Assignment 4

PUNDI BINDUSREE
CS21BTECH11048
Papoulis Chapter3 Example 3.10

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Outline

Question

Question

We place at random n points in the interval (0,T). What is the probability that k of these n points are in the interval (t_1, t_2)?

solution

- This can be considered as a problem in repeated trials.

solution

- Given an interval (0,T) and a sub interval (t_1, t_2)
- In the given problem the experiment is S=placing a single point in the interval (0,T)
- In this experiment an event A=the single point got placed is in the interval (t_1, t_2).

solution

1. probability of event $A = P(A) = \frac{t_2 - t_1}{T}$
2. Let $P(A) = p = \frac{t_2 - t_1}{T}$
3. Let $q = P(A') = 1 - \frac{t_2 - t_1}{T}$

solution

- It is given that n random points are being placed in the interval (0,T).
- Among those n points k points are in the sub interval (t_1, t_2).

solution

- Let P(B)= probability of event B.
  where B is the event of k points being placed in the sub interval (t_1, t_2) among n points in (0,T).

solution

1. $P(B) = \binom{n}{k} p^k q^{n-k}$

$$\text{where } p = \frac{t_2 - t_1}{T}$$

$$q = 1 - \frac{t_2 - t_1}{T}$$

solution

2. Substituting p,q in P(B) gives

$$P(B) = \binom{n}{k} \left(\frac{t_2 - t_1}{T}\right)^k \left(1 - \frac{t_2 - t_1}{T}\right)^{n-k}$$

solution

- Therefore the probability of k points being placed in the sub interval (t_1, t_2) among n points in (0,T) = $\binom{n}{k}$ $(\frac{t_2-t_1}{T})^k(1-\frac{t_2-t_1}{T})^{n-k}$.