Bispectrum Analisis

MSc. Jordany Vieira (Ph.D. Student at USP)
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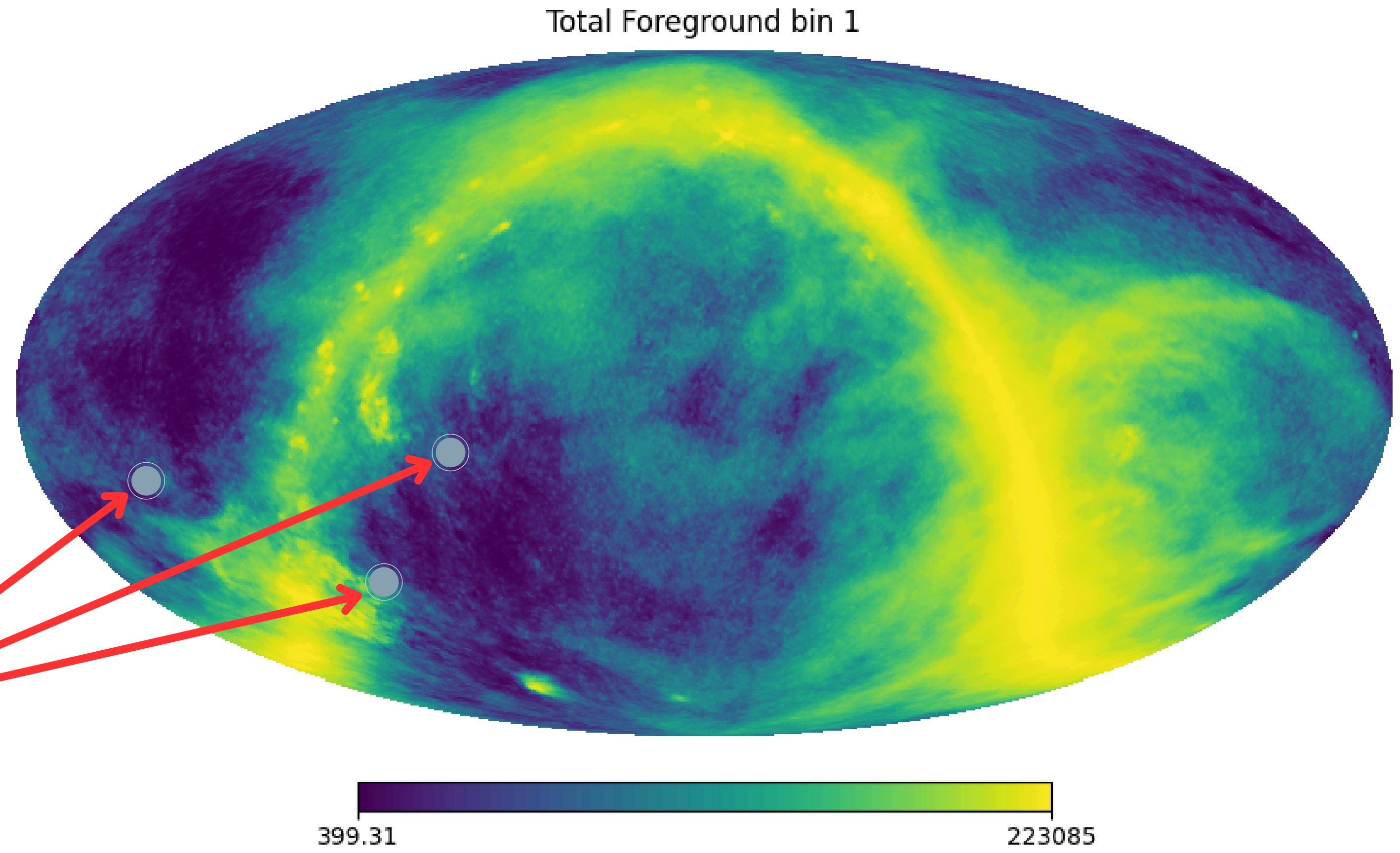
Participants: Abdalla F., J. Ojaswi, Zhang J., Sang Y., Marins A., Fornazier K., Barreto J.

Outline

1. **What is Bispectrum? Why Study?**
2. **Bispectrum Module**
3. **Results**
4. **3D-cubes to HEALPix maps**
5. **CIC Method**
6. **Results**
7. **Next Steps**

What is Bispectrum? Why Study?

- Bispectrum is the third order of the Angular n-points power spectrum.
- Study the asymmetry of distribution in statistical field theory.



Correlation between this 3 points in the sky.

- Search for traces of non-Gaussianities in 21-cm fields associated with the inflationary period.
- Study the fluctuations in the matter power spectrum.
- Use to study the interactions between Dark Energy and Dark Matter and alternative cosmology models.

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Bispectrum Module

BINGO

$$B_{\ell_1\ell_2\ell_3} = \int d\Omega M_{\ell_1}(\Omega) M_{\ell_2}(\Omega) M_{\ell_3}(\Omega)$$

Bispectrum from sky maps

$$B_{\ell_1\ell_2\ell_3} = \int d\Omega \sum_{m_1,m_2,m_3} Y_{\ell_1 m_1}(\Omega) Y_{\ell_2 m_2}(\Omega) Y_{\ell_3 m_3}(\Omega) a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}$$

After spherical harmonic transformations

$$\int d\Omega Y_{\ell_1 m_1}(\Omega) Y_{\ell_2 m_2}(\Omega) Y_{\ell_3 m_3}(\Omega) = \sqrt{N_\Delta^{\ell_1\ell_2\ell_3}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

Gaunt Integral

$$B_{\ell_1\ell_2\ell_3} = \sqrt{N_\Delta^{\ell_1\ell_2\ell_3}} \sum_{m_1,m_2,m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}$$

Bispectrum to be calculated

$$N_\Delta^{\ell_1\ell_2\ell_3} \equiv \frac{(2\ell_1+1)+(2\ell_2+1)+(2\ell_3+1)}{4\pi} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} -\sqrt{(\ell_3 \mp m_3)(\ell_3 \pm m_3 + 1)} \begin{pmatrix} \ell_1 & \ell_1 & \ell_1 \\ m_1 & m_2 & m_3 \pm 1 \end{pmatrix} = \\ \sqrt{(\ell_1 \mp m_1)(\ell_1 \pm m_1 + 1)} \begin{pmatrix} \ell_1 & \ell_1 & \ell_1 \\ m_1 \pm 1 & m_2 & m_3 \end{pmatrix} + \\ + \sqrt{(\ell_2 \mp m_2)(\ell_2 \pm m_2 + 1)} \begin{pmatrix} \ell_1 & \ell_1 & \ell_1 \\ m_1 & m_2 \pm 2 & m_3 \end{pmatrix} \end{aligned}$$

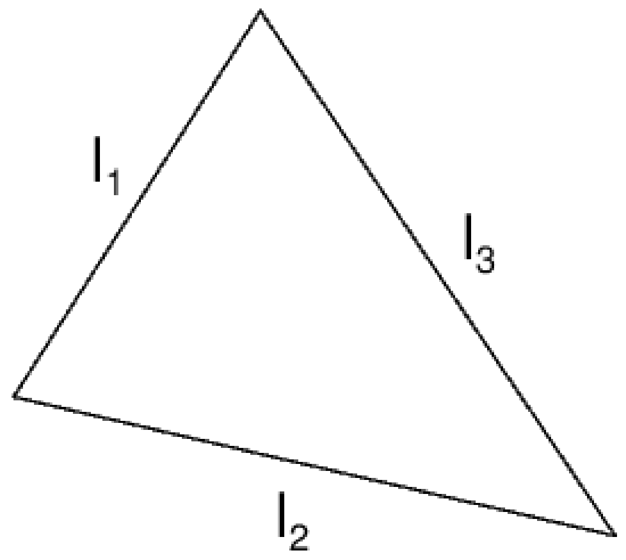
$$Cw = v$$

Known Wigner-3j

Matrix of Wigner-3j coefficients

Wigner-3j to be calculated

To solve the matrix equation we use LU decomposition.



Bispectrum constraints

$$|\ell_1 - \ell_2| \leq \ell_3 \leq \ell_1 + \ell_2$$

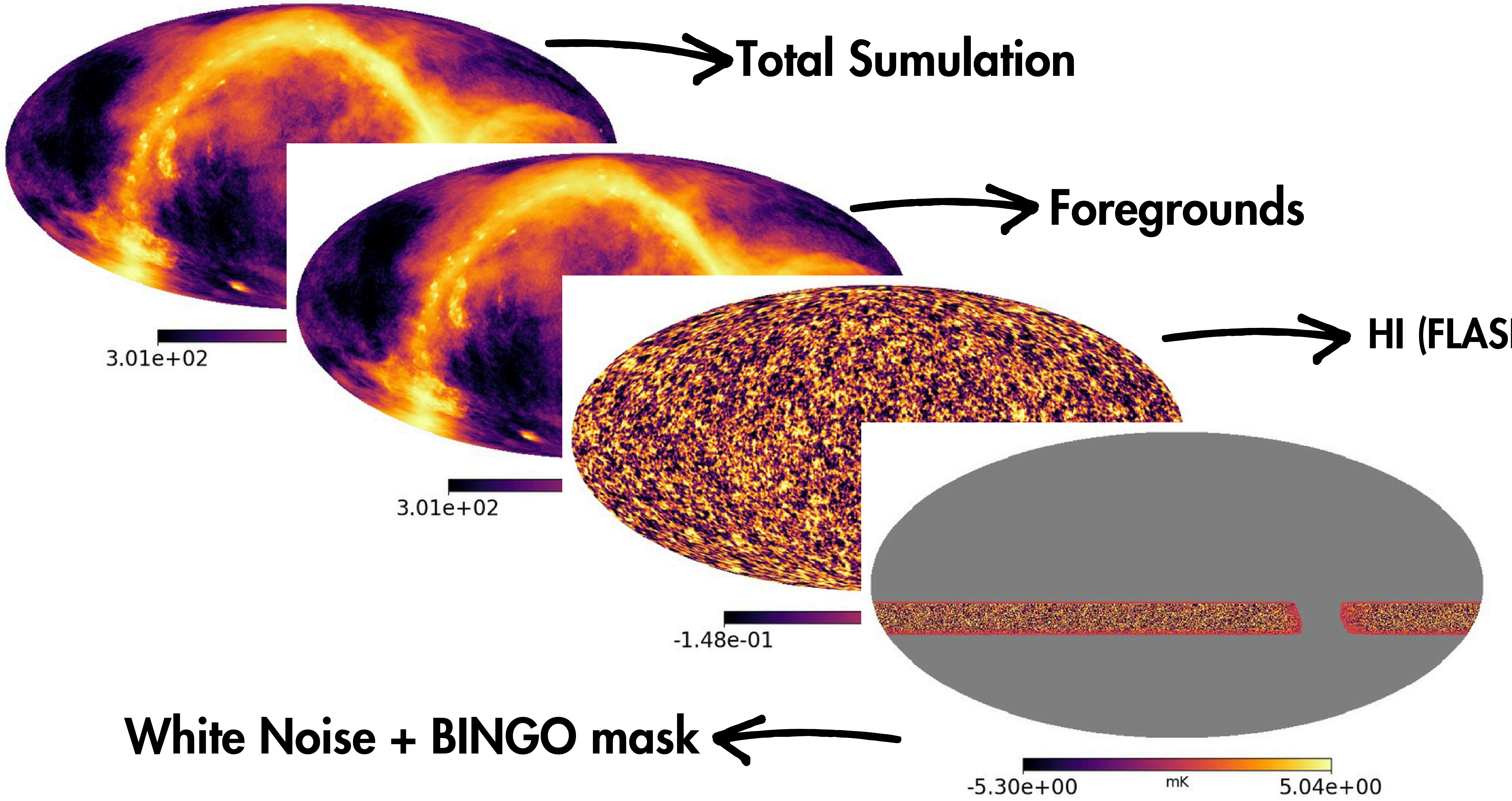
triangle inequality

$\ell_1 + \ell_2 + \ell_3$ **even**

parity invariance

$$m_1 + m_2 + m_3 = 0$$

rotation invariance


Total Sumulation
Foregrounds
HI (FLASK ans N-body)
White Noise + BINGO mask
3.01e+02
3.01e+02
-1.48e-01
-5.30e+00
mK
5.04e+00

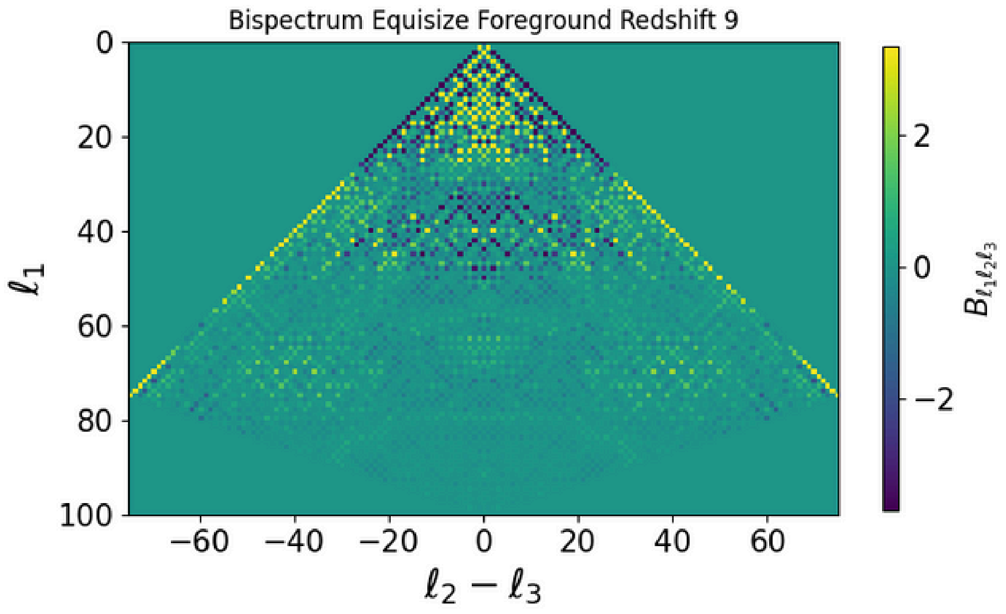
We study 3 cases in bispectrum, which imposes new constraints

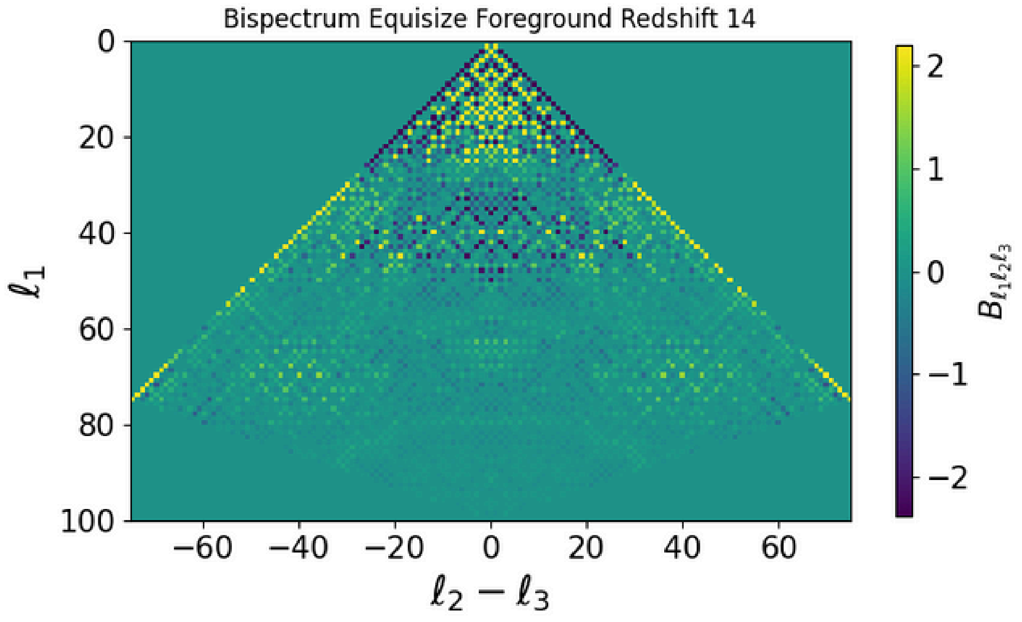
Equisize $\longrightarrow$ $\ell_1 + \ell_2 + \ell_3 = \ell_0$ $\longrightarrow$ $\ell_{max} = 300$, $\ell_0 = 300$

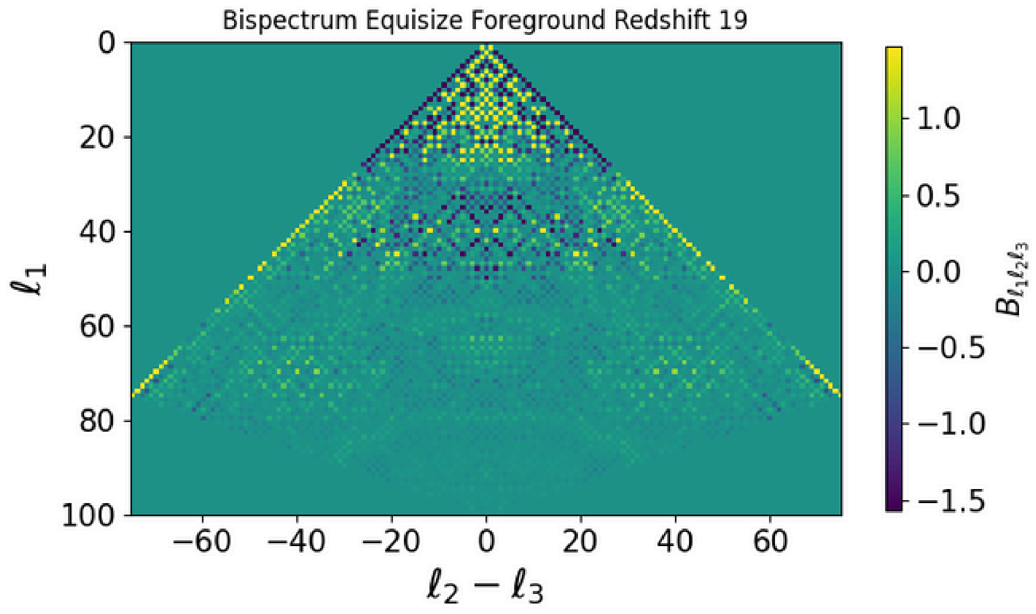
Isosceles $\longrightarrow$ $\ell_1 = \ell_2 \leq \ell_3$ $\longrightarrow$ $\ell_{max} = 150$

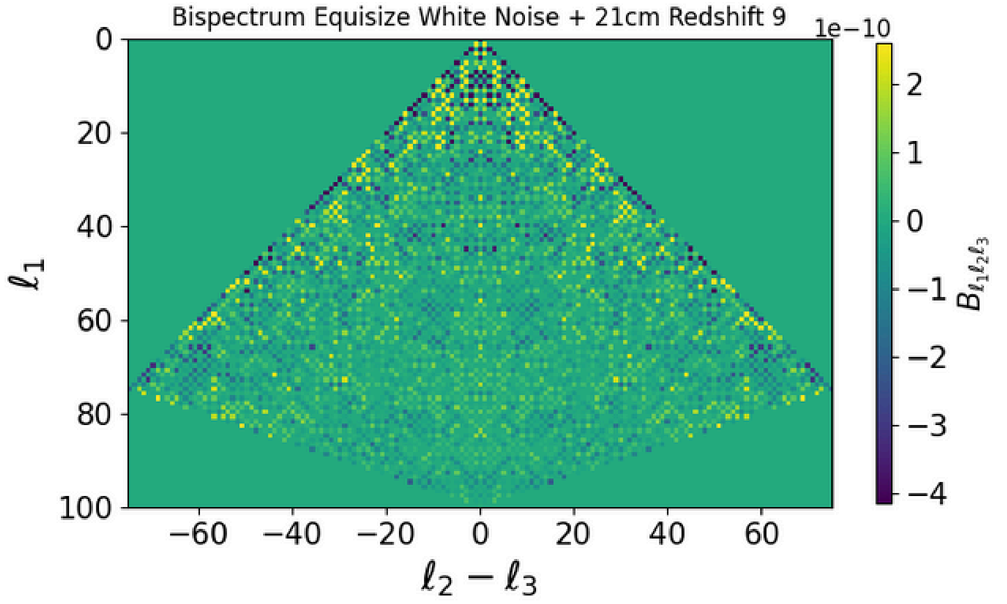
Equilateral $\longrightarrow$ $\ell_1 = \ell_2 = \ell_3$ $\longrightarrow$ $\ell_{max} = 300$

