BIO165/265 - Course Notes

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Introduction

Resources and References

This class assumes introductory mathematical knowledge, building on this to cover concepts in Statistics, Stochasticity, and Dynamical Systems. We list here some resources for you to consult and a few key concepts that will show up throughout the course.

Materials recapping assumed knowledge

1. **Khan Academy** has lots of free videos available that are great for class materials and prerequisite knowledge! We recommend the following - intro to calculus: derivatives and integrals.

2. **Three Blue One Brown** has online video courses on calculus and differential equations. These courses include many visual animations that provide good intuition for the core concepts of these two math disciplines.

3. **WolframAlpha** is a great tool for symbolic math; it automates simplifying mathematical terms. Highly recommend!

Suggested introductory textbooks; materials covered in class

1. **Computer Age Statistical Inference**

2. **Stats 101** all on YouTube

3. **Think Stats, Allen B. Downey** - this is an introductory textbook with examples in Python, and designed to be easy to read and is beginner-friendly. It is however less through and formal than *All of Statistics*. Published under a creative commons license, and for free here.

4. **Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering, Steven Strogatz**: this is a great introduction to dynamical systems, starting from the basics that we cover in this class and covering the most important topics in this field from an applied perspective. Designed to be easy to read, assumes some familiarity with mathematical notation.

Further reading

1. **All of Statistics, Larry A. Wasserman (2004)** - this is a very through but somewhat notation-heavy and technical textbook that is a really good reference for commonly used statistical methods. This assumes some mathematical knowledge and comfort with mathematical notation. Electronic version available through the Stanford library

2. **Algorithms from the Book, Kenneth Lange** A through and advanced treatment of the most useful algorithms for statistics and optimization; assumes a fair bit of mathematical background. Open access version available here.

1 Key mathematical notation and concepts

We review a few symbols that are commonly used in math and will show up throughout this class.

- **In**: The $\in$ symbol means "belongs to" or "is in". For instance, if we have a set $A := \{2, 3, 6, 10\}$, then $x \in A$ means that x is an element of A; so it is one of the values $2, 3, 6, 10$.

- **Limit**: The limit of a term is the value that the function approaches as the argument approaches some value. For instance, $\lim_{x\to\infty} \frac{1}{x}$ is the value that $\frac{1}{x}$ (the function) approaches as x (the argument) tends to ∞. Can you see why $\lim_{x\to\infty} \frac{1}{x} = 0$?

- **Derivatives**; Derivatives are conceptually the values of the slope of a function. If we have a function $y = f(x)$, then we write the derivative as $\frac{dy}{dx}$ or $f'(x)$, which is a function of its own. We can also think about this value as the rate of change of y with respect to x. To evaluate the value of the derivative at a specific value of x (let's say $x = 5$), we could write $\frac{dy}{dx}|_{x=5}$ or $f'(5)$.

  We don't have space here for a detailed treatment of calculus, but recommend looking at some of the resources listed above if this doesn't feel familiar. We won't have too many complicated derivatives in this class, but **WolframAlpha** is a great tool for simplifying terms, including derivatives and integrals!

  Finally, we will keep coming back to using derivatives to find the minimum or maximum of a function. The key idea here is that any value of x at which a function $f(x)$ reaches a local minimum or maximum meets the condition that $f(x) = 0$. Thus, we can find mimina and maxima by solving for values of x where $f(x) = 0$. A more detailed explanation is here.

- **Integrals** are the opposite operation of derivatives (aka "anti-derivatives". This is the idea of "the fundamental theorem of calculus" (more on this here). Intuitively they can be understood to be the area under the curve of a function, and are denoted by $\int f(x)$. If we are only considering a specific range of x values, let's say from 0 to 5, we can write the "definite integral" as $\int_{x=0}^{5}$. For this class a conceptual understanding of integrals is enough; we will primarily be using numerical methods for integration.

- **Sum**: The $\sum$ sign is the notation for a sum. It is often indexed from a starting points to an end point, and has a term inside the sign, like so: $\sum_{x=0}^{5} x$ has the starting index 0 and ending index 5. This is a shorthand for writing $0 + 1 + 2 + 3 + 4 + 5$. Often expressions can be more complicated; for instance, $\sum_{i=1}^{3} x^i$ is shorthand for $x^1 + x^2 + x^3$. Finally, instead of having indexes from one integer to another, we often refer to a set of values that the index i can take; for instance let's define a set, $A := \{2, 3, 6, 10\}$. If we write $\sum_{i\in A} x^i$, this is equivalent to $x^2 + x^3 + x^6 + x^{10}$.

- **Product:** The $\prod$ symbol is used to indicate a product, and is analogous to the $\sum$ sign. For instance, $\prod_{i=1}^{3} x^i$ is shorthand for $x^1 \cdot x^2 \cdot x^3 = x^6$.

One last note on formal mathematical writing: use full sentences and punctuation when writing proofs; mathematical language should be thought of as an extension of the English (or any other) language!

References