

Random vs Fixed

—

Fixed effects

Fixed effects are terms (parameters) in a statistical model which are fixed, or non-random, quantities (e.g., treatment group's mean response). For the same treatment, we expect this quantity to be the same from experiment to experiment.

Random effects

Random effects are terms (parameters) in a statistical model which are considered as random quantities or variables (e.g., block id). Specifically, terms whose levels are a representative sample from a population, and where the **variance of the population is of interest**.

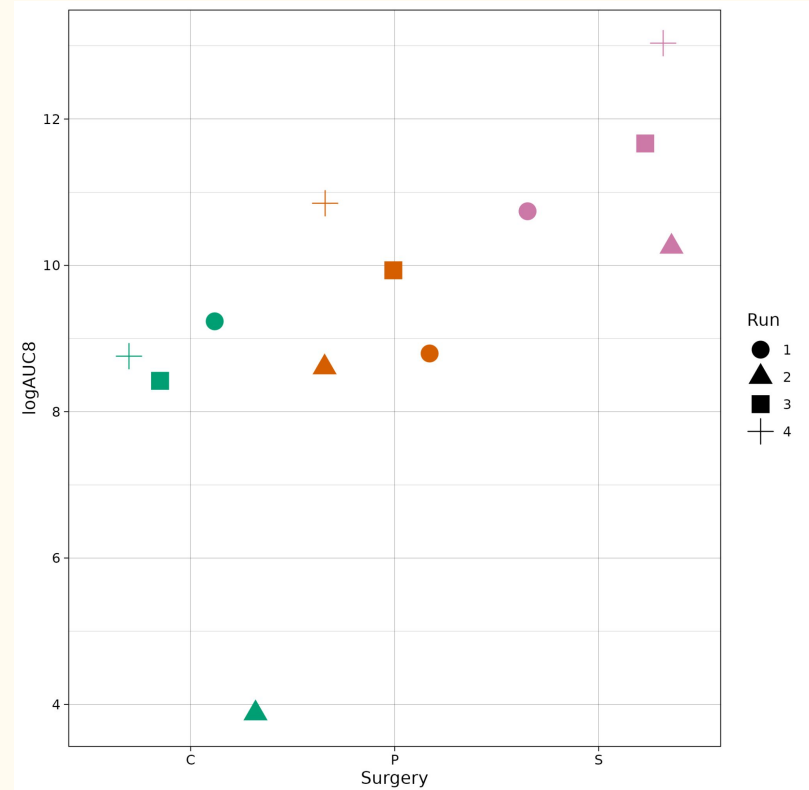
For example, setting a **block** as a random effect allows us to **infer variation between blocks** as well as the **variation between experimental units within blocks**.

Why differentiate fixed and random effects?

Partition known sources of variation that are unimportant to key scientific question(s) to **improve precision** of comparisons between treatment means.

A Randomised Controlled Block Design (RCBD)

Run	Surgery	Rat	logAUC8
1	C	1	9.24
1	P	2	8.81
1	S	3	10.75
2	C	4	3.89
2	P	5	8.62
2	S	6	10.24
3	C	7	8.42
3	P	8	9.93
3	S	9	11.68
4	C	10	8.77
4	P	11	10.86
4	S	12	13.05