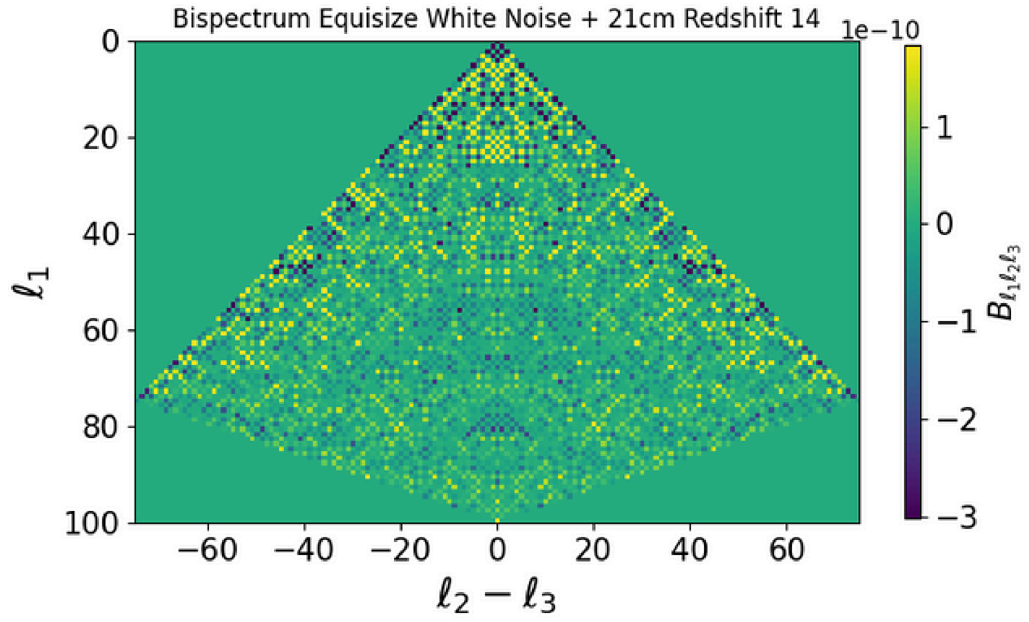
Results

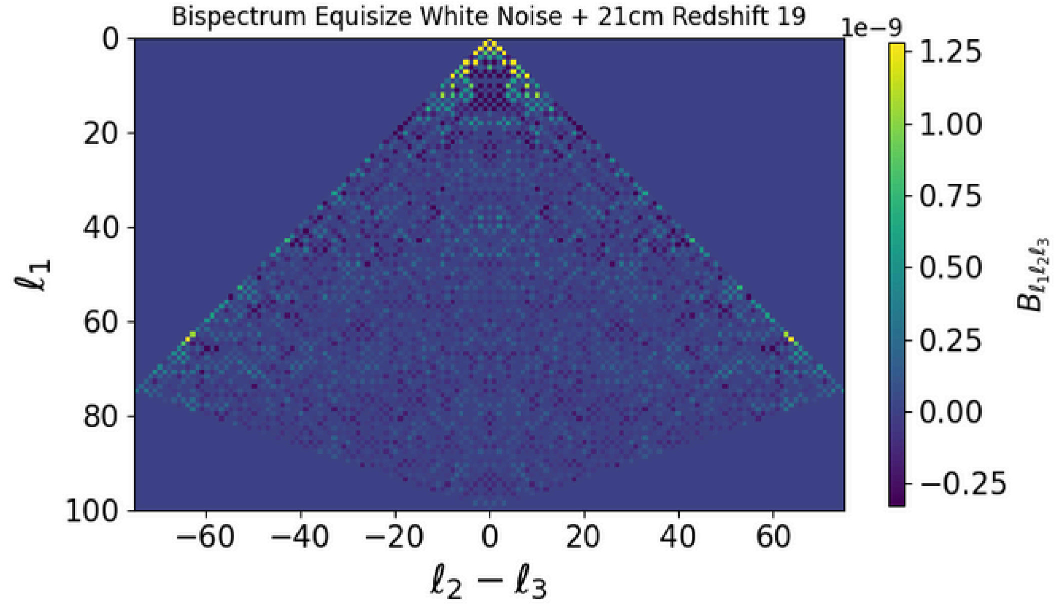

Bispectrum Equisize Foreground Redshift 9
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$

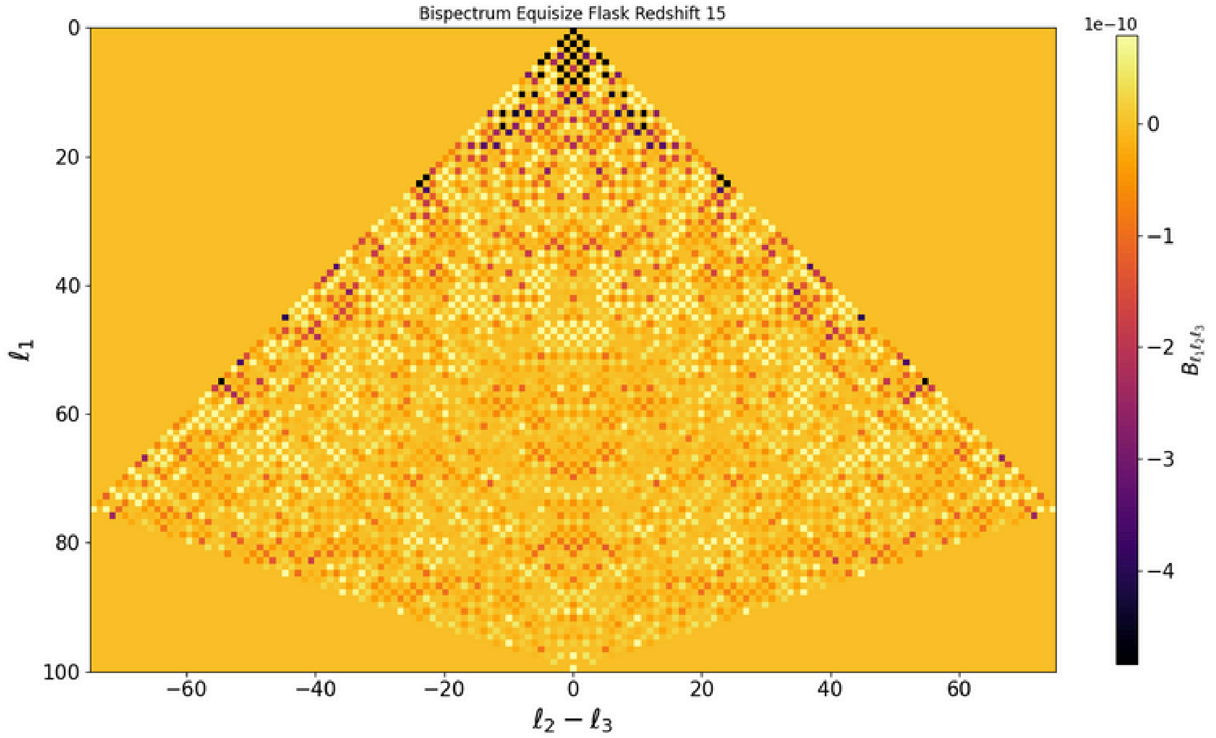

Bispectrum Equisize Foreground Redshift 14
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$

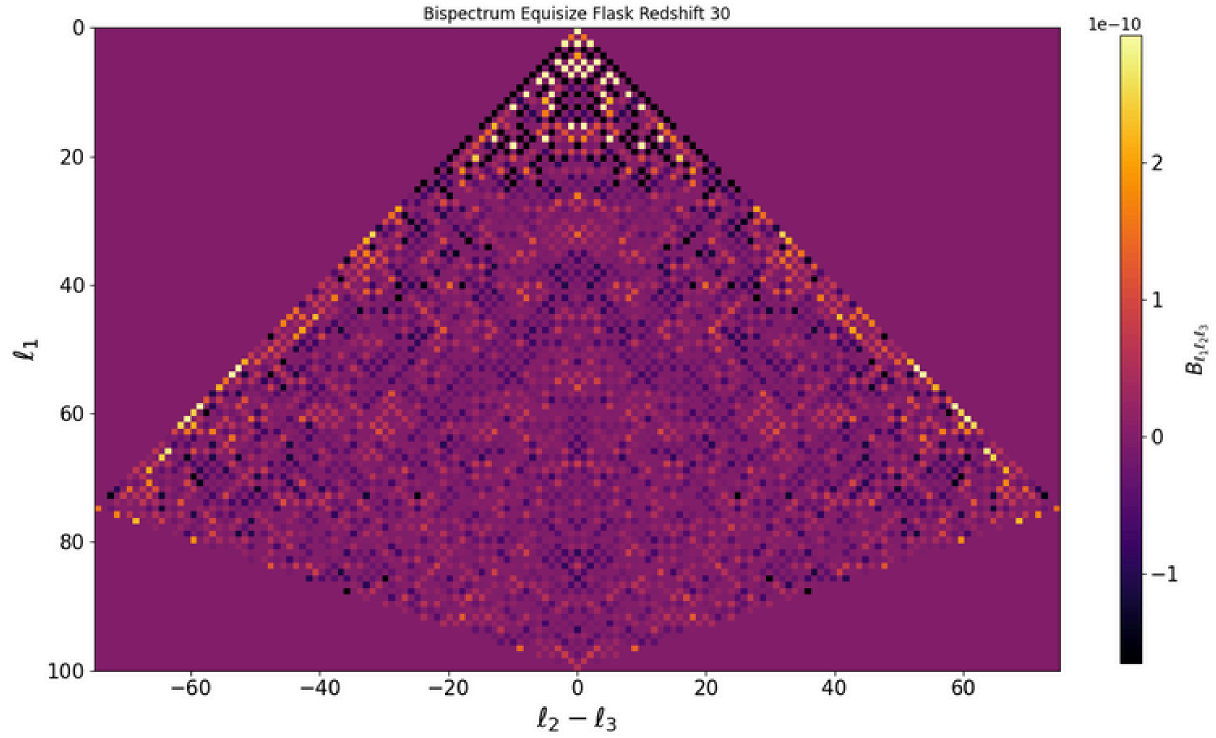

Bispectrum Equisize Foreground Redshift 19
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$

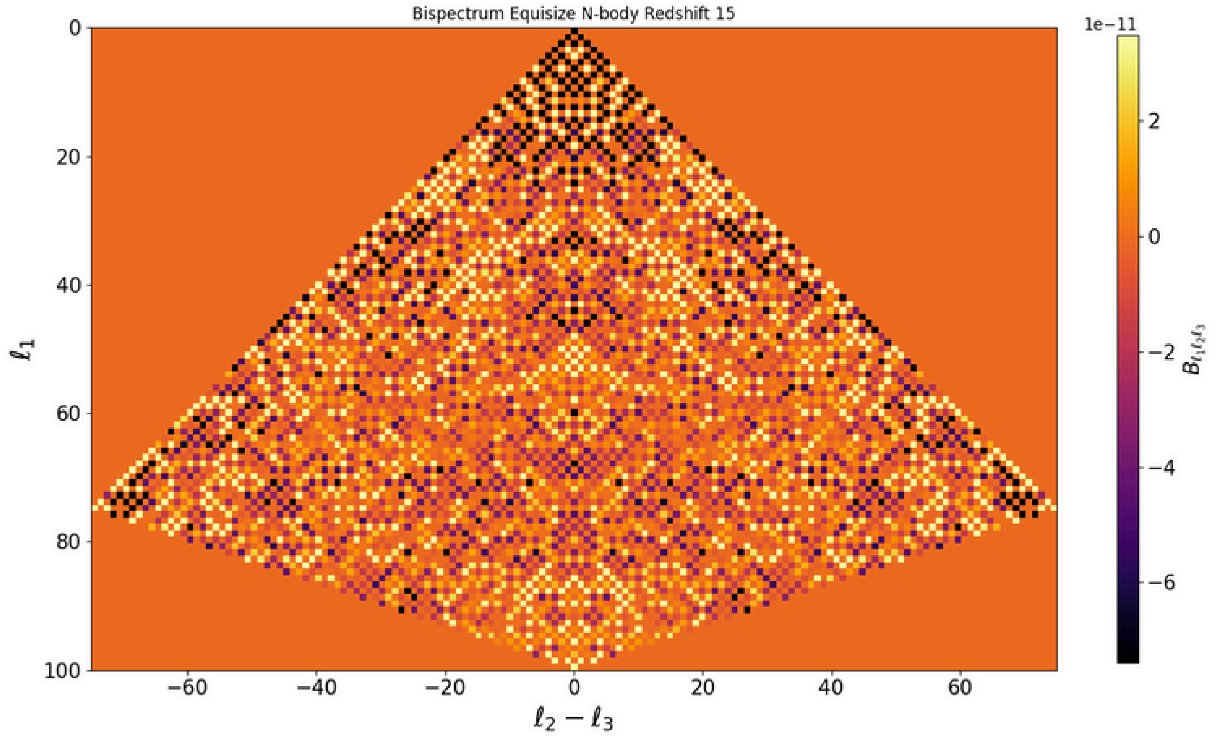

Bispectrum Equisize White Noise + 21cm Redshift 9
1e−10
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$

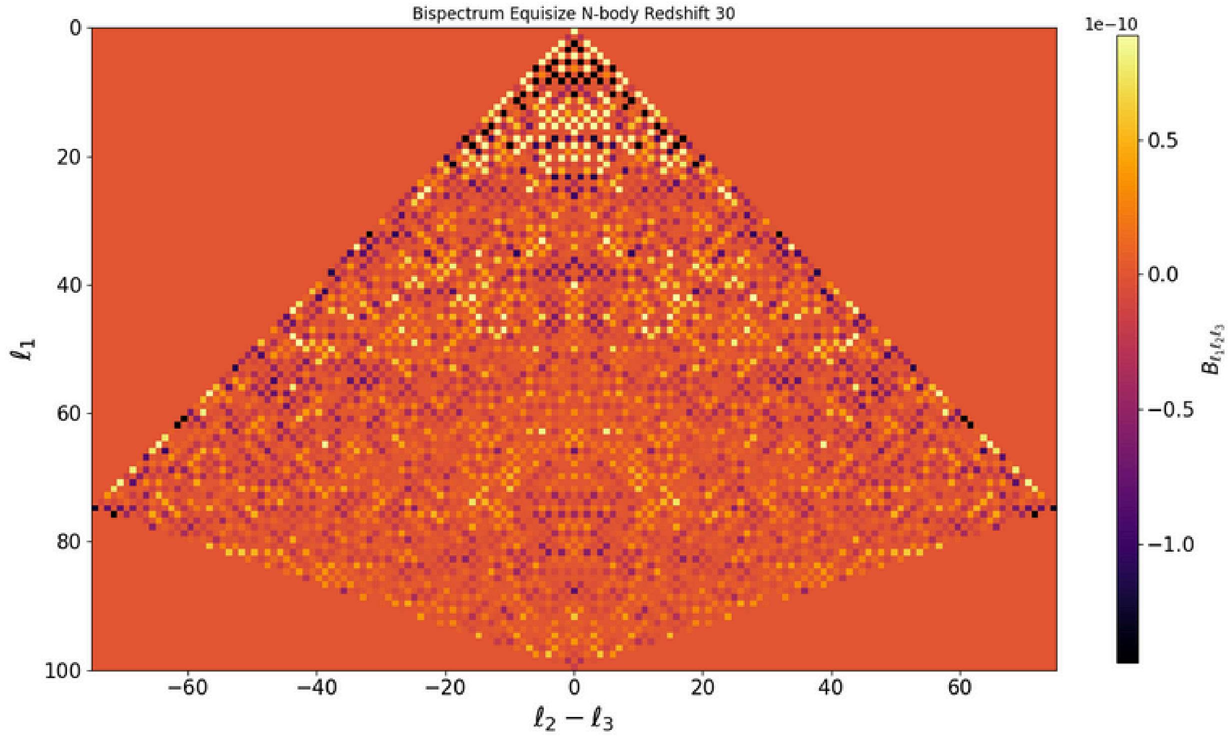

Bispectrum Equisize White Noise + 21cm Redshift 14
1e−10
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$


Bispectrum Equisize White Noise + 21cm Redshift 19
1e−9
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$


Bispectrum Equisize Flask Redshift 15
ℓ_1
$\ell_2 - \ell_3$
1e−10
$B_{\ell_1\ell_2\ell_3}$
0
20
40
60
80
100
−60
−40
−20
0
20
40
60
0
−1
−2
−3
−4


Bispectrum Equisize Flask Redshift 30
ℓ_1
$\ell_2 - \ell_3$
1e−10
$B_{\ell_1\ell_2\ell_3}$
0
20
40
60
80
100
−60
−40
−20
0
20
40
60
2
1
0
−1


Bispectrum Equisize N-body Redshift 15
ℓ_1
$\ell_2 - \ell_3$
1e−11
$B_{\ell_1\ell_2\ell_3}$
0
20
40
60
80
100
−60
−40
−20
0
20
40
60
2
0
−2
−4
−6


Bispectrum Equisize N-body Redshift 30
ℓ_1
$\ell_2 - \ell_3$
1e−10
$B_{\ell_1\ell_2\ell_3}$
0
20
40
60
80
100
−60
−40
−20
0
20
40
60
0.5
0.0
−0.5
−1.0

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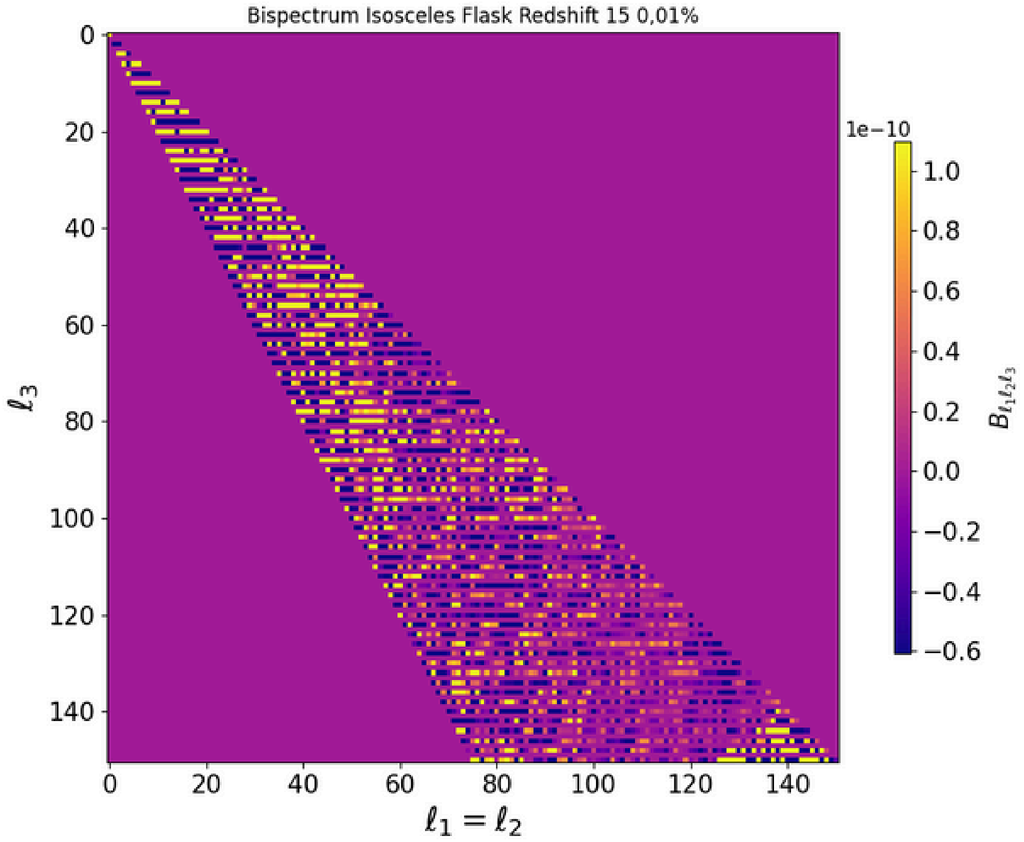

