BINGO

POLARIZATION LEAKAGE

GABRIEL COSTA

10/07/2024

BINGO III

The bingo project: III. optical design and optimization of the focal plane.

- Cross Polarization
- Stokes Parameters
- GRASP

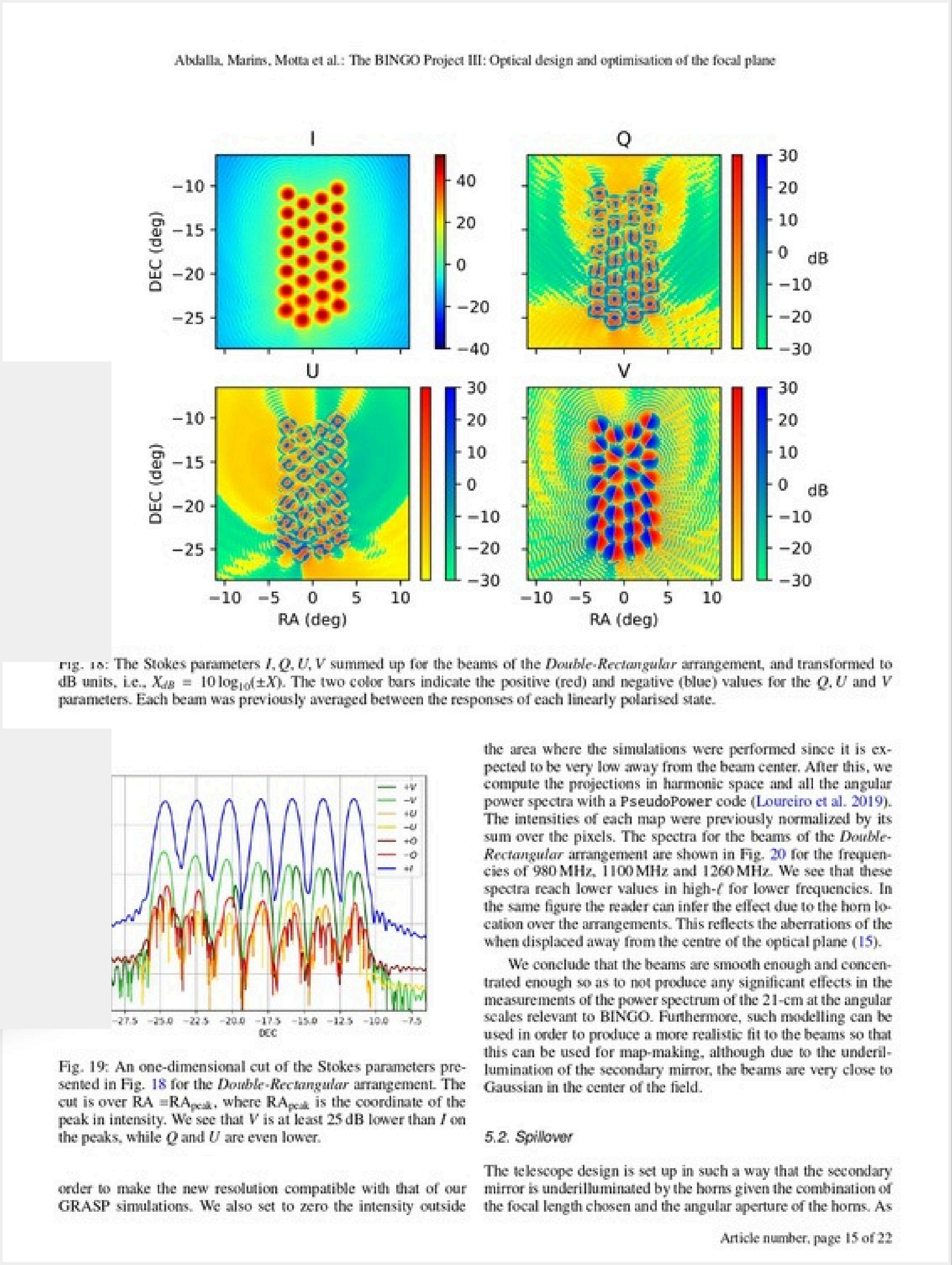


Fig. 18: The Stokes parameters I, Q, U, V summed up for the beams of the *Double-Rectangular* arrangement, and transformed to dB units, i.e., $X_{dB} = 10\log_{10}(\pm X)$. The two color bars indicate the positive (red) and negative (blue) values for the Q, U and V parameters. Each beam was previously averaged between the responses of each linearly polarised state.

Fig. 19: An one-dimensional cut of the Stokes parameters presented in Fig. 18 for the *Double-Rectangular* arrangement. The cut is over RA $=\mathrm{RA}_{\mathrm{peak}}$, where $\mathrm{RA}_{\mathrm{peak}}$ is the coordinate of the peak in intensity. We see that V is at least 25 dB lower than I on the peaks, while Q and U are even lower.

order to make the new resolution compatible with that of our GRASP simulations. We also set to zero the intensity outside the area where the simulations were performed since it is expected to be very low away from the beam center. After this, we compute the projections in harmonic space and all the angular power spectra with a PseudoPower code (Loureiro et al. 2019). The intensities of each map were previously normalized by its sum over the pixels. The spectra for the beams of the *Double-Rectangular* arrangement are shown in Fig. 20 for the frequencies of 980 MHz, 1100 MHz and 1260 MHz. We see that these spectra reach lower values in high-ℓ for lower frequencies. In the same figure the reader can infer the effect due to the horn location over the arrangements. This reflects the aberrations of the when displaced away from the centre of the optical plane (15).

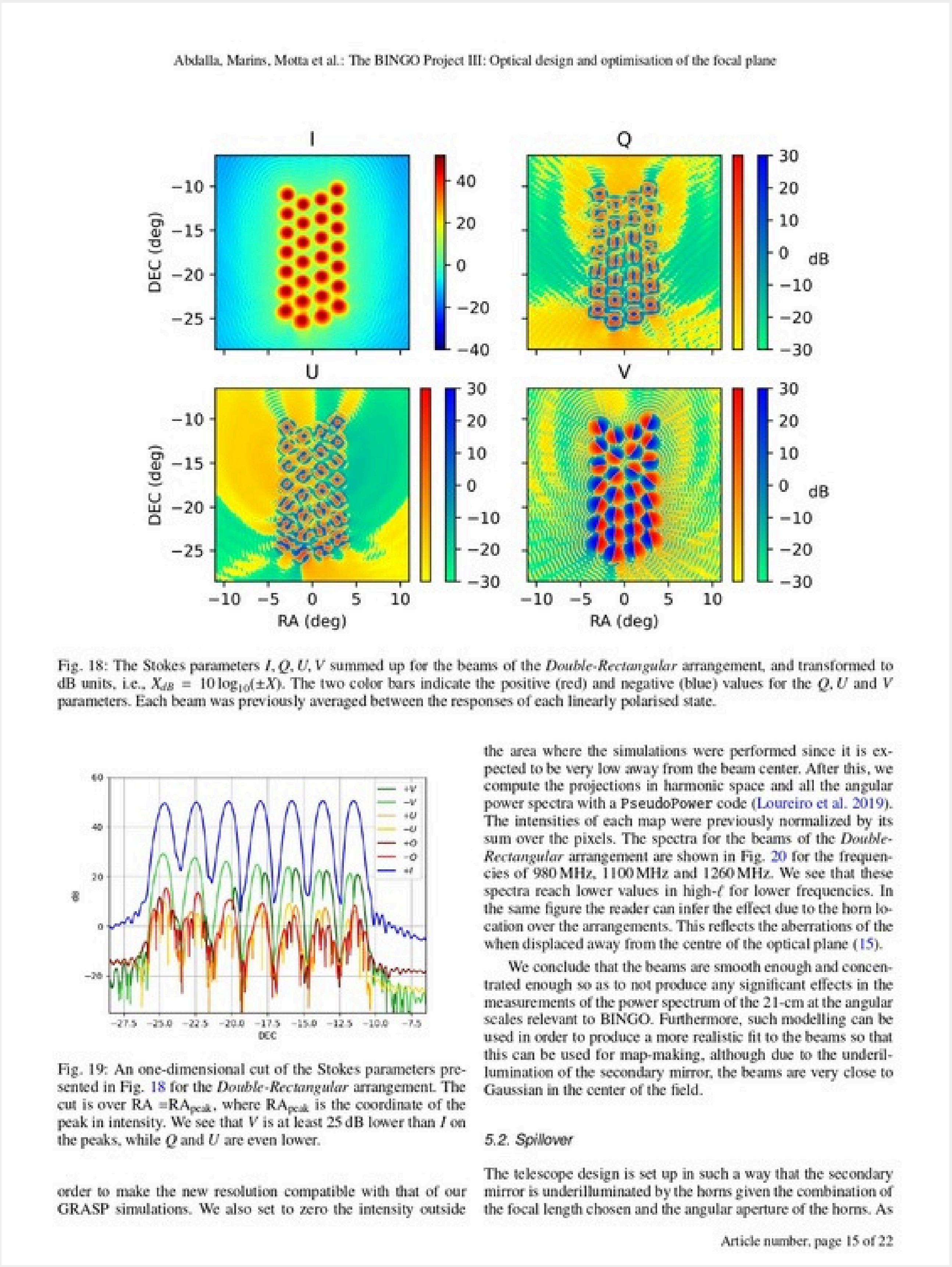
We conclude that the beams are smooth enough and concentrated enough so as to not produce any significant effects in the measurements of the power spectrum of the 21-cm at the angular scales relevant to BINGO. Furthermore, such modelling can be used in order to produce a more realistic fit to the beams so that this can be used for map-making, although due to the underillumination of the secondary mirror, the beams are very close to Gaussian in the center of the field.

