CP468 Assignment 1 Missionaries and Cannibals

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Abstraction

For convenience, in the remainder, each abbreviation has its own meaning:

Abbreviation	Meaning
$\mathbf{M}$	missionary
$\mathbf{C}$	cannibal
N	number of $\mathbf{M}$ or $\mathbf{C}$ (the same)
B	number of seats in boat
ml	number of $\mathbf{M}$ on left side
mb	number of $\mathbf{M}$ in boat
mr	number of $\mathbf{M}$ on right side
cl	number of $\mathbf{C}$ on left side
cb	number of $\mathbf{C}$ in boat
cr	number of $\mathbf{C}$ on right side
pos	position of boat, left side: 1, right side: 0
(ml, cl, pos)	one state when the boat is empty and on side pos, $ml\mathbf{M}$ and $cl\mathbf{C}$ are on the left side
$L(mb, cb)$	carry $mb\mathbf{M}$ and $cb\mathbf{C}$ to left side
$R(mb, cb)$	carry $mb\mathbf{M}$ and $cb\mathbf{C}$ to right side

Reality Constraints

Each number must be non-negative and less than the total number:

$$0 <= ml, mb, mr, cl, cb, cr <= N$$

$$0 <= mb, cb <= B$$

$$ml + mb + mr == cl + cb + cr == N$$

The boat can not across the river without someone in it:

$$0 < mb + cb <= B$$

Security Constraints

Whether on left, right or in boat, the number of $\mathbf{M}$ should not less than that of $\mathbf{C}$:

$$ml == 0 || ml >= cl$$

$$mb == 0 || mb >= cb$$

$$mr == 0 || mr >= cr$$

Initial State

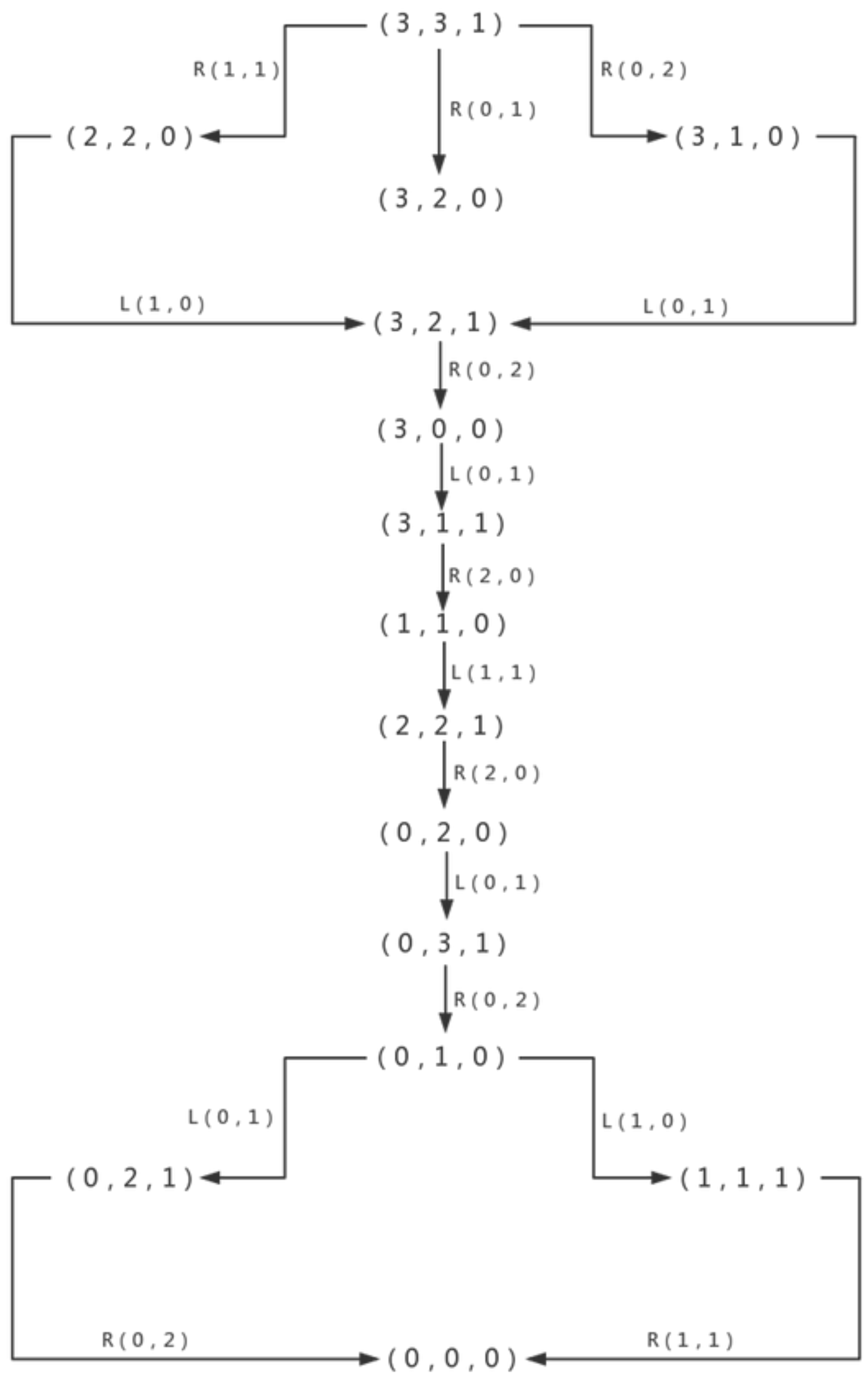
$N\mathbf{M}$ and $N\mathbf{C}$ are on one side, let's say left side, with a boat at the same side, described as $(N, N, 1)$.

