Bispectrum in BINGO Radio Telescope

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Outline

1. What is Bispectrum?
2. Why Study Bispectrum?
3. Bispectrum Module.
4. Bispectrum Cases.
5. Results.
6. Next Steps.

What is Bispectrum?

- Bispectrum is the third order of the Angular n-points power spectrum.
- Study the asymmetry of distribution in statistical field theory.

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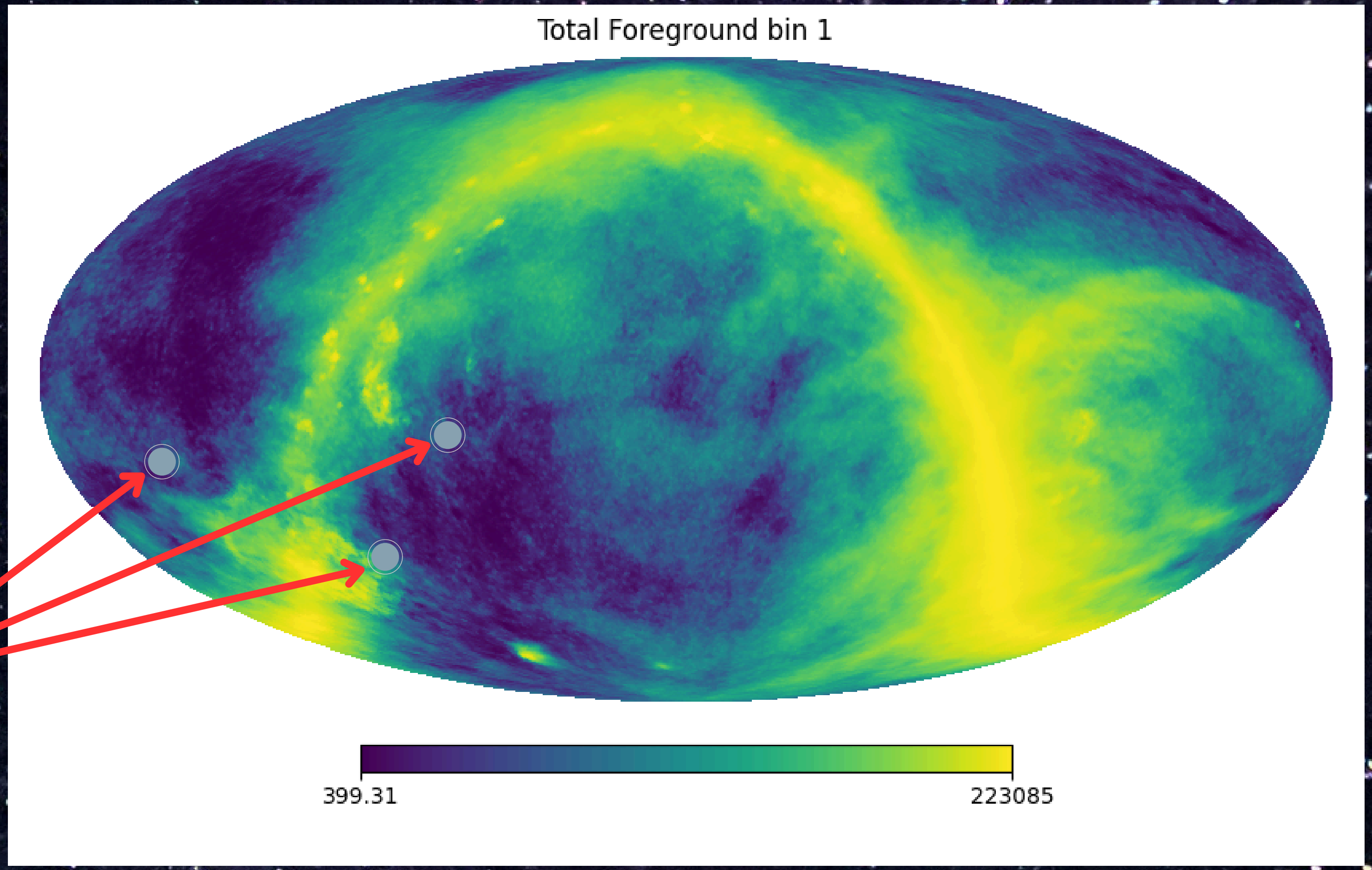


Correlation between this 3 points in the sky.

Why Study Bispectrum?

- Search for traces of non-Gaussianities in 21-cm fields associated with the inflationary period.
- Study the fluctuations in the matter power spectrum.
- Use to study the interactions between Dark Energy and Dark Matter and alternative cosmology models.

Bispectrum Module

$$B_{\ell_1\ell_2\ell_3} = \int d\Omega M_{\ell_1}(\Omega) M_{\ell_2}(\Omega) M_{\ell_3}(\Omega)$$

Bispectrum from sky maps

$$B_{\ell_1\ell_2\ell_3} = \int d\Omega \sum_{m_1,m_2,m_3} Y_{\ell_1 m_1}(\Omega) Y_{\ell_2 m_2}(\Omega) Y_{\ell_3 m_3}(\Omega) a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}$$

After spherical harmonic transformations

$$\int d\Omega Y_{\ell_1 m_1}(\Omega) Y_{\ell_2 m_2}(\Omega) Y_{\ell_3 m_3}(\Omega) = \sqrt{N_\Delta^{\ell_1\ell_2\ell_3}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

Gaunt Integral

$$B_{\ell_1\ell_2\ell_3} = \sqrt{N_\Delta^{\ell_1\ell_2\ell_3}} \sum_{m_1,m_2,m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}$$

$$N_\Delta^{\ell_1\ell_2\ell_3} \equiv \frac{(2\ell_1+1)+(2\ell_2+1)+(2\ell_3+1)}{4\pi} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}$$

Bispectrum to be calculated

Bispectrum Module

$$
\begin{aligned}
-\sqrt{(\ell_3 \mp m_3)(\ell_3 \pm m_3 + 1)} \begin{pmatrix} \ell_1 & \ell_1 & \ell_1 \\ m_1 & m_2 & m_3 \pm 1 \end{pmatrix} = \\
\sqrt{(\ell_1 \mp m_1)(\ell_1 \pm m_1 + 1)} \begin{pmatrix} \ell_1 & \ell_1 & \ell_1 \\ m_1 \pm 1 & m_2 & m_3 \end{pmatrix} + \\
+ \sqrt{(\ell_2 \mp m_2)(\ell_2 \pm m_2 + 1)} \begin{pmatrix} \ell_1 & \ell_1 & \ell_1 \\ m_1 & m_2 \pm 2 & m_3 \end{pmatrix}
\end{aligned}
$$

$$Cw = v$$

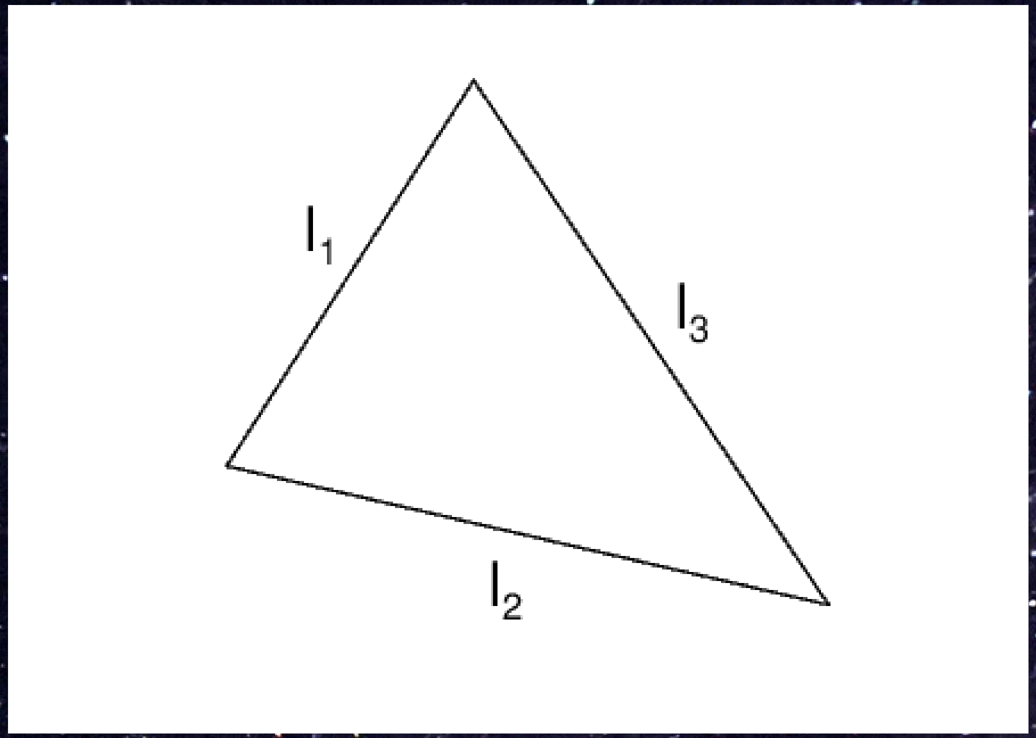
Matrix of Wigner-3j coefficients

Known Wigner-3j

Wigner-3j to be calculated

To solve the matrix equation we use LU decomposition.

