Stability of Reissner-Nordström black hole in de Sitter background under charges scalar perturbation.

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Solutions to Einstein field equations i.e. how we describe space time around these hypothetical compact objects

	Rotating	**Non-Rotating**
Charged	Kerr-Newman	Reissner-Nordström
Not-Charged	Kerr	Schwarzschild

Metric: Description of the space-time geometry around this object.
We use it to measure distances and angles in curved geometry

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q}{r^2} - \frac{\Lambda r^2}{3}$$

f(r) has 3 different roots, which we will denote:
Cauchy, Event and Cosmological horizons

Perturbations in the context of black holes refer to small disturbances or fluctuations in the fields or spacetime geometry surrounding a black hole. They can have different origins:

Astrophysical Processes: Objects like stars, gas clouds, or even other black holes interacting with a black hole can generate perturbations.

Quantum Fluctuations: Quantum effects near the event horizon can create tiny fluctuations in the field around the black hole.

Wave Scattering: Incident waves (like light, particles, or even gravitational waves) passing near a black hole can scatter off the curved spacetime, causing perturbations.

Black Hole Mergers: When two black holes merge, the violent interaction generates strong gravitational perturbations in the surrounding spacetime.

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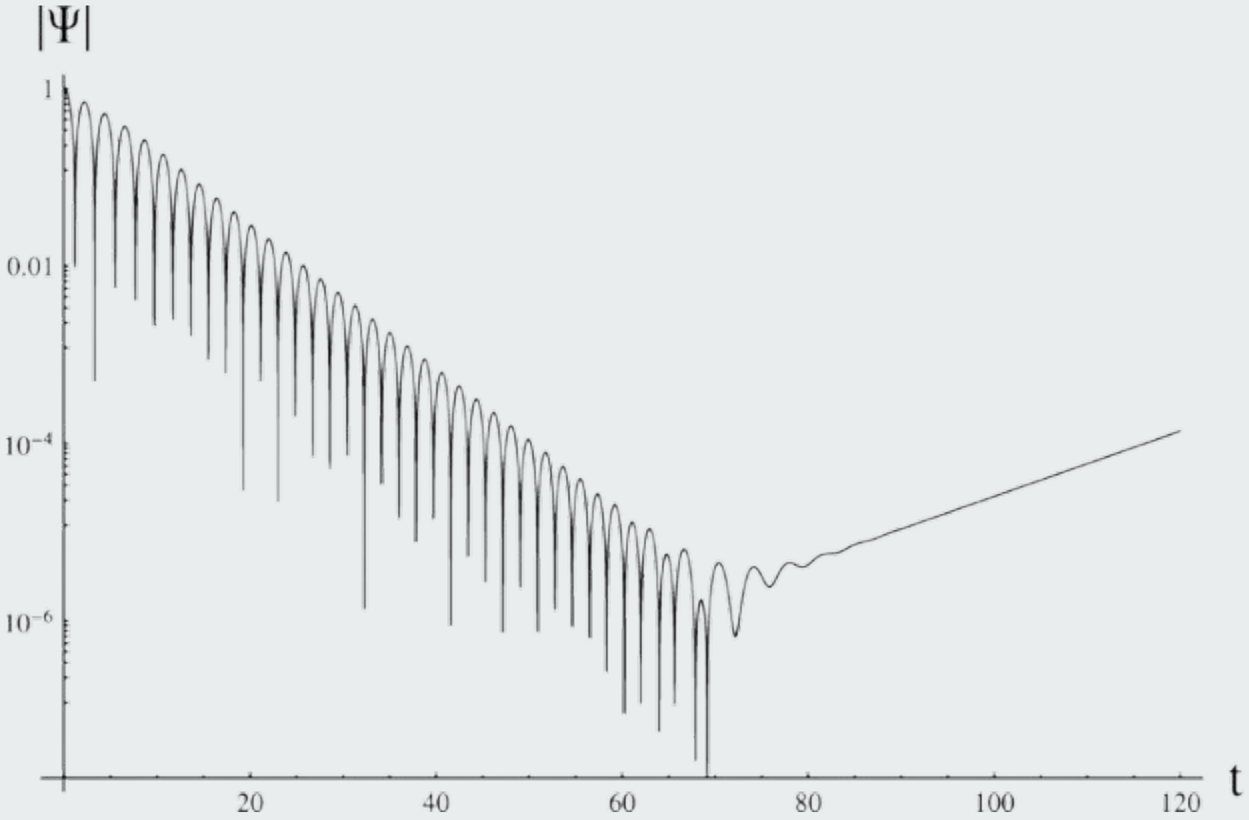
To study these perturbations we use the **Quasinormal Modes (QNMs)** which are specific patterns of oscillations that occur when a black hole or any other compact object, like a neutron star, is perturbed. These modes describe how the object responds to disturbances. Quasinormal modes represent the “ringing” of spacetime itself after it has been disturbed, similar to how a bell rings when struck.

Damped Oscillations: Quasinormal modes are characterized by oscillations that gradually decay over time. This decay happens because the energy of the perturbation is radiated away, often in the form of gravitational waves.

Complex Frequencies: The frequencies of QNMs are complex numbers. The real part of the frequency represents the oscillation rate (how fast the ringing happens), while the imaginary part represents the damping rate (how quickly the ringing fades away). The higher the imaginary part, the faster the oscillation dies out.

Time-domain profile for vector type of gravitational perturbations



Quasinormal modes, stability and shadows of a black hole in the 4D Einstein–Gauss–Bonnet gravity.

And how to do this?

Starting from the relativistic Klein-Gordon (wave) Equation and using our metric inside the covariant derivative

$$\left[\left(\nabla^{\nu}-iqA^{\nu}\right)\left(\nabla_{\nu}-iqA_{\nu}\right)-\mu^{2}\right]\varphi=0$$

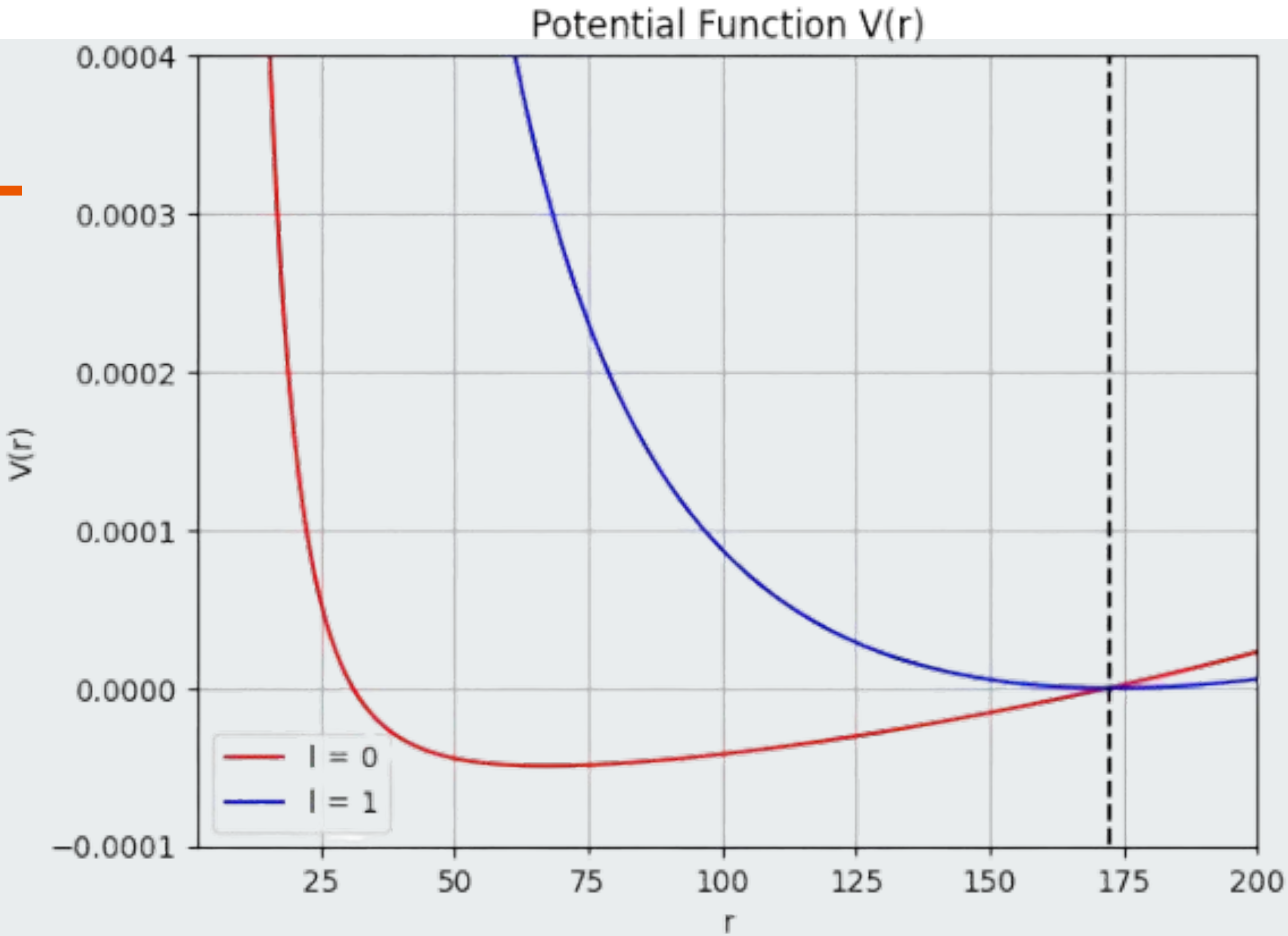
We'll get an equation (sum of every component) which we then apply the spherical harmonic to finally get the Schrödinger-type equation and a potential for each value of the angular momentum l of the perturbation scalar field