Goal State

$N\mathbf{M}$, $N\mathbf{C}$ and the boat are on the right side, described as $(0, 0, 0)$.

State Space

Take $N == 3, B == 2$ as an example, below is the complete state space graph:



There are 15 states, 10 actions and 16*2=32 transition models in total. For short we just show half of these transition models:

Transition Models
$Result\big((3,3,1), R(1,1)\big) = (2,2,0)$
$Result\big((3,3,1), R(0,1)\big) = (3,2,0)$
$Result\big((3,3,1), R(0,2)\big) = (3,1,0)$
$Result\big((2,2,0), L(1,0)\big) = (3,2,1)$
$Result\big((3,1,0), L(0,1)\big) = (3,2,1)$
$Result\big((3,2,1), R(0,2)\big) = (3,0,0)$
$Result\big((3,0,0), L(0,1)\big) = (3,1,1)$
$Result\big((3,1,1), R(2,0)\big) = (1,1,0)$
$Result\big((1,1,0), L(1,1)\big) = (2,2,1)$
$Result\big((2,2,1), R(2,0)\big) = (0,2,0)$
$Result\big((0,2,0), L(0,1)\big) = (0,3,1)$
$Result\big((0,3,1), R(0,2)\big) = (0,1,0)$
$Result\big((0,1,0), L(0,1)\big) = (0,2,1)$
$Result\big((0,1,0), L(1,0)\big) = (1,1,1)$
$Result\big((0,2,1), R(0,2)\big) = (0,0,0)$
$Result\big((1,1,1), R(1,1)\big) = (0,0,0)$

Path Cost

Path cost increased by one each time when the boat cross the river.

Algorithm Analysis

We considered two algorithms for this problem:
iterative deepening depth-first search(IDS) and iterative deepening a* search(IDA*).

	IDS	IDA*
Complete	Y	Y
Optimal	Y	Y
Time	$O(b^d)$	$O(b^d)$
Space	$O(bd)$	$O(bd)$

IDS and IDA* are both complete in that they will continue searching until find a solution or exhaust all states. They are optimal in that the cost is increasing during iteration, in other words, the first solution found must cost least.
IDS and IDA* are also the same in structure: outside is a iteration of limit and inside is a recursion of search function.
But IDS uses $g(n)$ (cost from start state to current state n) as limit, IDA* uses $f(n) = g(n) + h(n)$ (global estimated cost) instead.

Moreover, IDS does DFS within the limit from one child state; while IDA* will expand at the child state with the minimum value of $f(n)$.