5.2. Spillover

The telescope design is set up in such a way that the secondary mirror is underilluminated by the horns given the combination of the focal length chosen and the angular aperture of the horns. As

BINGO III

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} \langle e_x e_x^* \rangle + \langle e_y e_y^* \rangle \\ \langle e_x e_x^* \rangle - \langle e_y e_y^* \rangle \\ \langle e_x e_y^* \rangle + \langle e_y e_x^* \rangle \\ -\imath(\langle e_x e_y^* \rangle - \langle e_y e_x^* \rangle) \end{bmatrix}$$



Abdalla, Marins, Motta et al.: The BINGO Project III: Optical design and optimisation of the focal plane

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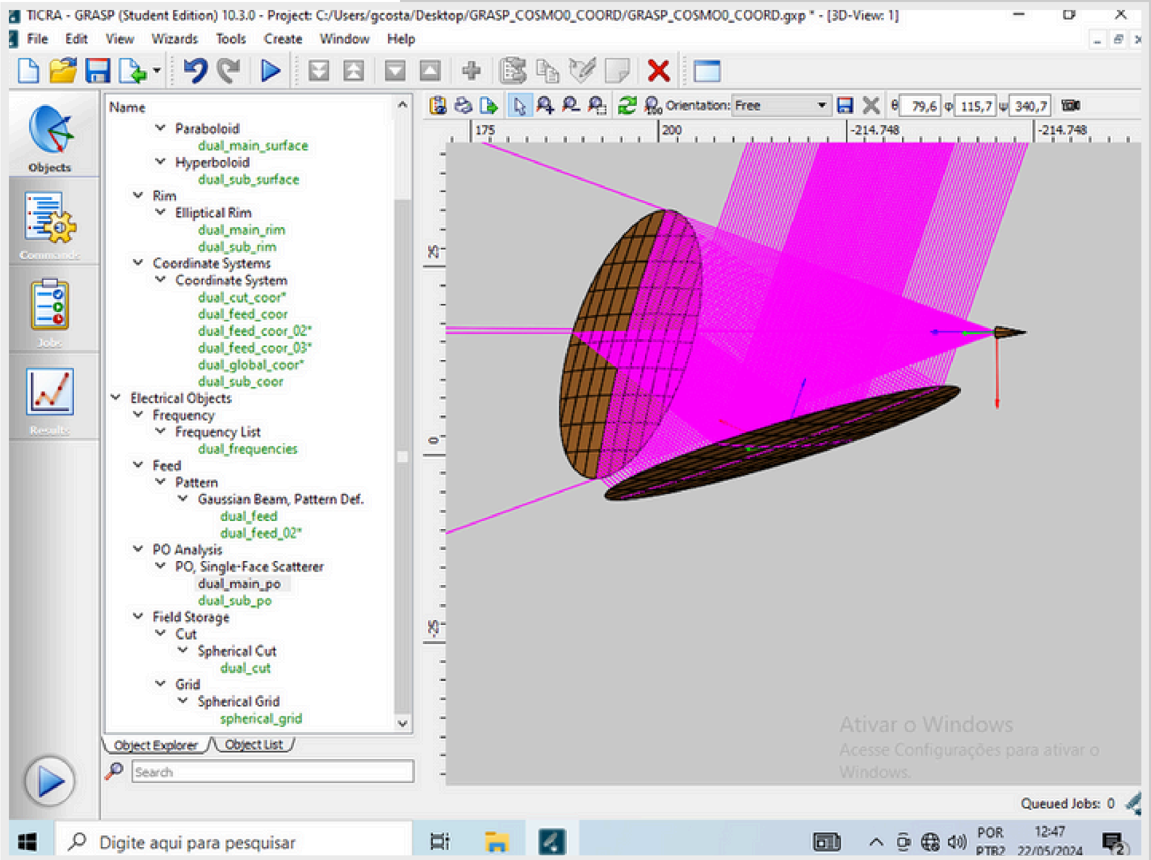
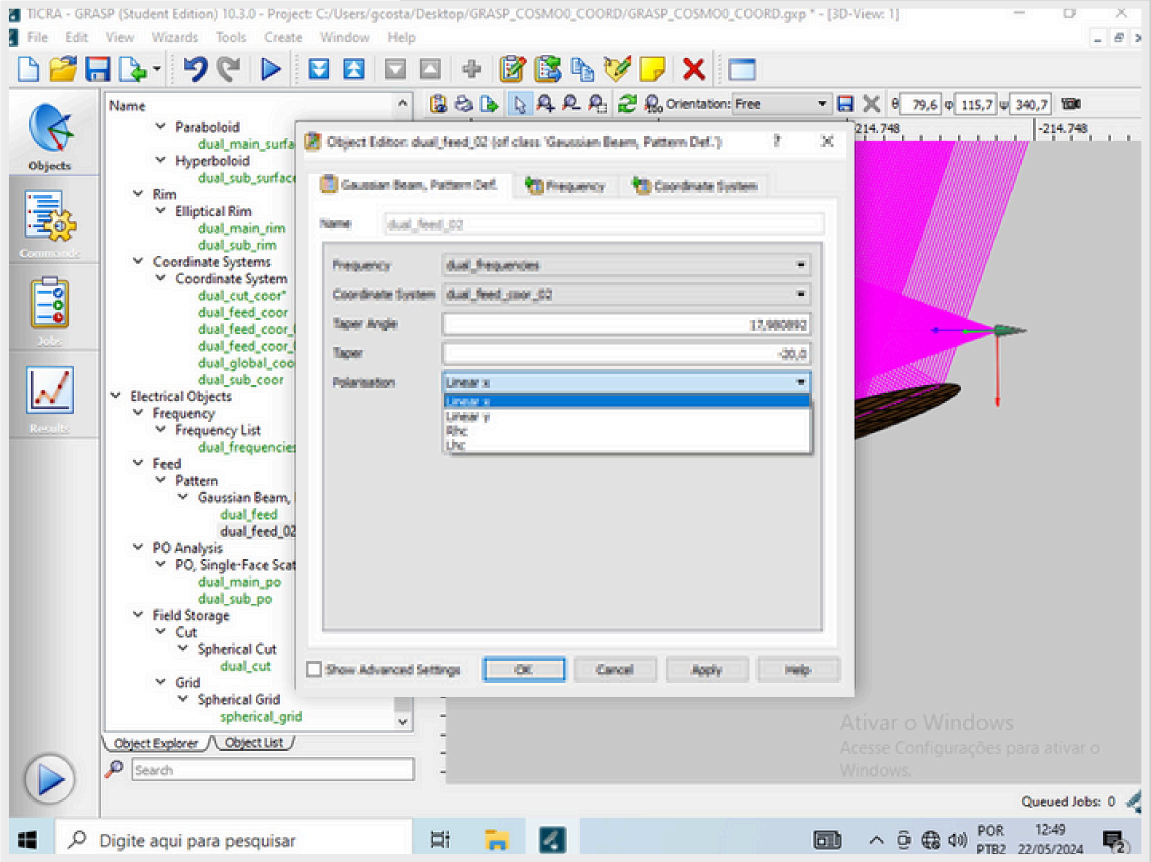
BINGO III

GRASP (General Reflector Antenna Software Package)

- Analysis and design of reflector antennas;
- Physical Optics method to calculate the reflected electromagnetic field;
- BINGO arrangement.

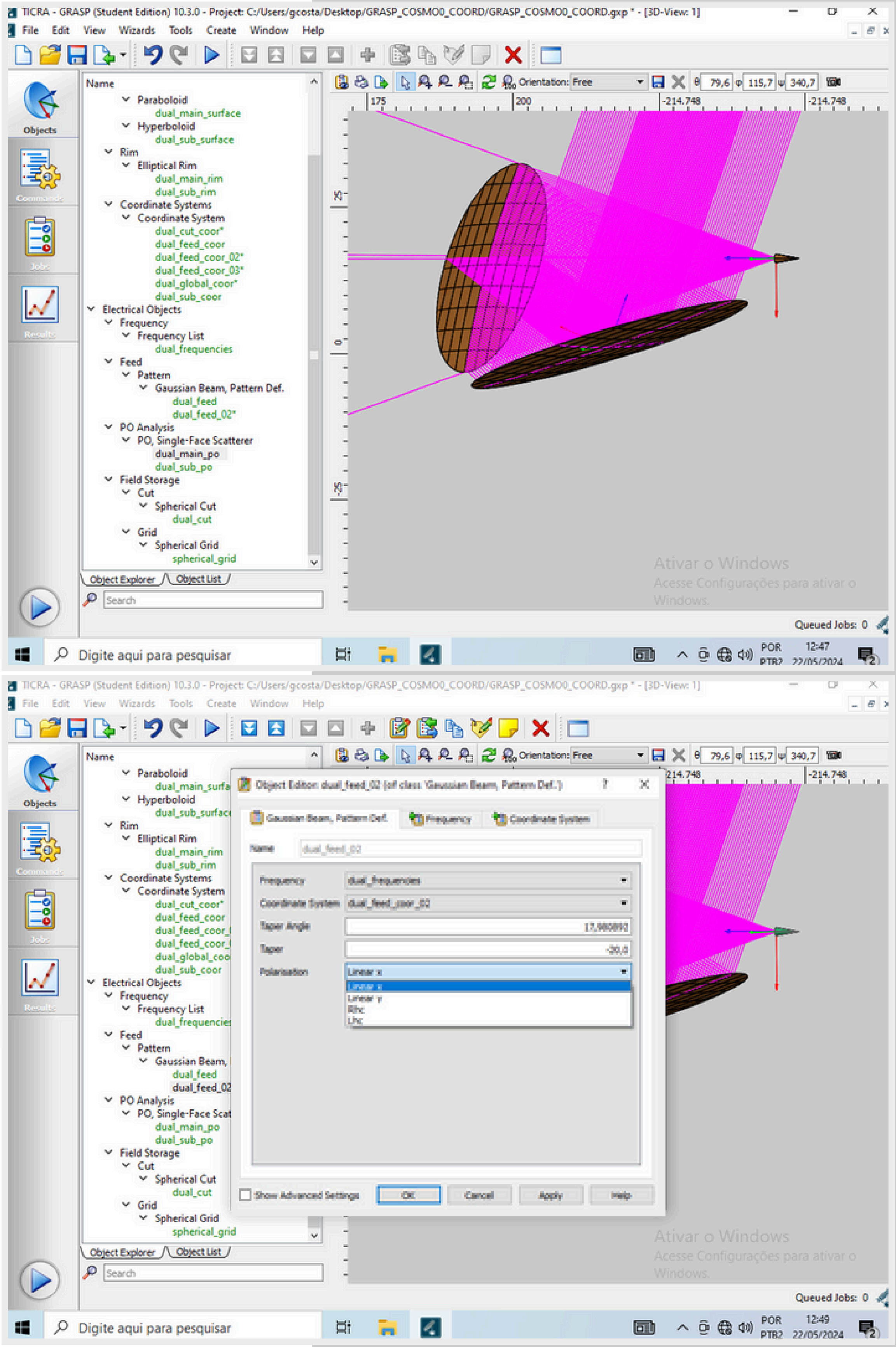




BINGO III

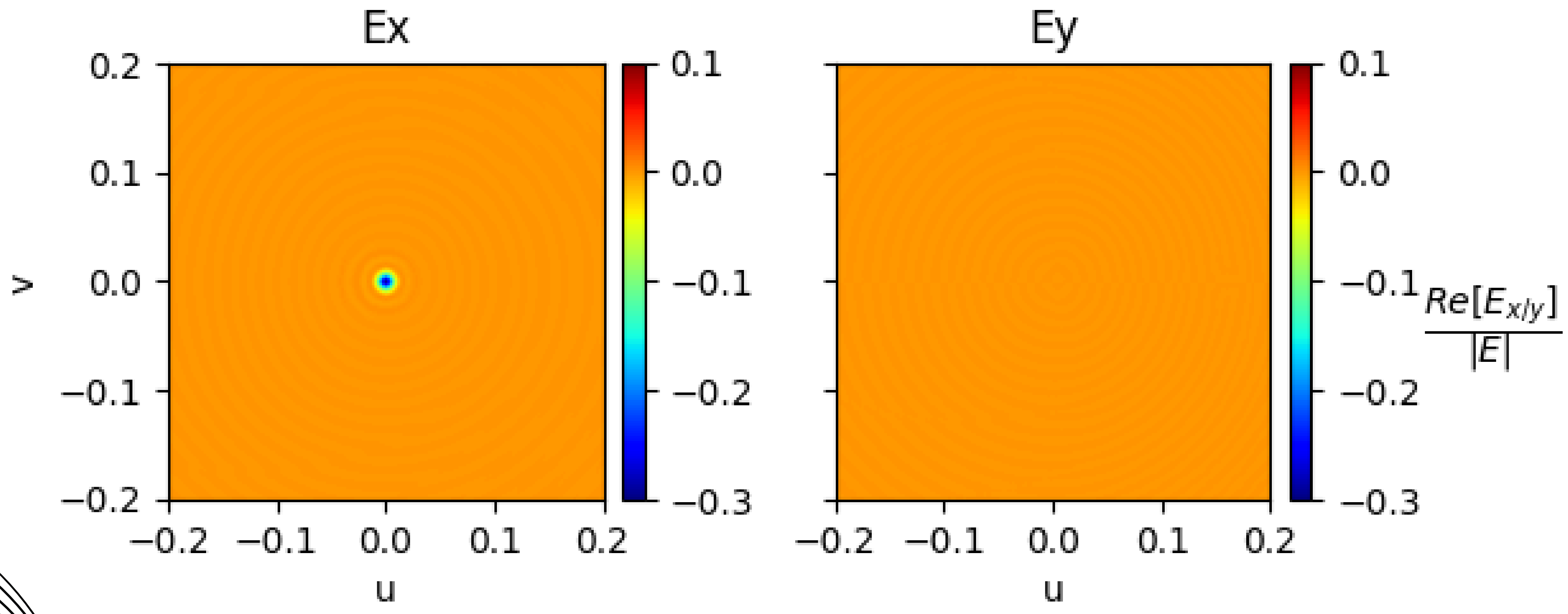
GRASP (General Reflector Antenna Software Package)

