Matrix Factorization Approaches for Demixing fMRI Data

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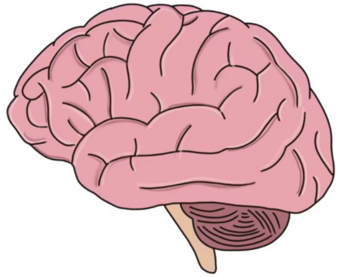
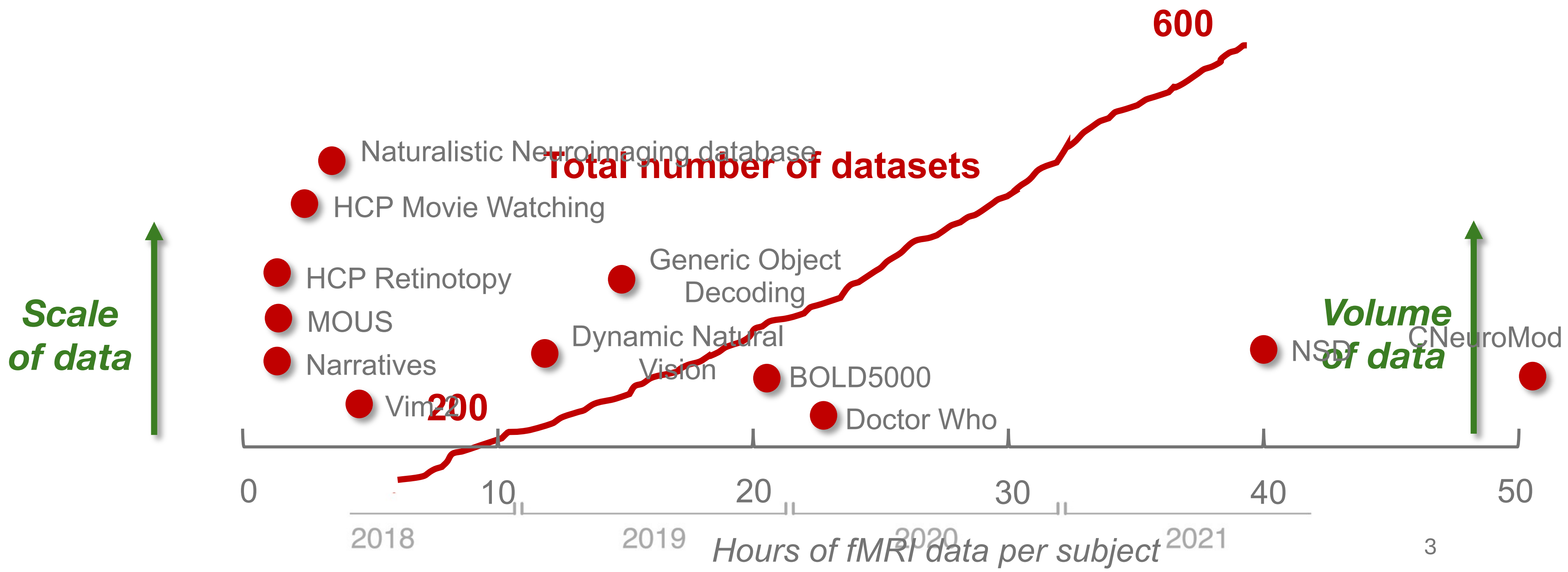


Lecture Overview

- Example from research: how does our brain represent our visual world?
- High-level introduction to Matrix Factorization Methods
- Another research example: how does our brain represent sounds?
- Diverse applications of matrix factorization in neuroscience
- Practical Demonstration: Demixing fMRI data using matrix factorization