Bispectrum Module



Bispectrum constraints

$|\ell_1 - \ell_2| \leq \ell_3 \leq \ell_1 + \ell_2$

triangle inequality

$\ell_1 + \ell_2 + \ell_3$ even

parity invariance

$m_1 + m_2 + m_3 = 0$

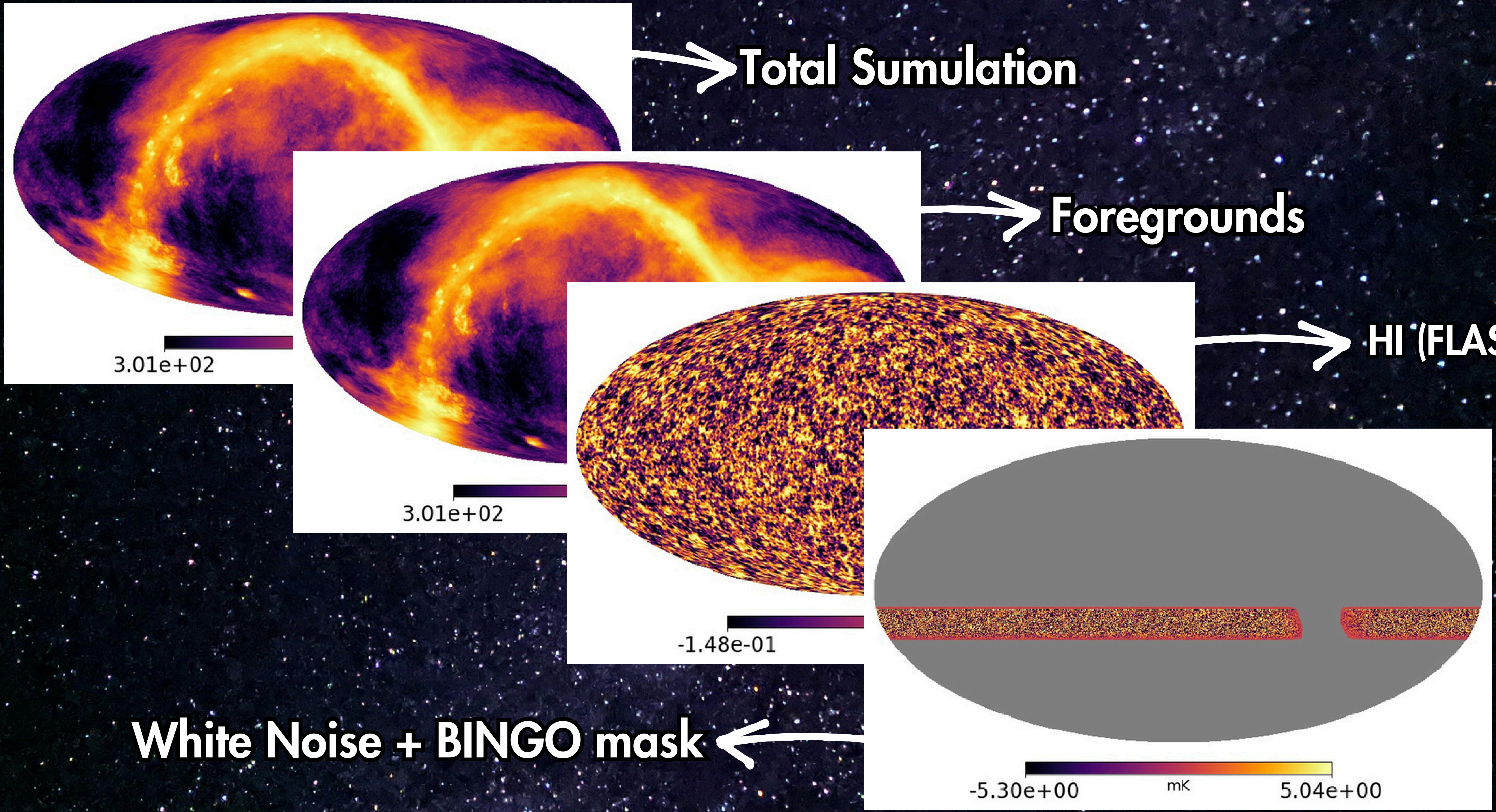
rotation invariance

Bispectrum Module

Total Sumulation

Foregrounds

HI (FLASK ans N-body)



White Noise + BINGO mask

Bispectrum Cases

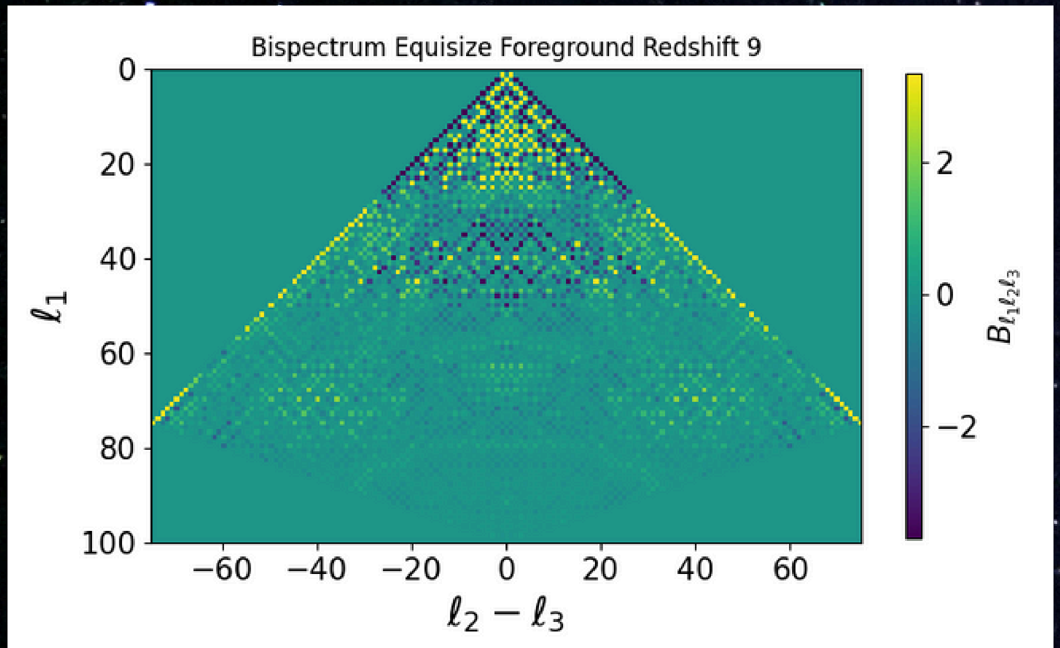
We study 3 cases in bispectrum, which imposes new constraints

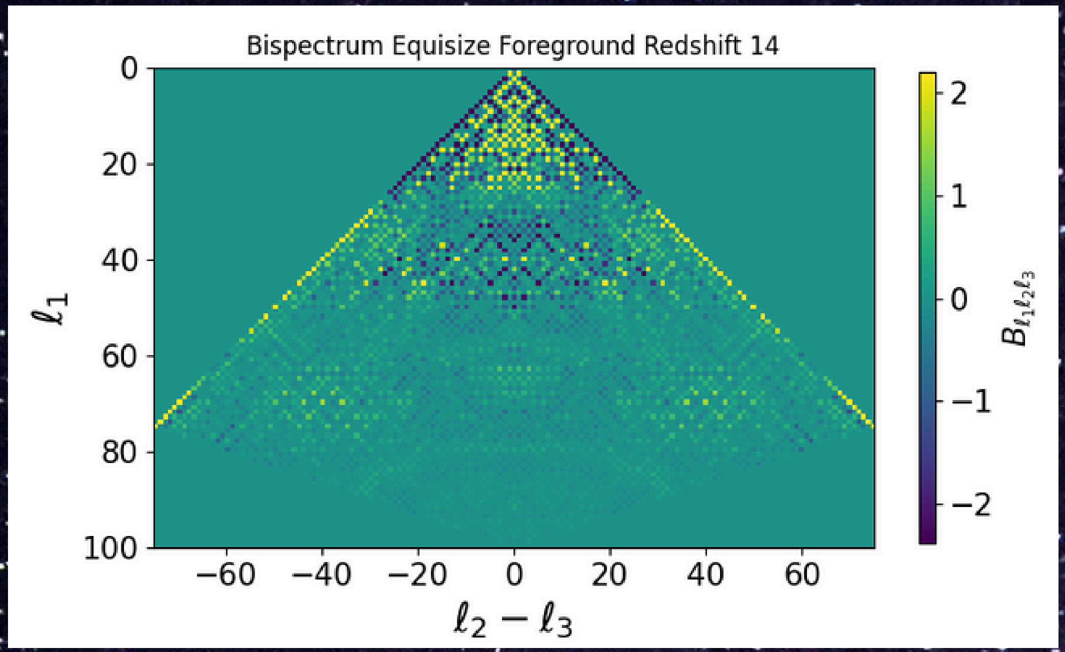
Equisize $\longrightarrow$ $\ell_1 + \ell_2 + \ell_3 = \ell_0$ $\longrightarrow$ $\ell_{max} = 300$, $\ell_0 = 300$

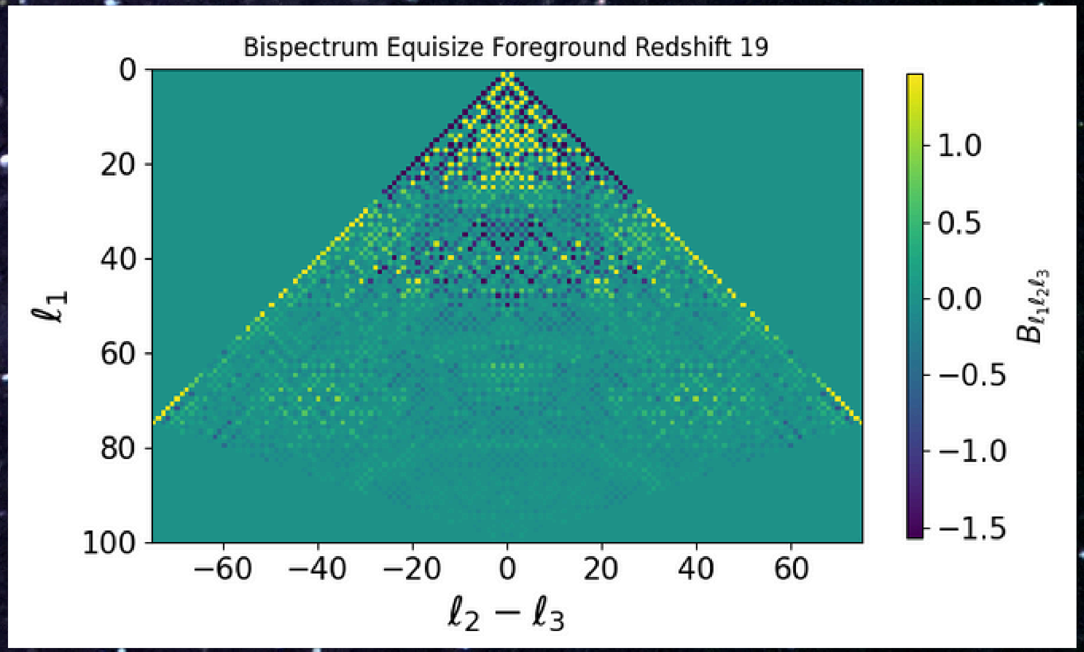
Isosceles $\longrightarrow$ $\ell_1 = \ell_2 \leq \ell_3$ $\longrightarrow$ $\ell_{max} = 150$