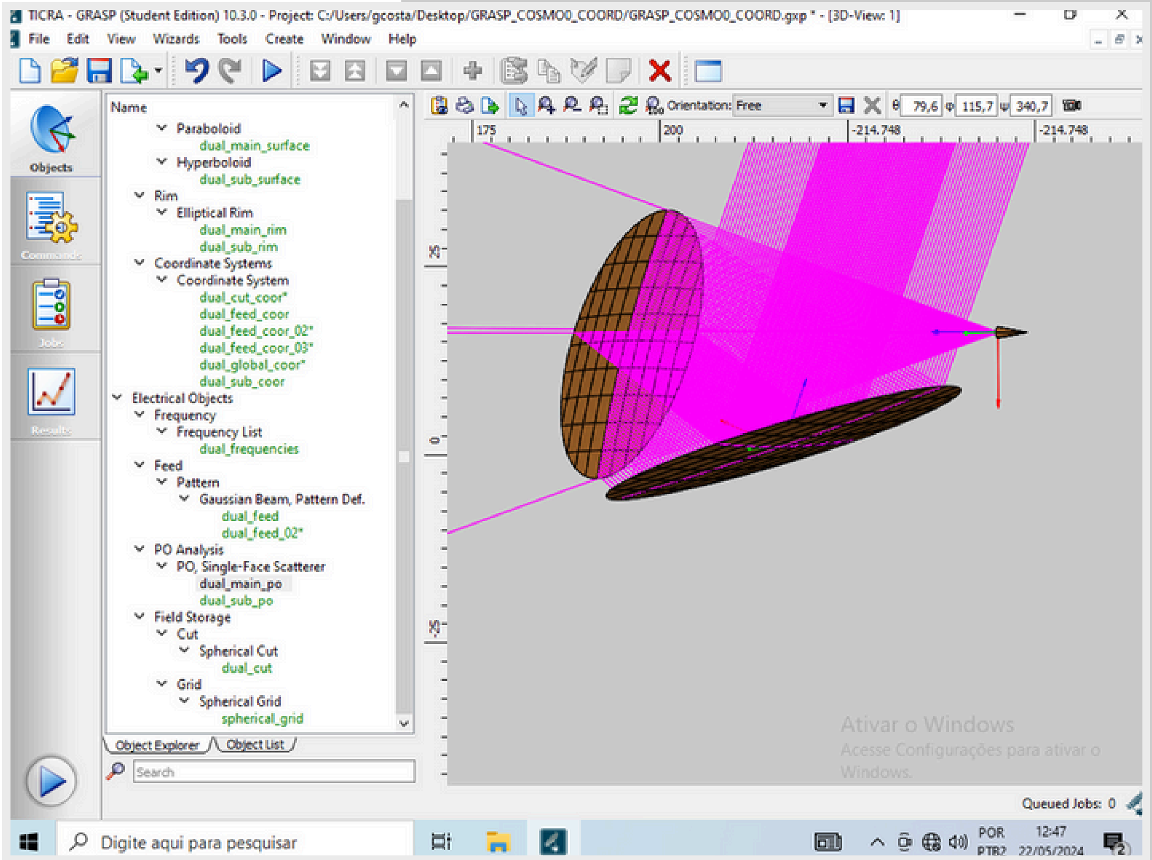
- Monochromatic Gaussian beam.
- Choose the input polarization.
- Four polarization inputs.

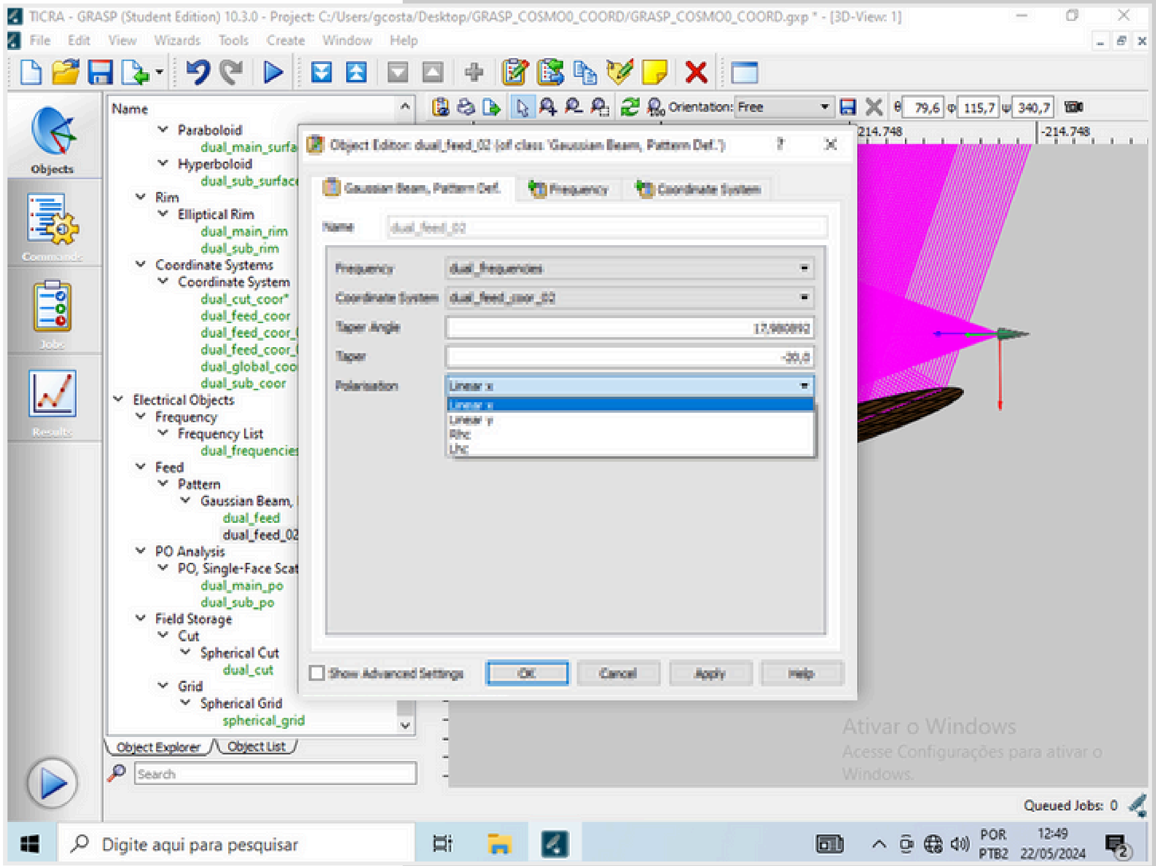


BINGO III

GRASP output for a beam with linear x polarization (Real part):







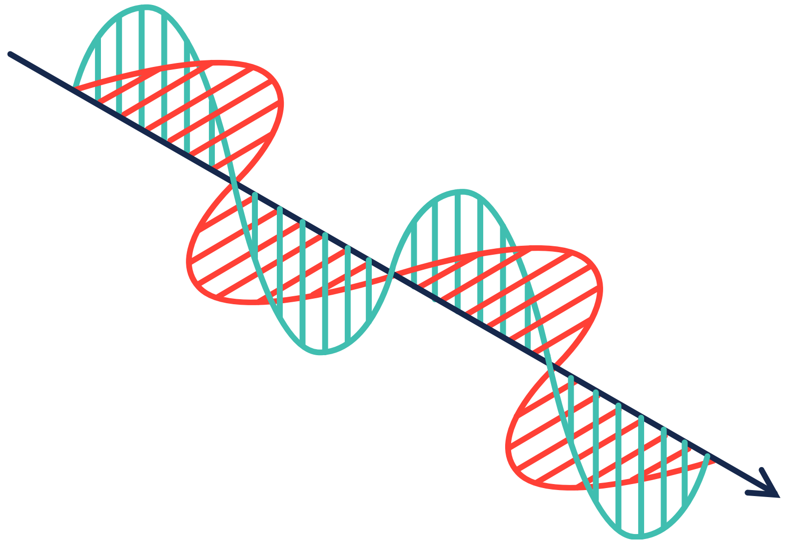
STOKES PARAMETERS

Eletromagnetic field:

$$\mathbf{E} = (\hat{\mathbf{x}} E_1 + \hat{\mathbf{y}} E_2) e^{-i\omega t} = \mathbf{E}_0 e^{-i\omega t}.$$