$$-\frac{\partial^2\Psi}{\partial t^2}+\frac{\partial^2\Psi}{\partial r_\star^2}-2iq\frac{Q}{r}\frac{\partial\Psi}{\partial t}-V\left(r\right)=0$$

$$V\left(r\right)=-q^2\left(\frac{Q}{r}\right)^2+f\left(r\right)\left(\frac{l\left(l+1\right)}{r^2}+\mu^2+\frac{f'\left(r\right)}{r}\right)$$

r_star = tortoise coordinate: transformation that simplifies the radial direction, it makes easier to study the behavior when near the horizon l is the angular momentum of the perturbing field, not the black hole itself. This angular momentum is related to the shape and behavior of the wave (or field) as it interacts with the black hole's spacetime. It comes from the spherical harmonics.

l=0: The perturbation is spherically symmetric (no angular variation).
l=1: The perturbation has a dipole structure (one node).
l=2: The perturbation has a quadrupole structure (two nodes), and so on.

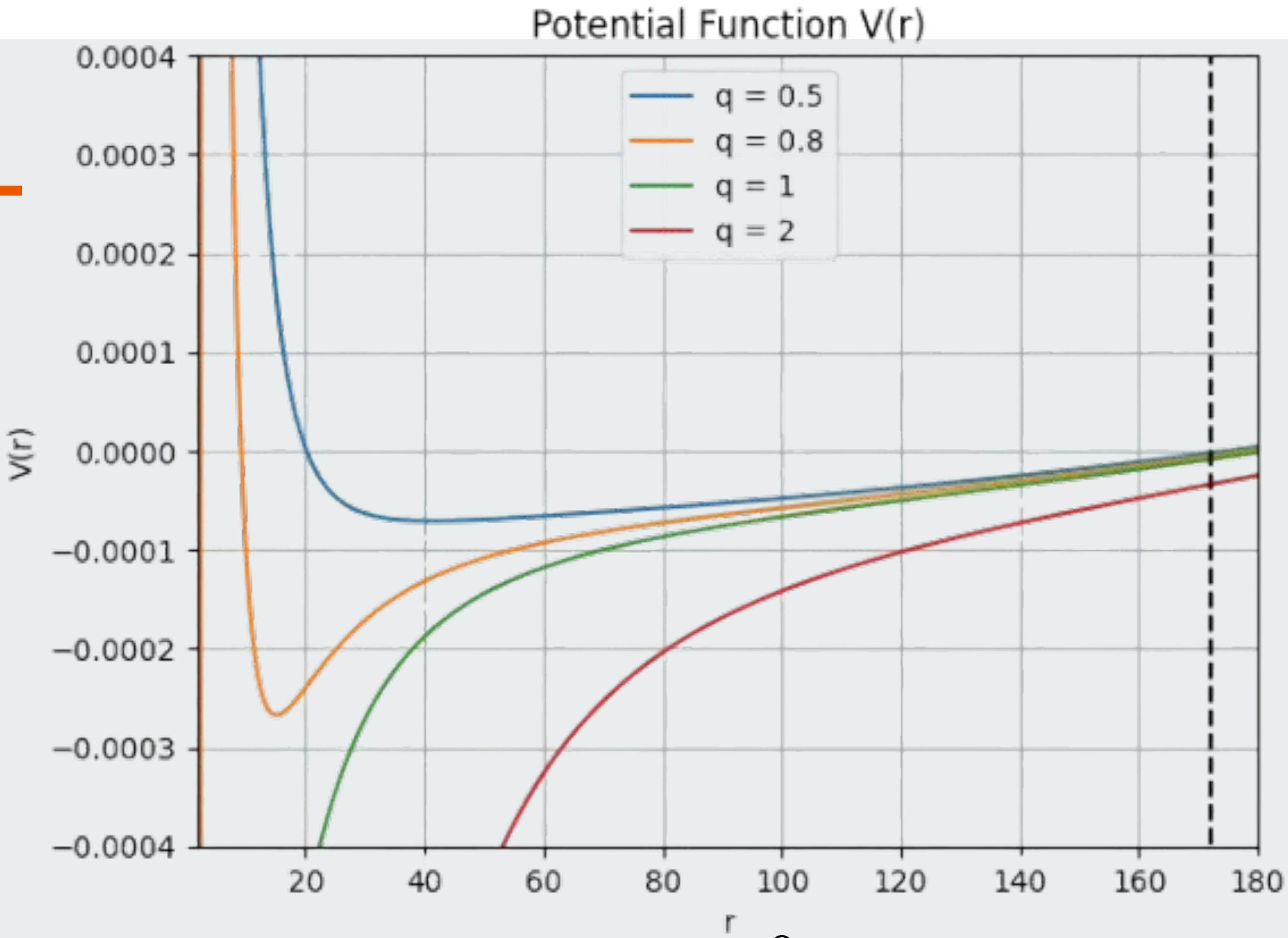

Potential Function V(r)
0.0004
0.0003
0.0002
0.0001
0.0000
−0.0001
V(r)
25
50
75
100
125
150
175
200
r
l = 0
l = 1