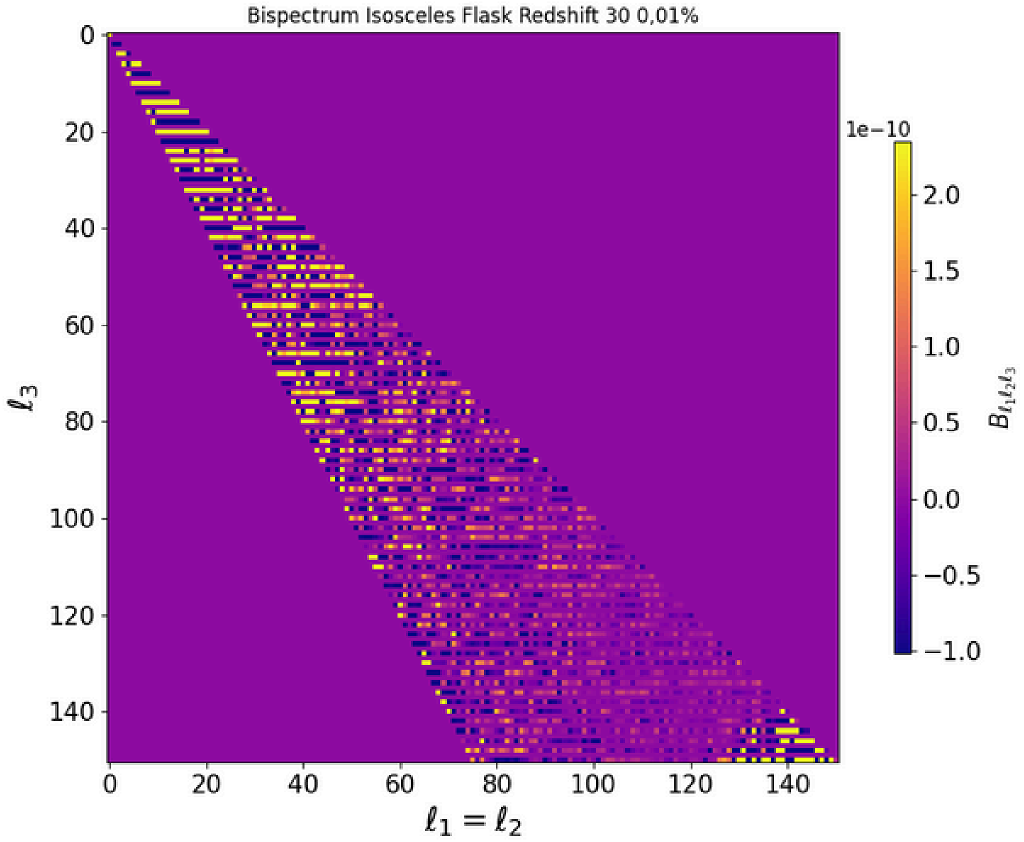

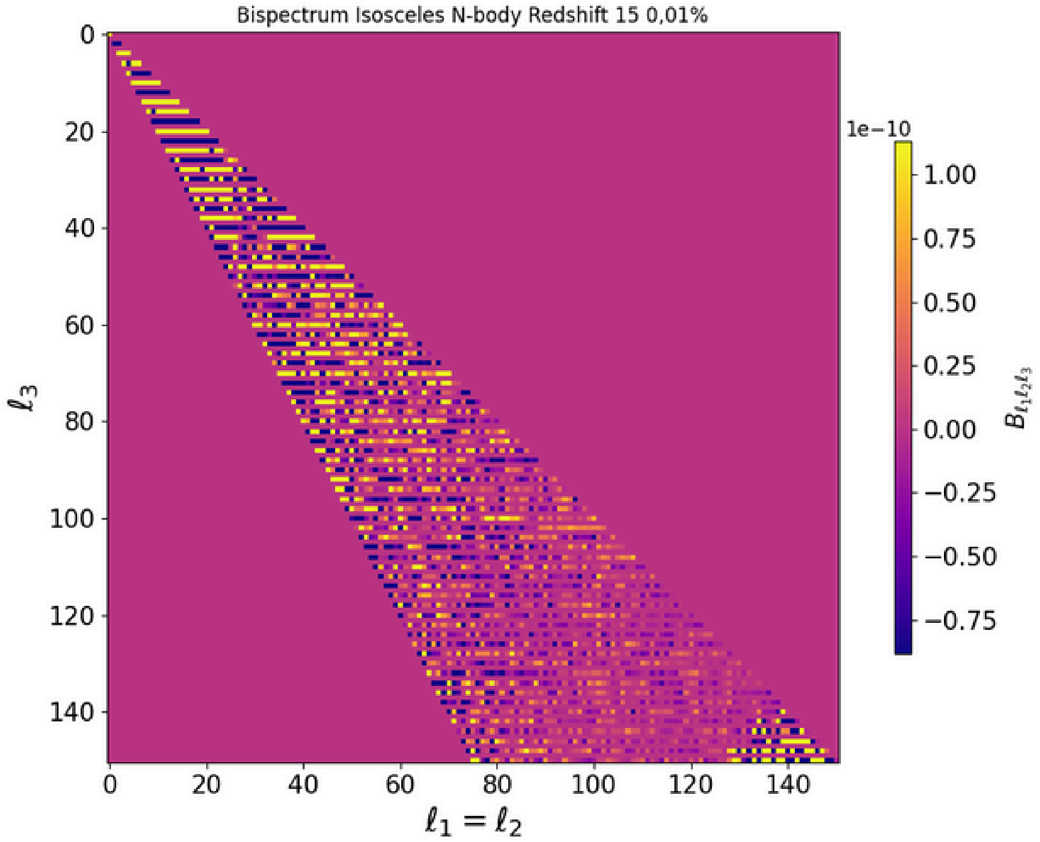

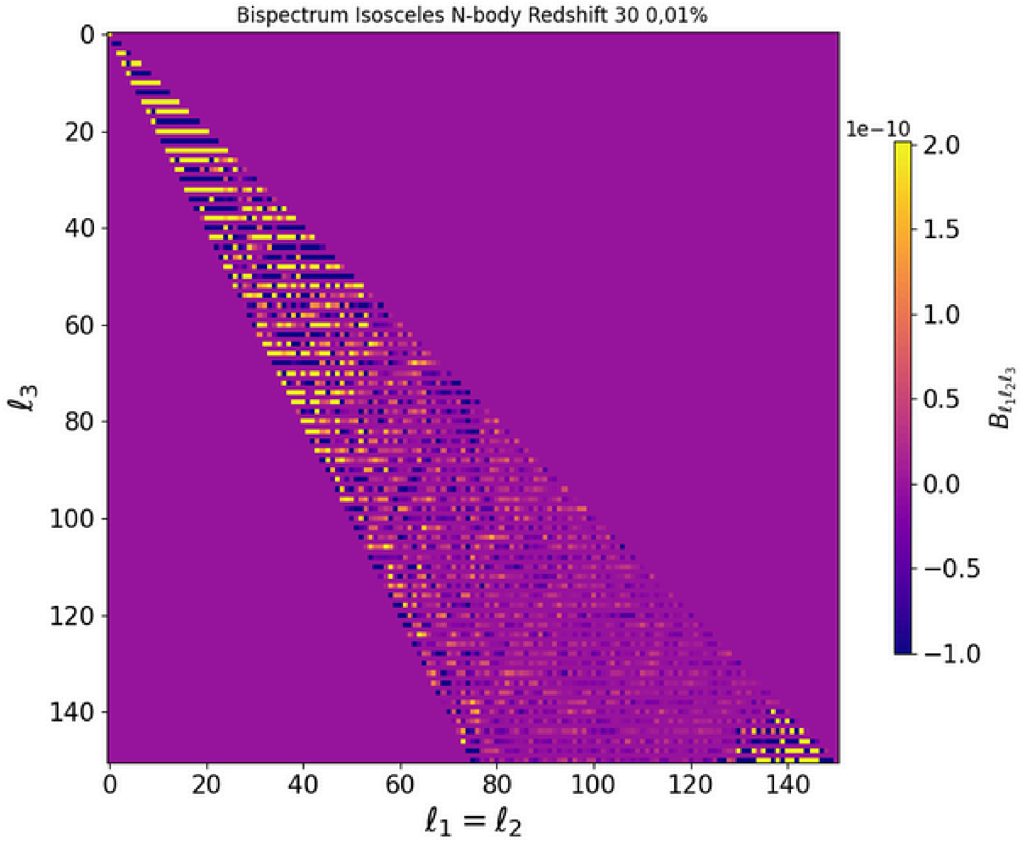

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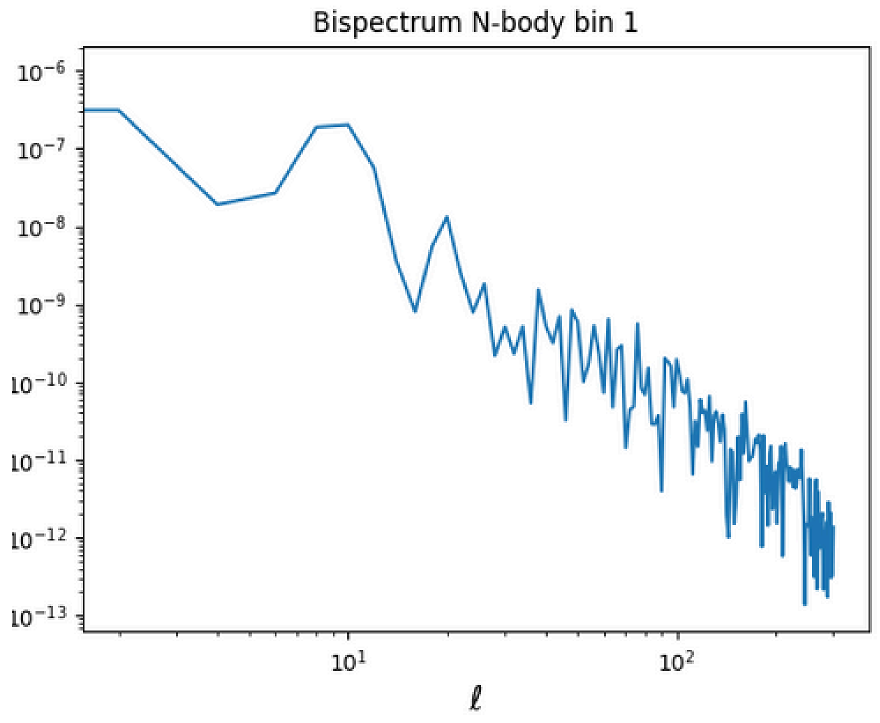
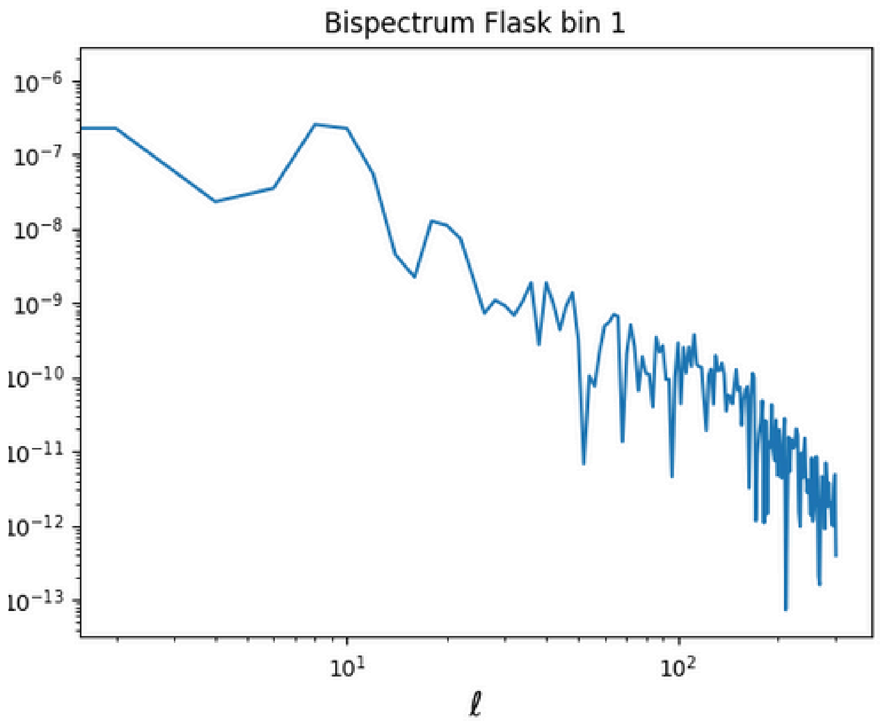
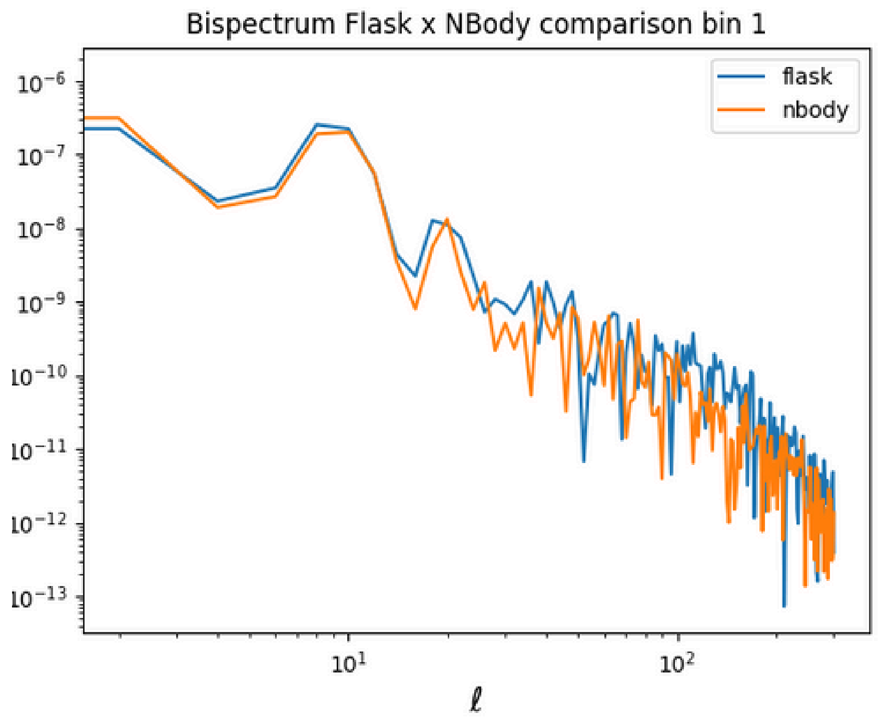
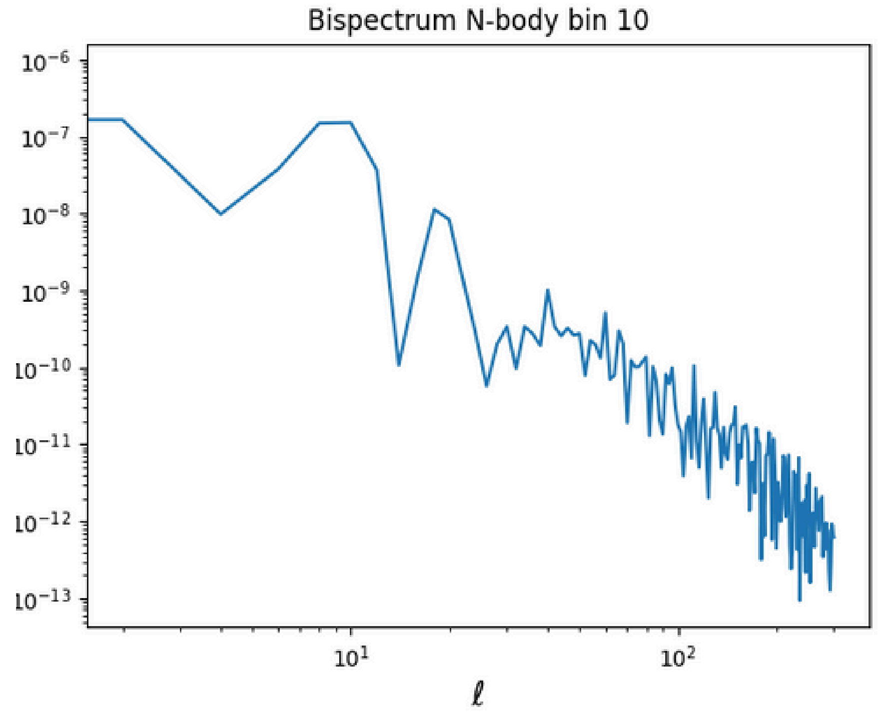
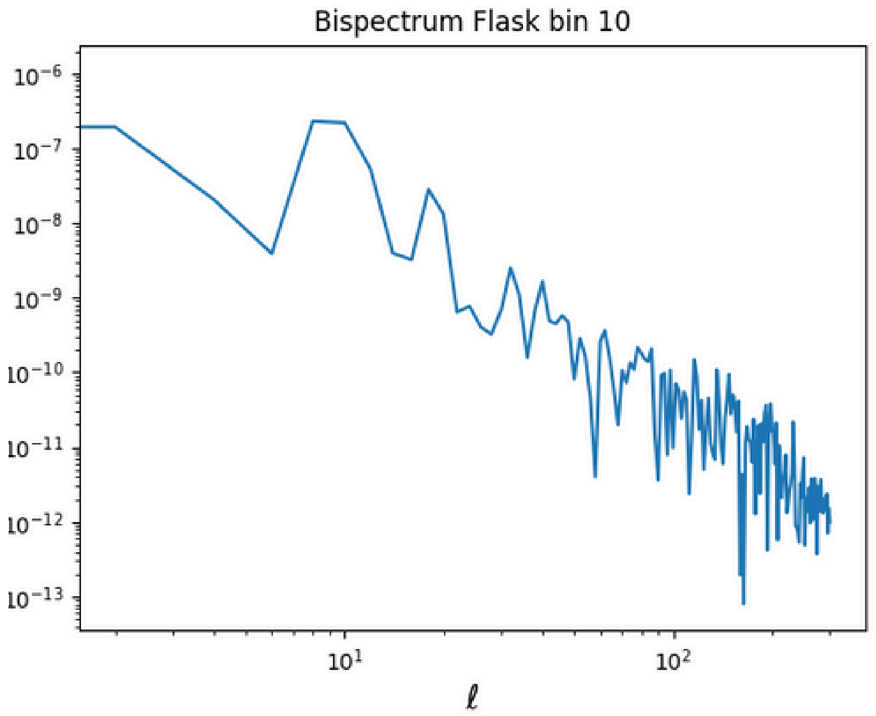
BINGO