Towards data-driven modeling in **large-scale naturalistic neuroscience**

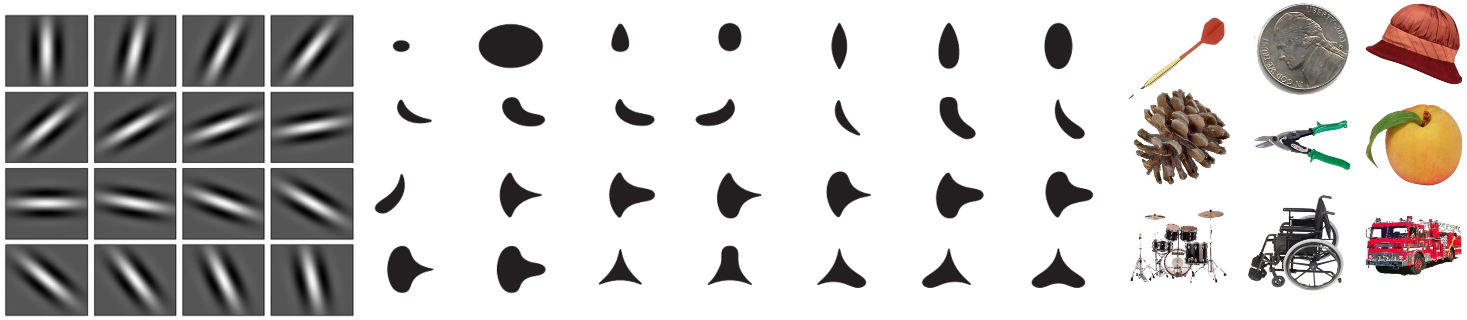


Towards data-driven modeling in **large-scale naturalistic neuroscience**

Volume of data

Scale of data

Complexity of data



Towards data-driven modeling in **large-scale naturalistic neuroscience**

Volume of data

Scale of data

Complexity of data

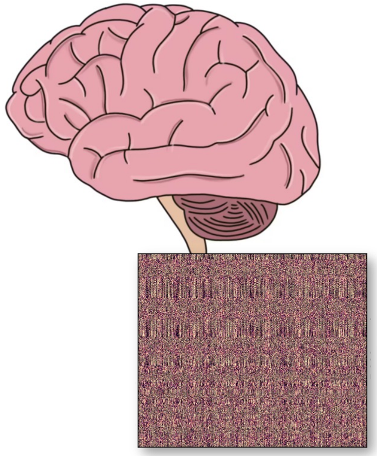
Complex images

Video clips

Movies

Towards **data-driven modeling** in large-scale naturalistic neuroscience

Stimuli



Measured brain activity

Computational model → Model interpretability →

Models bridge large-scale experiments with scientific insights

Towards **data-driven modeling** in large-scale naturalistic neuroscience

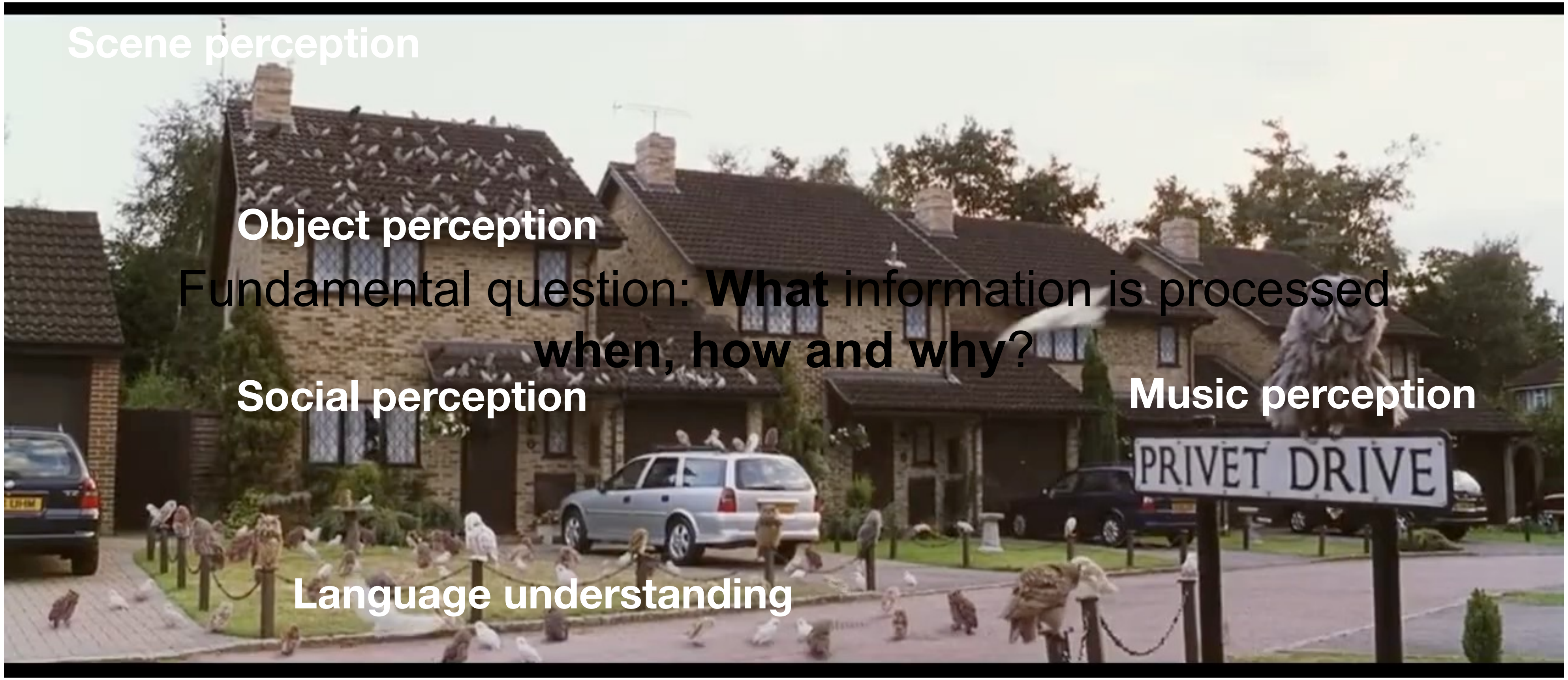
> "*When any field amasses enough quantitative data, it turns to mathematical modeling to make sense of it. Models find structure in large piles of numbers; they can stitch together disparate findings and show how they arise from a unified process.*"