$\mu = 0;\ q = 0$

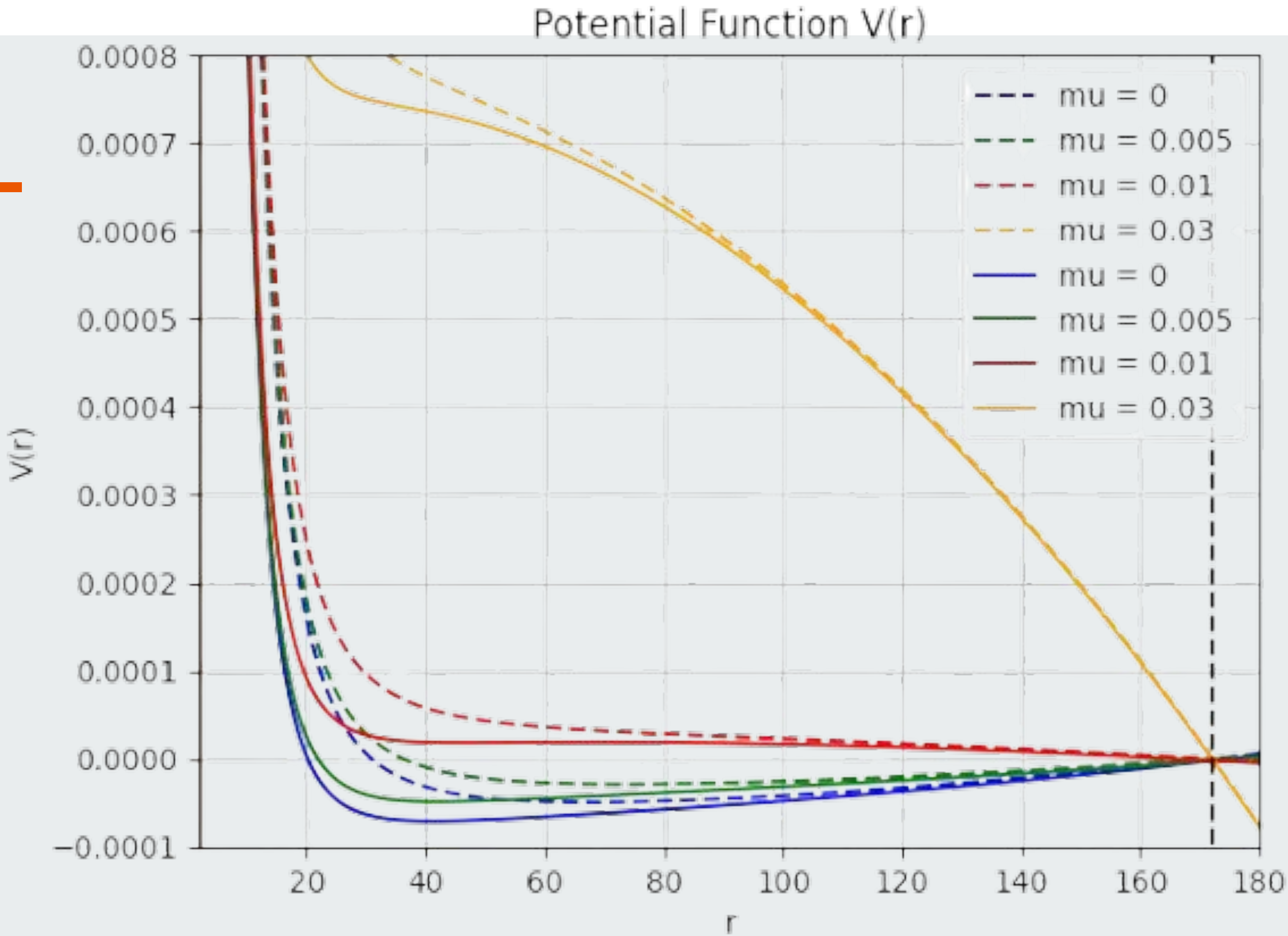
The negative well indicates a region where the potential is attractive, potentially trapping perturbations near the black hole. This trapped region can lead to instabilities, as perturbations might be amplified instead of decaying.

If there is a absence of a significant negative well, it means perturbations are less likely to be trapped or amplified, suggesting that this mode is more stable compared to

The dashed vertical line marks the location of the cosmological horizon, which is where the potential approaches zero. Beyond this point, the spacetime expansion dominates, and perturbations are effectively pushed away from the black hole


Potential Function V(r)
q = 0.5
q = 0.8
q = 1
q = 2
0.0004
0.0003
0.0002
0.0001
0.0000
−0.0001
−0.0002
−0.0003
−0.0004
V(r)
20
40
60
80
100
120
140
160
180
r

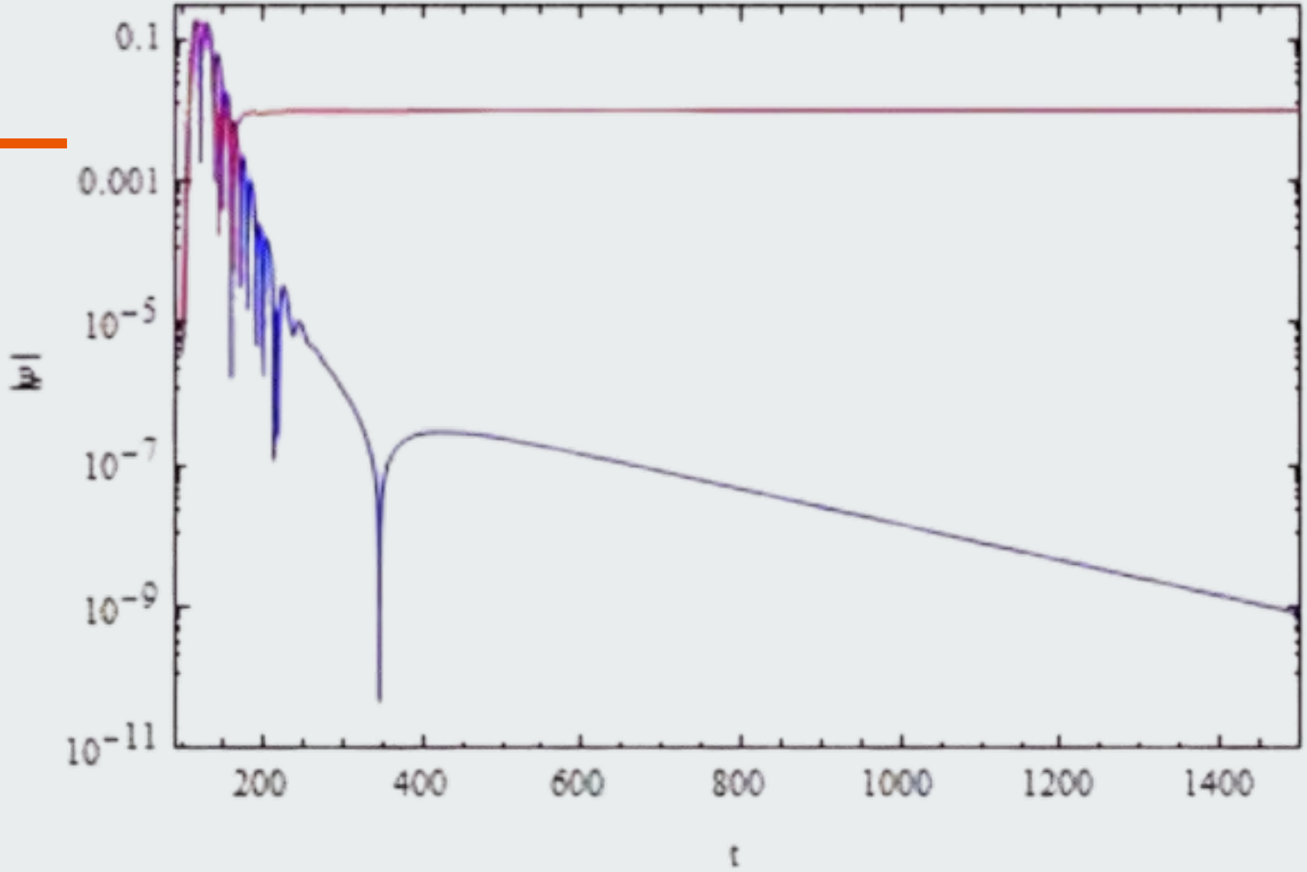
$$\mu = 0$$



$$q = 0\ (dashed);\ q = 0.5\ (solid)$$

$$-\frac{\partial^2\Psi}{\partial t^2}+\frac{\partial^2\Psi}{\partial r_\star^2}-2iq\frac{Q}{r}\frac{\partial\Psi}{\partial t}-V(r)=0$$

We don't have analytic solutions to this wave equation, therefore, we must use a numerical method called The **Finite Difference Method (FDM)** which replaces the continuous derivatives in the equations with discrete differences, allowing for an approximate solution to be calculated. Thus we can derive the evolution of Ψ



$\mu = 0;\ q = 0$

The **time-domain profile** is a plot of how a perturbation evolves over time..