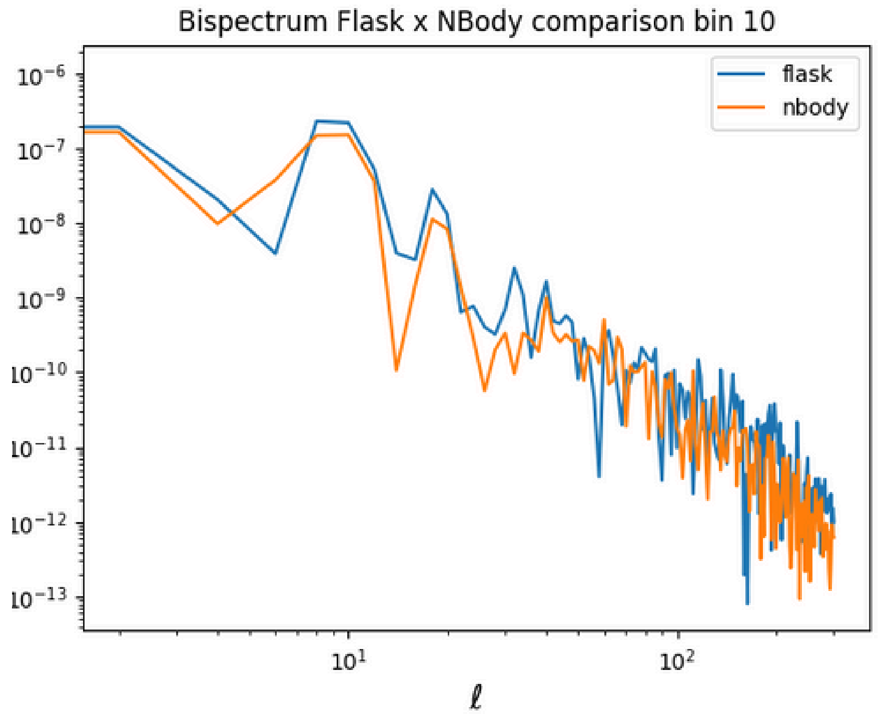


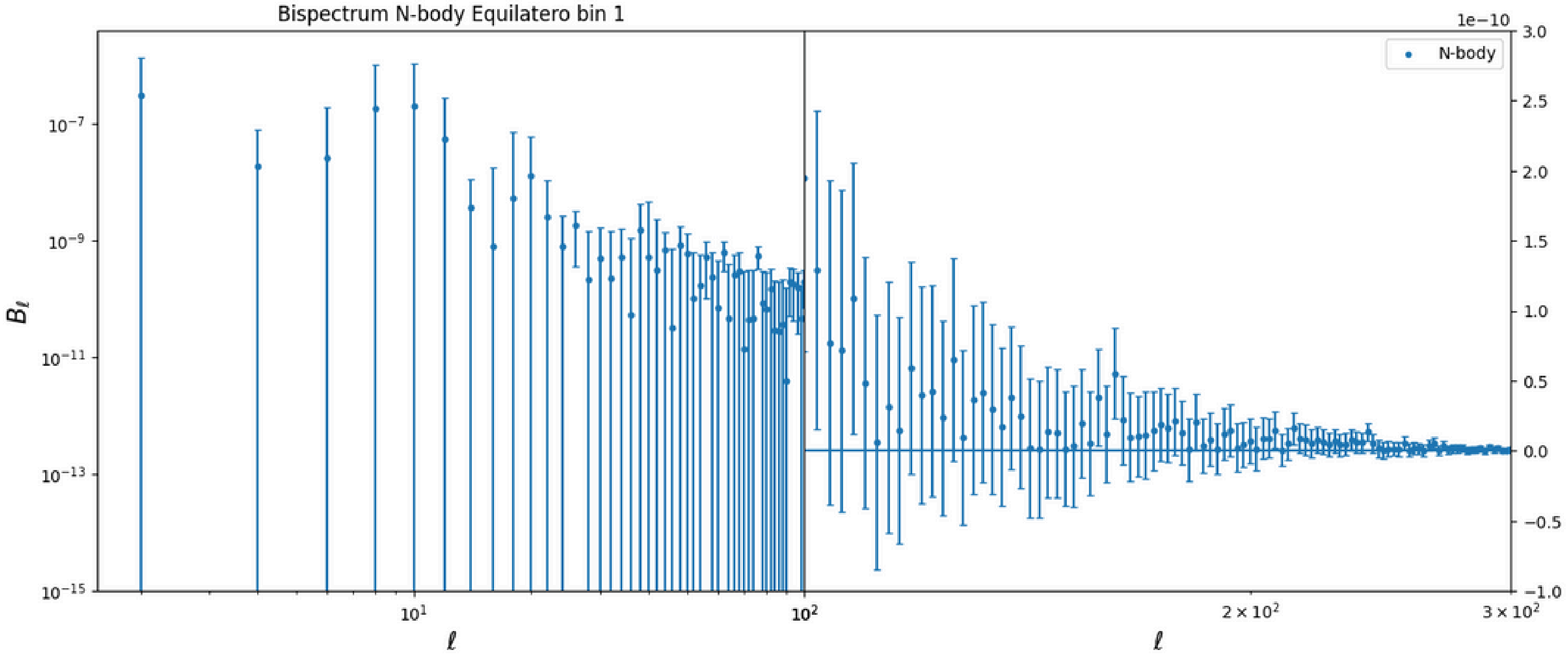


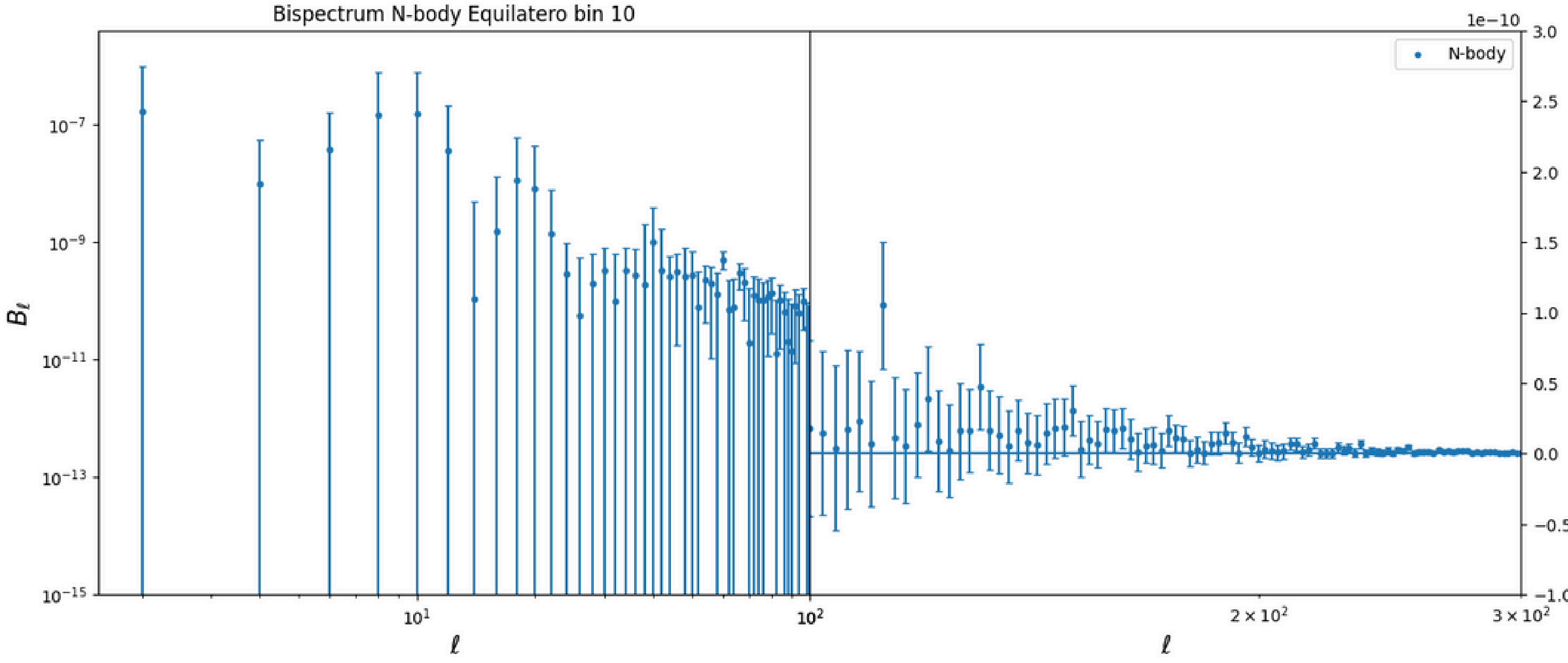


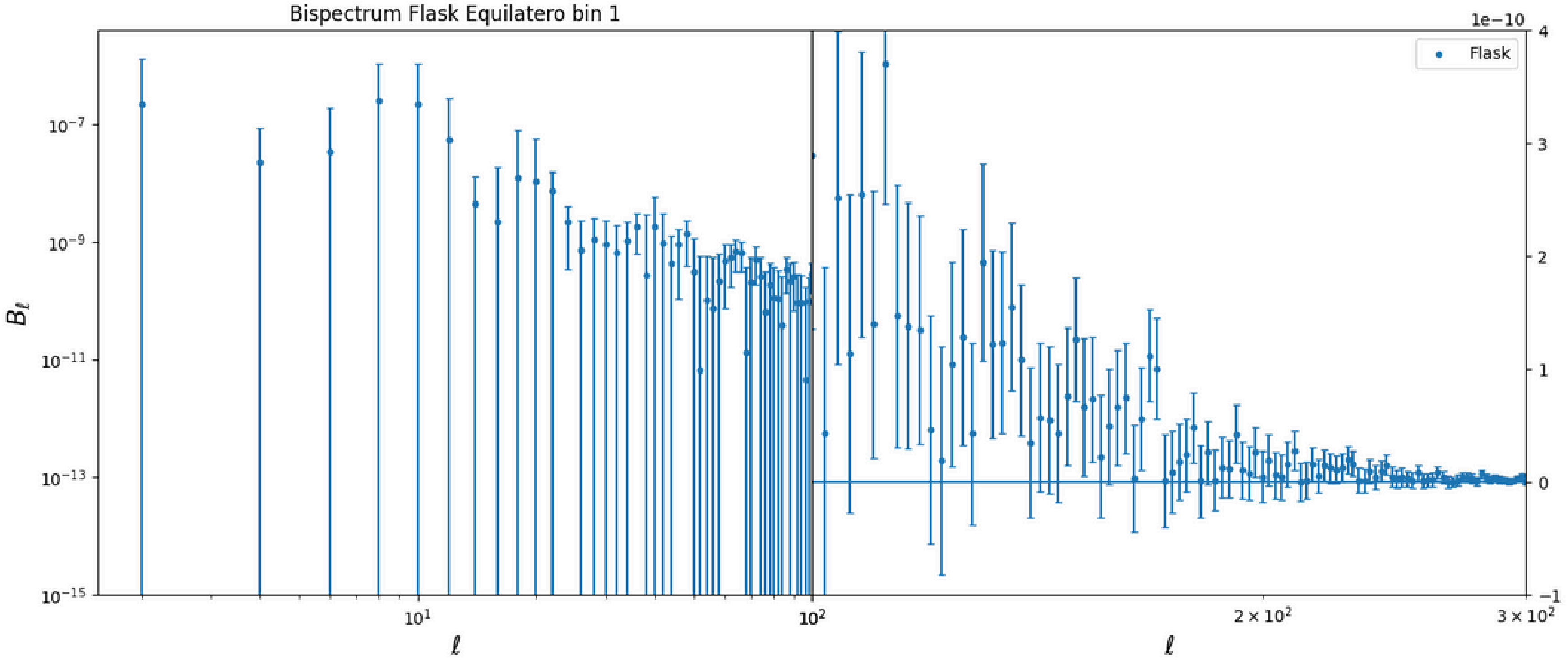


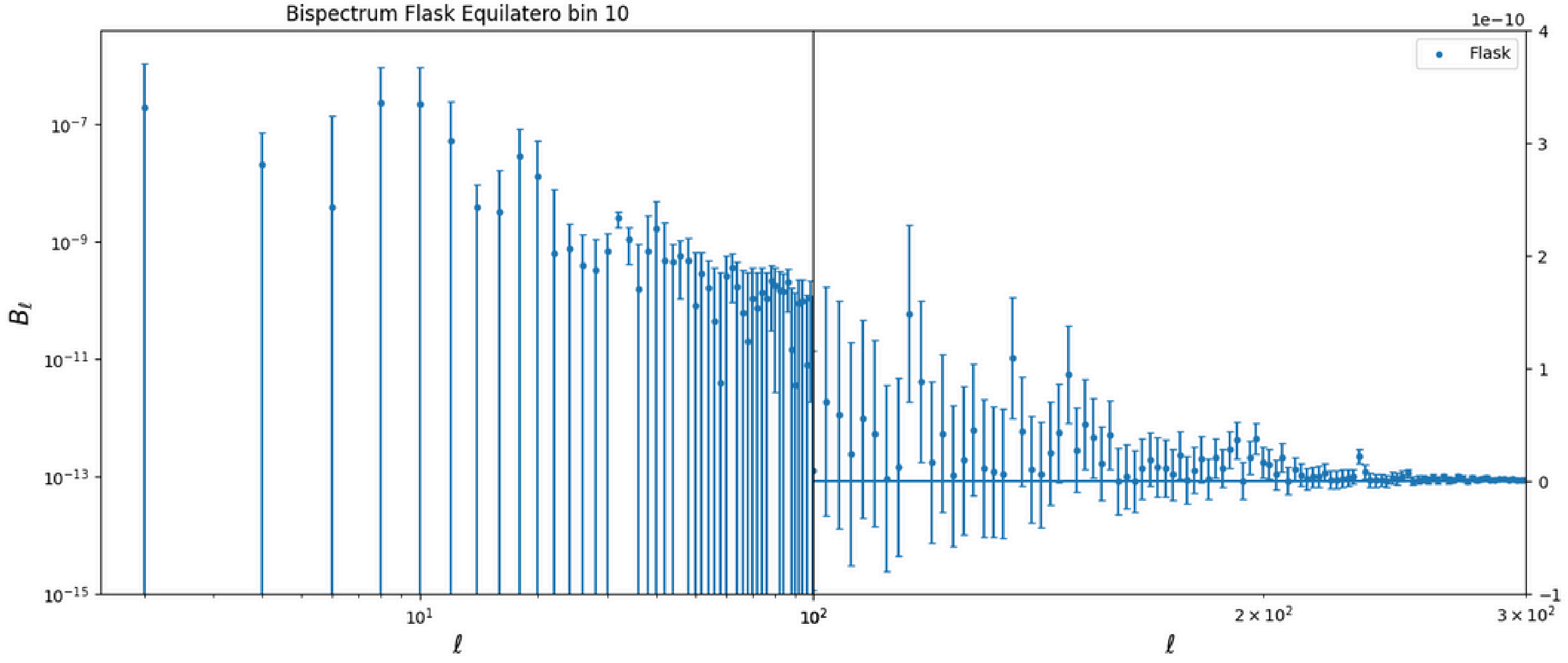




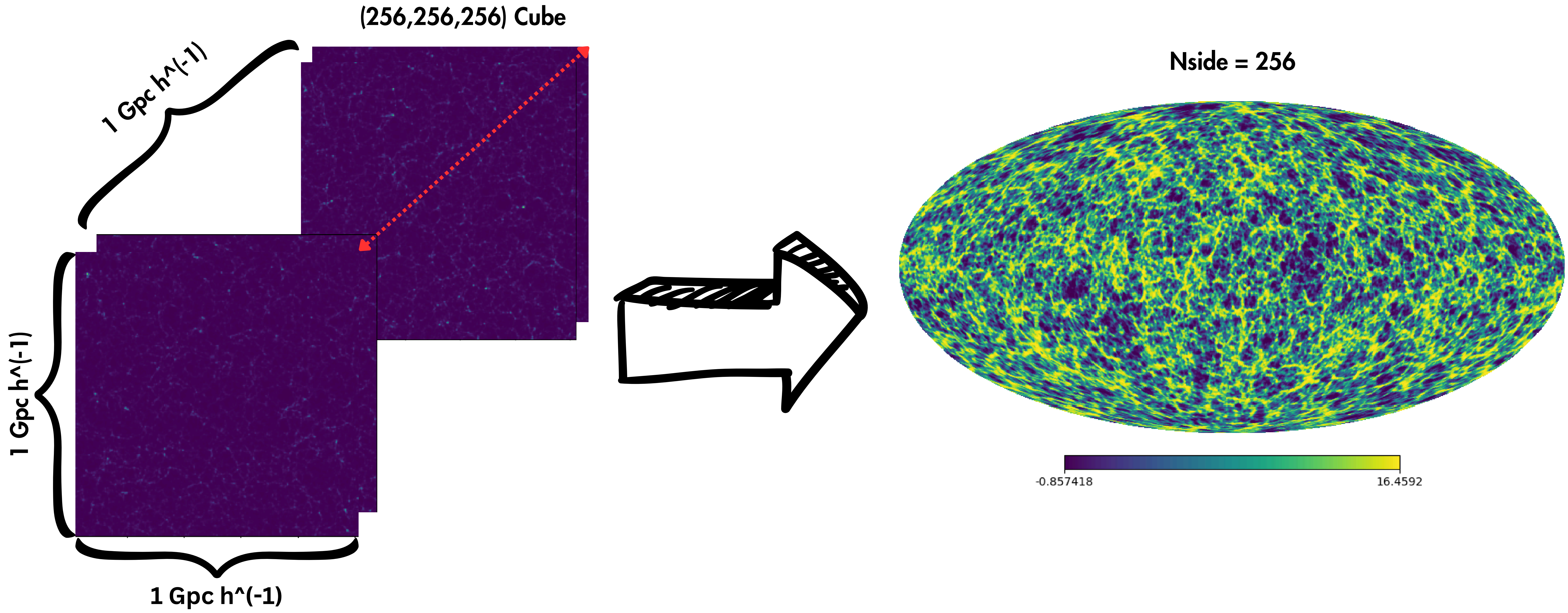

Bispectrum N-body Equilatero bin 1
N-body
B_ℓ
ℓ
1e−10


Bispectrum N-body Equilatero bin 10
N-body
B_ℓ
ℓ
1e−10


Bispectrum Flask Equilatero bin 1
Flask
B_ℓ
ℓ
1e−10


Bispectrum Flask Equilatero bin 10
Flask
B_ℓ
ℓ
1e−10

3D-cubes to HEALPix maps



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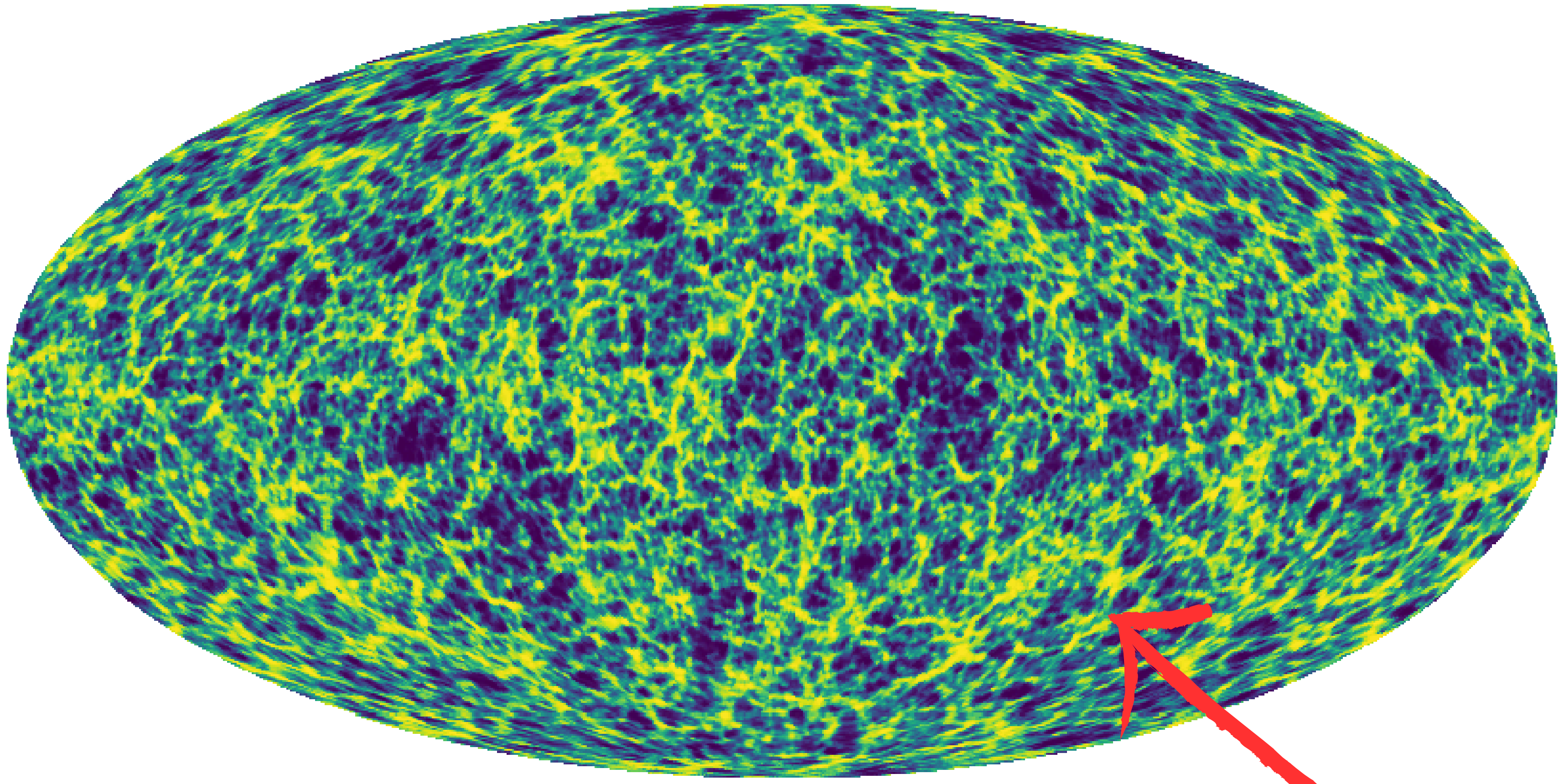
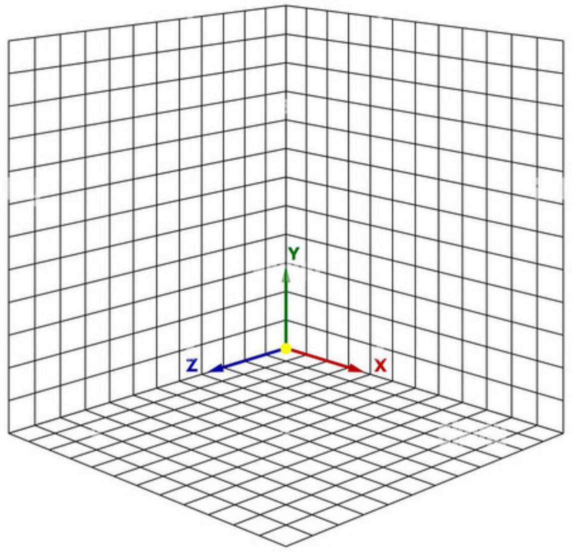




(x, y, z)

Transform the each pixel position in 3D grid position

The Algorithm

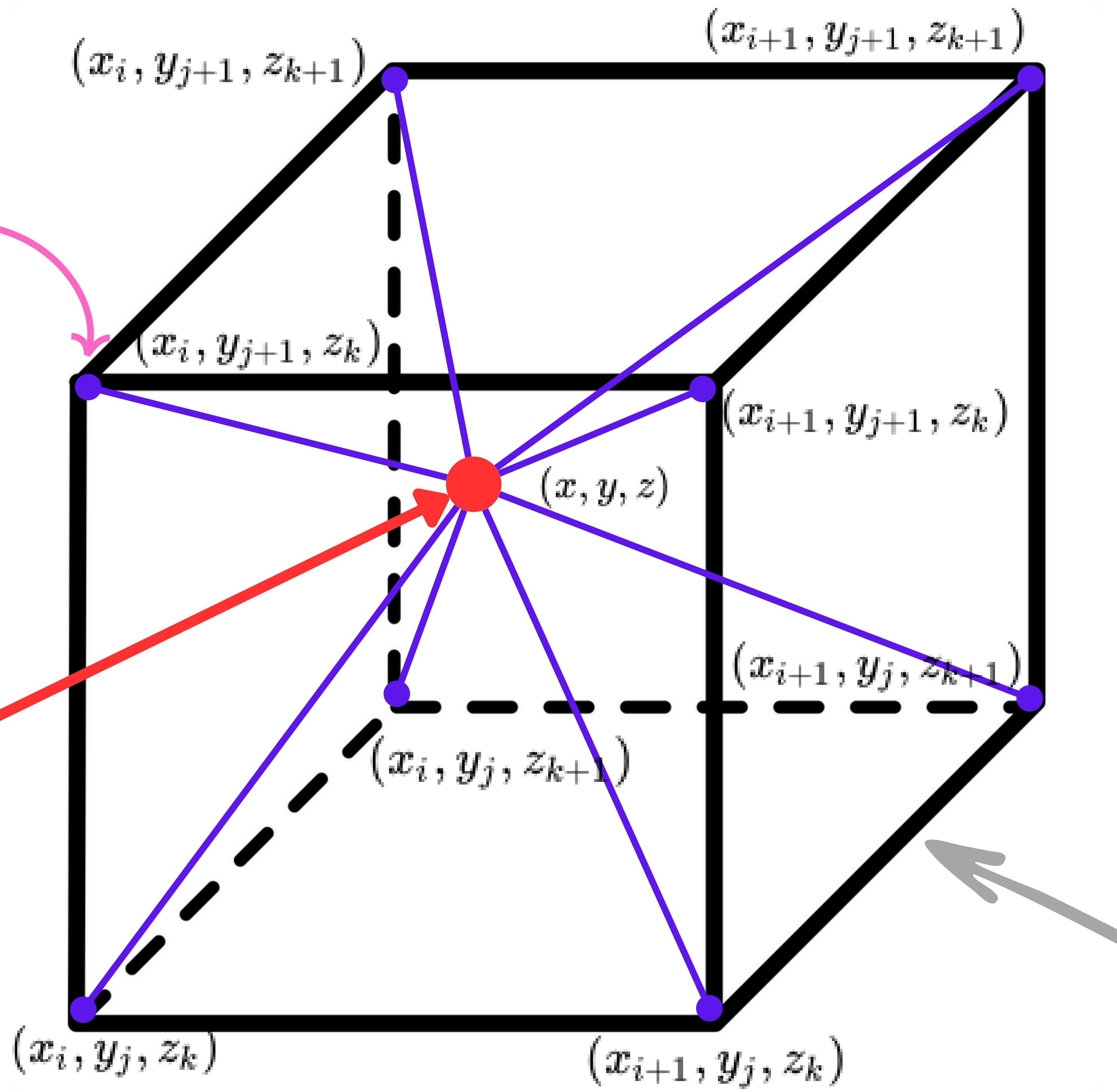
1. **Choose the bin Distance (Mpc.h^(-1))**
2. **Number of Slices (1, 2, 5 and 15)**
3. **Calculate the vector position with distances**
4. **Use a CIC method to get the information in the cube**