-- Models of the Mind (2020)

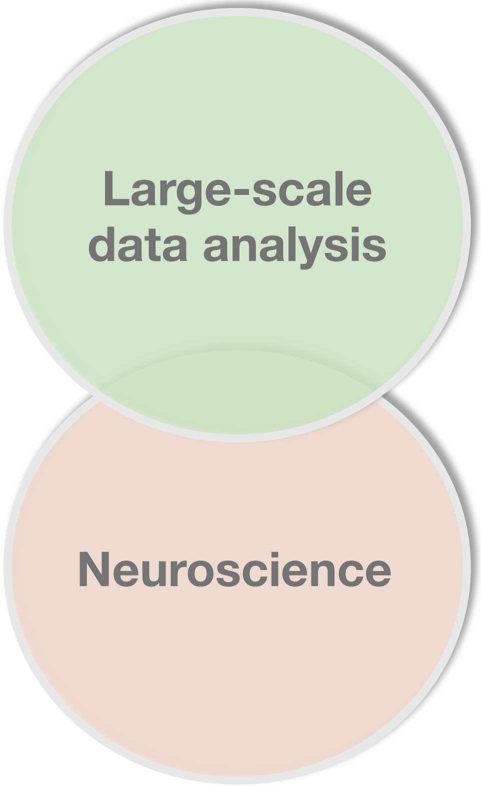
How do we process the external world?

Large-scale data analysis

Neuroscience



Fundamental question: **What** information is processed **when, how and why**?



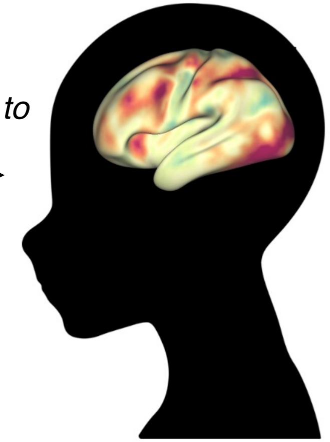
*How is the brain **representing**?* *world?*

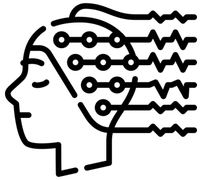
Stimuli

The girl went outside to get a coffee but the café was closed

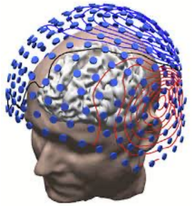
presented to subjects

Measured brain responses



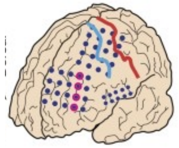


EEG



MEG

fMRI



ECoG

How were these regions discovered?