- **Decreasing Curve**: If the amplitude of the perturbation decreases over time, it means that the perturbation is **dissipating** or **decaying**. This implies that the system is stable, as any disturbance introduced to the black hole fades away rather than growing uncontrollably

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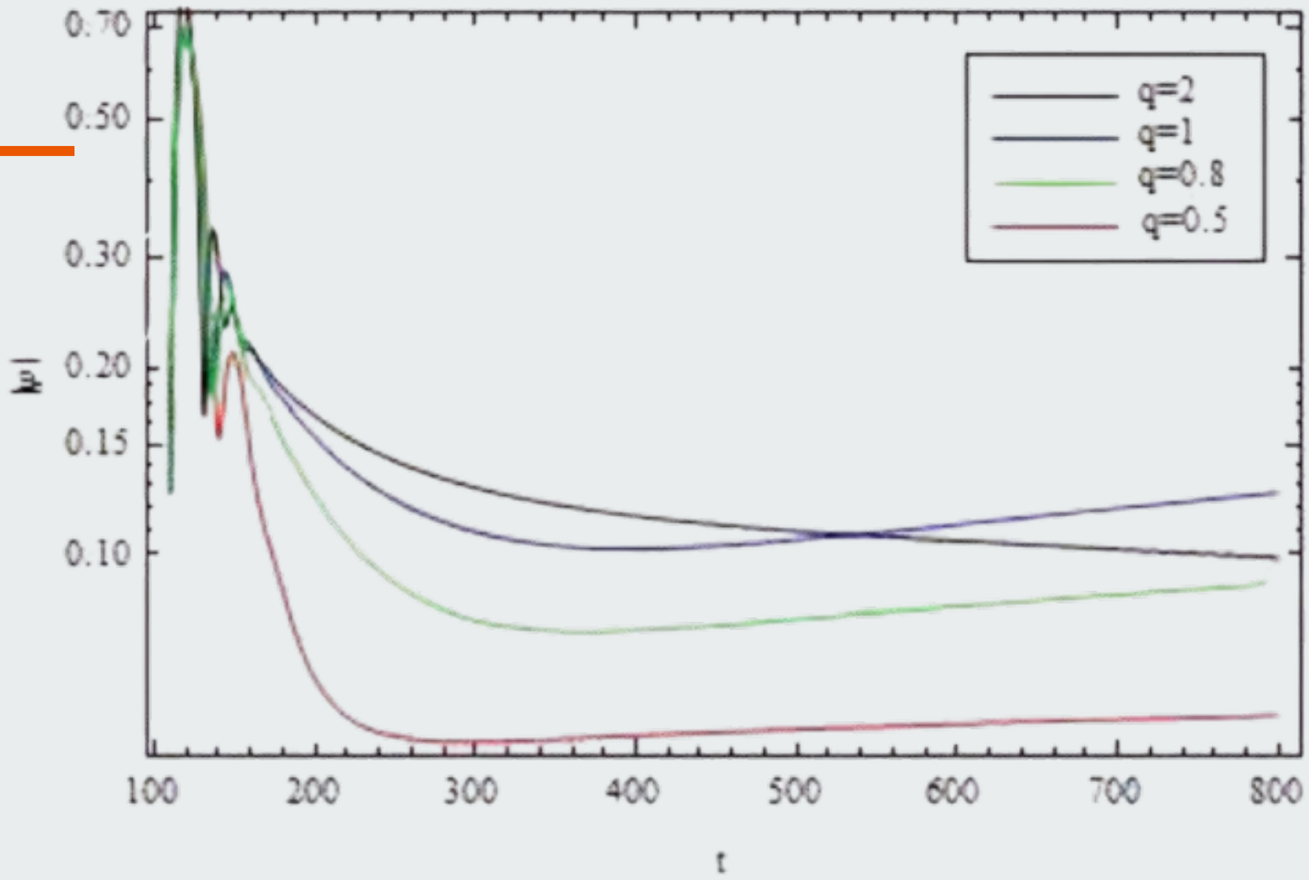
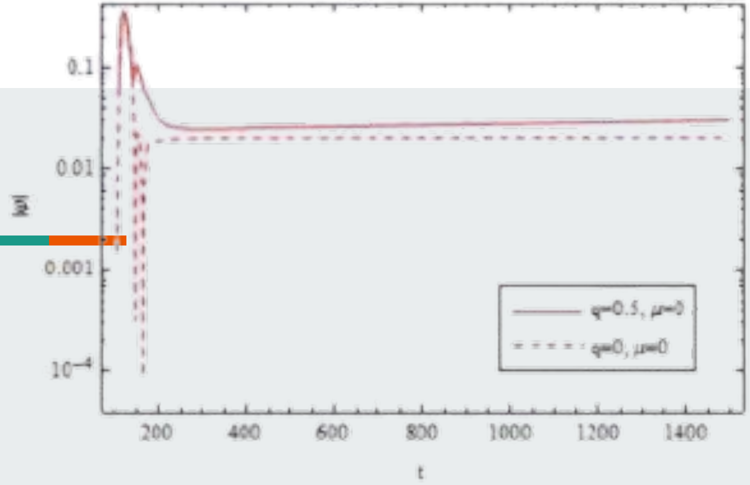
- **Growing Curve**: If the amplitude of the perturbation increases over time, it suggests that the perturbation is becoming **larger**. This kind of growth indicates an **instability**. Instead of decaying, the perturbation amplifies, potentially leading to significant changes in the black hole's structure or behavior.



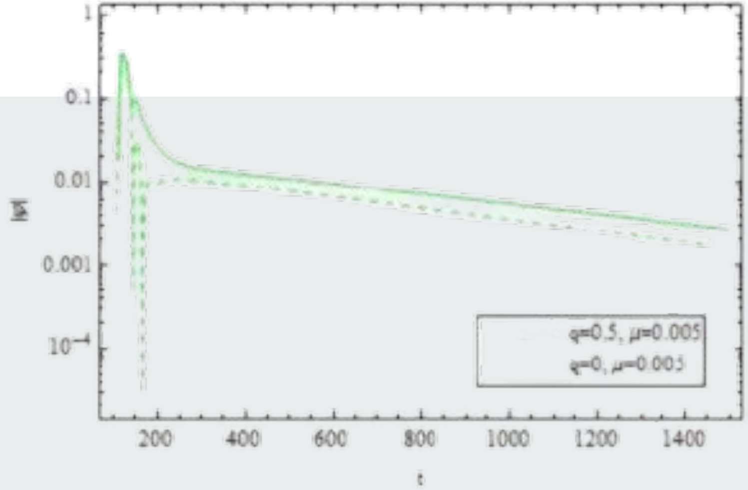
0.70
0.50
0.30
0.20
0.15
0.10
$|\psi|$
100
200
300
400
500
600
700
800
t
q=2
q=1
q=0.8
q=0.5

$\mu = 0$



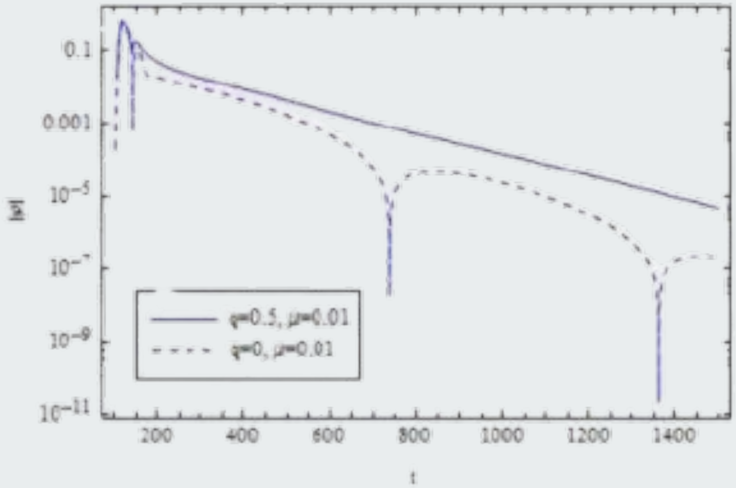
q=0.5, μ=0
q=0, μ=0
|ψ|
t

(a)