5. **Put the information in the pixel vector**
6. **Take the mean between each slice produced**
7. **Create the map with the pixel vextor of the mean**

CIC Method

The data information is in the corners of the voxel

Pix vector in the cube

The voxel



$$(x_i, y_i, z_i) < (x, y, z) < (x_{i+1}, y_{i+1}, z_{i+1})$$

$$n_{i,j,k} = \delta_{i,j,k}(x_{i+1} - x)(y_{j+1} - y)(z_{k+1} - z)$$

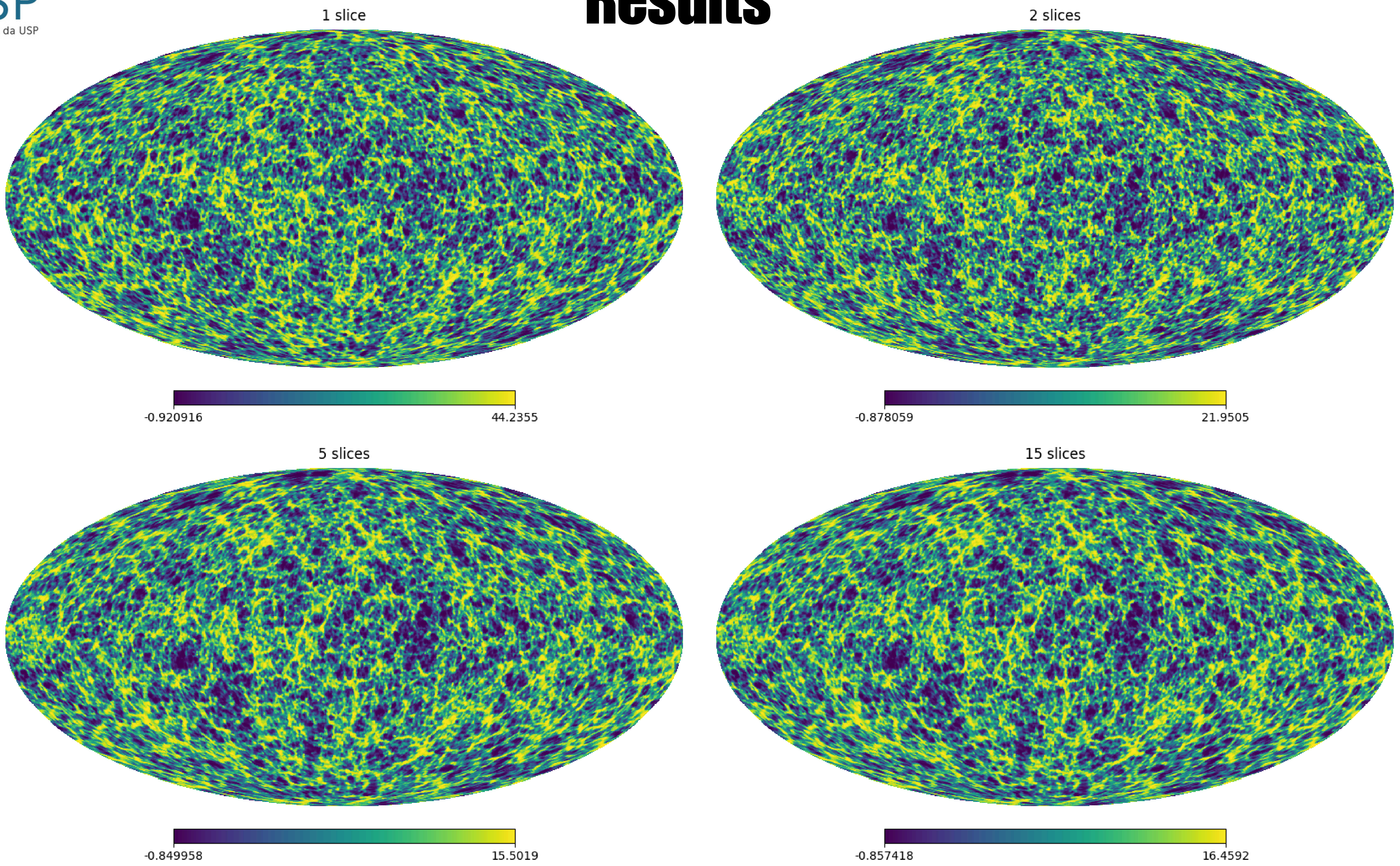
$$\downarrow$$

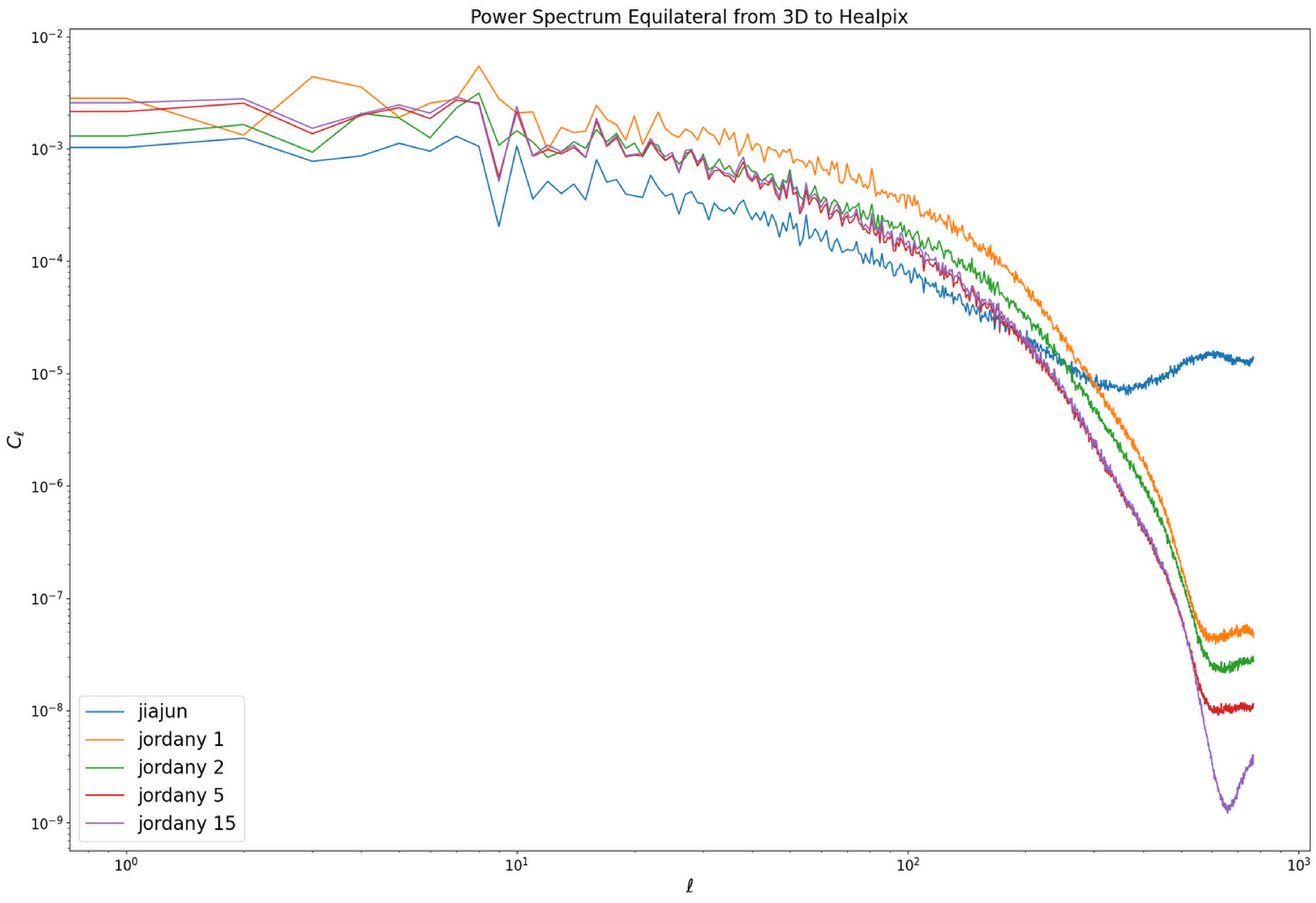
8 combinations

$$\downarrow$$

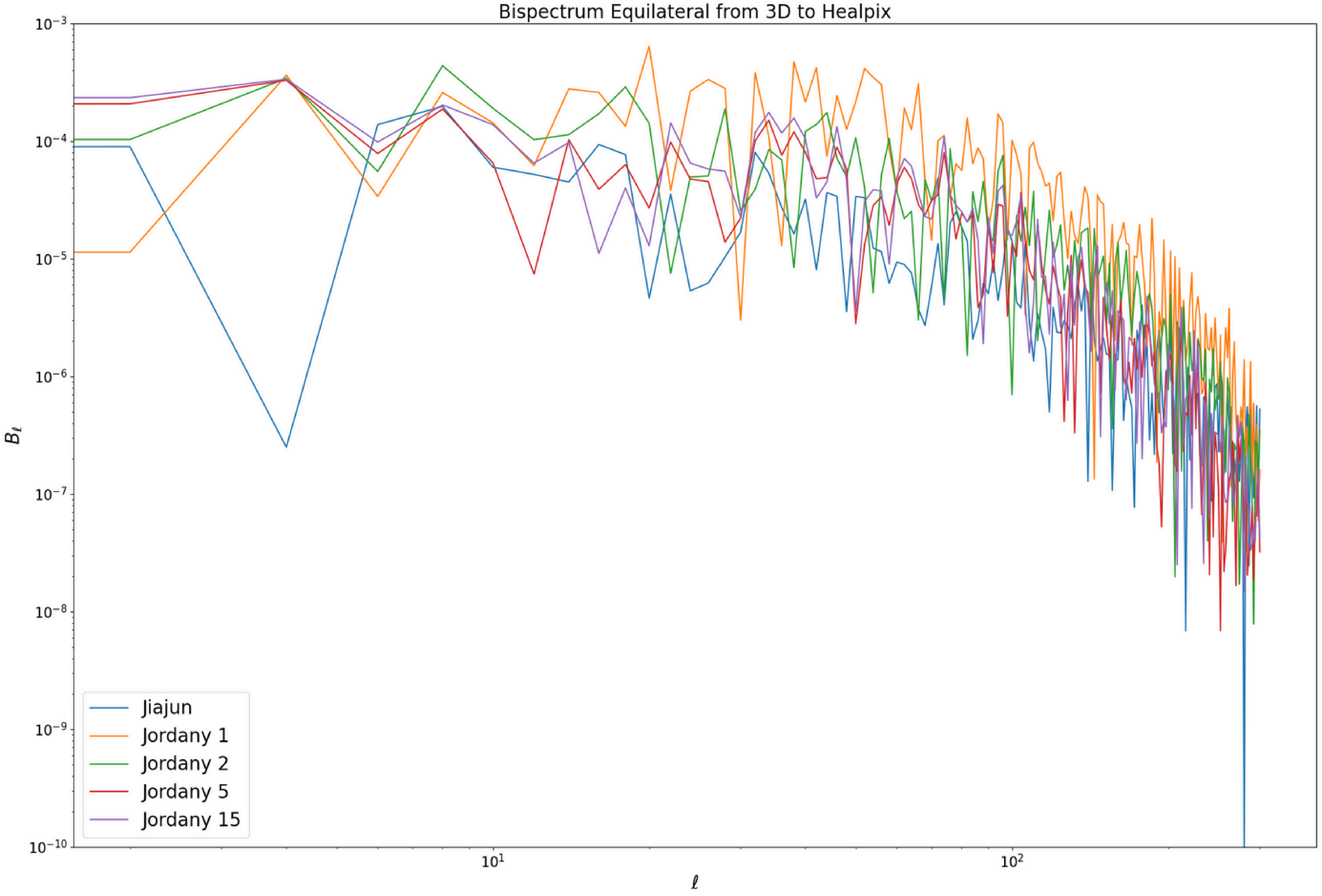
Sum all n’s to the total information for (x,y,z)

Results


1 slice
-0.920916
44.2355
2 slices
-0.878059
21.9505
5 slices
-0.849958
15.5019
15 slices
-0.857418
16.4592



Power Spectrum Equilateral from 3D to Healpix
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
C_ℓ
10^{0}
10^{1}
10^{2}
10^{3}
ℓ
jiajun
jordany 1
jordany 2
jordany 5
jordany 15