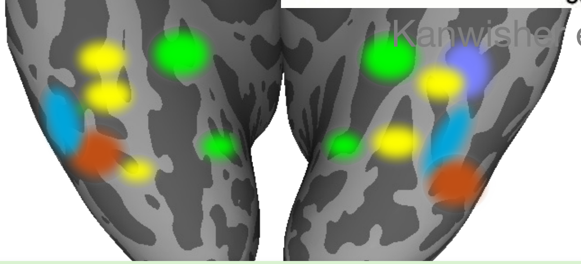
Human brain fMRI is a teeming atlas of

Generate hypothesis

e.g. are there neurons selectively responsive to faces?

li to

Measure brain response and compare activity



Kanwisher et al., 1997

Motion

Reaching

1959-

1995-

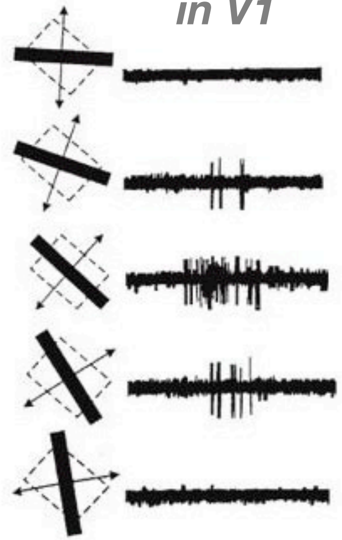
Places

Faces

Objects

Bodies

Orientation selectivity in V1



[Hubel & Wiesel (1962)]

gratings in V4

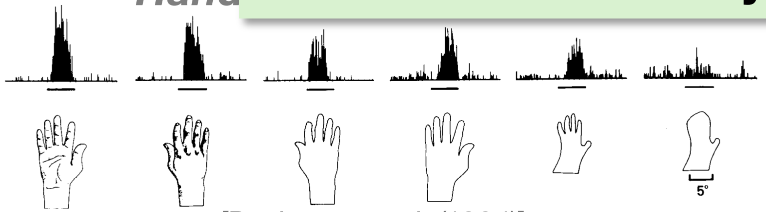
[Gallant et al. (1993)]

Kanwisher et al., 1997

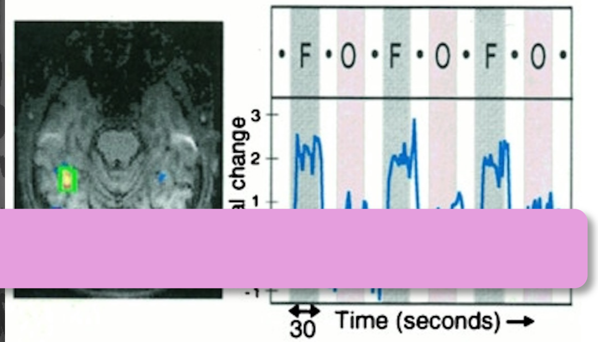
sic

Langu

Hand



[Desimone et al. (1984)]



All of the major neuroscientific discoveries were enabled by the above **hypothesis-driven process**

What are the limitations of this approach?

Generate hypothesis → ***Carefully design stimuli to test hypothesis*** → ***Measure brain response and compare activity***

Limited by human intuitions

- Limited generalization
- Lack ethological validity

Low spatial resolution of fMRI

What have we missed? Are there other important dimensions?

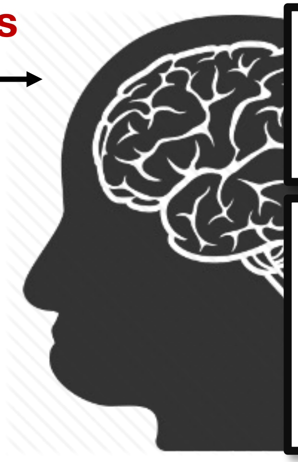
What are the ***fundamental dimensions*** along which the cortex is organized?

A new approach:

Large-scale datasets + Data-driven modeling

Naturalistic images

shown to subjects

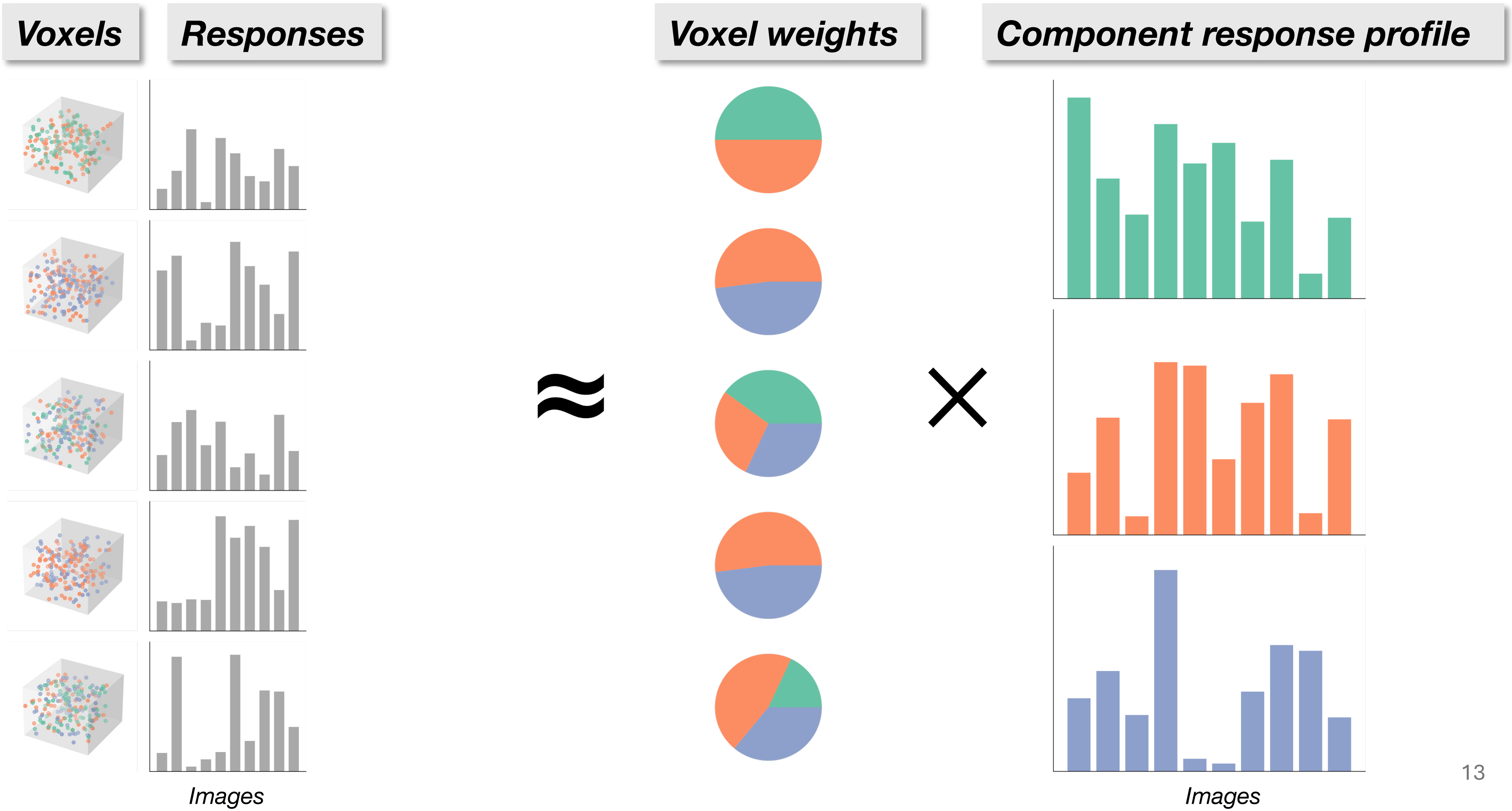


What have we missed? Are there other important dimensions?

What are the ***fundamental dimensions*** along which the cortex is organized?

