat{k}\end{aligned}$$

$$\overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k} \quad (3)$$

Scalar triple product of vectors

$$\begin{aligned}\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD} &= \overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = \\ &= (-4\hat{i} - 6\hat{j} - 2\hat{k}) \cdot ((-\hat{i} + 4\hat{j} + 3\hat{k}) \times (-8\hat{i} - \hat{j} + 3\hat{k})) \\ &= (-4\hat{i} - 6\hat{j} - 2\hat{k}) \cdot (15\hat{i} + 21\hat{j} + 33\hat{k}) \\ &= 60 - 126 - 66 \\ &= 0\end{aligned}$$

As the scalar triple product value is the condition for coplanar vectors is satisfied therefore the given four positional vectors are coplanar.

Hence it is proved that the given positional vectors are coplanar vectors.