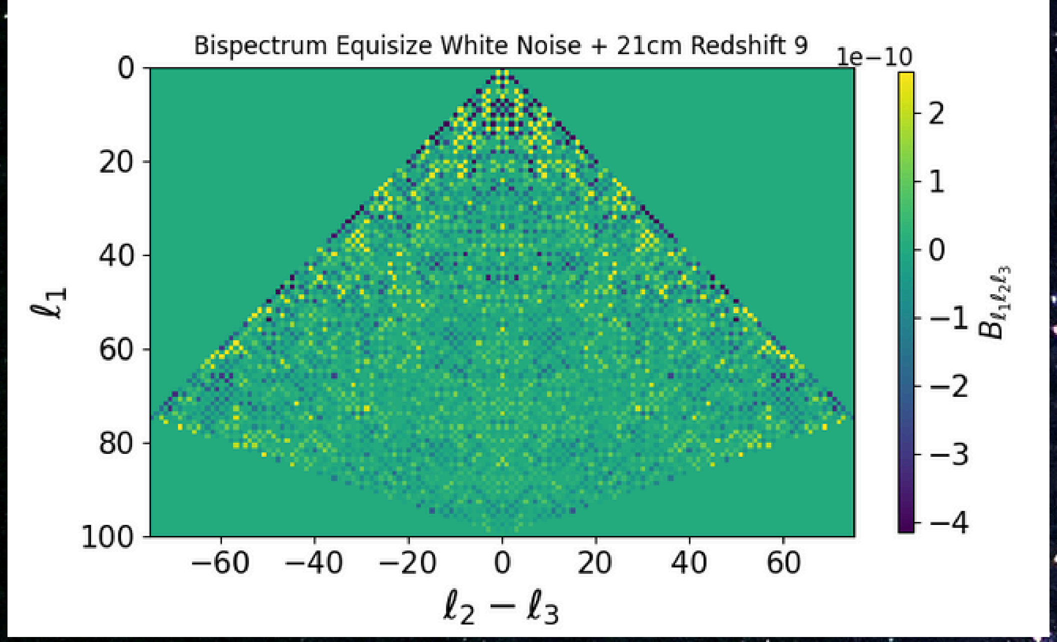
Equilateral $\longrightarrow$ $\ell_1 = \ell_2 = \ell_3$ $\longrightarrow$ $\ell_{max} = 300$

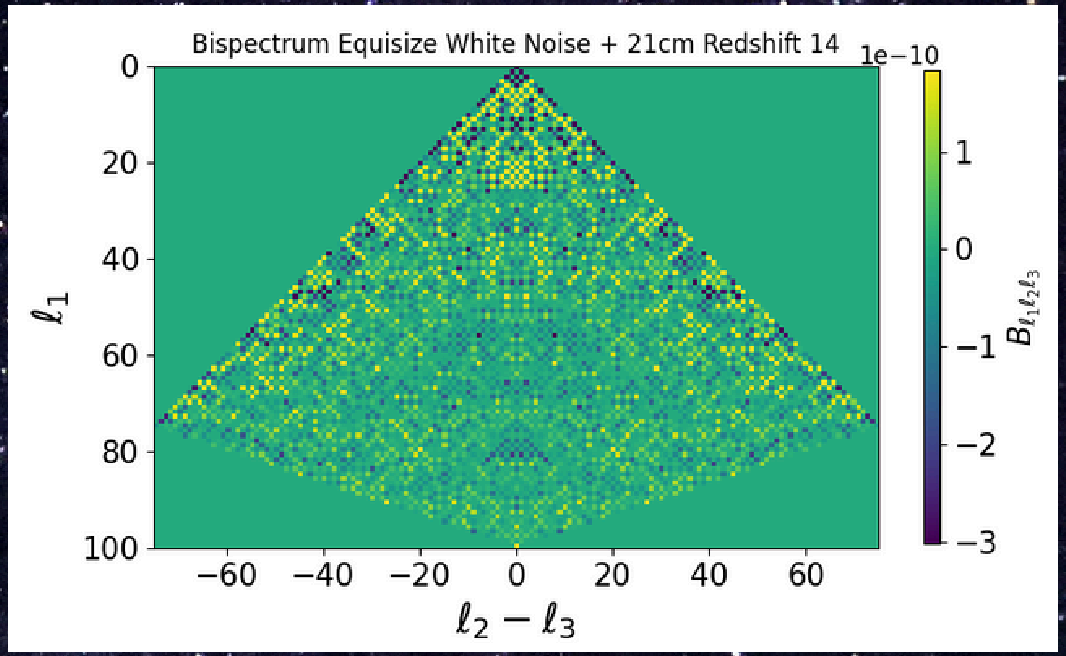
Results

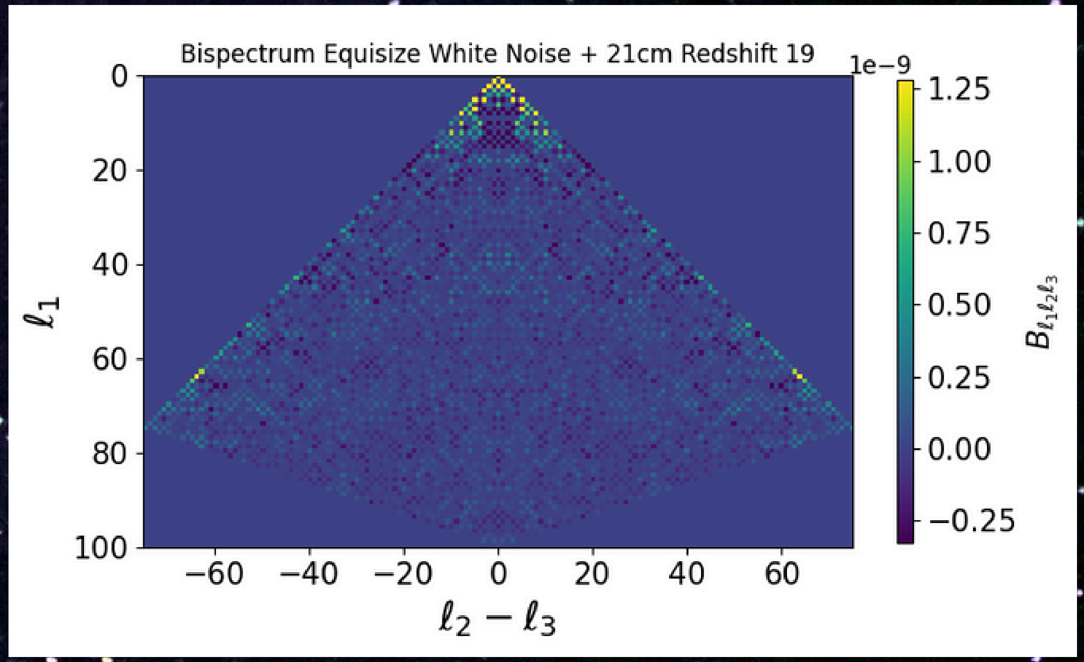

Bispectrum Equisize Foreground Redshift 9
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$


Bispectrum Equisize Foreground Redshift 14
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$


Bispectrum Equisize Foreground Redshift 19
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$


Bispectrum Equisize White Noise + 21cm Redshift 9
1e−10
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$


Bispectrum Equisize White Noise + 21cm Redshift 14
1e−10
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$


Bispectrum Equisize White Noise + 21cm Redshift 19
1e−9
ℓ_1
$\ell_2 - \ell_3$
$B_{\ell_1\ell_2\ell_3}$