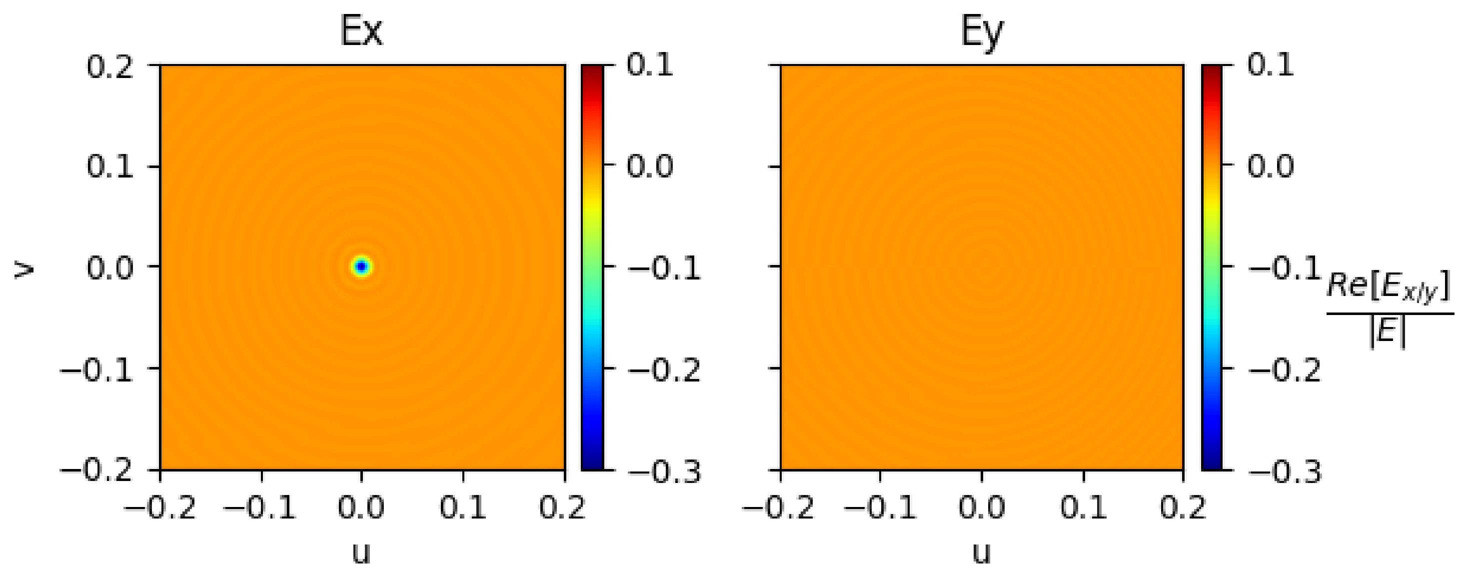
Complex aplitudes:

$$E_1 = \mathcal{E}_1 e^{i\phi_1}, \qquad E_2 = \mathcal{E}_2 e^{i\phi_2}.$$



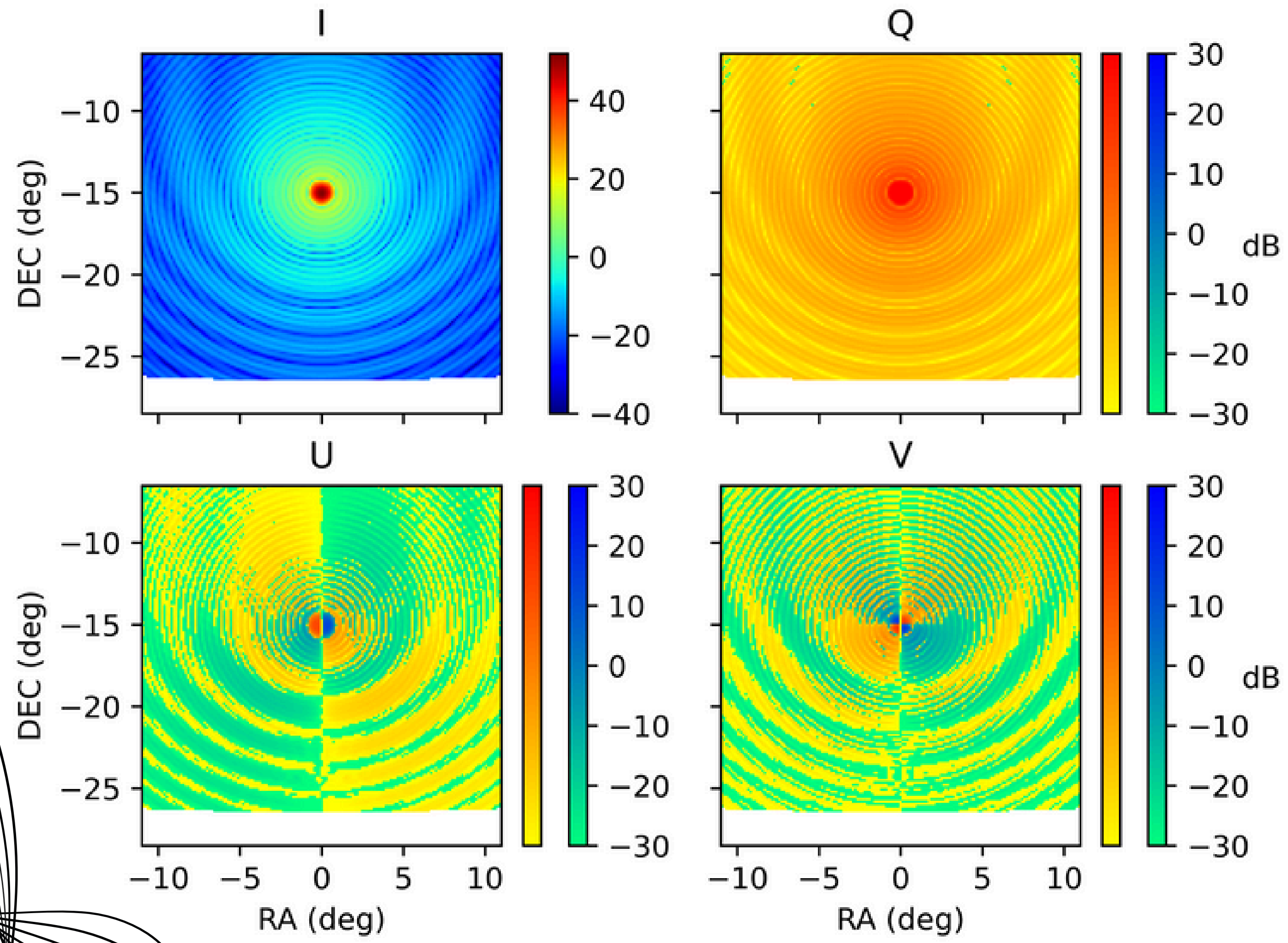
Stokes Vector are defined as:

$$\boldsymbol{s} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} \langle e_x e_x^* \rangle + \langle e_y e_y^* \rangle \\ \langle e_x e_x^* \rangle - \langle e_y e_y^* \rangle \\ \langle e_x e_y^* \rangle + \langle e_y e_x^* \rangle \\ -\imath(\langle e_x e_y^* \rangle - \langle e_y e_x^* \rangle) \end{bmatrix}$$



STOKES PARAMETERS

Measured Stokes Vector:



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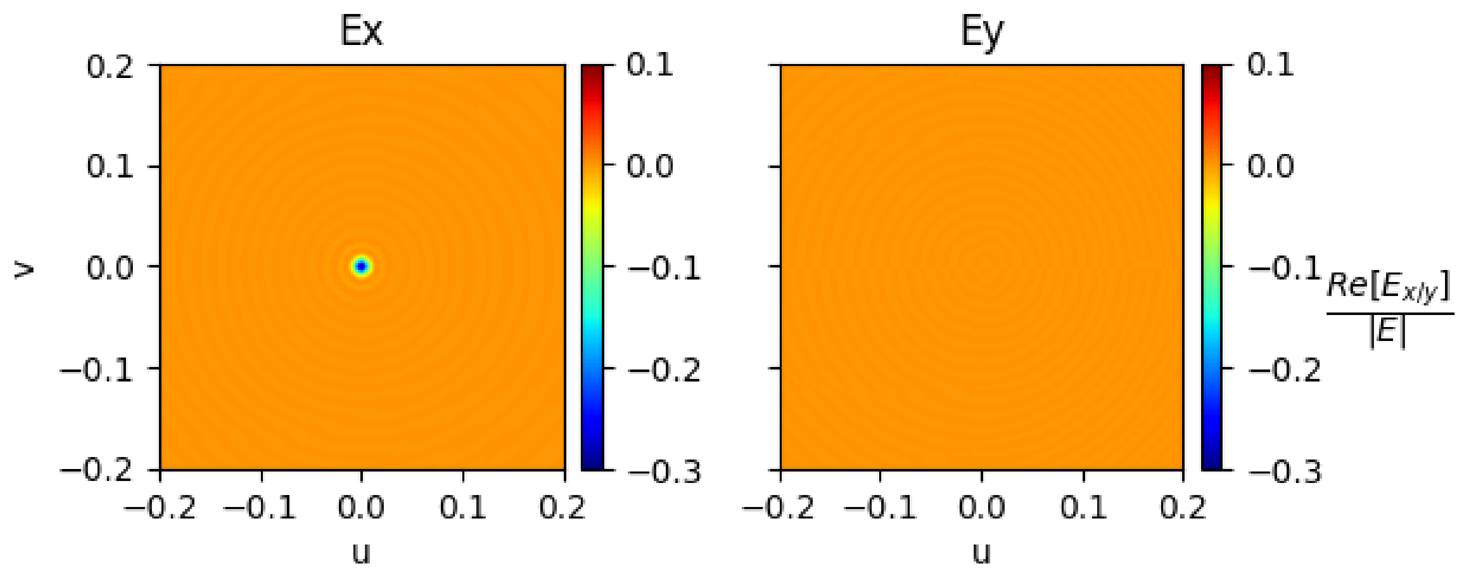

STOKES PARAMETERS

Stokes vector for each polarization state:

$$S_{\mathrm{LHP}} = I_0 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad S_{\mathrm{LVP}} = I_0 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \quad S_{\mathrm{L+45P}} = I_0 \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

$$S_{\mathrm{L-45P}} = I_0 \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \quad S_{\mathrm{RCP}} = I_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad S_{\mathrm{LCP}} = I_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix},$$

$$\boldsymbol{s} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} \langle e_x e_x^* \rangle + \langle e_y e_y^* \rangle \\ \langle e_x e_x^* \rangle - \langle e_y e_y^* \rangle \\ \langle e_x e_y^* \rangle + \langle e_y e_x^* \rangle \\ -\imath(\langle e_x e_y^* \rangle - \langle e_y e_x^* \rangle) \end{bmatrix} .$$