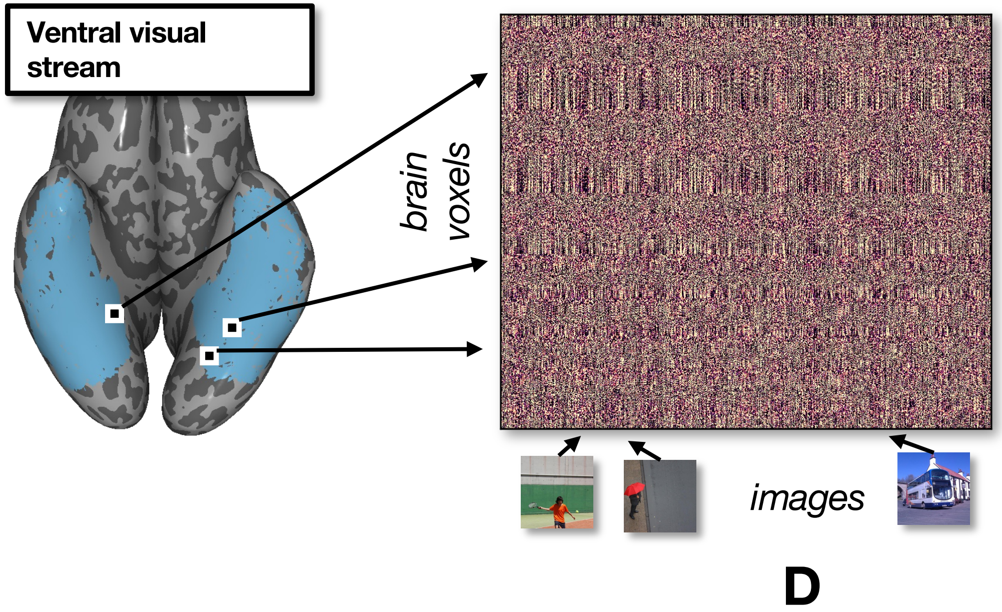
Allen et al., (2022). Nat. Neuroscience

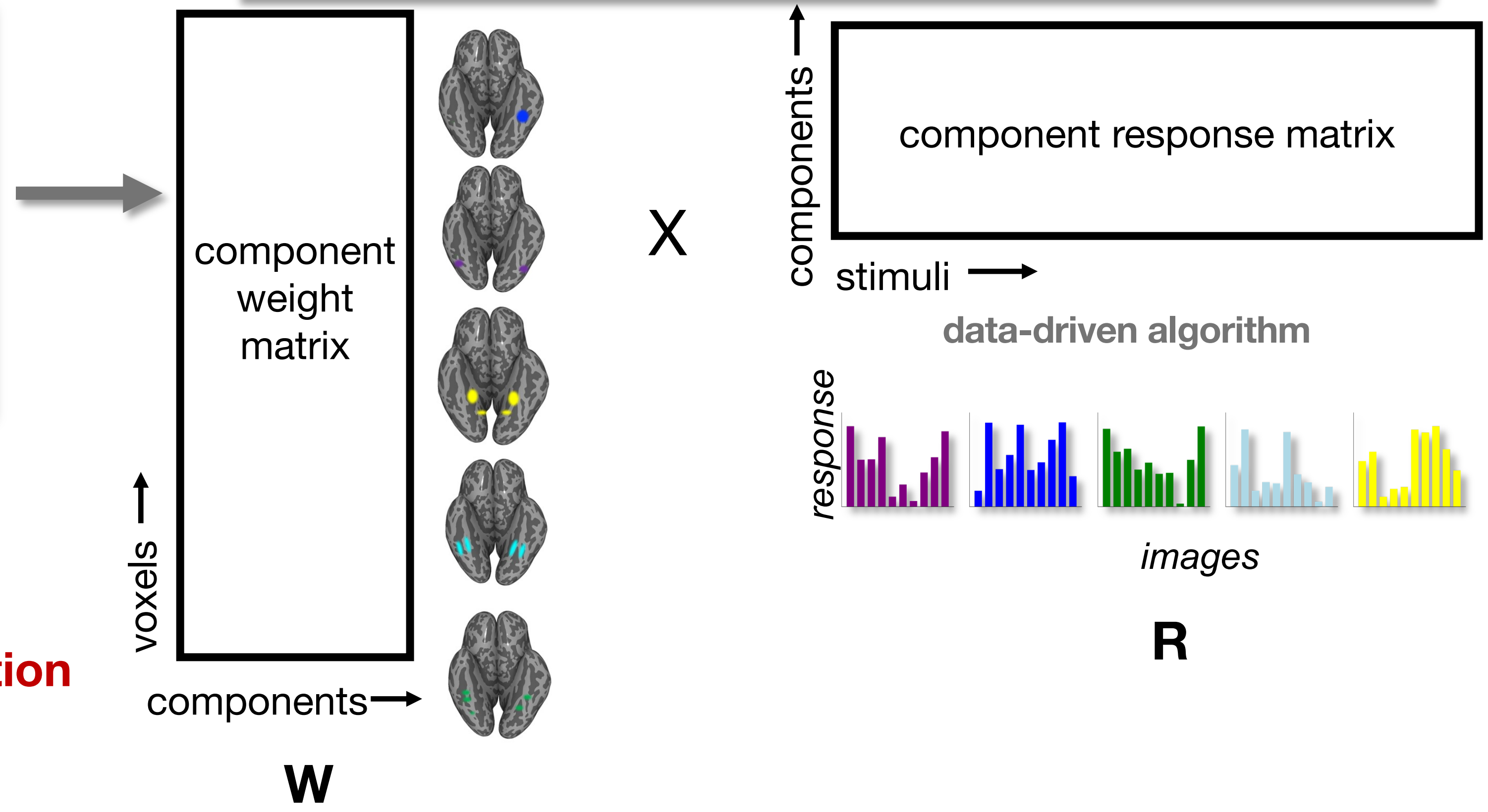
Large-scale datasets + Data-driven modeling