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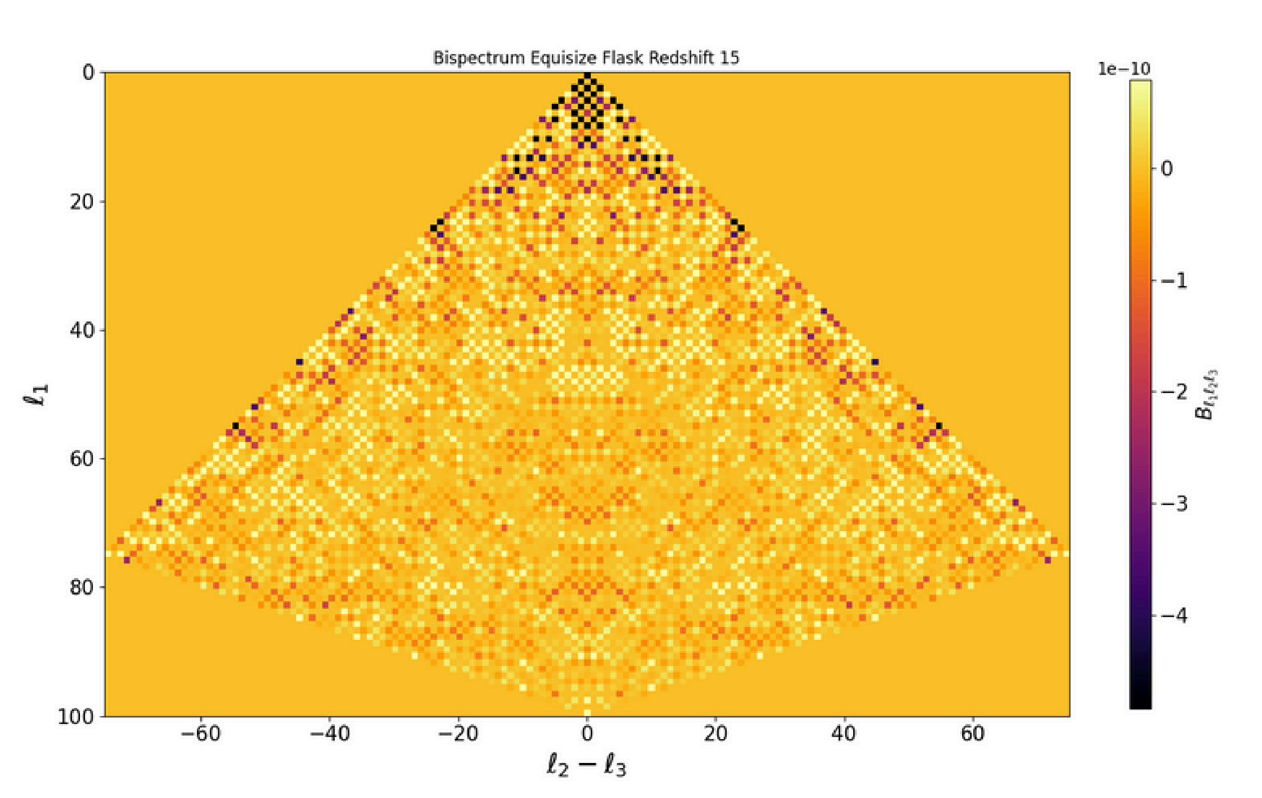
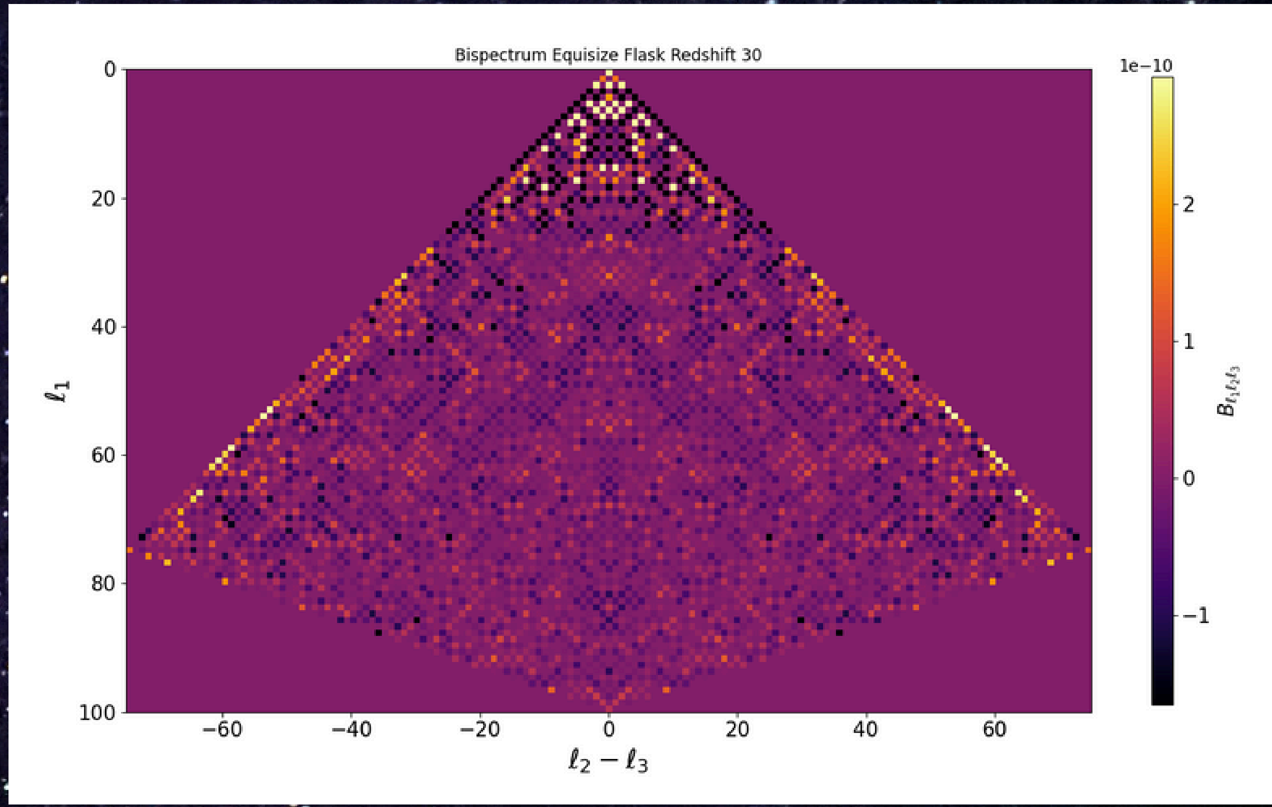
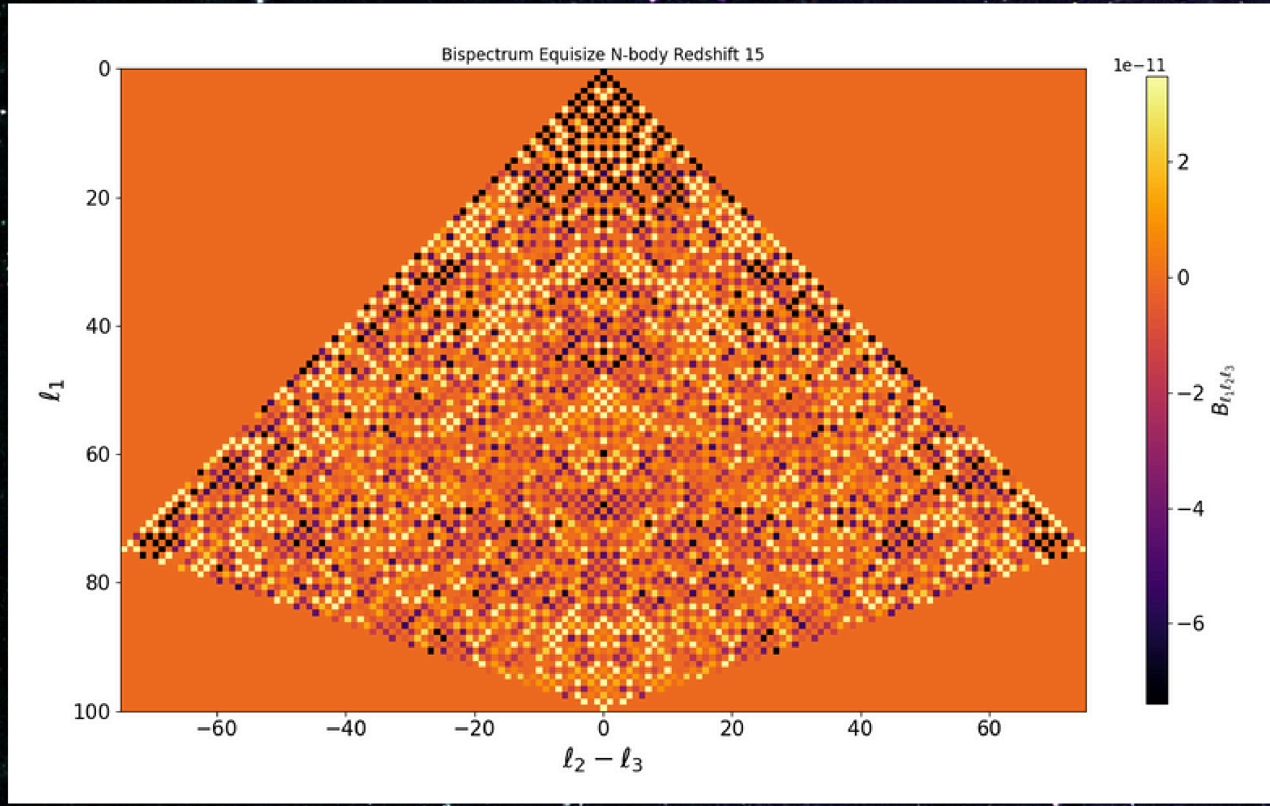
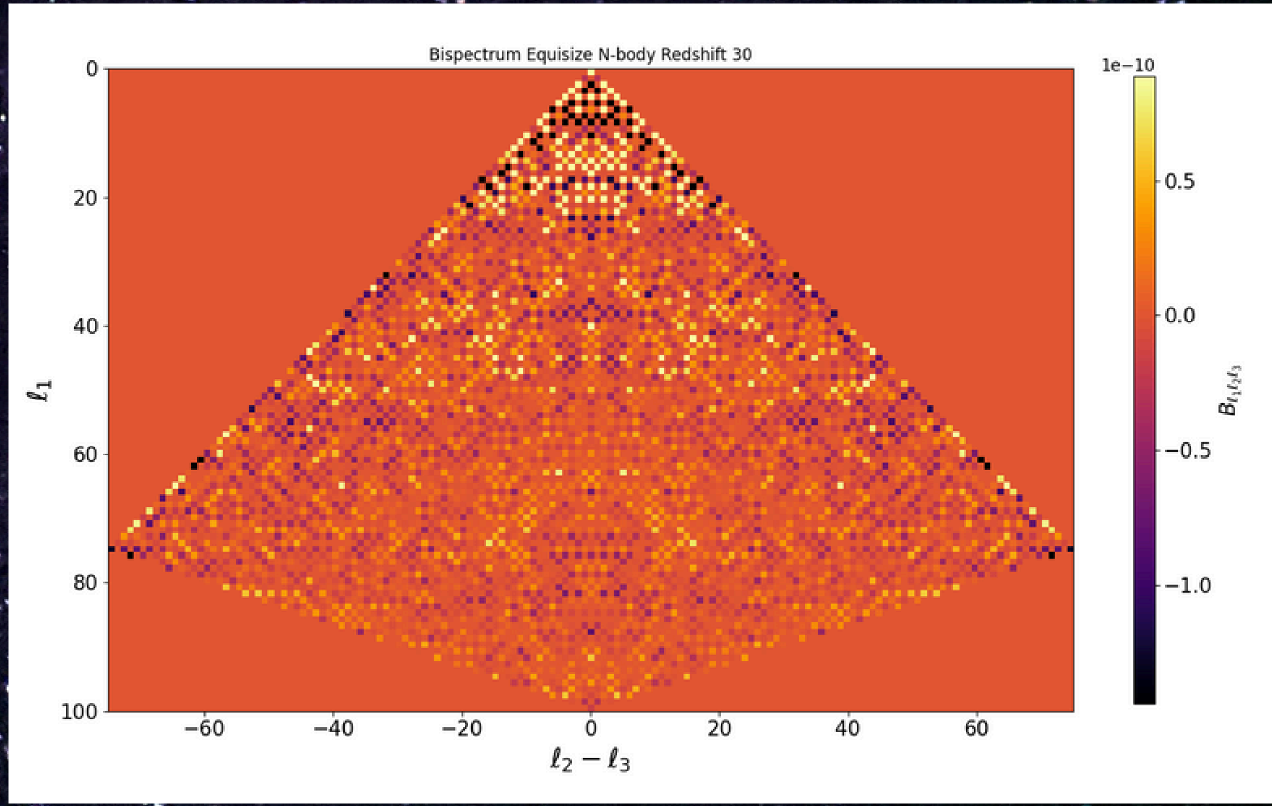
Results