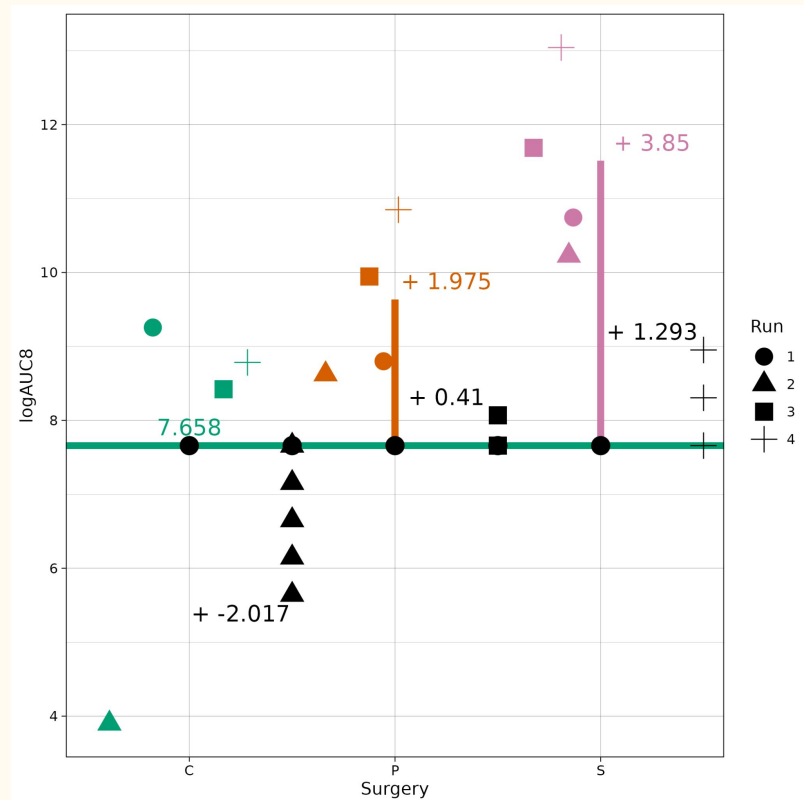


Run as a fixed effect

```
lm <- lm(logAUC8 ~ Run + Surgery, data = rcdbd)
```

```
## Coefficients:
```

##	Estimate	Std. Error	t value	Pr(> t)	
## (Intercept)	7.6583	0.8506	9.004	0.000105	***
## Run2	-2.0167	0.9822	-2.053	0.085854	.
## Run3	0.4100	0.9822	0.417	0.690882	
## Run4	1.2933	0.9822	1.317	0.235963	
## SurgeryP	1.9750	0.8506	2.322	0.059293	.
## SurgeryS	3.8500	0.8506	4.526	0.003991	**



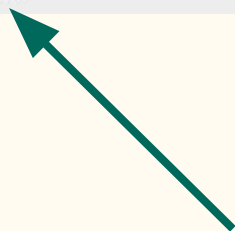
Run as a fixed effect

```
anova(lm)
```

Analysis of Variance Table

Response: logAUC8

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Run	3	17.6496	5.8832	4.0658	0.06797 .
Surgery	2	29.6517	14.8258	10.2458	0.01162 *
Residuals	6	8.6821	1.4470		



Within Run (residual) variance

Follow [this section](#) of the course guide

```
lmerTest_mod <- lmerTest::lmer(logAUC8 ~ Surgery + (1|Run), data = rcdbd)
```

or



Run as a random effect

```
lmer4_mod <- lme4::lmer(logAUC8 ~ Surgery + (1|Run), data = rcdbd)
```

```
##
## Random effects:
## Groups   Name      Variance Std.Dev.
## Run      (Intercept) 1.479    1.216
## Residual                1.447    1.203
## Number of obs: 12, groups: Run, 4
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)    7.5800     0.8552   8.863
## SurgeryP       1.9750     0.8506   2.322
## SurgeryS       3.8500     0.8506   4.526
```

Between Run (groups)
variance

Within Run (residual) variance


```
lmerTest_mod <- lmerTest::lmer(logAUC8 ~ Surgery + (1|Run), data = rcbd)
```

or



Run as a random effect

```
lmer4_mod <- lme4::lmer(logAUC8 ~ Surgery + (1|Run), data = rcbd)
```

Specifying **Run** as random effect changes our estimated baseline (i.e., **Intercept** coefficient) as now and effect due to **Run** is attributed to the structural component of the model.