Large-scale datasets + Data-driven modeling

Goal: Discover the components ('neural populations') that produce the data





Non-Negative Matrix Factorization

$$\mathbf{D} \approx \mathbf{WR}$$

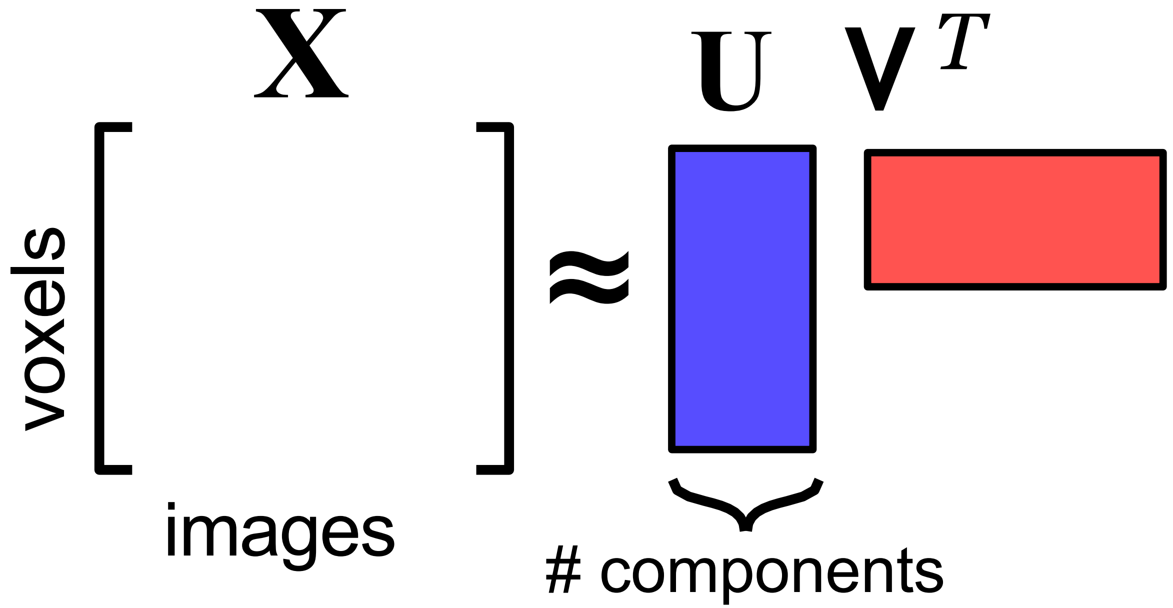
Interlude: Demixing in neuroscience

- Many brain signals reflect linear mixtures of multiple sources
  - EEG, fMRI, calcium imaging

- Often want to recover underlying sources from the mixed signals
  - Matrix Factorization Methods like Non-negative Matrix Factorization (NMF) or Independent Component Analysis (ICA) provide general-purpose statistical machinery