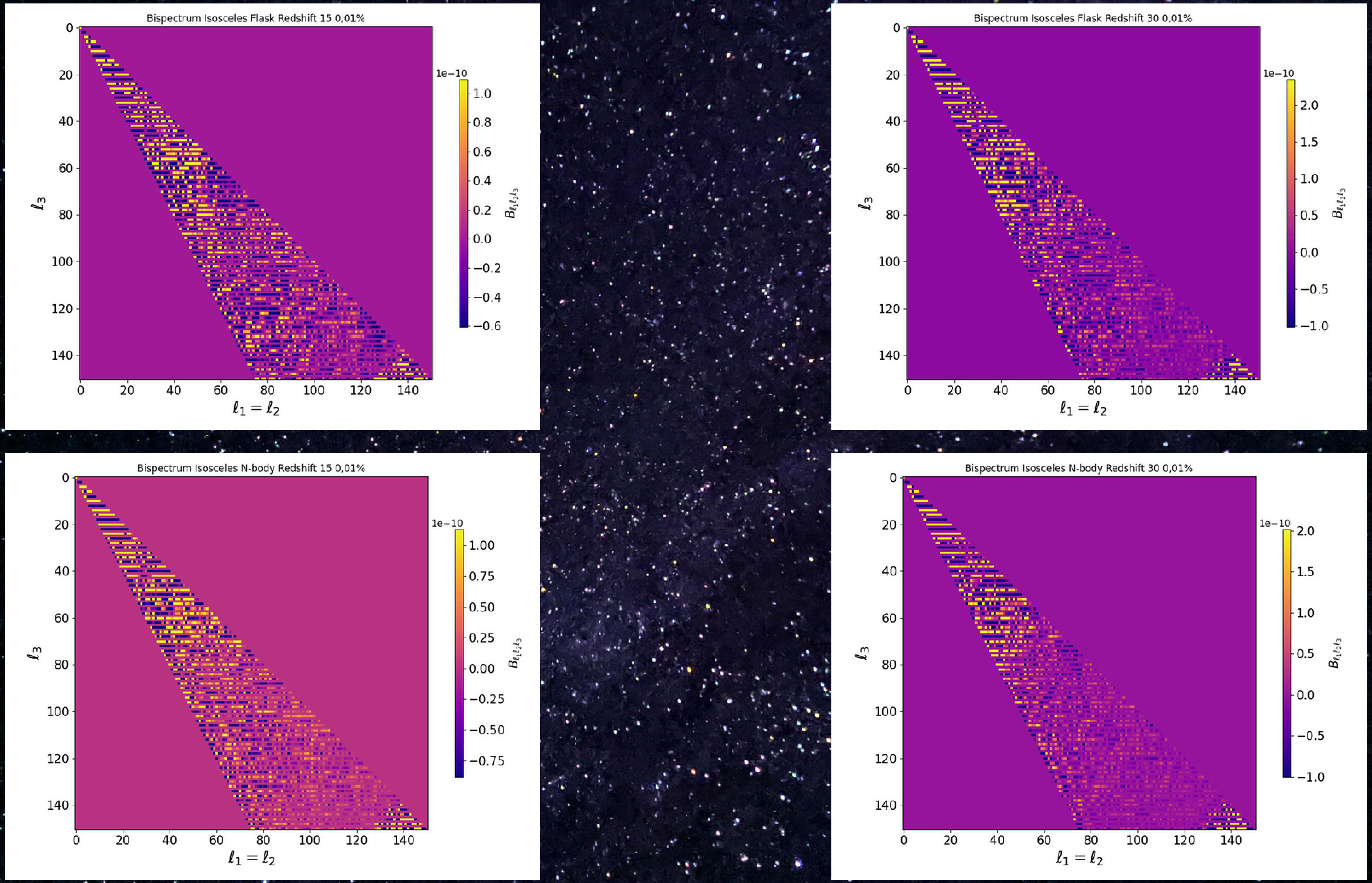




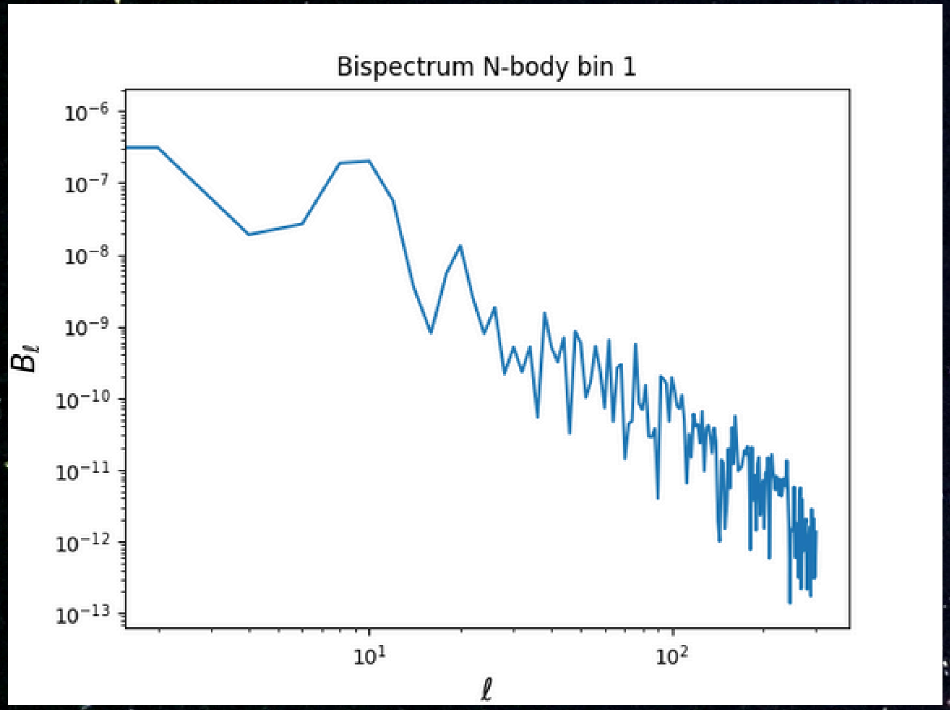


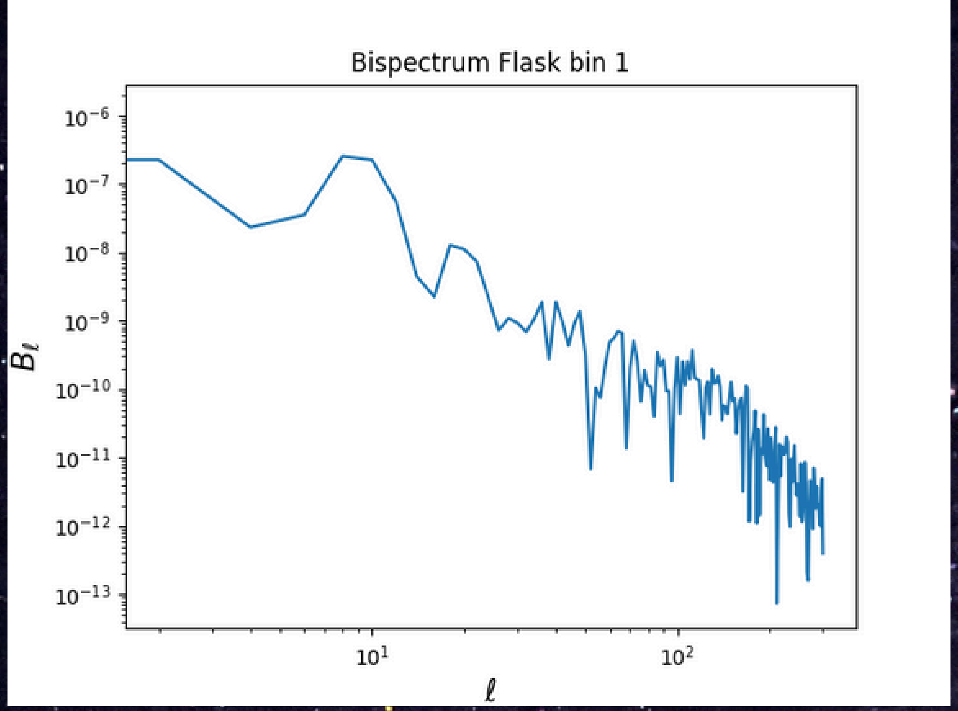


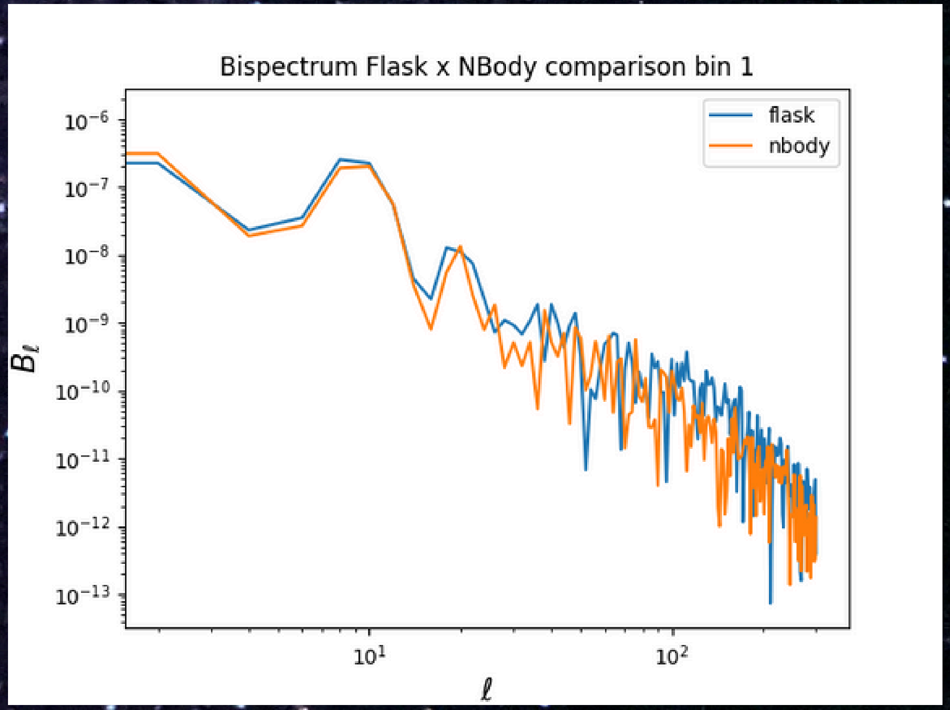
Results

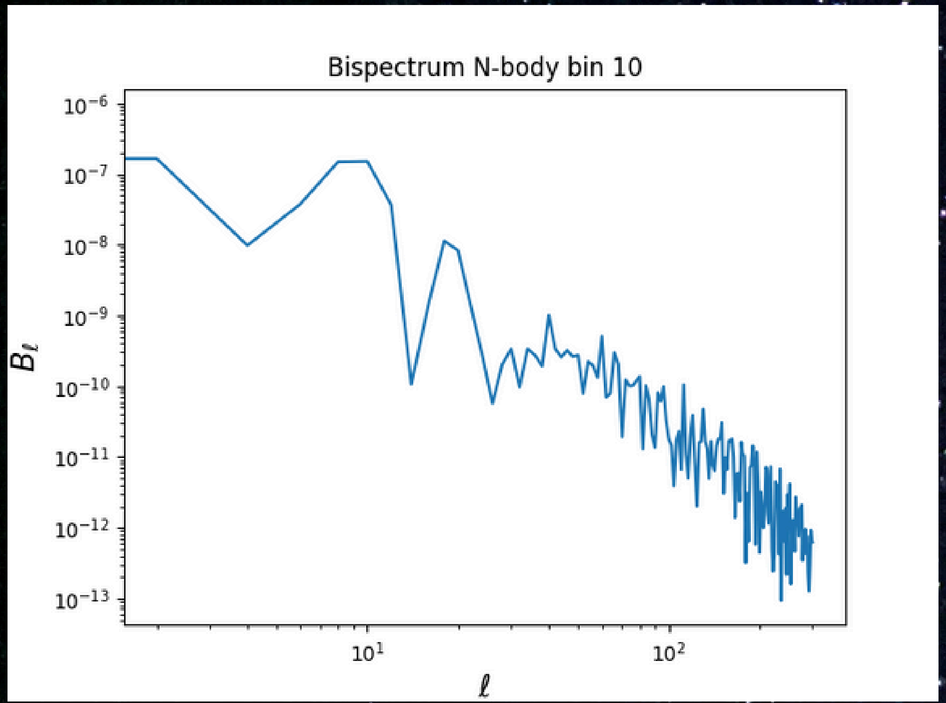

Bispectrum Isosceles Flask Redshift 15 0,01%
Bispectrum Isosceles Flask Redshift 30 0,01%
Bispectrum Isosceles N-body Redshift 15 0,01%
Bispectrum Isosceles N-body Redshift 30 0,01%
ℓ_3
$\ell_1 = \ell_2$
$B_{\ell_1 \ell_2 \ell_3}$
1e−10