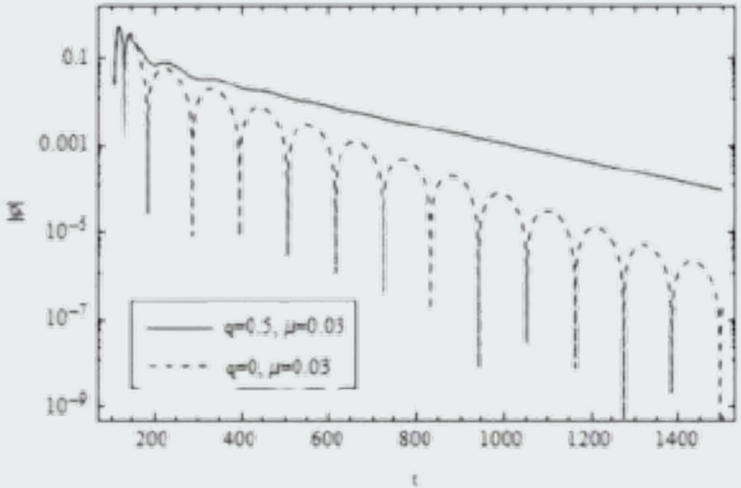


q=0.5, μ=0.005
q=0, μ=0.005
|ψ|
t

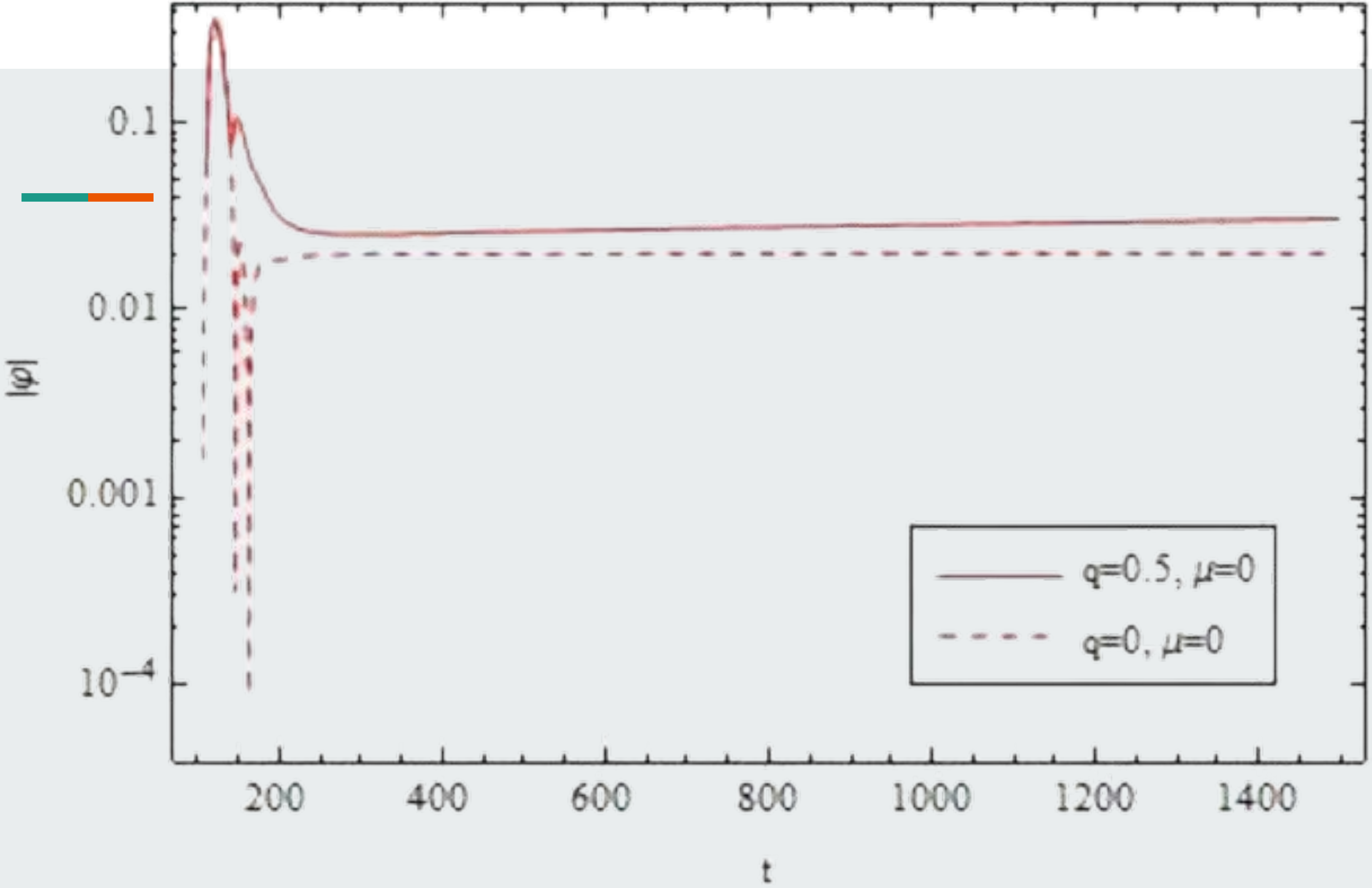
(b)



q=0.5, μ=0.01
q=0, μ=0.01
|ψ|
t



q=0.5, μ=0.03
q=0, μ=0.03
|ψ|
t



0.1
0.01
0.001
10^{-4}
$|\varphi|$
200
400
600
800
1000
1200
1400
t
q=0.5, μ=0
q=0, μ=0

Why do we want to look for instabilities?

An unstable black hole may evolve into something different or even cease to exist as a black hole. When instabilities occur, they can lead to drastic changes such as:

- The collapse of the black hole.
- **Emission of large amounts of energy. (Gravitational Waves, FRBs)**
- The formation of other objects, like a naked singularity.

In practical terms, if black holes were inherently unstable under common conditions we might not observe black holes in the universe in the same way we do. Understanding stability is thus crucial to confirming whether theoretical models of black holes match what is observed in reality.

In this paper they related the found instability (when $l=0$, $q=0$, $\mu=0$) with a phenomenon called:

Superradiance

which occurs when waves (electromagnetic, gravitational, or scalar waves) are amplified as they interact with the black hole. When conditions are right, the energy of these waves can increase as they scatter, extracting rotational or electromagnetic energy from the object. In some cases, this amplification process can lead to what is called **instabilities**, where the object becomes unstable under continuous amplification of these waves.

What’s the connection to our group?

- If waves are super radiantly amplified near a black hole, this process could lead to a highly energetic burst of radiation.
- Black holes with accretion disks can develop instabilities. These instabilities can potentially cause the black hole to release sudden bursts of energy, which might be detectable as an FRB.
- When two black holes merge, the event could cause instabilities in the surrounding environment, potentially leading to FRBs.

Studying **Fast Radio Bursts (FRBs)** can potentially provide valuable information about the physical properties of the objects that produce them. Although the exact mechanisms behind FRBs are not yet fully understood.
However, we will observe them through ABDUS which will identify individual fast radio sources.

My Research Work

Instead of a scalar perturbation, our goal is to look for instabilities using fermions, which are perturbations caused by these fundamental class of particles in the Standard Model of particle physics, defined by their intrinsic properties and their half-integer spin.

Fermions include particles such as quarks and leptons, which are the building blocks of matter. For example:

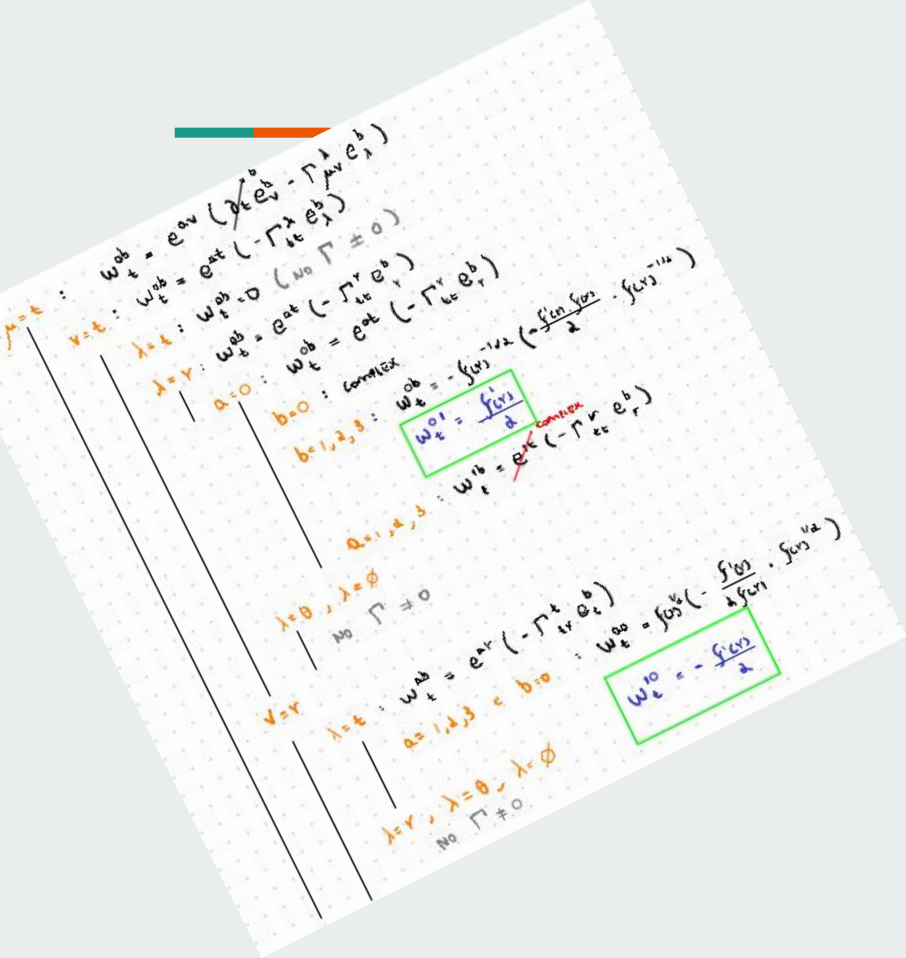
- Quarks combine to form protons and neutrons.
- Leptons include electrons, which orbit the nucleus of an atom, as well as neutrinos, which are often produced in nuclear reactions.