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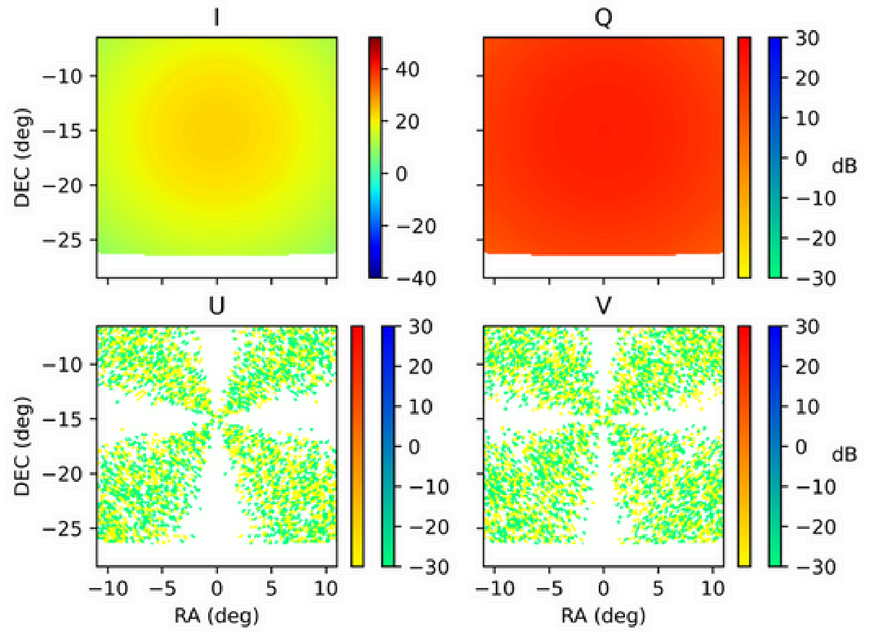
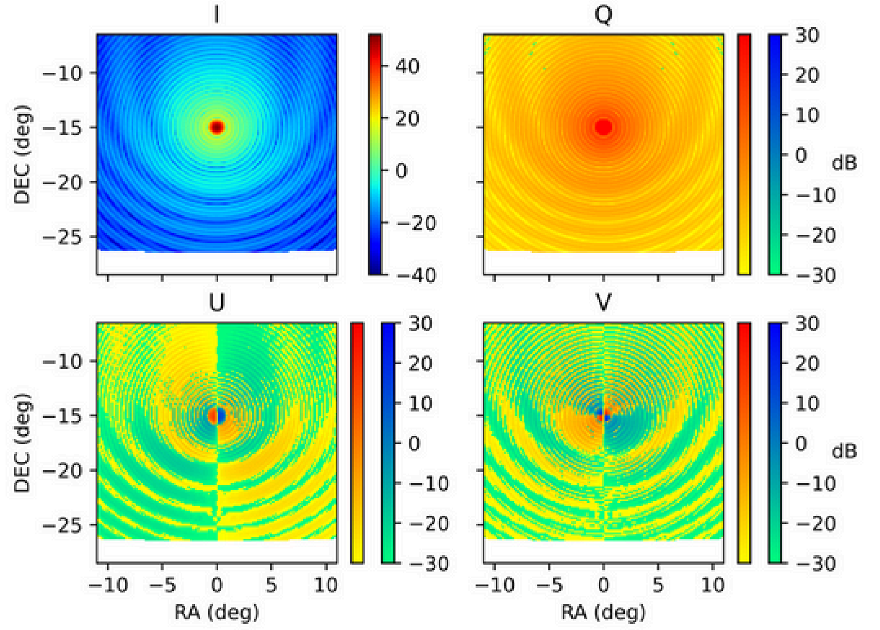
Source Stokes Vector:



Measured Stokes Vector:



MUELLER MATRIX

If a beam of light is initially in the state S and then passes through an optical element M and comes out in a state S’, then it is written:

$$\mathrm{s}' = \mathrm{M}\mathrm{s}$$

Mueller Matrix:

$$\mathbf{M} = \begin{bmatrix} I \to I' & Q \to I' & U \to I' & V \to I' \\ I \to Q' & Q \to Q' & U \to Q' & V \to Q' \\ I \to U' & Q \to U' & U \to U' & V \to U' \\ I \to V' & Q \to V' & U \to V' & V \to V' \end{bmatrix}.$$

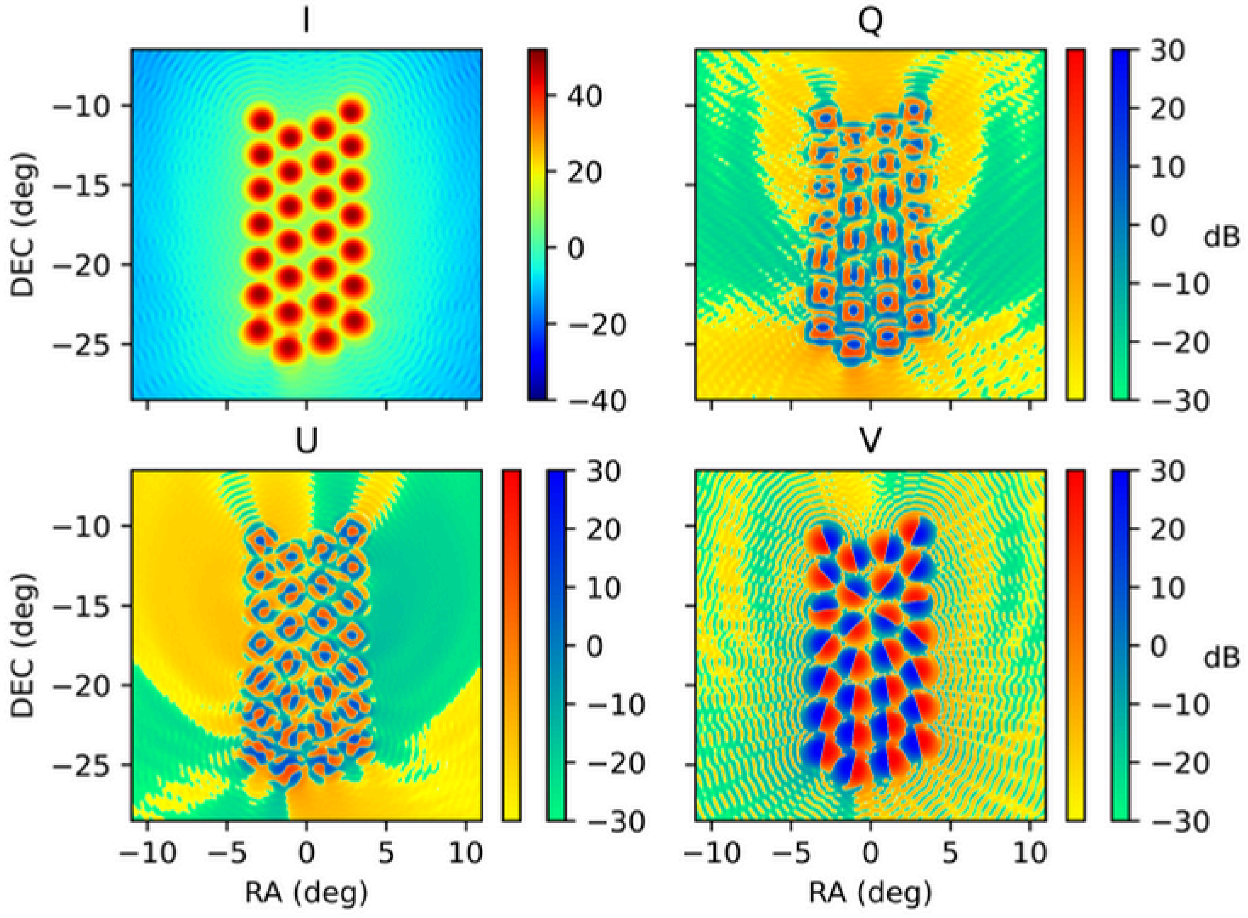
$$\mathbf{S}_1' = \mathbf{M}\mathbf{S}_1 = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{pmatrix} I_0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} I_0 \cdot M_{11} \\ I_0 \cdot M_{12} \\ I_0 \cdot M_{13} \\ I_0 \cdot M_{14} \end{pmatrix} = I_0 \begin{pmatrix} M_{11} \\ M_{12} \\ M_{13} \\ M_{14} \end{pmatrix} = \mathbf{S}_1'$$

$$\mathbf{S}_1 = I_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Unpolarized source

$$\mathbf{M} = \begin{bmatrix} I \to I' & Q \to I' & U \to I' & V \to I' \\ I \to Q' & Q \to Q' & U \to Q' & V \to Q' \\ I \to U' & Q \to U' & U \to U' & V \to U' \\ I \to V' & Q \to V' & U \to V' & V \to V' \end{bmatrix}.$$

As did in BINGO III paper: $\mathbf{S}_1' =$



CALCULATING BINGO MUELLER MATRIX

Defining the following vectors

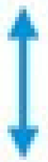
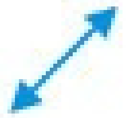
$$\frac{1}{2}(\mathbf{S_{Lx}} + \mathbf{S_{Ly}}) = I_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \equiv \mathbf{S_1}$$

$$\frac{1}{2}(\mathbf{S_{Lx}} - \mathbf{S_{Ly}}) = I_0 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \equiv \mathbf{S_2}$$

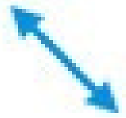
$$\frac{1}{2}(\mathbf{S_{L45}} - \mathbf{S_{L-45}}) = I_0 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \equiv \mathbf{S_3}$$

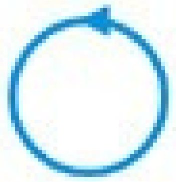
$$\frac{1}{2}(\mathbf{S_{CRP}} - \mathbf{S_{CLP}}) = I_0 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \equiv \mathbf{S_4}$$

$$S_{\mathrm{LHP}} = I_0 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad S_{\mathrm{LVP}} = I_0 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \quad S_{\mathrm{L+45P}} = I_0 \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

$$S_{\mathrm{L-45P}} = I_0 \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \quad S_{\mathrm{RCP}} = I_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad S_{\mathrm{LCP}} = I_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix},$$





CALCULATING BINGO MUELLER MATRIX

$$\frac{1}{2}(\mathbf{S_{Lx}} + \mathbf{S_{Ly}}) = I_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \equiv \mathbf{S_1}$$

$$\mathbf{MS_1} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{pmatrix} I_0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} I_0 \cdot M_{11} \\ I_0 \cdot M_{12} \\ I_0 \cdot M_{13} \\ I_0 \cdot M_{14} \end{pmatrix} = I_0 \begin{pmatrix} M_{11} \\ M_{12} \\ M_{13} \\ M_{14} \end{pmatrix} = \mathbf{S_1'}$$

$$\frac{1}{2}(\mathbf{S_{Lx}} - \mathbf{S_{Ly}}) = I_0 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \equiv \mathbf{S_2}$$

$$\mathbf{MS_2} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{pmatrix} 0 \\ I_0 \\ 0 \\ 0 \end{pmatrix} = I_0 \begin{pmatrix} M_{21} \\ M_{22} \\ M_{23} \\ M_{24} \end{pmatrix} = \mathbf{S_2'}$$

$$\frac{1}{2}(\mathbf{S_{L45}} - \mathbf{S_{L-45}}) = I_0 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \equiv \mathbf{S_3}$$

$$\mathbf{MS_3} = I_0 \begin{pmatrix} M_{31} \\ M_{32} \\ M_{33} \\ M_{34} \end{pmatrix} = \mathbf{S_3'}$$

$$\frac{1}{2}(\mathbf{S_{CRP}} - \mathbf{S_{CLP}}) = I_0 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \equiv \mathbf{S_4}$$

$$\mathbf{MS_4} = I_0 \begin{pmatrix} M_{41} \\ M_{42} \\ M_{43} \\ M_{44} \end{pmatrix} = \mathbf{S_4'}$$

CALCULATING BINGO MUELLER MATRIX

As the Mueller matrix is a linear transformation:

$$\frac{1}{2}(\mathbf{M}\mathbf{s}_{\mathbf{Lx}} + \mathbf{M}\mathbf{s}_{\mathbf{Ly}}) = \frac{1}{2}\mathbf{M}(\mathbf{s}_{\mathbf{Lx}} + \mathbf{s}_{\mathbf{Ly}}) = \mathbf{s}'_{\mathbf{1}} = \frac{1}{2}(\mathbf{s}'_{\mathbf{Lx}} + \mathbf{s}'_{\mathbf{Ly}})$$

So S′ can be written as:

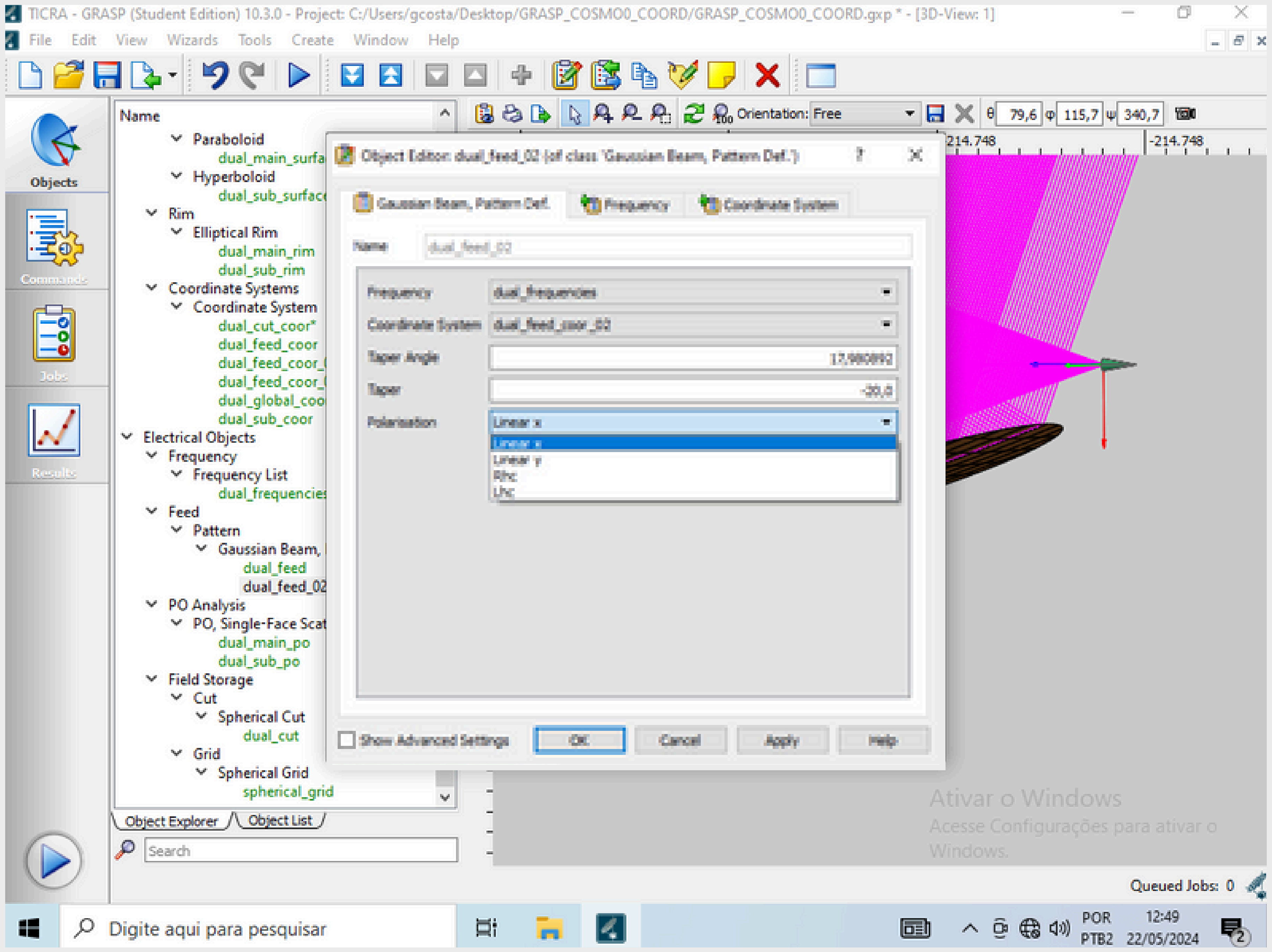
$$\mathbf{S}'_{\mathbf{1}} = \frac{1}{2}(\mathbf{S}'_{\mathbf{Lx}} + \mathbf{S}'_{\mathbf{Ly}}) = I_0 \begin{pmatrix} M_{11} \\ M_{12} \\ M_{13} \\ M_{14} \end{pmatrix}$$

$$\mathbf{S}'_{\mathbf{3}} = \frac{1}{2}(\mathbf{S}'_{\mathbf{L45}} - \mathbf{S}'_{\mathbf{L-45}}) = I_0 \begin{pmatrix} M_{31} \\ M_{32} \\ M_{33} \\ M_{34} \end{pmatrix}$$

$$\mathbf{S}'_{\mathbf{2}} = \frac{1}{2}(\mathbf{S}'_{\mathbf{Lx}} - \mathbf{S}'_{\mathbf{Ly}}) = I_0 \begin{pmatrix} M_{21} \\ M_{22} \\ M_{23} \\ M_{24} \end{pmatrix}$$

$$\mathbf{S}'_{\mathbf{4}} = \frac{1}{2}(\mathbf{S}'_{\mathbf{CRP}} - \mathbf{S}'_{\mathbf{CLP}}) = I_0 \begin{pmatrix} M_{41} \\ M_{42} \\ M_{43} \\ M_{44} \end{pmatrix}$$