Bispectrum Equilateral from 3D to Healpix
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
B_ℓ
10^{1}
10^{2}
ℓ
Jiajun
Jordany 1
Jordany 2
Jordany 5
Jordany 15

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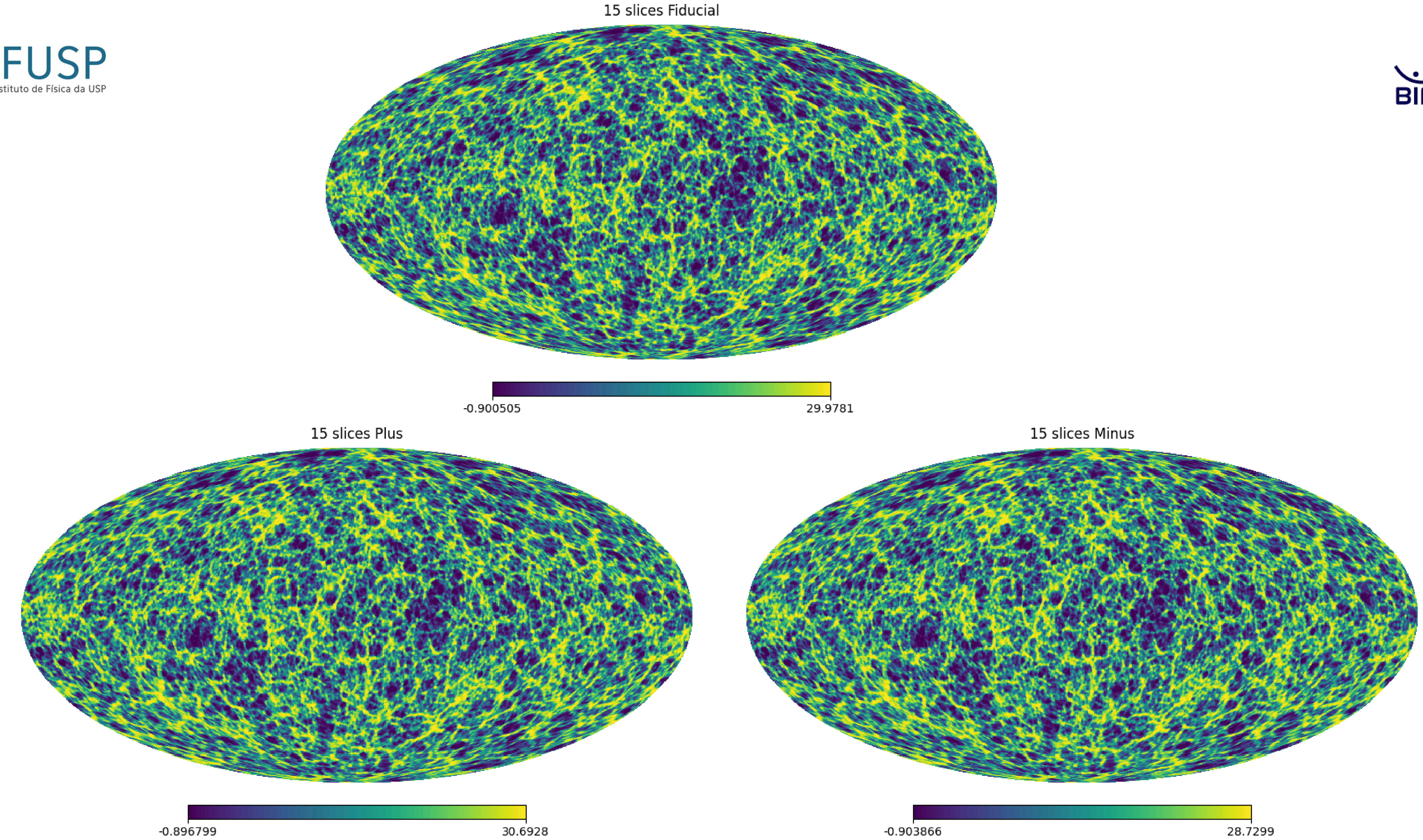
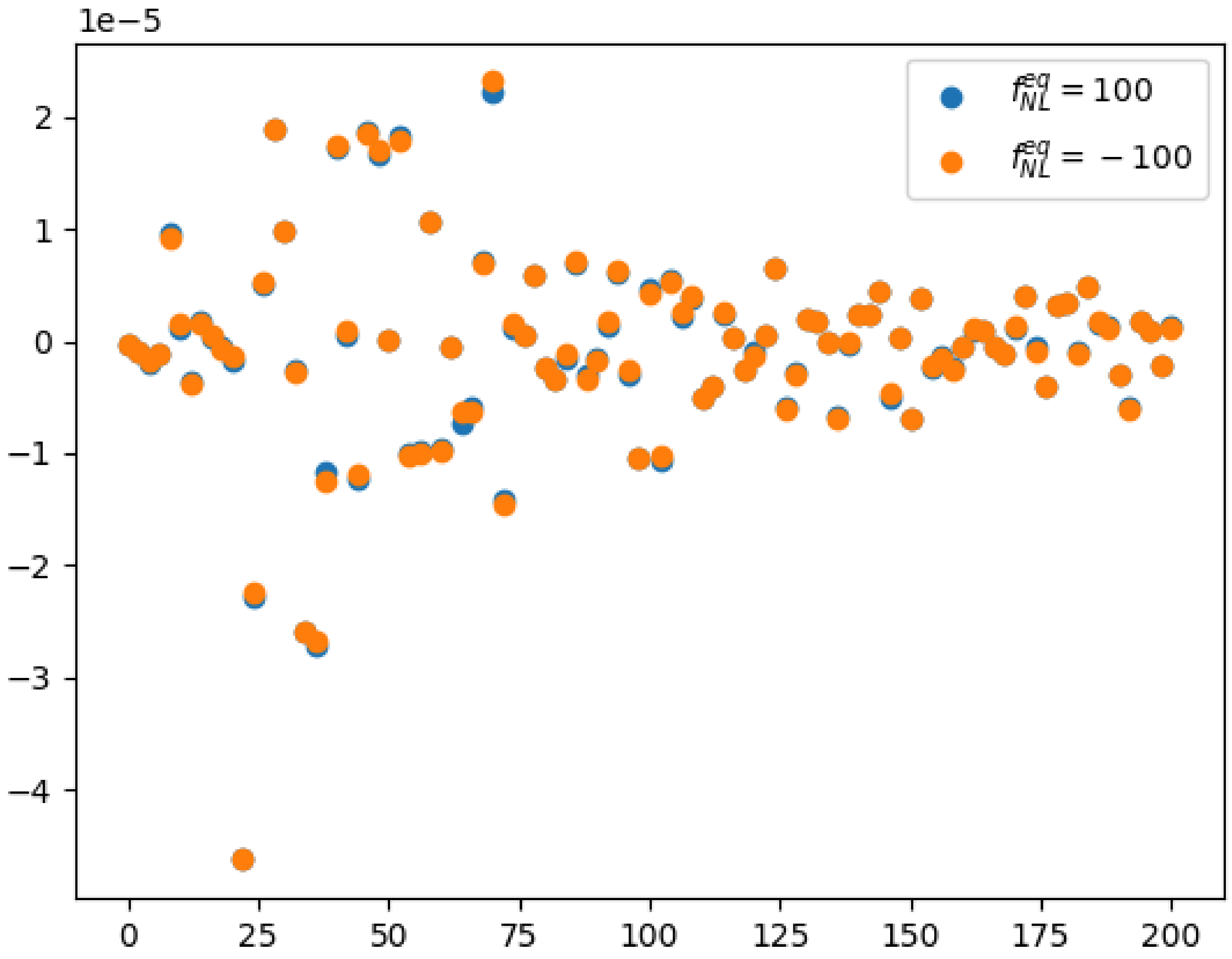
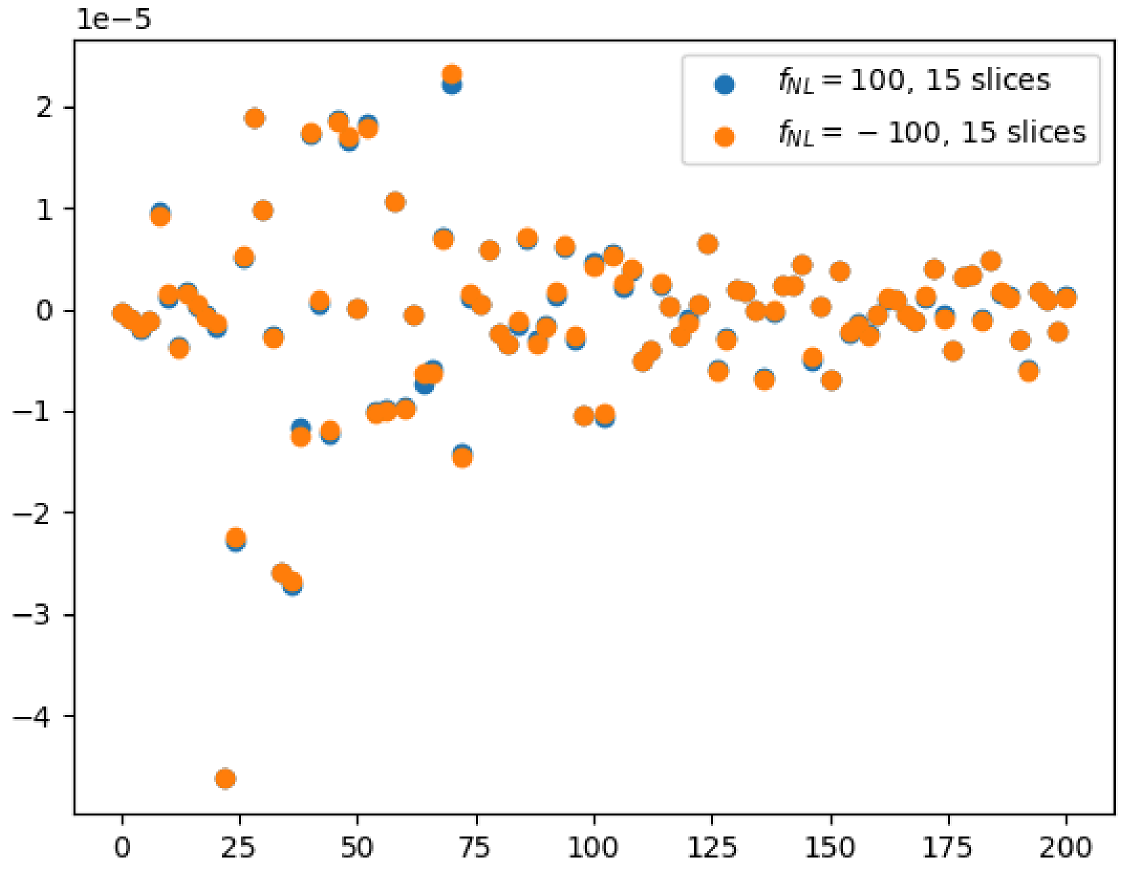
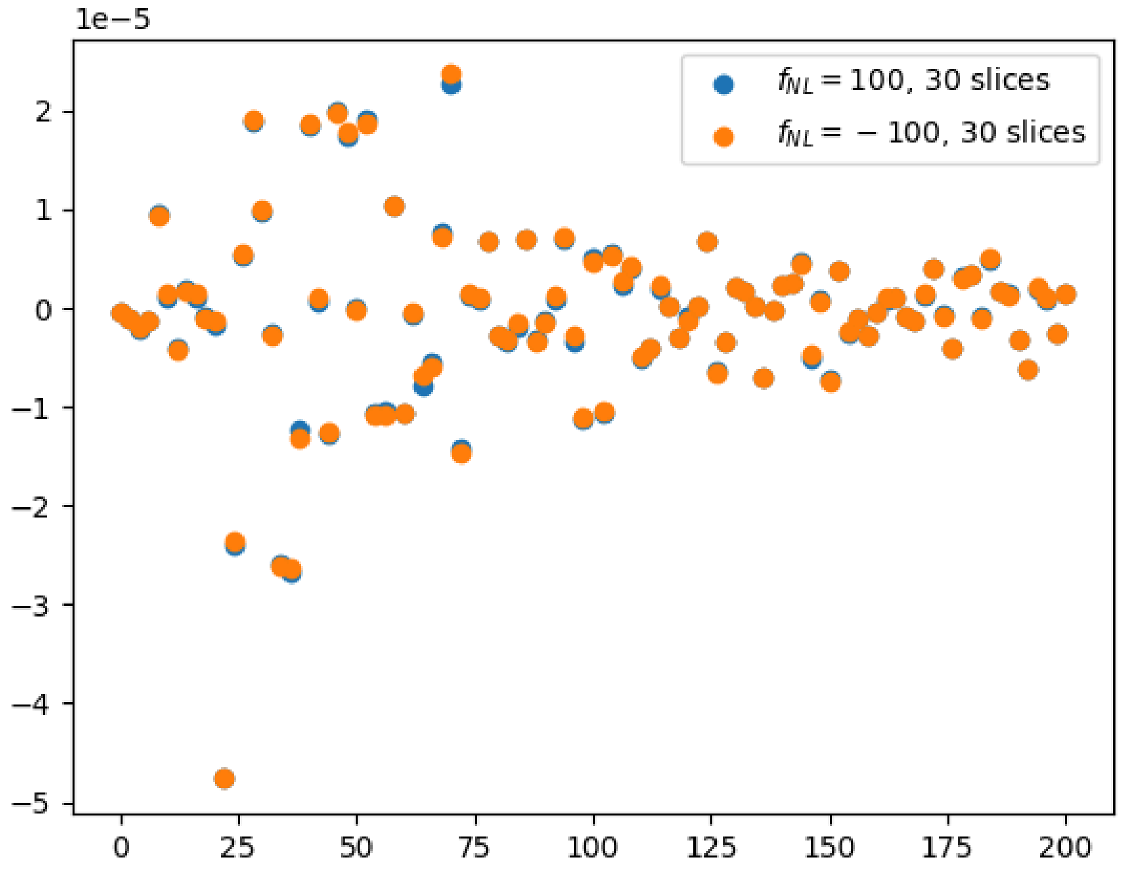
BINGO

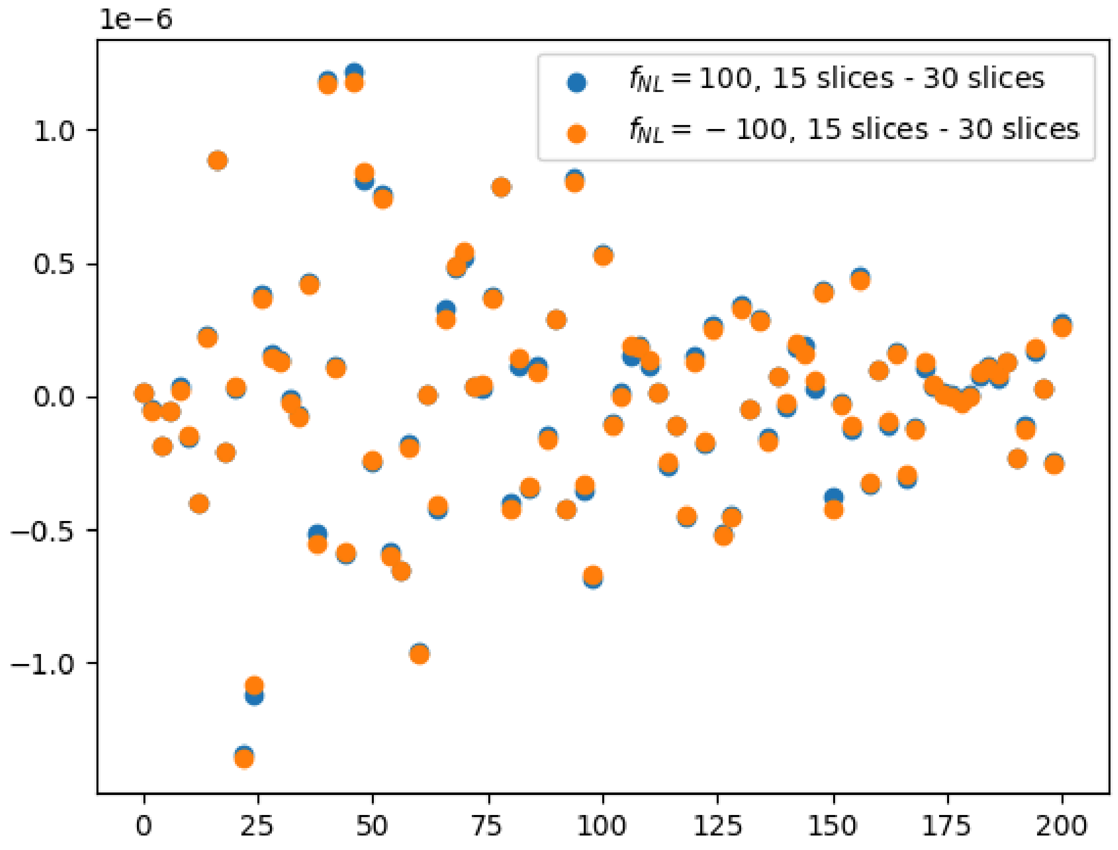




1e−5
$f_{NL}^{eq} = 100$
$f_{NL}^{eq} = -100$
2
1
0
−1
−2
−3
−4
0
25
50
75
100
125
150
175
200


1e−5
$f_{NL} = 100$, 15 slices
$f_{NL} = -100$, 15 slices


1e−5
$f_{NL} = 100$, 30 slices
$f_{NL} = -100$, 30 slices


1e−6
$f_{NL} = 100$, 15 slices - 30 slices
$f_{NL} = -100$, 15 slices - 30 slices

Next Steps

- Study the bispectrum in foreground residuals for BINGO simulations
- Analyze different types of beams
- Verify all possibilities of bispectrum changes in maps from cube
- Do a complete analysis in Quijote simulations (equilateral case)
- Compare with theoretical results (equilateral case)
- Start the studies in interact models with 3D cubes.

Thank You!

Obrigado!