Heuristic for IDA*

Suppose a relaxed situation without the security constraint: $\mathbf{M}$ and $\mathbf{C}$ can stay together with any combinations.
In order to cost least, we fill the boat with at most B people every time from left to right but only let one man come back to pick up remaining people.
Follow this way, in the last time we send B people to right, while in other rounds(turn right then come back to left) we can send at most $B-1$ people each time across the river.
In states when the boat is on the left, $(ml, cl, 1)$, it costs at least:

$$2 * \left\lceil \frac{ml + cl - B}{B - 1} \right\rceil + 1$$

In states when the boat is on the right, $(ml, cl, 0)$, since just one person can come to the left, we can transform state from $(ml, cl, 0)$ to $(ml+1, cl, 1)$ or $(ml, cl+1, 1)$ with one step, thus it costs at least:

$$2 * \left\lceil \frac{ml + cl + 1 - B}{B - 1} \right\rceil + 2$$

Since the two polynomials together describe an optimal cost of the relaxed problem, they are consistent heuristic for the origin $\mathbf{M}$ and $\mathbf{C}$ problem.

Outputs

Below is the output of two algorithms when $N == 3$ and $B == 2$. Solutions of two algorithms are the same because they traverse all actions in the same sequence provided by Action.java. One difference is that IDA*'s limit starts directly from 9, owing its efficiency to the heuristic function.

```
IDS N=3 B=2
try depth limit 1
try depth limit 2
try depth limit 3
try depth limit 4
try depth limit 5
try depth limit 6
try depth limit 7
try depth limit 8
try depth limit 9
try depth limit 10
try depth limit 11
optimal solution found
step 1: (3,3,1) => (0,2)
step 2: (3,1,0) <= (0,1)
step 3: (3,2,1) => (0,2)
step 4: (3,0,0) <= (0,1)
step 5: (3,1,1) => (2,0)
step 6: (1,1,0) <= (1,1)
step 7: (2,2,1) => (2,0)
step 8: (0,2,0) <= (0,1)
step 9: (0,3,1) => (0,2)
step 10: (0,1,0) <= (0,1)
step 11: (0,2,1) => (0,2)

IDA* N=3 B=2
try depth limit 9
try depth limit 11
optimal solution found
step 1: (3,3,1) => (0,2)
```

```
step 2: (3,1,0) <= (0,1)
step 3: (3,2,1) => (0,2)
step 4: (3,0,0) <= (0,1)
step 5: (3,1,1) => (2,0)
step 6: (1,1,0) <= (1,1)
step 7: (2,2,1) => (2,0)
step 8: (0,2,0) <= (0,1)
step 9: (0,3,1) => (0,2)
step 10: (0,1,0) <= (0,1)
step 11: (0,2,1) => (0,2)
```

Both algorithms are able to solve N missionaries, N cannibals and B seats problems. Below are optimal costs when $N <= 10$ and $B <= 10$:

B N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	N	N	N	N	N	N	N	N	N	N
2	1	5	11	N	N	N	N	N	N	N
3	1	3	5	9	11	13	N	N	N	N
4	1	1	3	5	7	9	11	13	15	17
5	1	1	3	3	5	7	7	9	9	11
6	1	1	1	3	3	5	5	7	7	9
7	1	1	1	3	3	3	5	5	7	7
8	1	1	1	1	3	3	3	5	5	5
9	1	1	1	1	3	3	3	3	5	5
10	1	1	1	1	1	3	3	3	3	5

Source Codes

Source codes can be find here: https://github.com/BIOTONIC/MissionariesAndCannibals