At the end I could NEVER scape QUANTUM MECHANICS

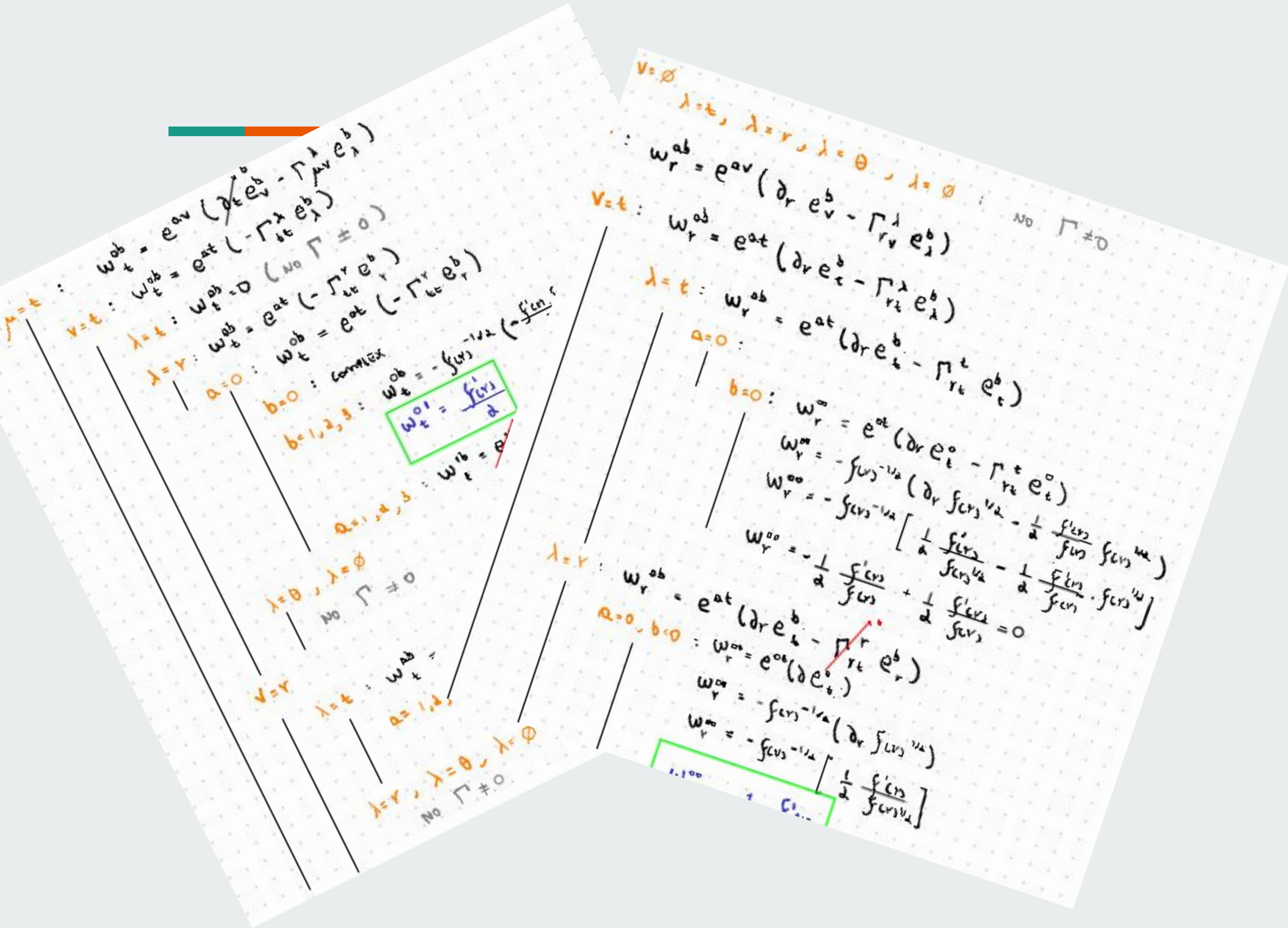
My Research Work

$$(i\hbar\gamma^{\mu}D_{\mu} - mc)\psi = 0$$

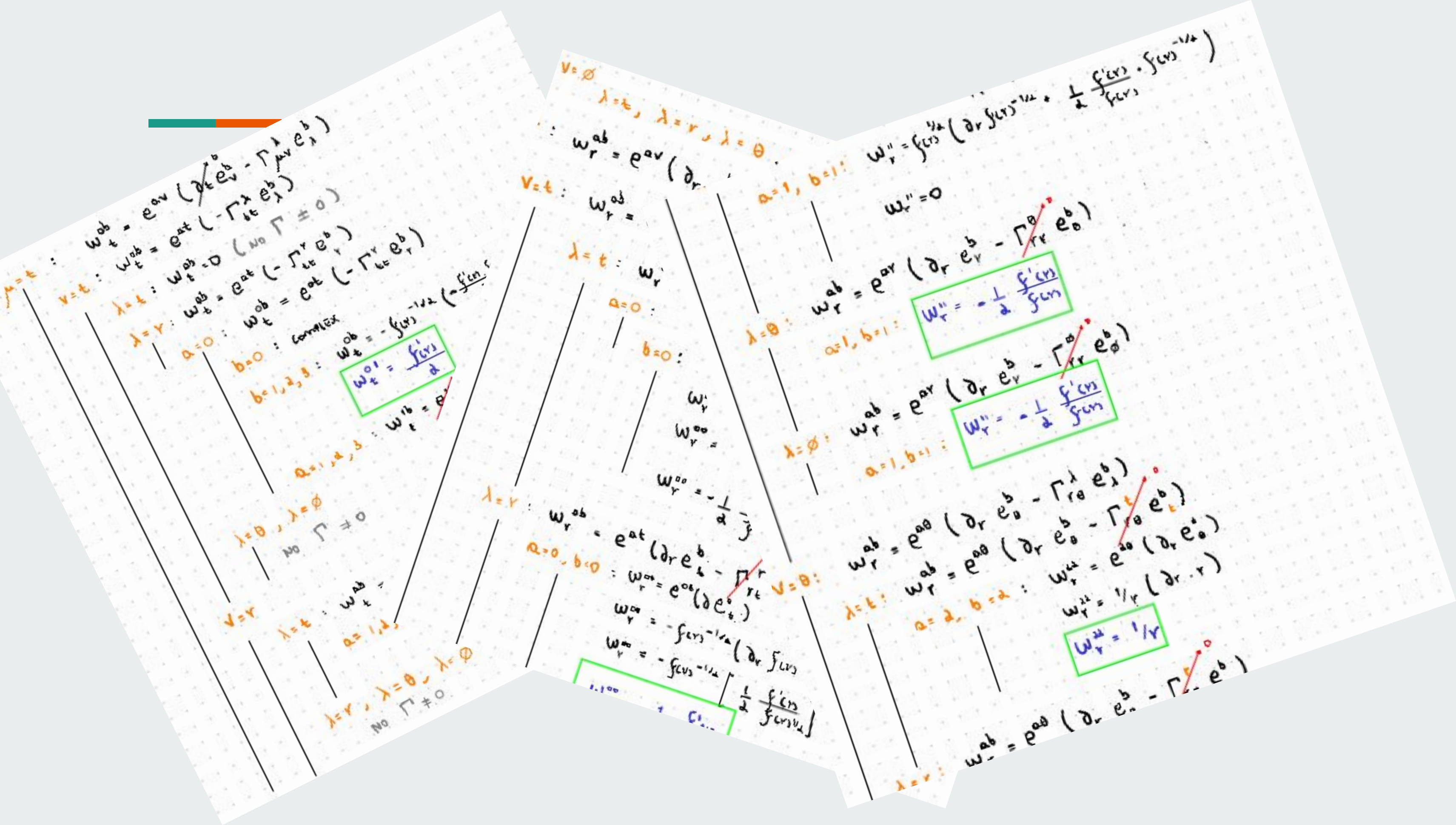
My Research Work