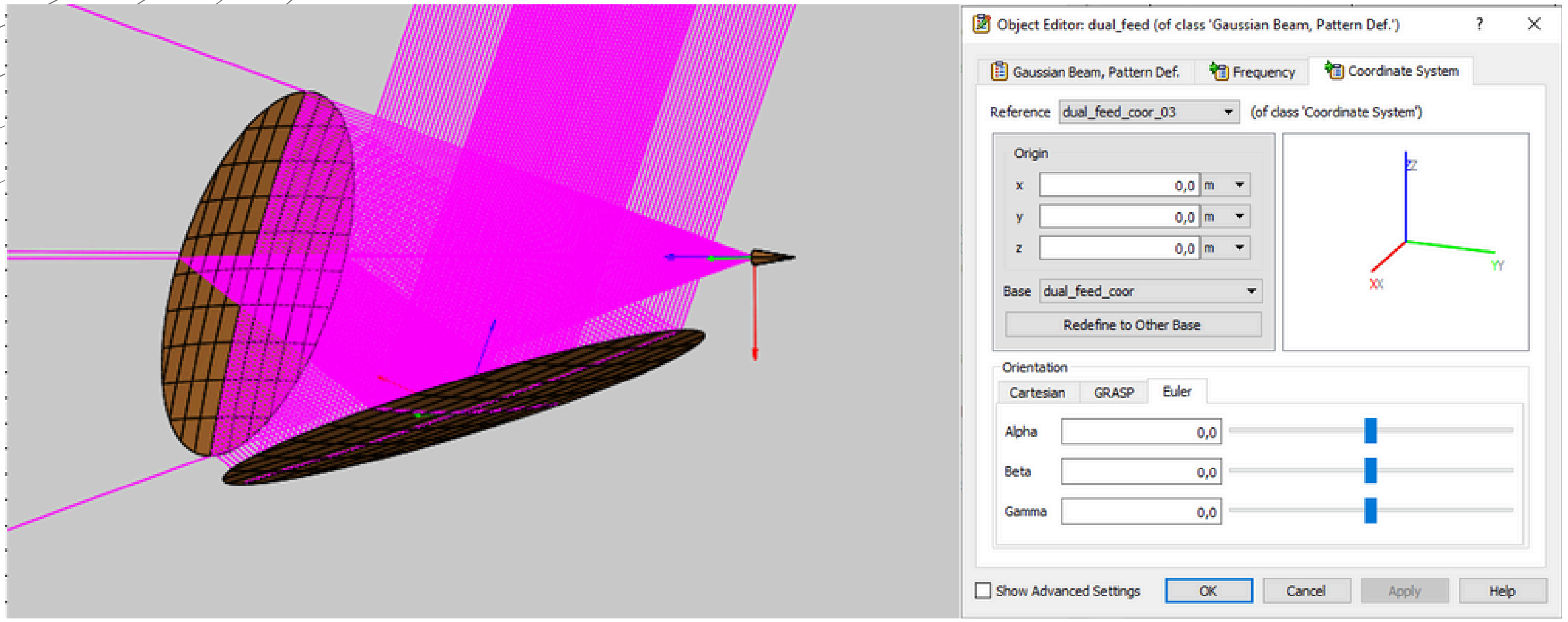
CALCULATING BINGO MUELLER MATRIX

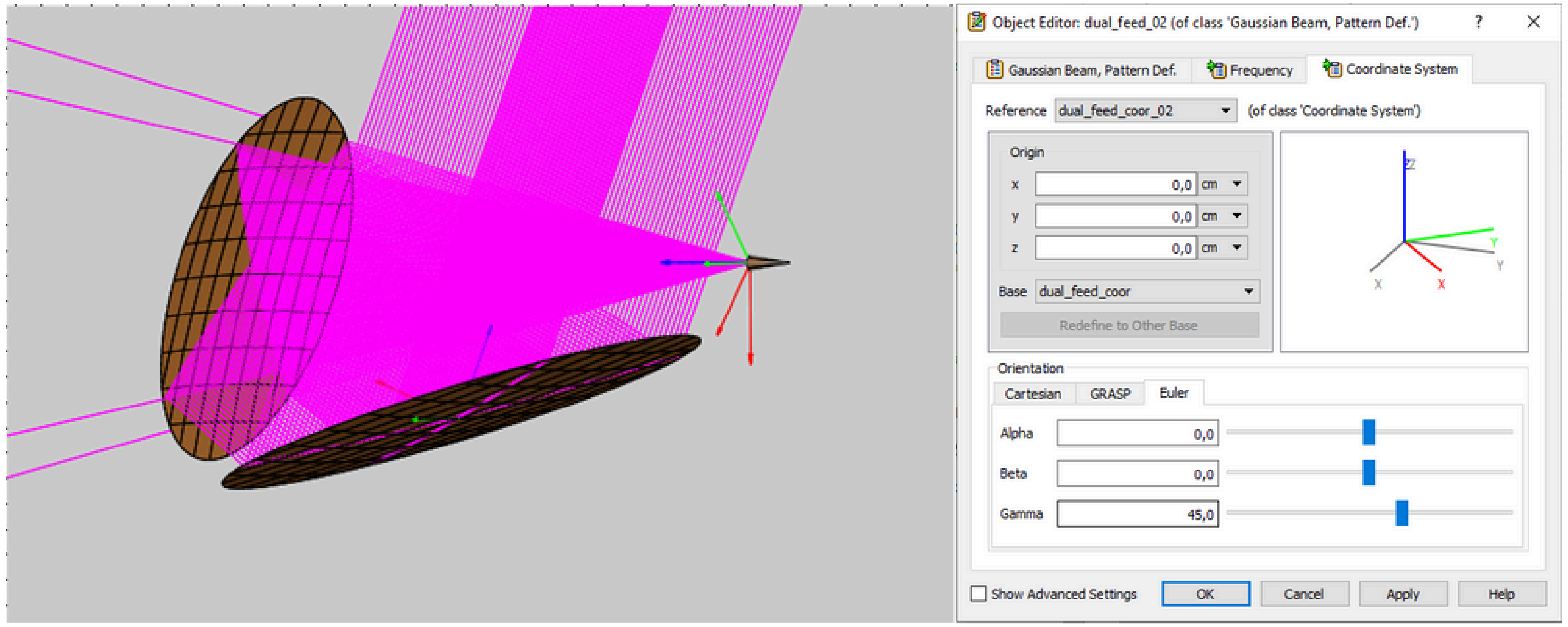


There isn't a 45° polarization in GRASP

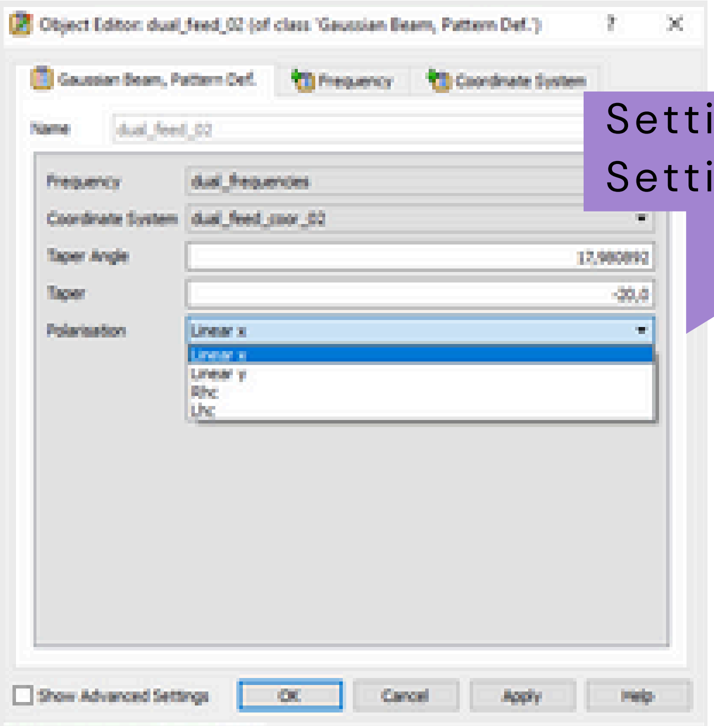
$$S'_3 = \frac{1}{2}(S'_{L45} - S'_{L-45}) = I_0 \begin{pmatrix} M_{31} \\ M_{32} \\ M_{33} \\ M_{34} \end{pmatrix}$$

CALCULATING BINGO MUELLER MATRIX





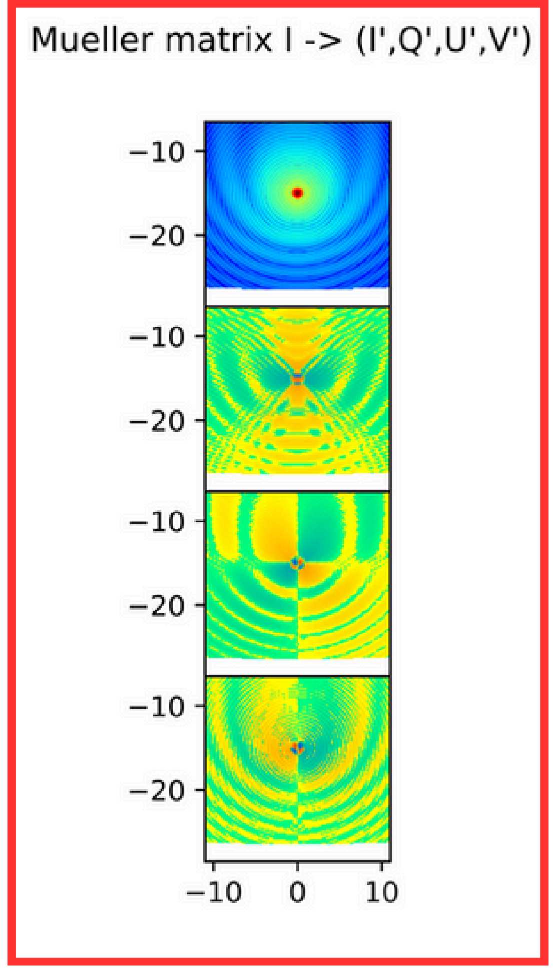
As GRASP calculates the electric field as if the horn were the light source, we can rotate the horn by 45 degrees.

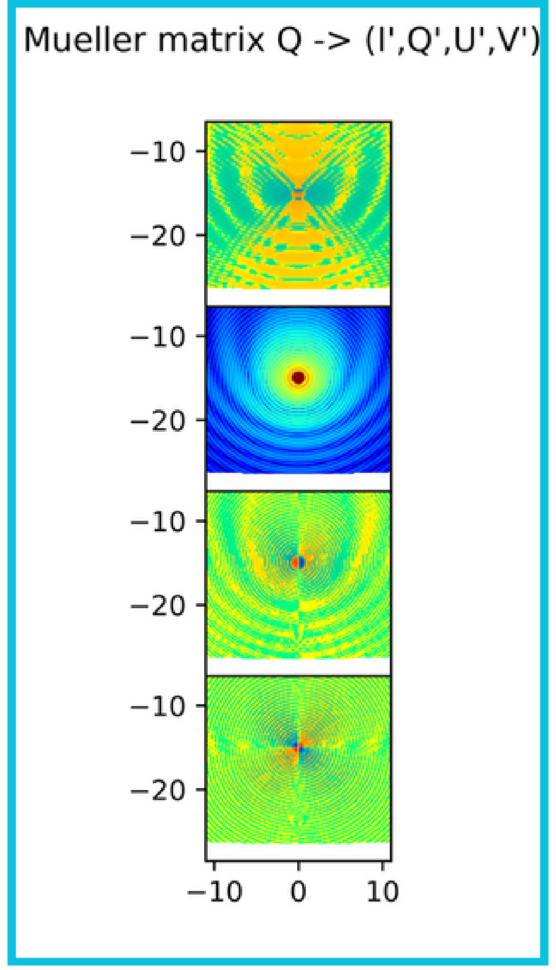


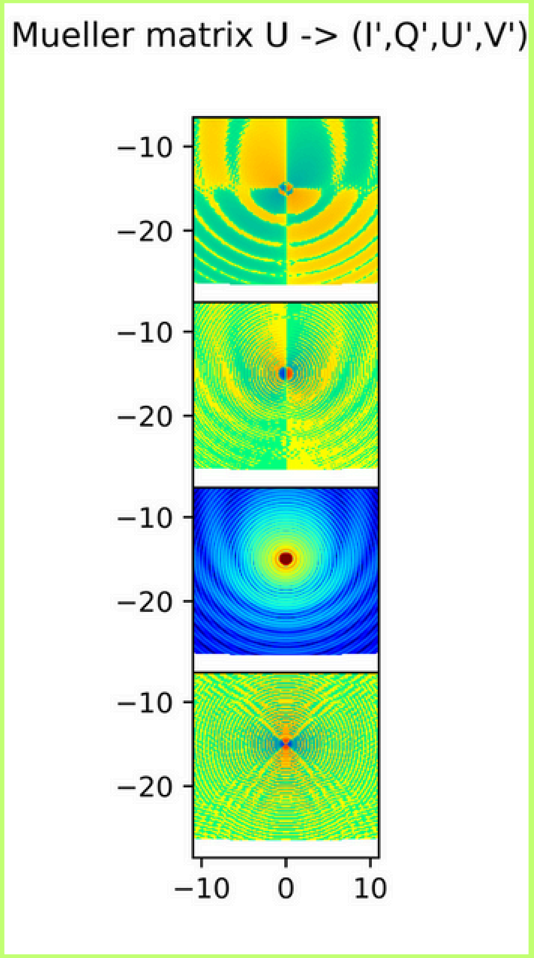
Setting Linear x ⟶ 45° polarisation
Setting Linear y ⟶ -45° polarisation

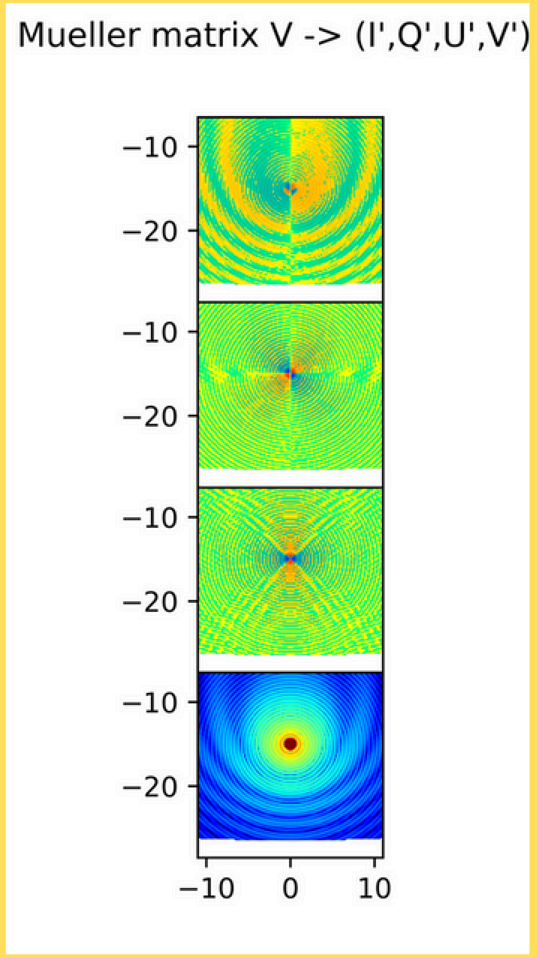
CALCULATING BINGO MUELLER MATRIX

$$I_0 \begin{pmatrix} M_{11} \\ M_{12} \\ M_{13} \\ M_{14} \end{pmatrix} = \mathbf{S}'_1 \qquad I_0 \begin{pmatrix} M_{21} \\ M_{22} \\ M_{23} \\ M_{24} \end{pmatrix} = \mathbf{S}'_2 \qquad I_0 \begin{pmatrix} M_{31} \\ M_{32} \\ M_{33} \\ M_{34} \end{pmatrix} = \mathbf{S}'_3 \qquad I_0 \begin{pmatrix} M_{41} \\ M_{42} \\ M_{43} \\ M_{44} \end{pmatrix} = \mathbf{S}'_4$$