Test.java	State.java	Action.java	IDS.java	IDAStar.java
call two algorithms	store state's info & provide functions related to state	use two lists to store all valid arrangements of $\mathbf{M}$ and $\mathbf{C}$ on boat	IDS algorithm	IDA* algorithm

```
// Test.java
// call two algorithms

public class Test {
    public static void main(String[] args) {
        new IDS(3, 2);
        new IDAStar(3,2);
    }
}
```

```
// State.java
// store state's info & provide functions related to state

class State {
    int N; // numbers of missionaries or cannibals
```

```
int B; // number of seats on boat
int ml; // numbers of missionaries on the left
int cl; // numbers of cannibals on the left
int pos; // current position of the boat, 0: on the right, 1: on the left

// start state: ml=N, cl=N, pos=1
// goal state: ml=0, cl=0, pos=0

State(int N, int B) {
    this.N = N;
    this.B = B;
    this.ml = N;
    this.cl = N;
    this.pos = 1;
}

State(int N, int B, int ml, int cl, int pos) {
    this.N = N;
    this.B = B;
    this.ml = ml;
    this.cl = cl;
    this.pos = pos;
}

State doAction(int mb, int cb) {
    if (pos == 0) {
        // current at right, to left
        //
        // 0<=ml+mb<=N
        // && 0<=cl+cb<=N
        // && (ml+mb==0||ml+mb==N||ml+mb==cl+cb)
        if (ml + mb >= 0 && ml + mb <= N
                && cl + cb >= 0 && cl + cb <= N
                && (ml + mb == 0 || ml + mb == N || ml + mb == cl + cb)) {
            return new State(N, B, ml + mb, cl + cb, 1);
        } else {
            return null;
        }
    } else {
        // current at left, to right
        //
        // 0<=ml-mb<=N
        // && 0<=cl-cb<=N
        // && (ml==mb||ml+mb==N||ml-mb==cl-cb)
        if (ml - mb >= 0 && ml - mb <= N
                && cl - cb >= 0 && cl - cb <= N
                && (ml == mb || ml + mb == N || ml - mb == cl - cb)) {
            return new State(N, B, ml - mb, cl - cb, 0);
        } else {
            return null;
        }
    }
}

@Override
public boolean equals(Object other) {
    if (this == other) {
        return true;
    }
    if (other == null) {
        return false;
    }
    if (getClass() != other.getClass()) {
        return false;
```

```java
        }
        State otherState = (State) other;
        if (otherState.N == this.N && otherState.B == this.B && otherState.ml == this.ml
                && otherState.cl == this.cl && otherState.pos == this.pos) {
            return true;
        } else {
            return false;
        }
    }

    @Override
    public String toString() {
        return "(" + this.ml + "," + this.cl + "," + this.pos + ")";
    }

    public String toPath(int mb, int cb) {
        return toString() + " " + (pos == 1 ? "=> " : "<= ") + "(" + mb + "," + cb + ")";
    }

    public String toPath(State child) {
        return toString() + " " + (pos == 1 ? "=> " : "<= ") + "("
                + Math.abs(child.ml - ml) + "," + Math.abs(child.cl - cl) + ")";
    }

    boolean isGoal() {
        if (ml == 0 && cl == 0 && pos == 0) {
            return true;
        } else {
            return false;
        }
    }

    int getHeuristic() {
        if (pos == 1) {
            return 1 + 2 * (int) Math.ceil((double) (ml + cl - B) / (B - 1));
        } else {
            return 2 + 2 * (int) Math.ceil((double) (ml + cl + 1 - B) / (B - 1));
        }
    }
}
```

```java
// Action.java
// use two lists to store all valid arrangements of m and c on boat

import java.util.ArrayList;

public class Action {
    public static ArrayList<Integer> mbs; // list of numbers of missionaries on boat
    public static ArrayList<Integer> cbs; // list of numbers of cannibals on boat

    Action(int N, int B) {
        mbs = new ArrayList<>();
        cbs = new ArrayList<>();
        for (int b = B; b > 0; b--) {
            for (int mb = 0; mb <= N; mb++) {
                if (mb <= b && b - mb >= 0 && b - mb <= N && (mb == 0 || 2 * mb >= b)) {
                    mbs.add(mb);
                    cbs.add(b - mb);
                }
            }
        }
    }
}
```

```java
// IDS.java

import java.util.ArrayList;
import java.util.Stack;

public class IDS {
    int N; // numbers of missionaries and cannibals are both N
    int B; // number of seats on boat

    Action action;
    ArrayList<State> explored;
    Stack<String> records; // records of every optimal step

    static final int SUCCESS = 0;
    static final int FAILURE = -1;
    static final int CUTOFF = -2;

    public IDS(int N, int B) {
        System.out.println("\nIDS N=" + N + " B=" + B);

        this.N = N;
        this.B = B;

        action = new Action(N, B);
        int limit = 0;
        int result;
        State state;
        while (true) {
            // iterate limit