We can interpret the fixed effects of a LMM as we might for a linear model (now the **Intercept** estimate changes depending on **Run** :

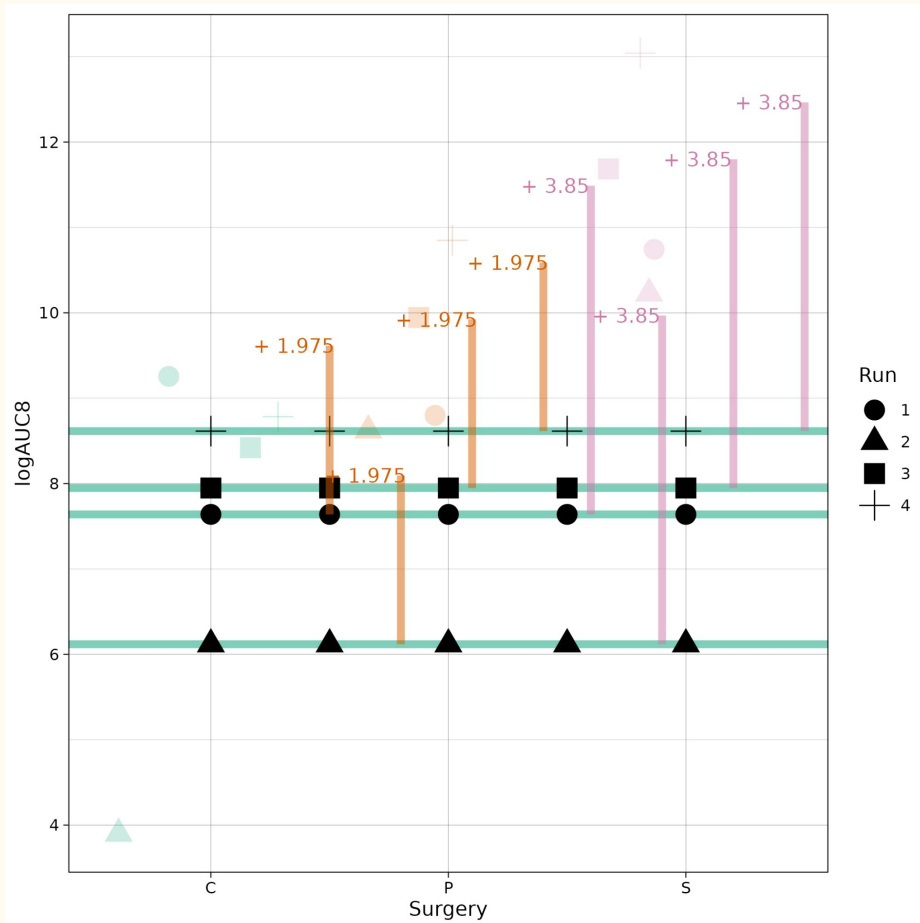
```
coefficients(lmer4_mod)
```

```
## $Run
##      (Intercept)  SurgeryP  SurgeryS
## 1      7.639067      1.975      3.85
## 2      6.118411      1.975      3.85
## 3      7.948225      1.975      3.85
## 4      8.614297      1.975      3.85
```

Follow [this section](#) of the course guide

Run as a random effect

```
## $Run
## (Intercept) SurgeryP SurgeryS
## 1 7.639067 1.975 3.85
## 2 6.118411 1.975 3.85
## 3 7.948225 1.975 3.85
## 4 8.614297 1.975 3.85
```



Follow [this section](#) of the course guide

Variance components

Between Run (groups) variance = $\hat{\sigma}^2 + 3 \hat{\sigma}_{\text{Run}}^2$

therefore

$$\hat{\sigma}_{\text{Run}}^2 = \frac{5.883 - \hat{\sigma}^2}{3} = \frac{5.883 - 1.447}{3} = 1.479$$

```
##
## Random effects:
## Groups   Name      Variance Std.Dev.
## Run      (Intercept) 1.479    1.216
## Residual                1.447    1.203
## Number of obs: 12, groups: Run, 4
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)   7.5800    0.8552   8.863
## SurgeryP      1.9750    0.8506   2.322
## SurgeryS      3.8500    0.8506   4.526
```

$\hat{\sigma}^2$ Within Run (residual)
variance

Analysis of Variance Table

Response: logAUC8

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Run	3	17.6496	5.8832	4.0658	0.06797 .
Surgery	2	29.6511	14.8258	10.2458	0.01162 *
Residuals	6	8.6821	1.4470		

Follow [this section](#) of the course guide