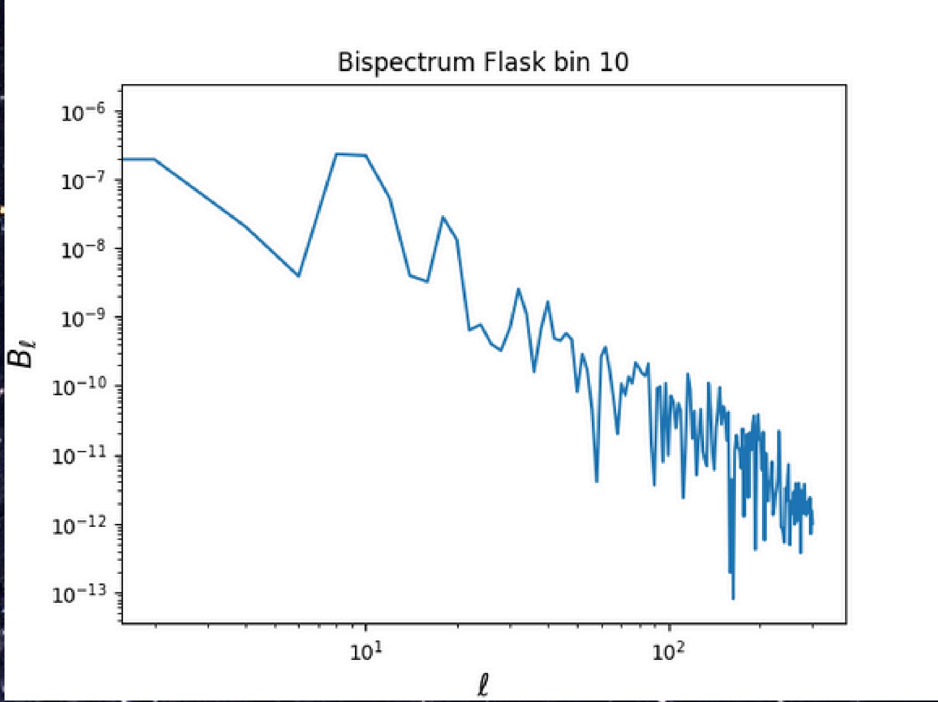
Results

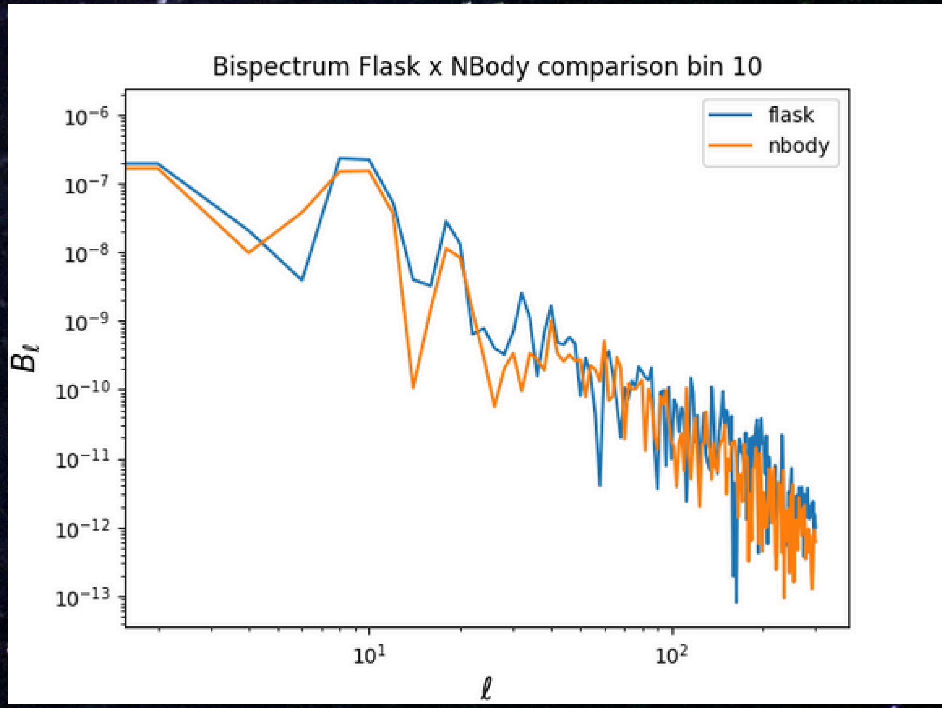

Bispectrum N-body bin 1
B_ℓ
ℓ


Bispectrum Flask bin 1
B_ℓ
ℓ

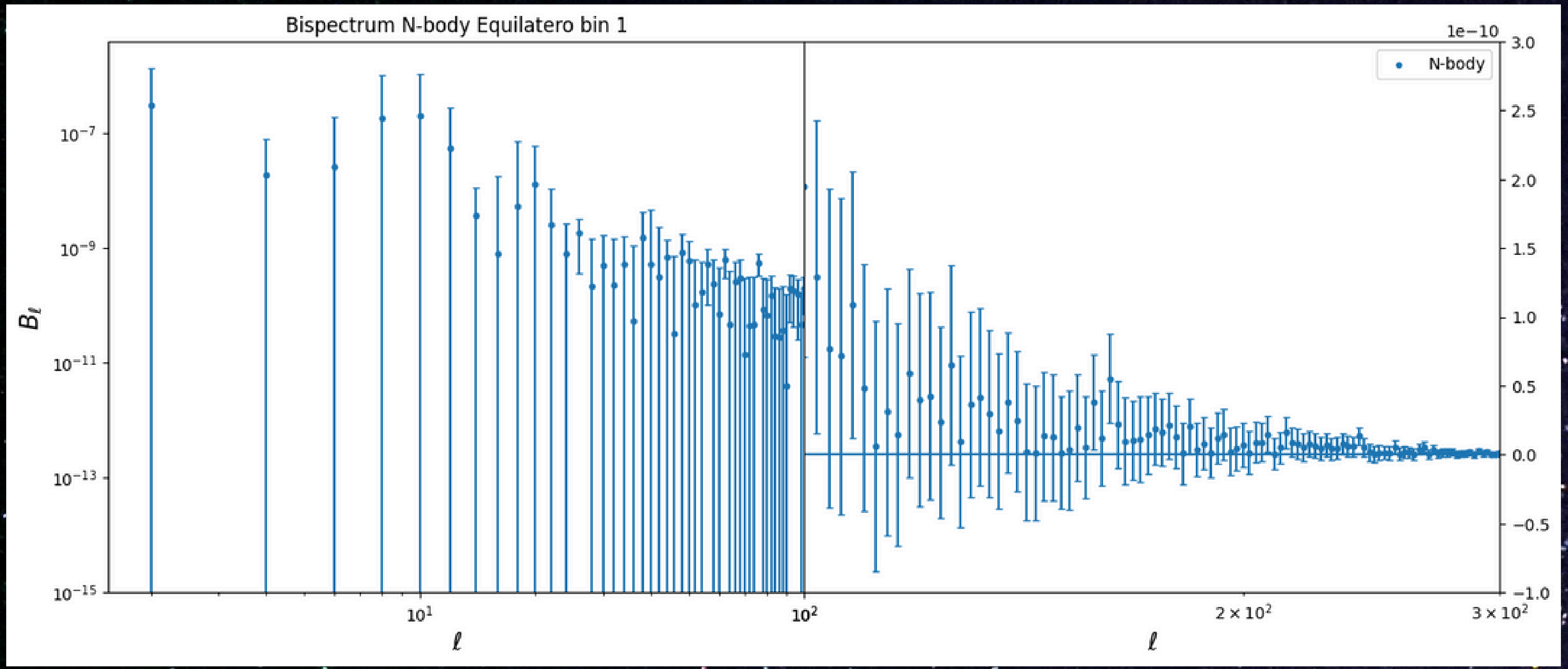

Bispectrum Flask x NBody comparison bin 1
flask
nbody
B_ℓ
ℓ

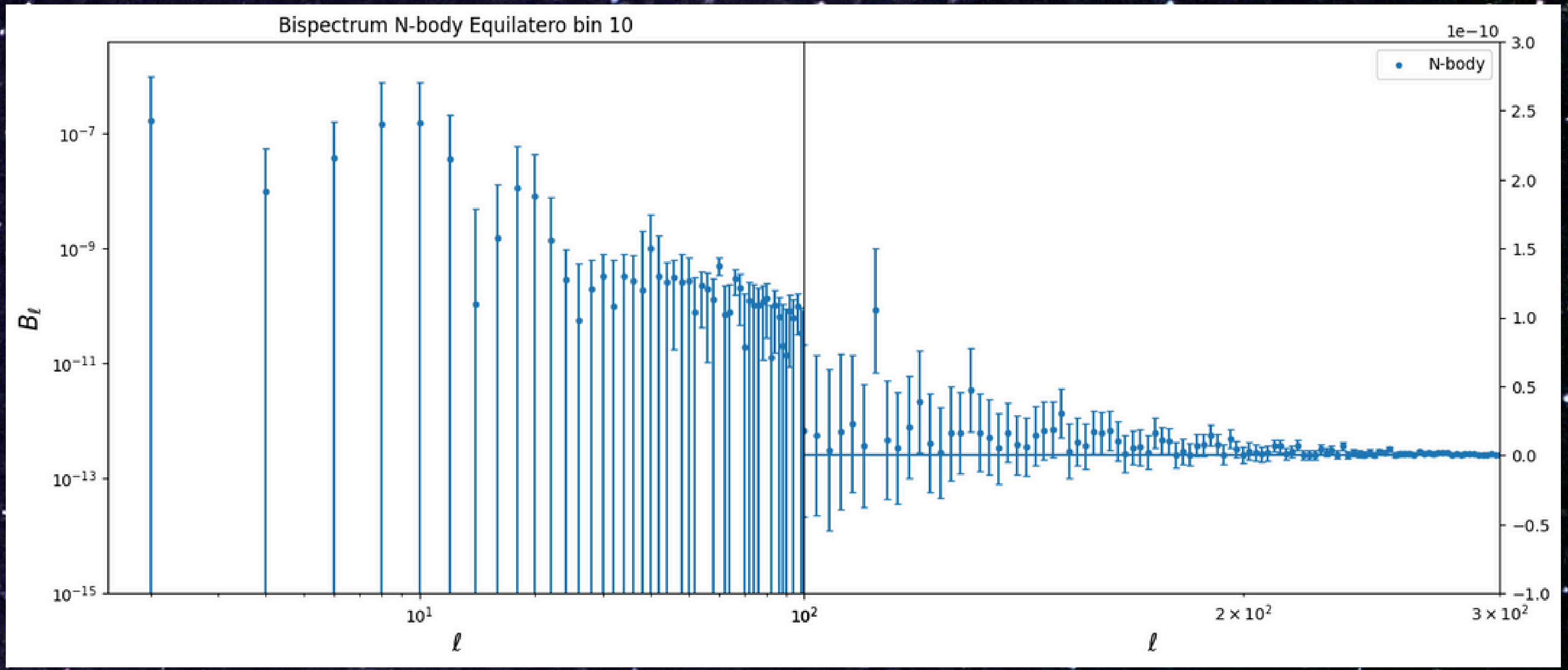

Bispectrum N-body bin 10
B_ℓ
ℓ

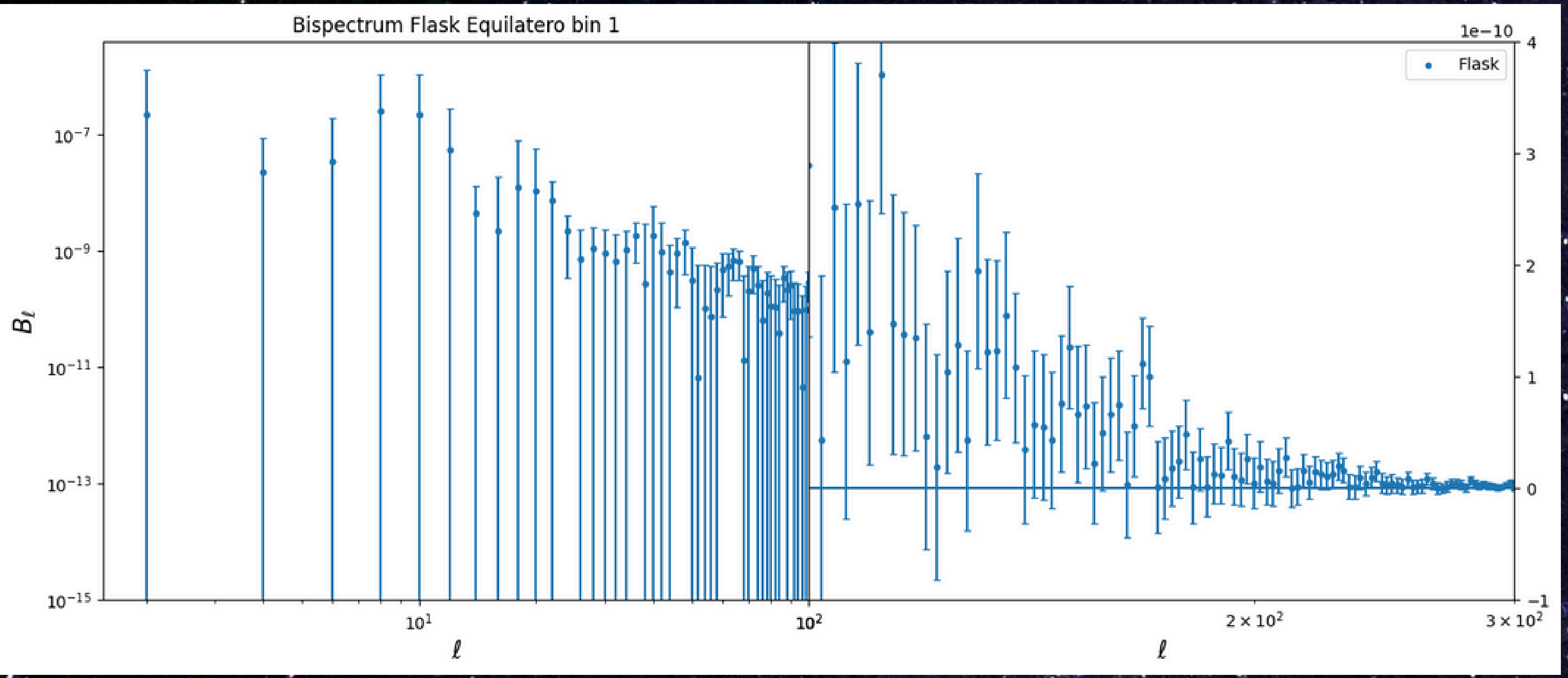

Bispectrum Flask bin 10
B_ℓ
ℓ

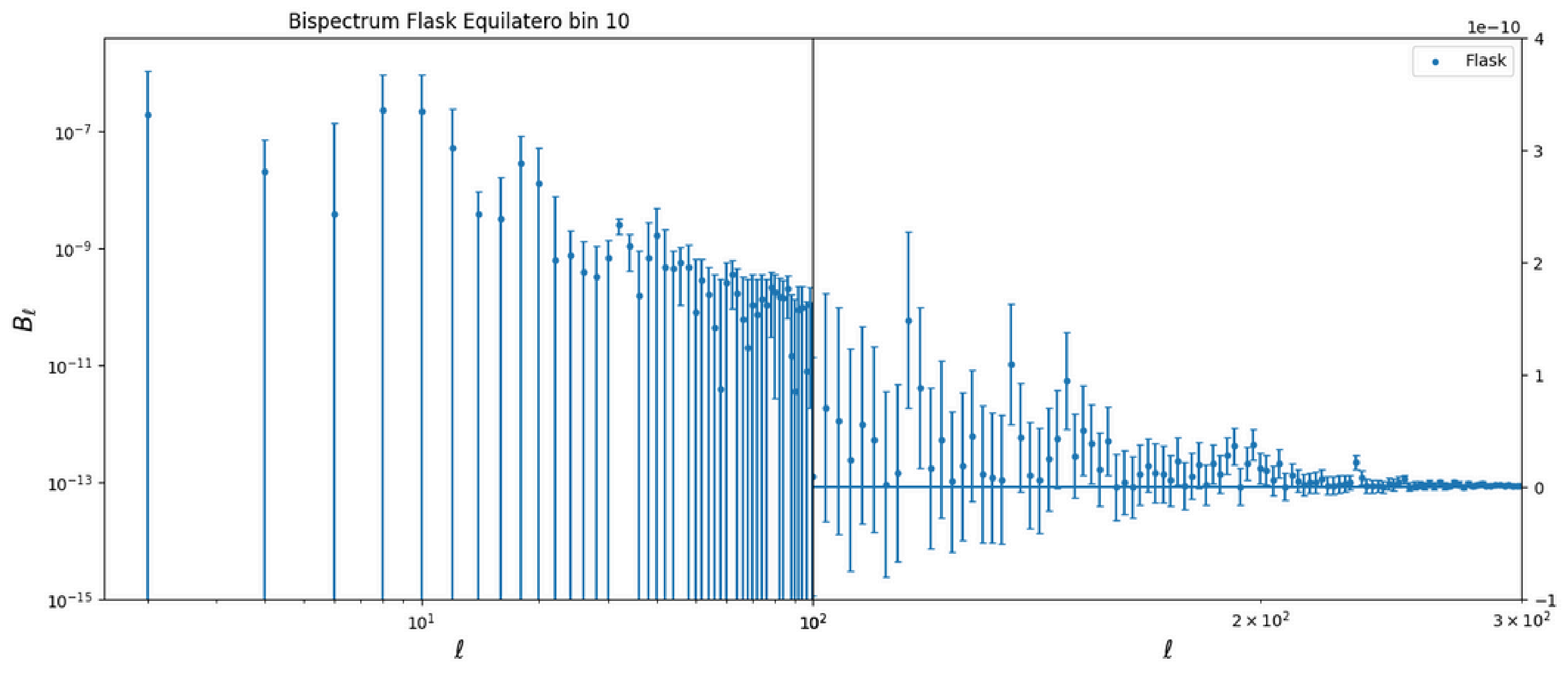

Bispectrum Flask x NBody comparison bin 10
flask
nbody