My Research Work



My Research Work



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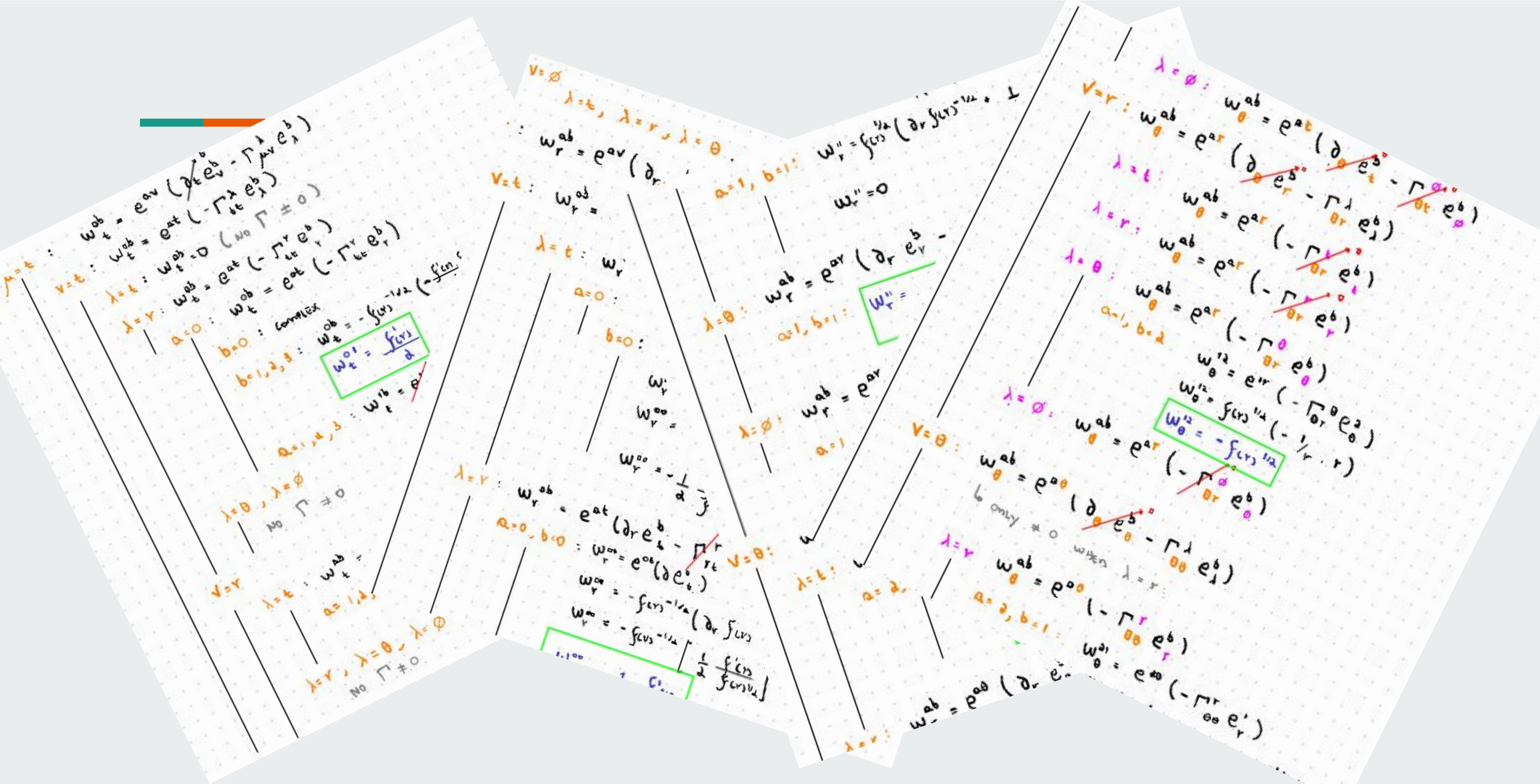

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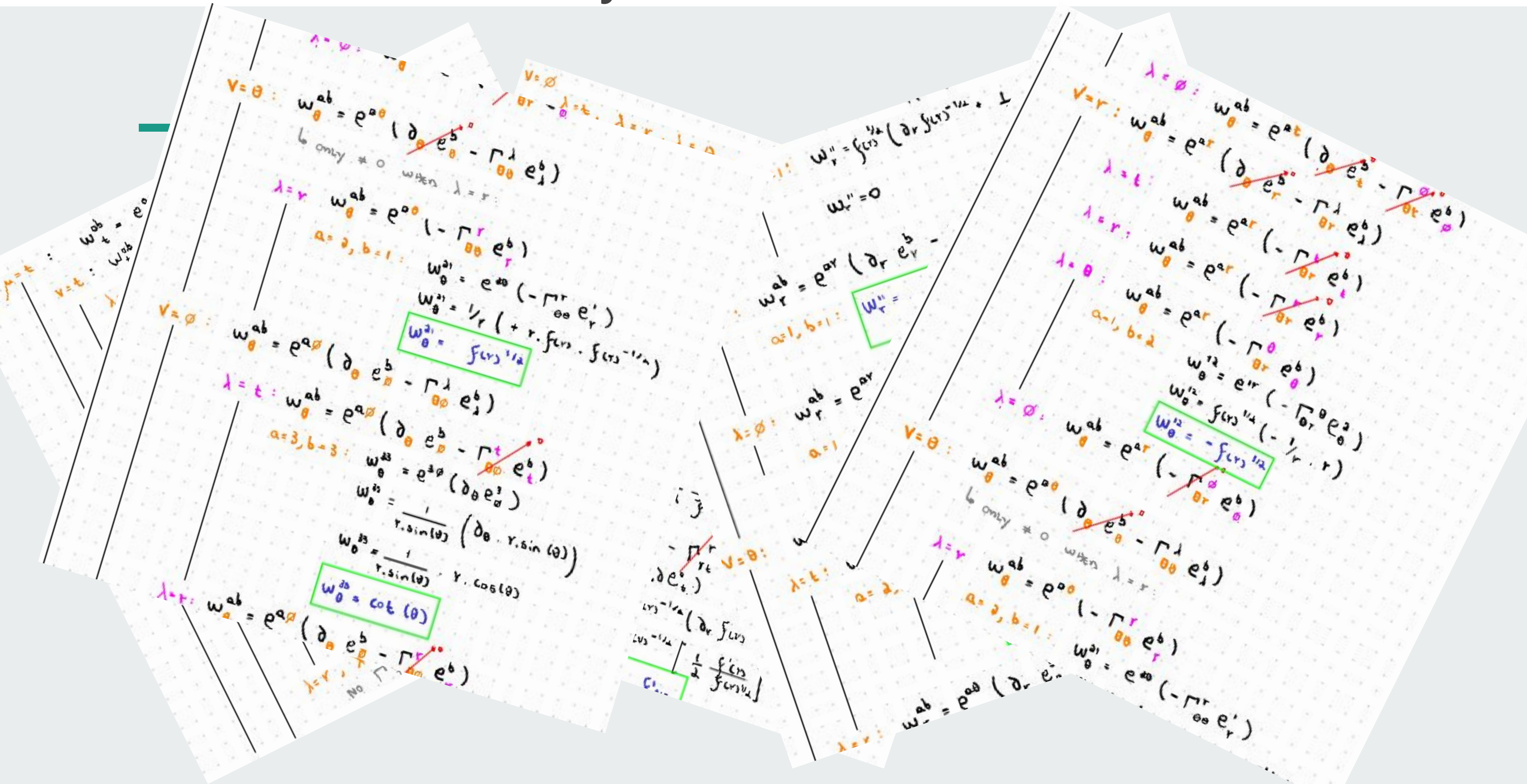