            limit++;
            System.out.println("try depth limit " + limit);
            // init state and explored list before every iteration
            state = new State(N, B);
            explored = new ArrayList<>();
            records = new Stack<>();
            result = search(state, limit);
            if (result == SUCCESS || result == FAILURE) {
                break;
            }
        }

        // print result
        if (result == SUCCESS) {
            int i = 1;
            while (!records.empty()) {
                System.out.println("step " + i + ": " + records.pop());
                i++;
            }
        } else {
            System.out.println("no solution");
        }
    }

    int search(State state, int limit) {
        explored.add(state);
        if (state.isGoal()) {
            explored.remove(state);
            System.out.println("optimal solution found");
            return SUCCESS;
        }
        if (limit == 0) {
            explored.remove(state);
            //System.out.println("cut off");
```

```
            return CUTOFF;
        }

        boolean isCutoff = false;
        // traversal all actions
        for (int i = 0; i < action.mbs.size(); i++) {
            State child = state.doAction(action.mbs.get(i), action.cbs.get(i));
            // graph search
            // only expand valid and non-repetitive states
            if (child != null && !explored.contains(child)) {
                //System.out.println(child);
                // recursion, decreasing limit
                int result = search(child, limit - 1);
                if (result == CUTOFF) {
                    isCutoff = true;
                } else if (result ==SUCCESS) {
                    // find a step
                    records.push(state.toPath(action.mbs.get(i), action.cbs.get(i)));
                    explored.remove(state);
                    return result;
                }
            }
        }
        explored.remove(state);
        if (isCutoff) {
            return CUTOFF;
        } else {
            return FAILURE;
        }
    }
}
```

```
// IDAStar.java

import java.util.Stack;

public class IDAStar {
    int N; // numbers of missionaries and cannibals are both N
    int B; // number of seats on boat

    Action action;
    Stack<State> path;
    Stack<String> records; // records of every optimal step

    static final int SUCCESS = 0;
    static final int FAILURE = -1;

    public IDAStar(int N, int B) {
        System.out.println("\nIDA* N=" + N + " B=" + B);

        this.N = N;
        this.B = B;

        action = new Action(N, B);
        records = new Stack<>();
        State state = new State(N, B);
        path = new Stack<>();
        path.push(state);
        int limit = state.getHeuristic();

        int result;
        while (true) {
            System.out.println("try depth limit " + limit);
```

```
            result = search(0, limit);
            if (result == SUCCESS || result == FAILURE) {
                break;
            }
            limit = result;
        }

        // print result
        if (result == SUCCESS) {
            State child = path.pop();
            while (!path.empty()) {
                records.push(path.peek().toPath(child));
                child = path.pop();
            }
            int i = 1;
            while (!records.empty()) {
                System.out.println("step " + i + ": " + records.pop());
                i++;
            }
        } else {
            System.out.println("no solution");
        }
    }

    int search(int g, int limit) {
        State state = path.peek();
        int f = g + state.getHeuristic();
        if (f > limit) {
            return f;
        } else if (state.isGoal()) {
            System.out.println("optimal solution found");
            return SUCCESS;
        }
        int min = Integer.MAX_VALUE;
        // traversal all actions
        for (int i = 0; i < action.mbs.size(); i++) {
            State child = state.doAction(action.mbs.get(i), action.cbs.get(i));
            // graph search
            // only expand valid and non-repetitive states
            if (child != null && !path.contains(child)) {
                //System.out.println(child);
                path.push(child);
                int result = search(g + 1, limit);
                if (result > 100) {
                    System.out.println(child);
                }
                if (result == SUCCESS) {
                    return SUCCESS;
                }
                if (result != FAILURE && result < min) {
                    min = result;
                }
                path.pop();
            }
        }
        return min == Integer.MAX_VALUE ? FAILURE : min;
    }
}
```