B_ℓ
ℓ

Results


Bispectrum N-body Equilatero bin 1
N-body
B_ℓ
ℓ
1e−10


Bispectrum N-body Equilatero bin 10
N-body
B_ℓ
ℓ
1e−10


Bispectrum Flask Equilatero bin 1
Flask
B_ℓ
ℓ
1e−10


Bispectrum Flask Equilatero bin 10
Flask
B_ℓ
ℓ
1e−10

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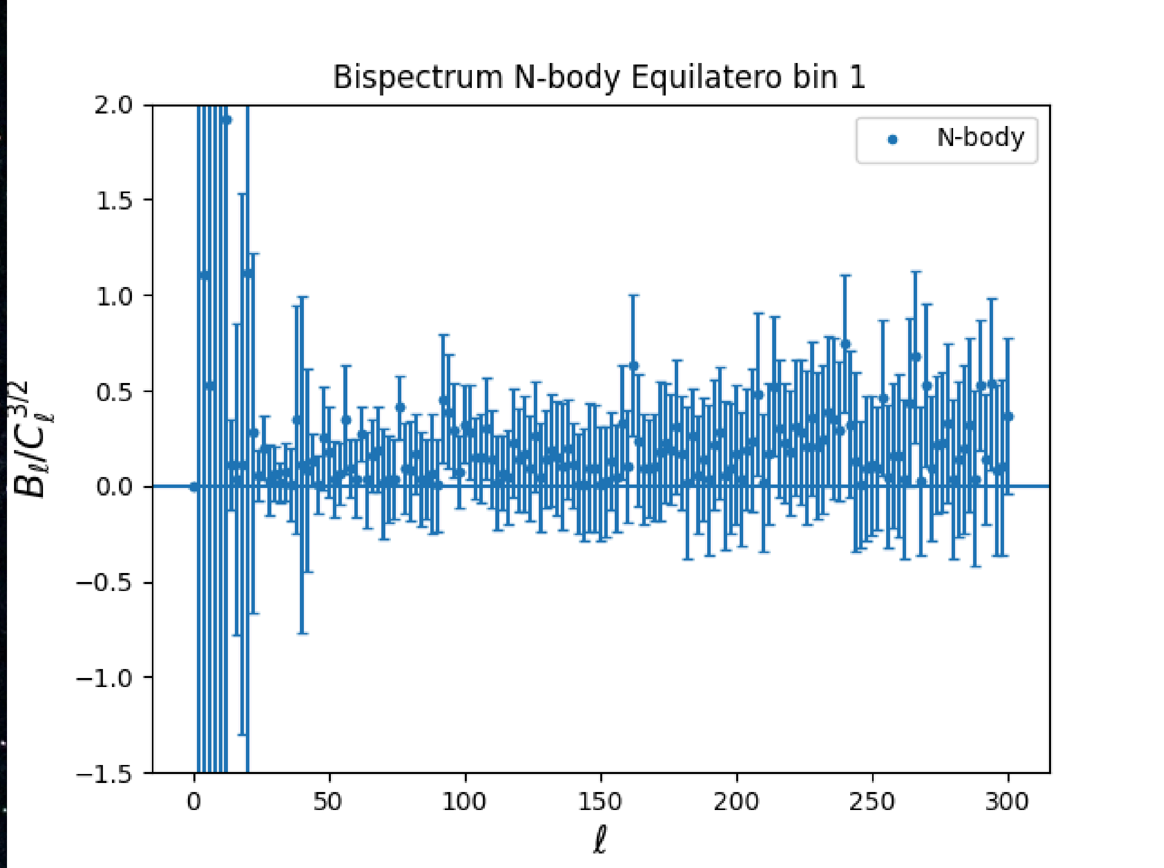
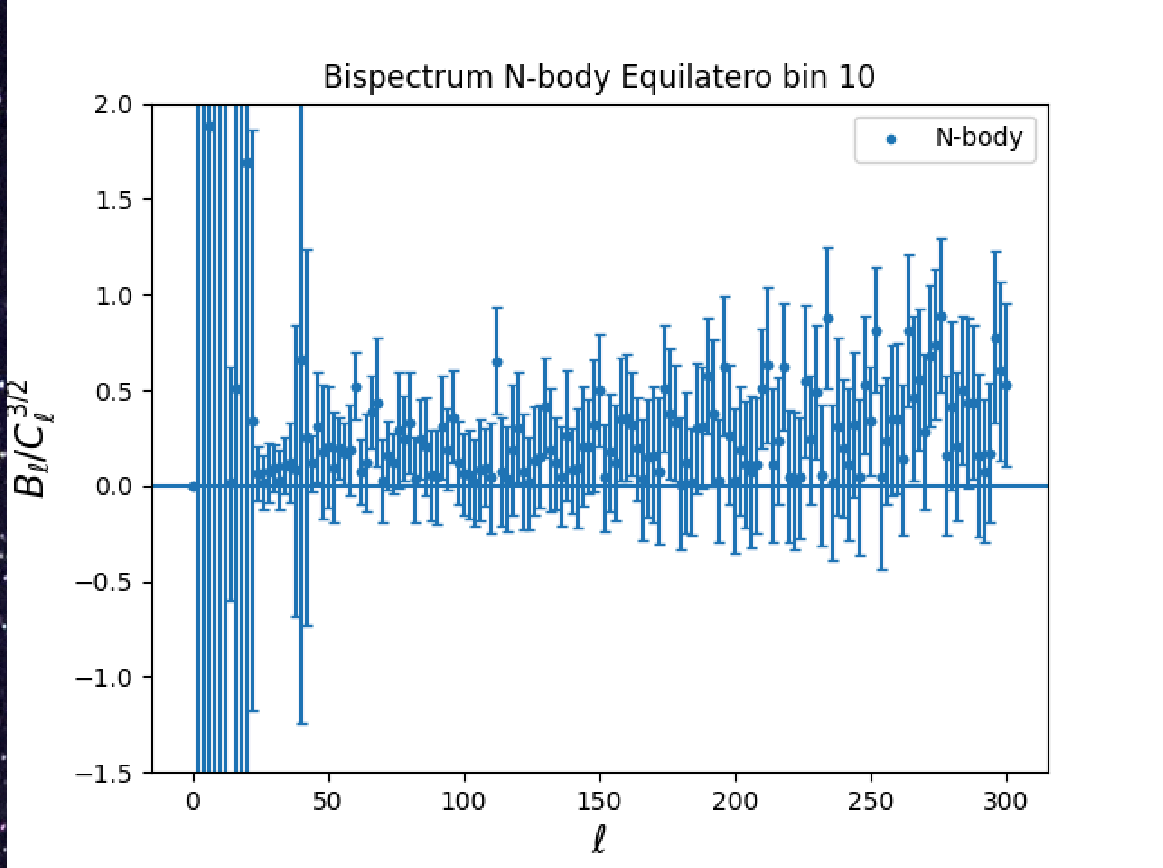
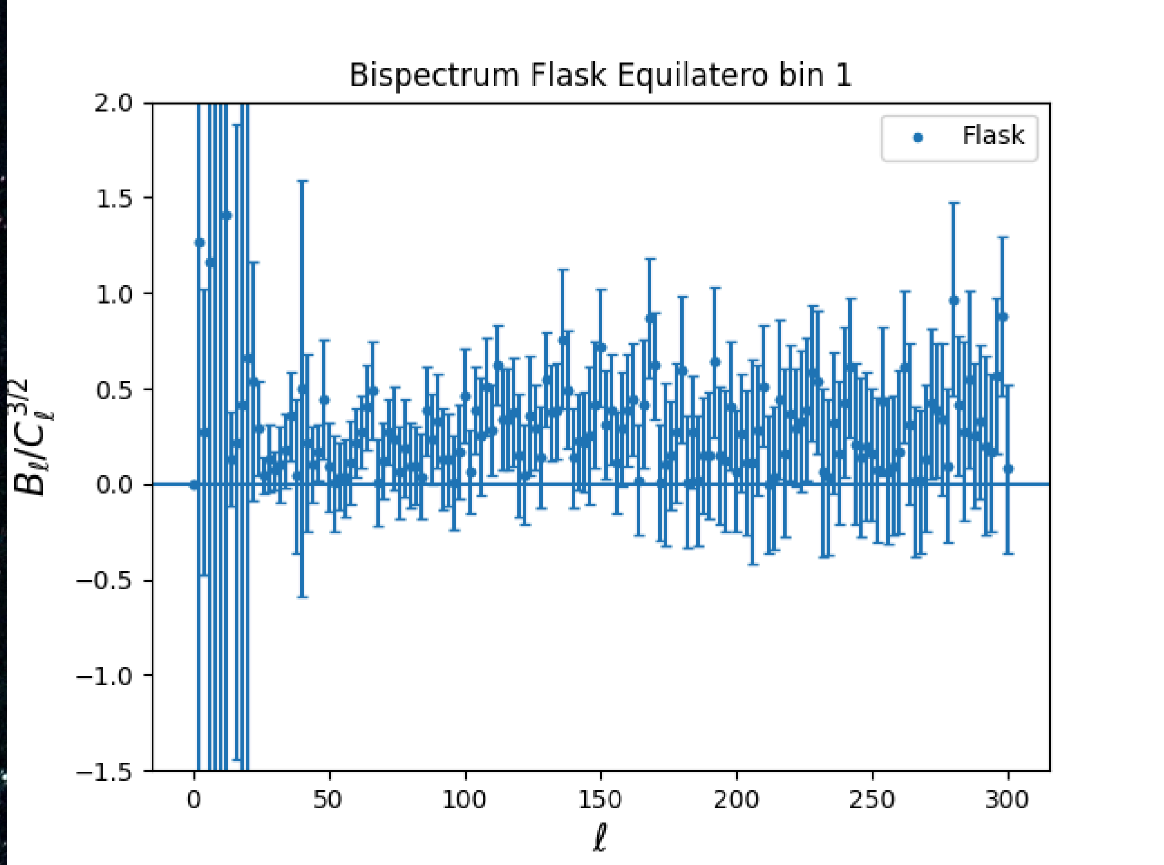
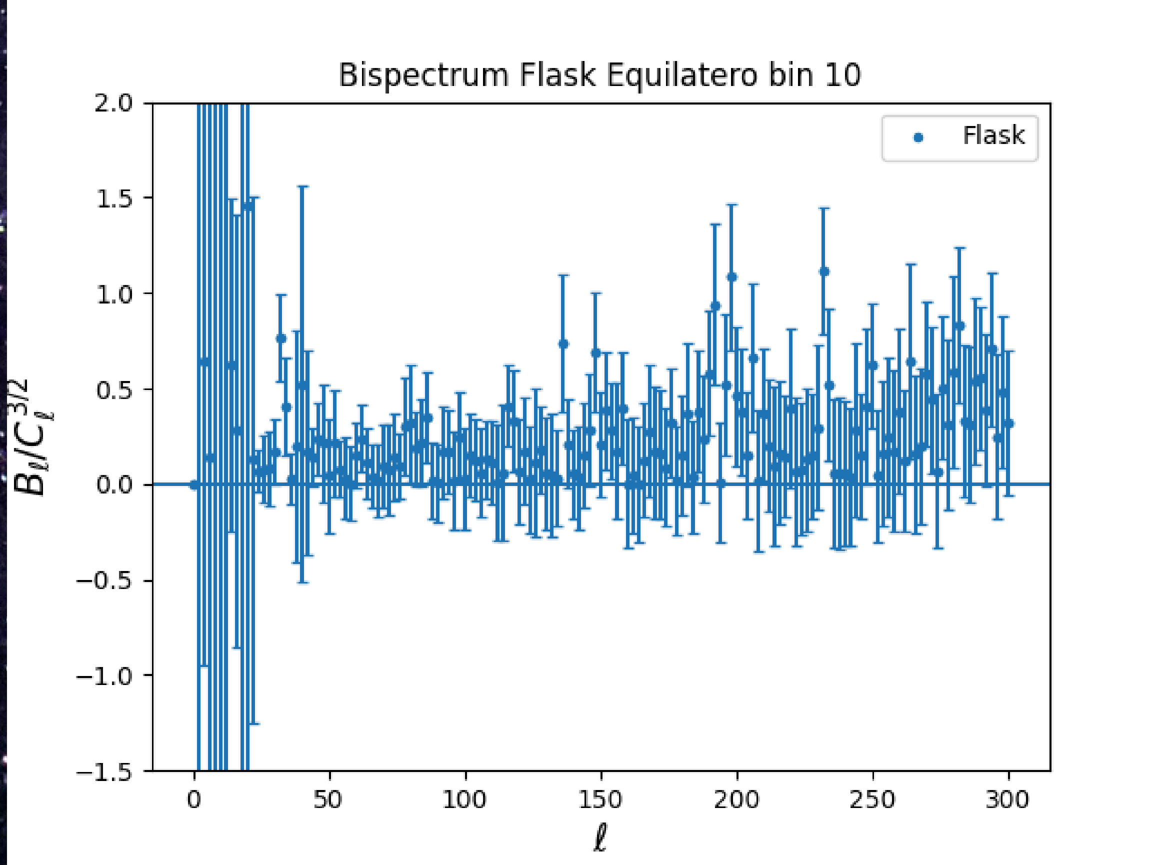
Results









Next Steps

- Finish the statistical results for the equilateral case (it depends on the SDumond queue)
- Cross-check the bispectrum module with the theory (primordial non-gaussianities).
- Use the bispectrum tool to estimate the non-gaussianity parameter (f_NL).
- Study the foreground residuals using the new versions of the bispectrum module and the module of foreground removal.
- Use the module in simulations with interaction models.

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Thank You!

Obrigado!

谢谢！

CAPES