谢谢！

Bispectrum Updates

MSc. Jordany Vieira (Ph.D. Student at USP)
Supervisor: Prof. Dr. Elcio Abdalla

Contact: jordanyv@gmail.com

Bispectrum Module

$$B_{\ell_1\ell_2\ell_3} = \int d\Omega M_{\ell_1}(\Omega) M_{\ell_2}(\Omega) M_{\ell_3}(\Omega)$$

Bispectrum from sky maps

$$B_{\ell_1\ell_2\ell_3} = \int d\Omega \sum_{m_1,m_2,m_3} Y_{\ell_1 m_1}(\Omega) Y_{\ell_2 m_2}(\Omega) Y_{\ell_3 m_3}(\Omega) a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}$$

After spherical harmonic transformations

$$\int d\Omega Y_{\ell_1 m_1}(\Omega) Y_{\ell_2 m_2}(\Omega) Y_{\ell_3 m_3}(\Omega) = \sqrt{N_\Delta^{\ell_1\ell_2\ell_3}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

Gaunt Integral

$$B_{\ell_1\ell_2\ell_3} = \sqrt{N_\Delta^{\ell_1\ell_2\ell_3}} \sum_{m_1,m_2,m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}$$

Bispectrum to be calculated

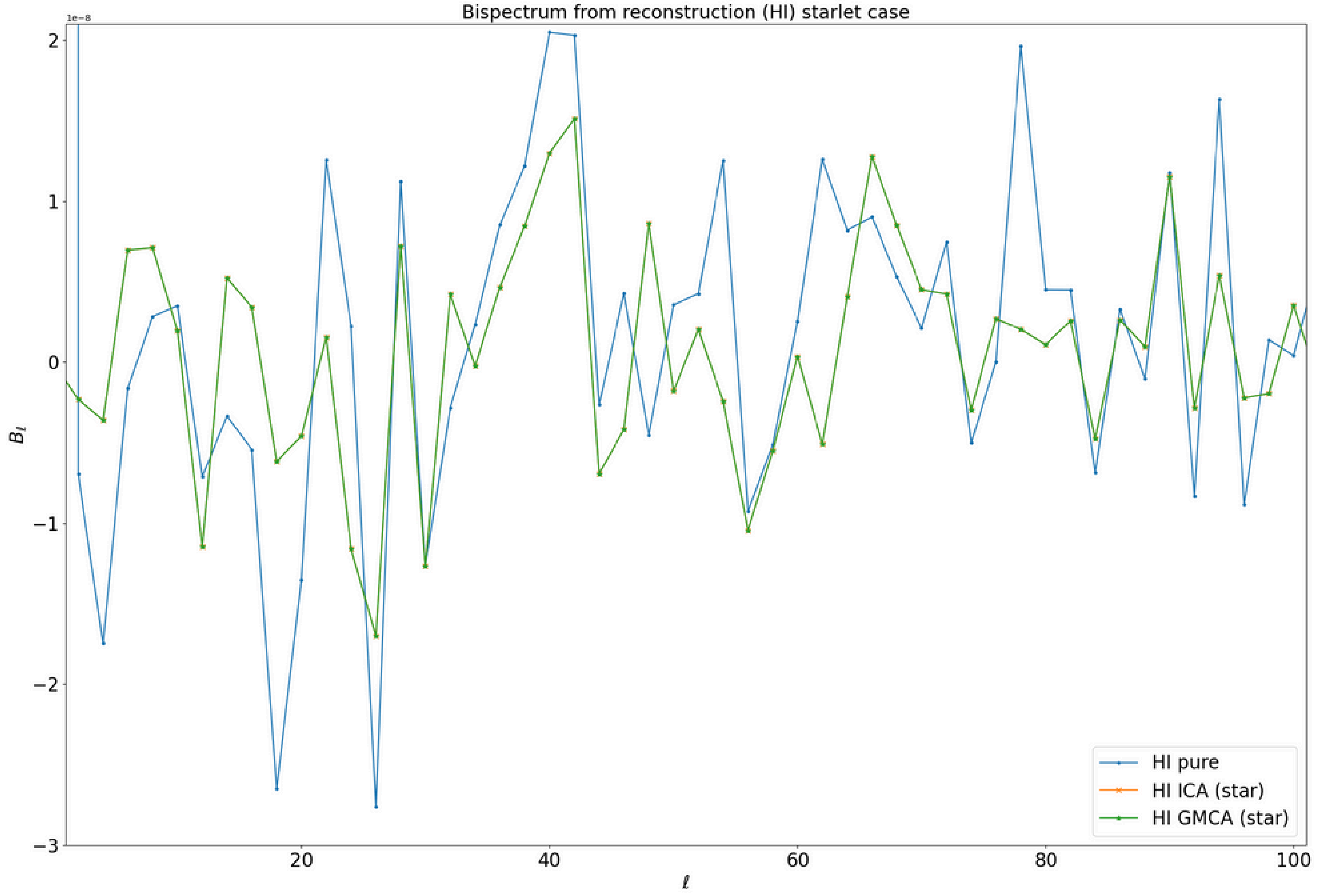
$$N_\Delta^{\ell_1\ell_2\ell_3} \equiv \frac{(2\ell_1+1)+(2\ell_2+1)+(2\ell_3+1)}{4\pi} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}$$

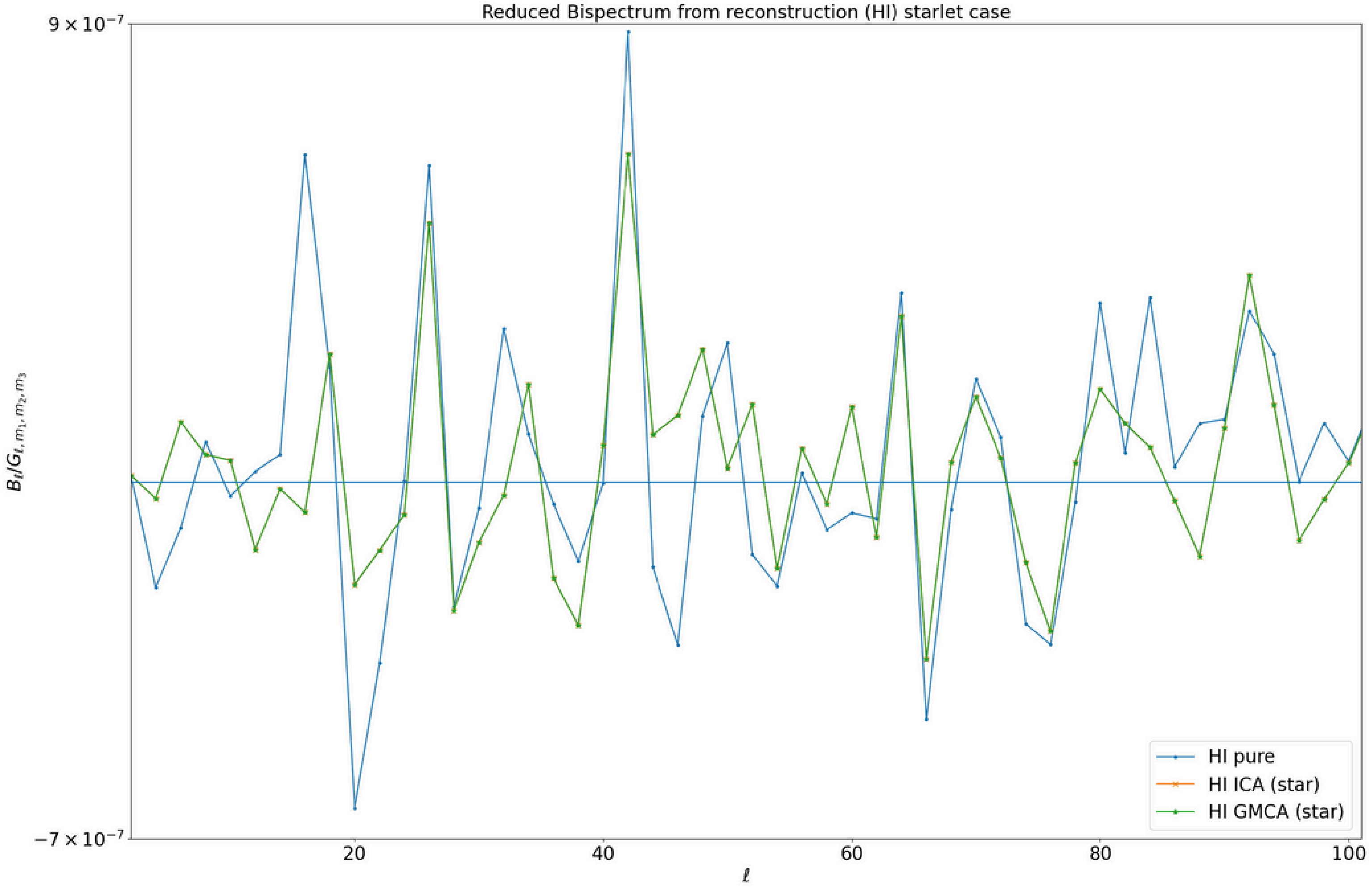
Bispectrum Module

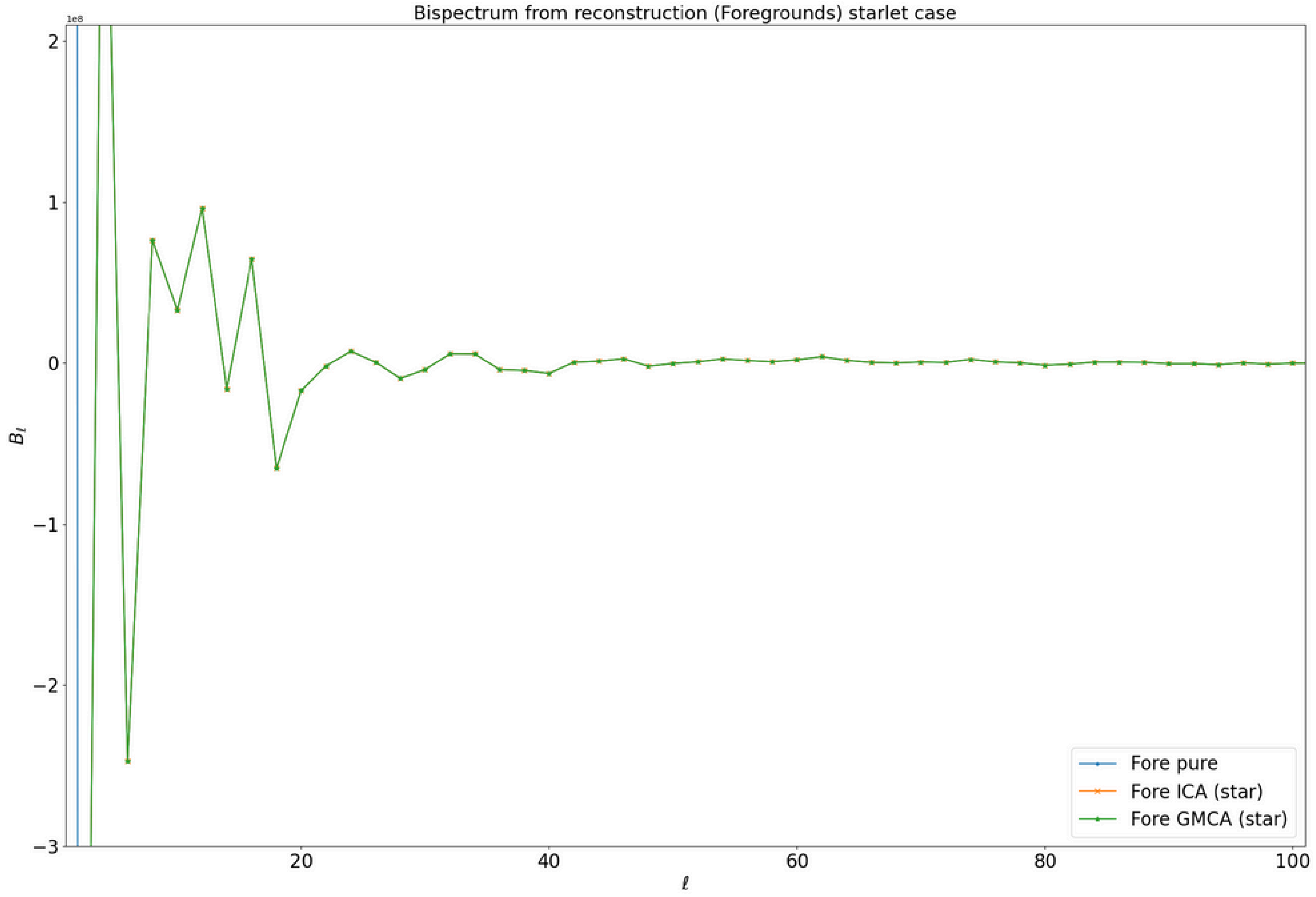
Bispectrum reduzido

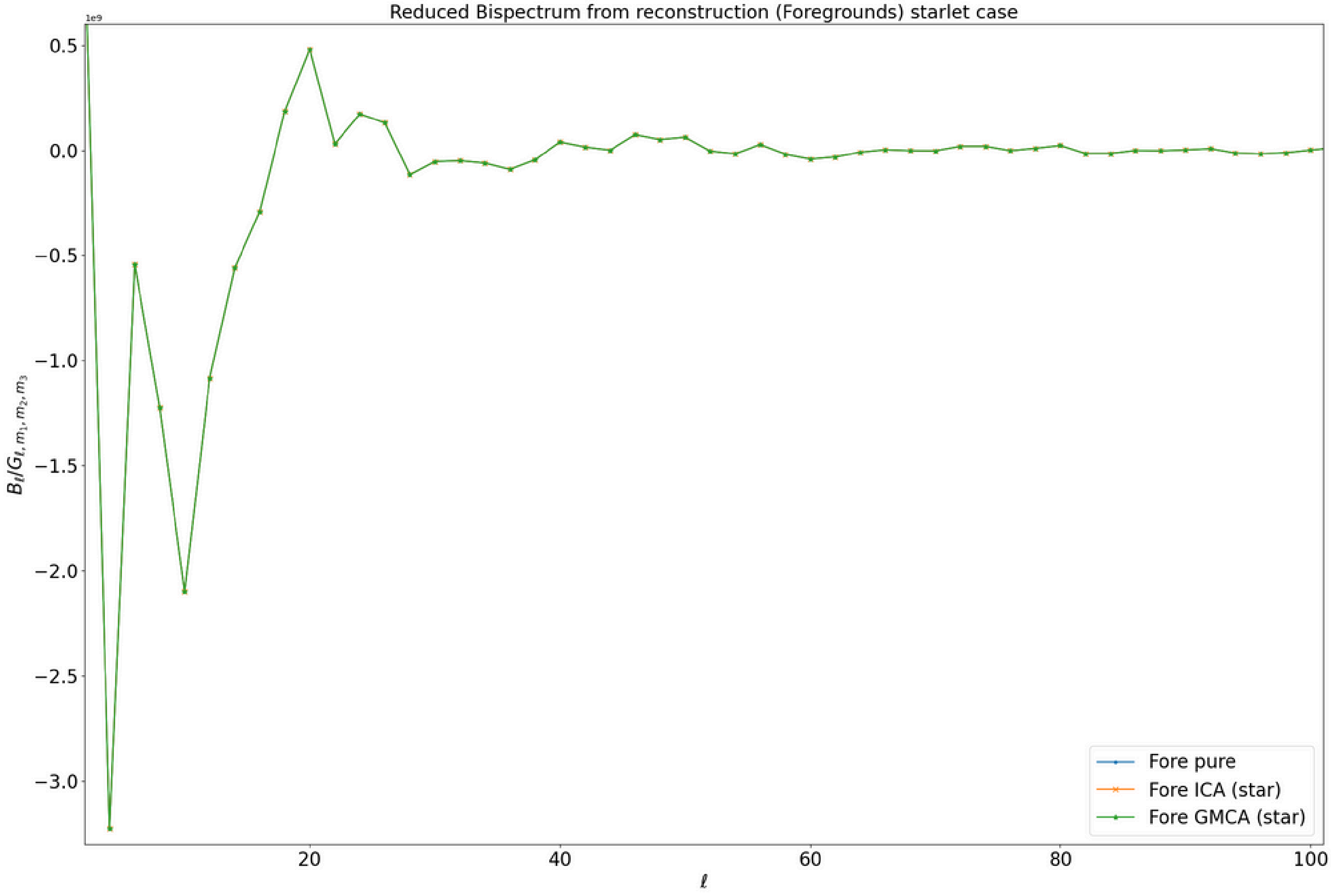
$$B_{\ell_1\ell_2\ell_3} = \sqrt{N_\Delta^{\ell_1\ell_2\ell_3}} \sum_{m_1,m_2,m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}$$

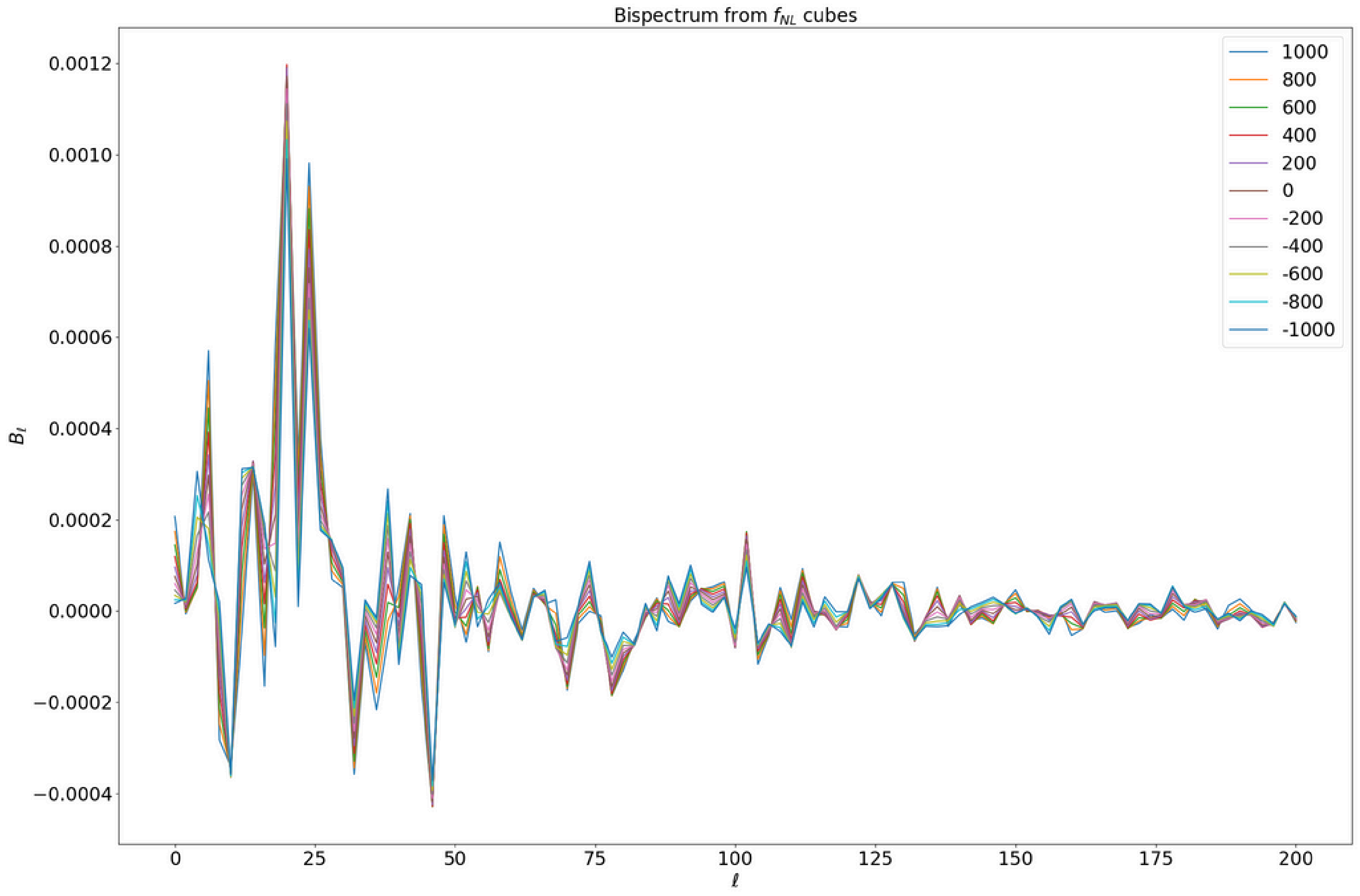
$$N_\Delta^{\ell_1\ell_2\ell_3} \equiv \frac{(2\ell_1+1)+(2\ell_2+1)+(2\ell_3+1)}{4\pi} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}$$