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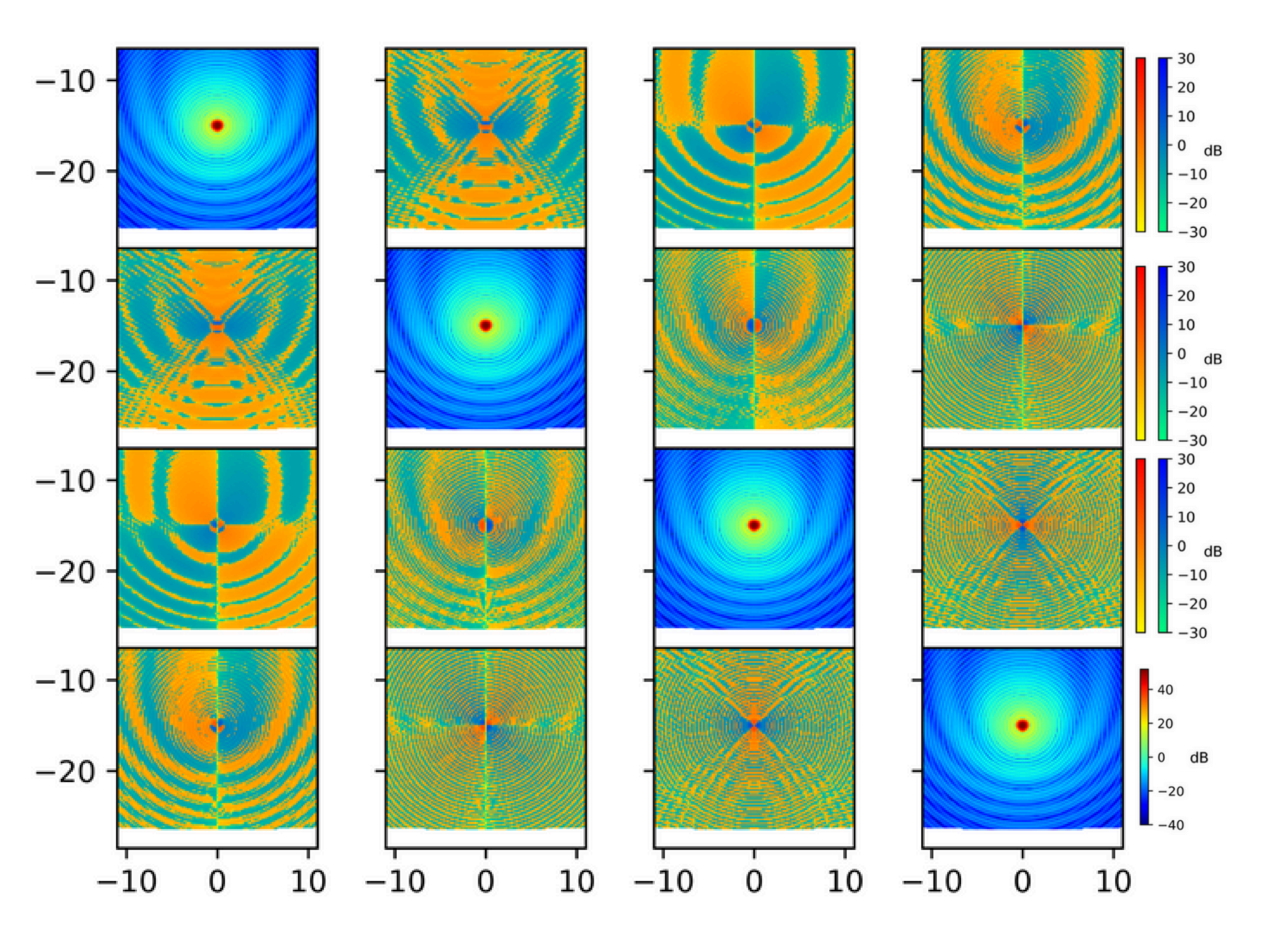
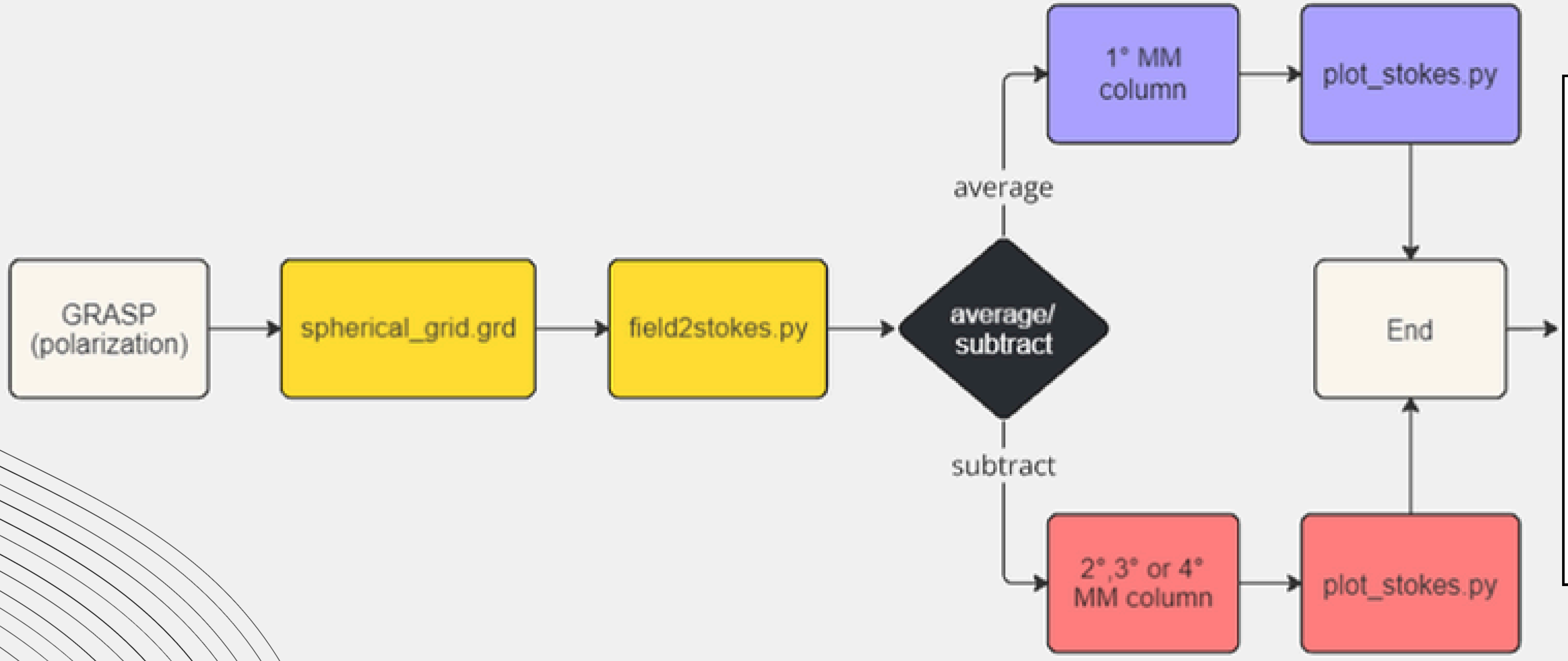
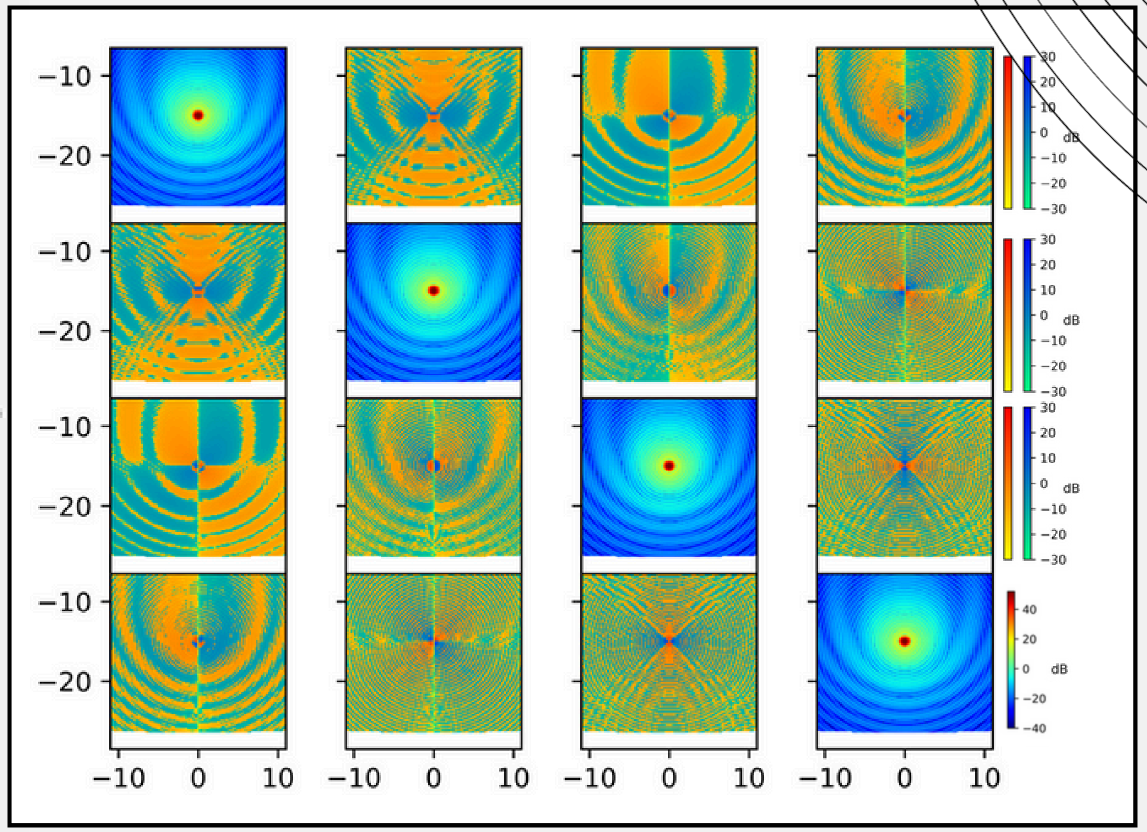
BINGO MUELLER MATRIX



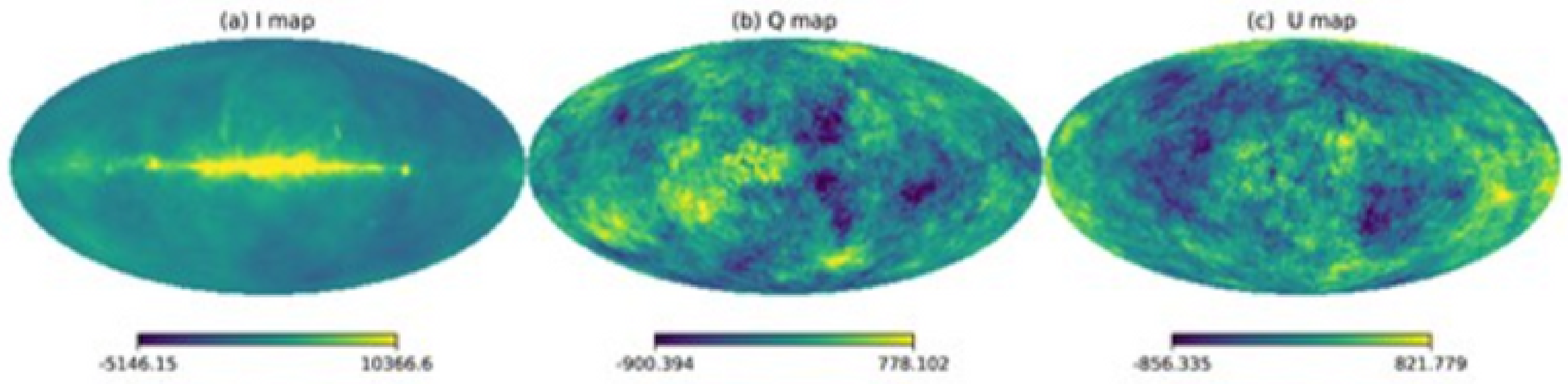
IN SUMMARY


GRASP
(polarization)
spherical_grid.grd
field2stokes.py
average/
subtract
average
subtract
1° MM
column
plot_stokes.py
2°,3° or 4°
MM column
plot_stokes.py
End



NEXT STEPS

Polarized Intensity Maps…



THANK'S FOR WATCHING

Abdalla, F. B. et al. The bingo project: III. optical design and optimization of the focal plane. Astronomy & Astrophysics.