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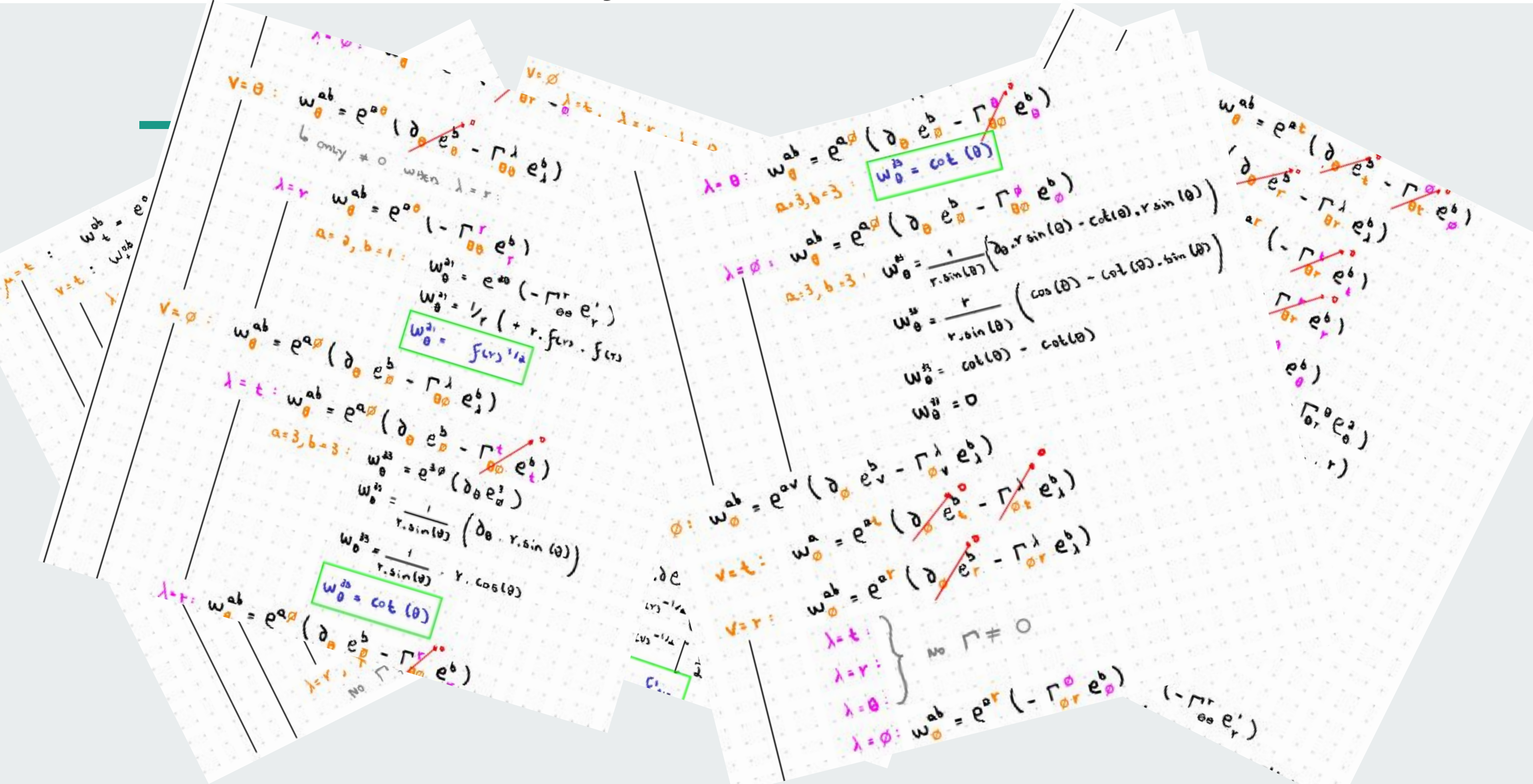
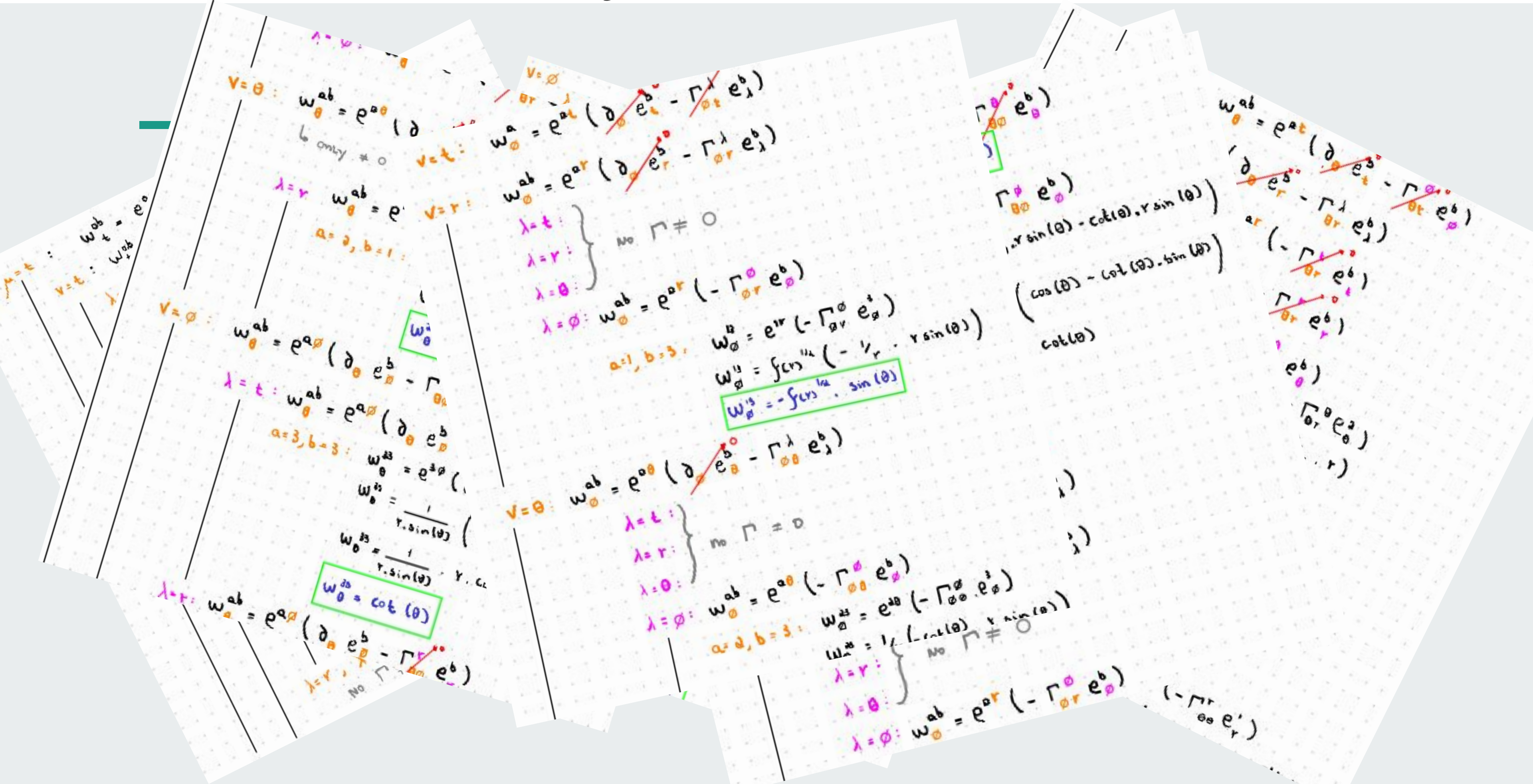


My Research Work



QUANTUM MECHANICS? GENERAL RELATIVITY?