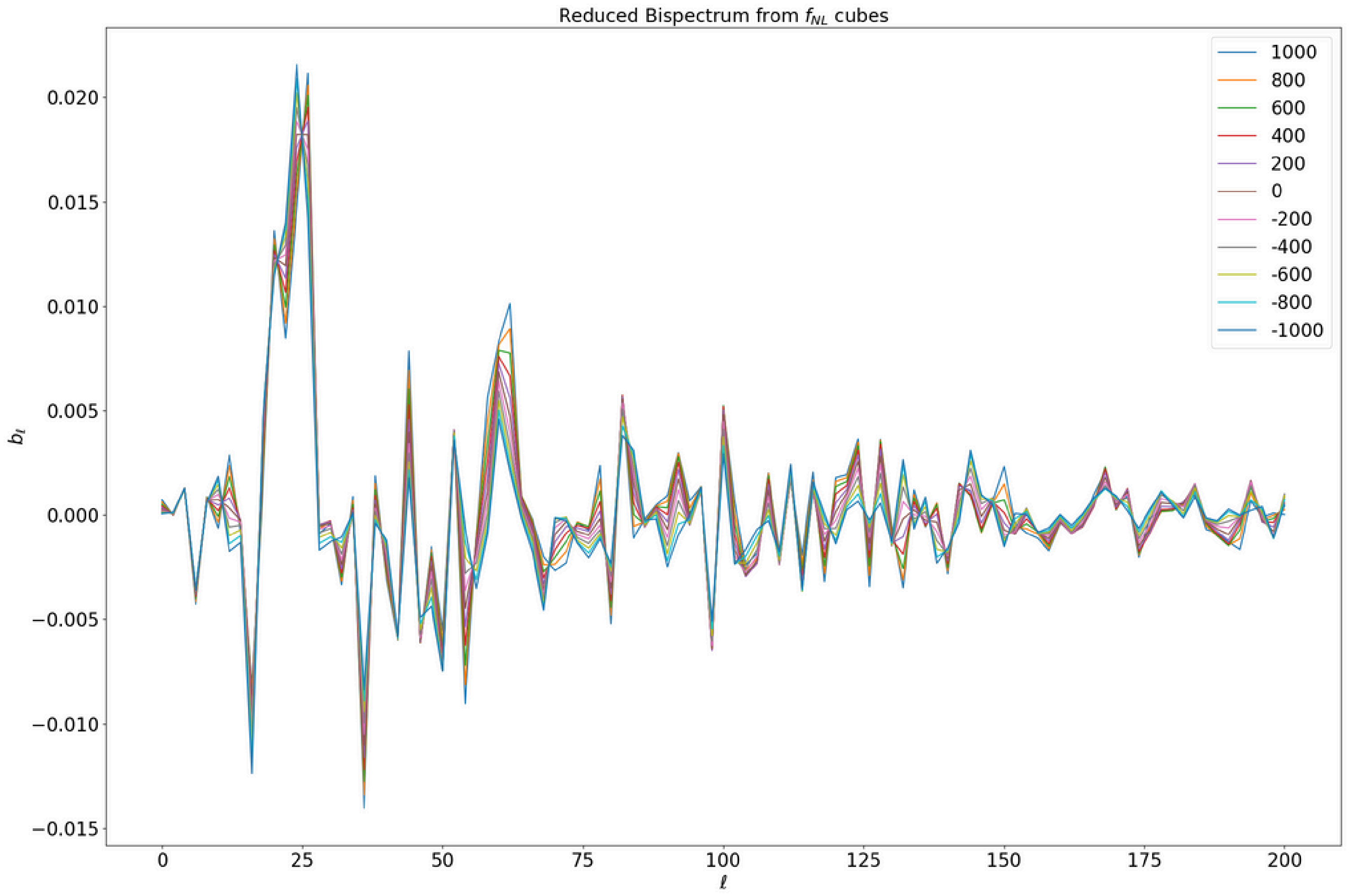

Bispectrum from reconstruction (HI) starlet case
1e−8
2
1
0
−1
−2
−3
20
40
60
80
100
B_ℓ
ℓ
HI pure
HI ICA (star)
HI GMCA (star)

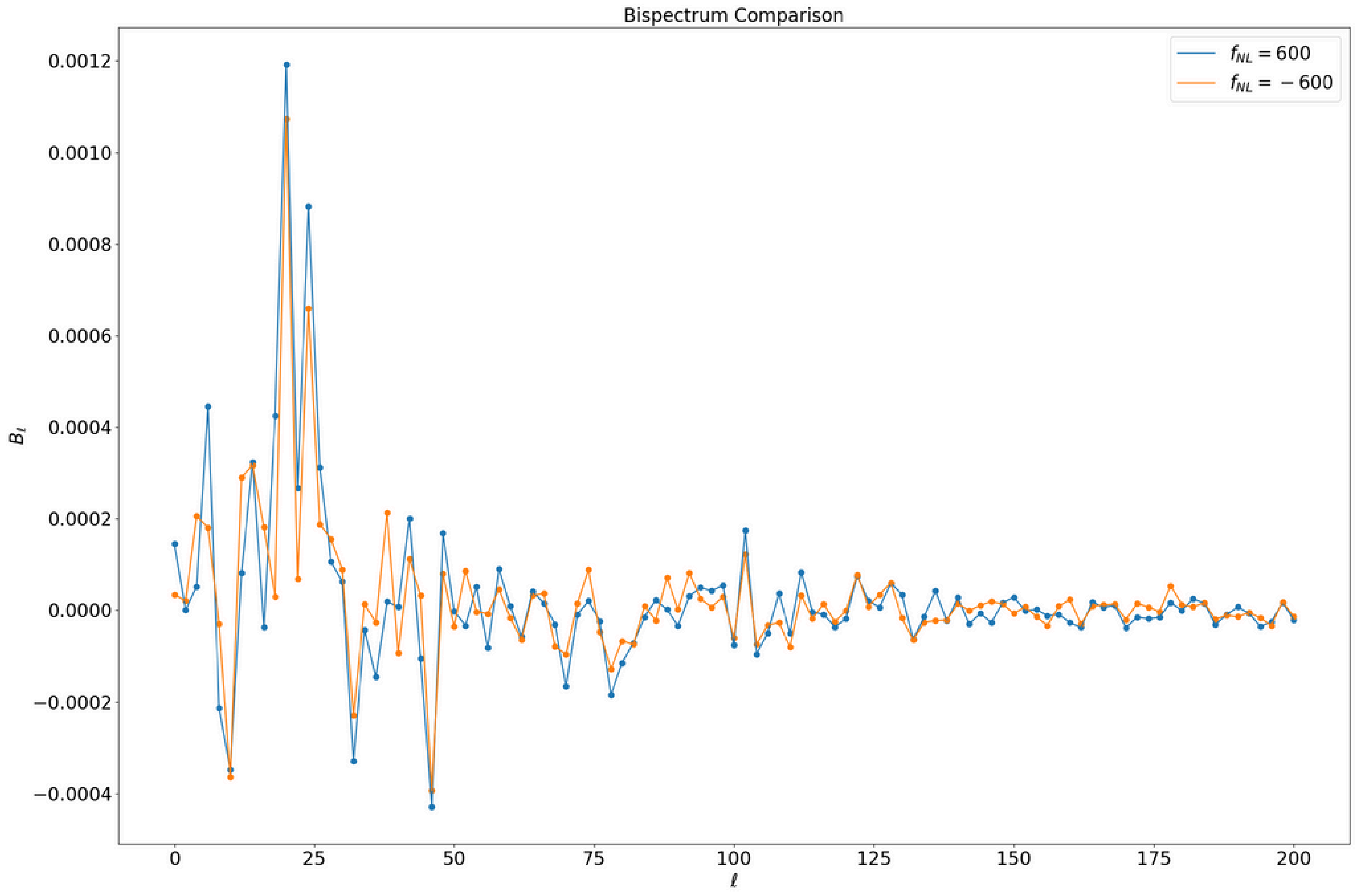

Reduced Bispectrum from reconstruction (HI) starlet case
9×10^{-7}
-7×10^{-7}
20
40
60
80
100
$B_\ell / G_{\ell, m_1, m_2, m_3}$
ℓ
HI pure
HI ICA (star)
HI GMCA (star)


Bispectrum from reconstruction (Foregrounds) starlet case
1e8
B_ℓ
ℓ
Fore pure
Fore ICA (star)
Fore GMCA (star)

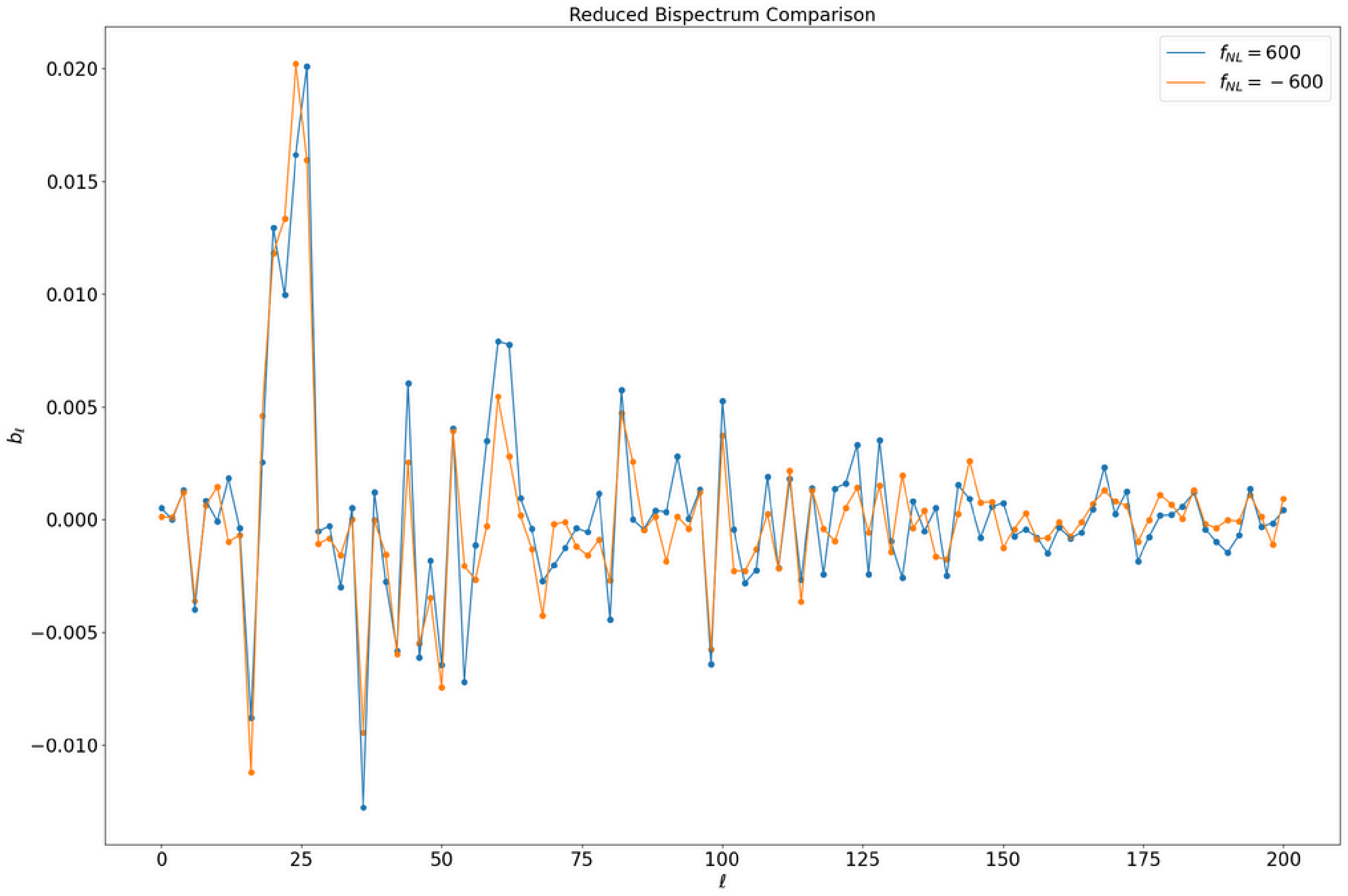

Reduced Bispectrum from reconstruction (Foregrounds) starlet case
1e9
$B_\ell / G_{\ell, m_1, m_2, m_3}$
ℓ
Fore pure
Fore ICA (star)
Fore GMCA (star)


Bispectrum from f_{NL} cubes
B_l
ℓ
1000
800
600
400
200
0
-200
-400
-600
-800
-1000

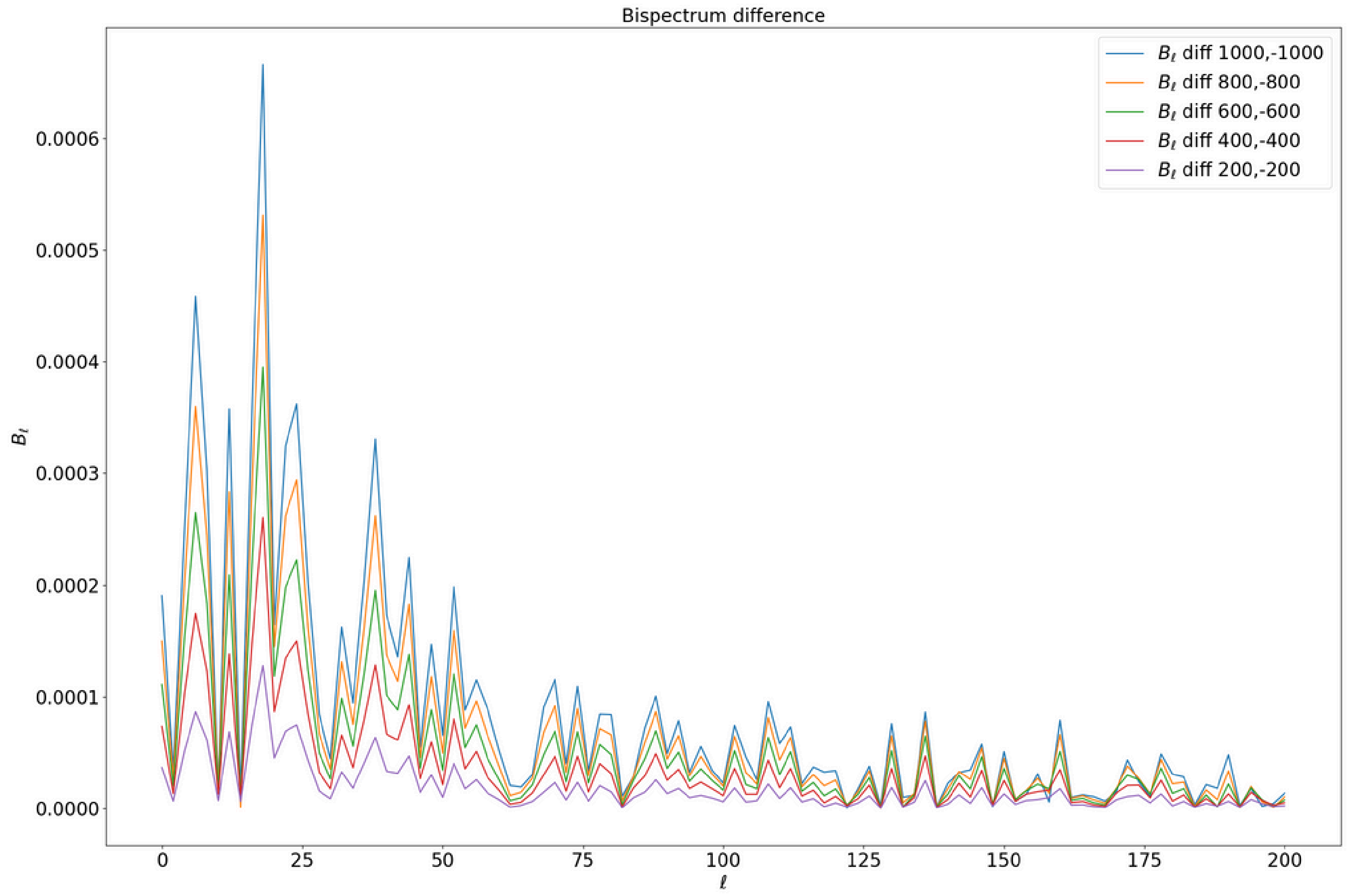

Reduced Bispectrum from f_{NL} cubes
b_l
ℓ
1000
800
600
400
200
0
-200
-400
-600
-800
-1000



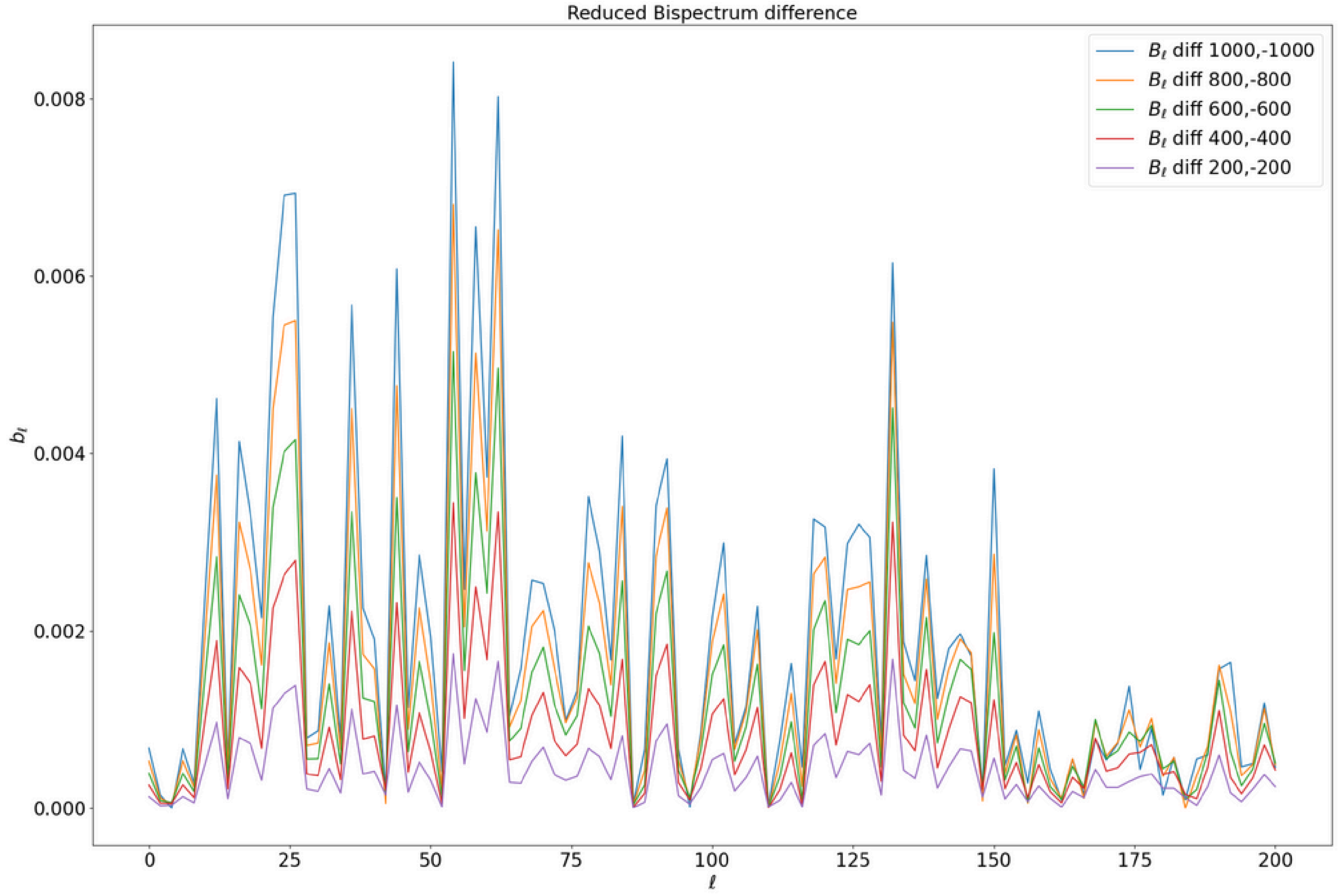
Bispectrum Comparison
$f_{NL} = 600$
$f_{NL} = -600$
B_l
ℓ



Reduced Bispectrum Comparison
$f_{NL} = 600$
$f_{NL} = -600$
b_l
ℓ



Bispectrum difference
B_ℓ diff 1000,-1000
B_ℓ diff 800,-800
B_ℓ diff 600,-600
B_ℓ diff 400,-400
B_ℓ diff 200,-200
B_l
ℓ



Reduced Bispectrum difference
B_ℓ diff 1000,-1000
B_ℓ diff 800,-800
B_ℓ diff 600,-600
B_ℓ diff 400,-400
B_ℓ diff 200,-200
b_l
ℓ