Interlude: Matrix Factorization

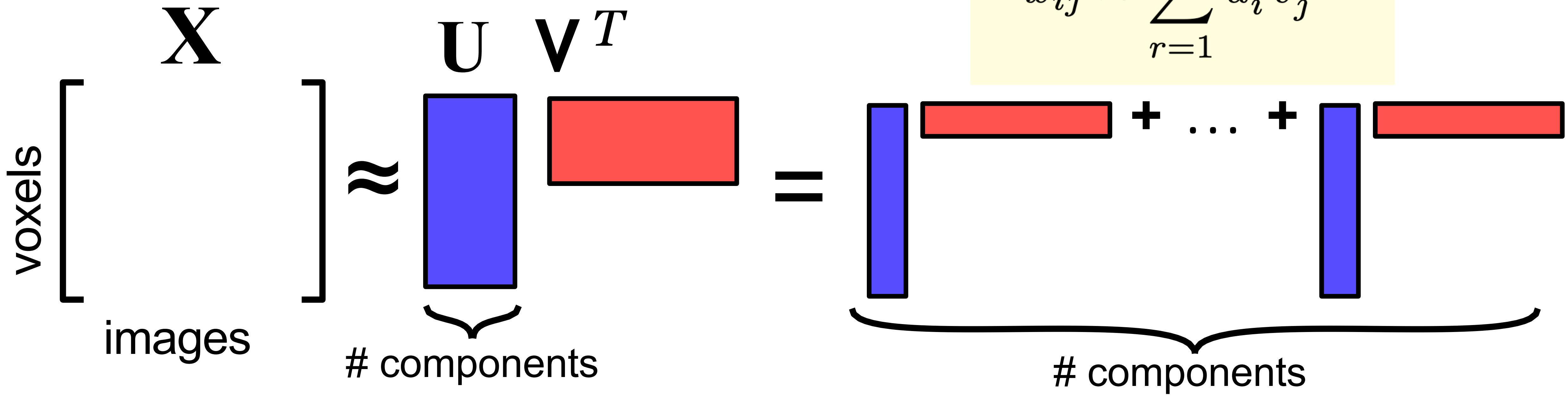
- A simple and general framework for extracting correlations and low-dimensional structure from matrix-coded datasets



Interlude: Matrix Factorization

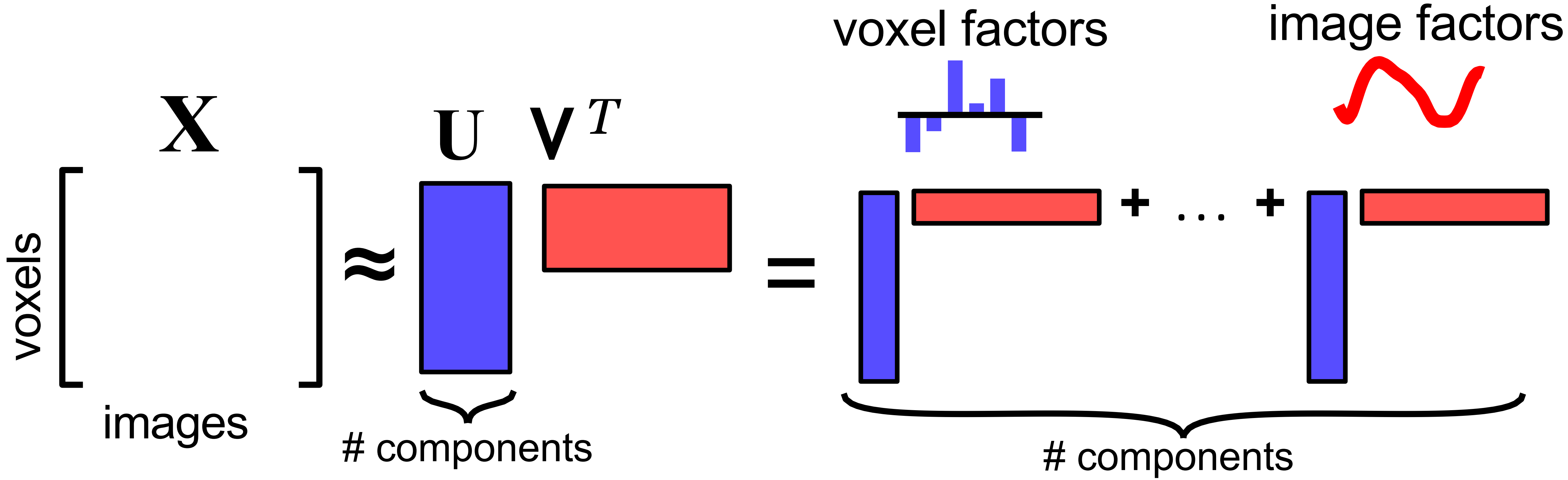
- A simple and general framework for extracting correlations and low-dimensional structure from matrix-coded datasets

$$x_{ij} \approx \sum_{r=1}^{R} u_i^r v_j^r$$

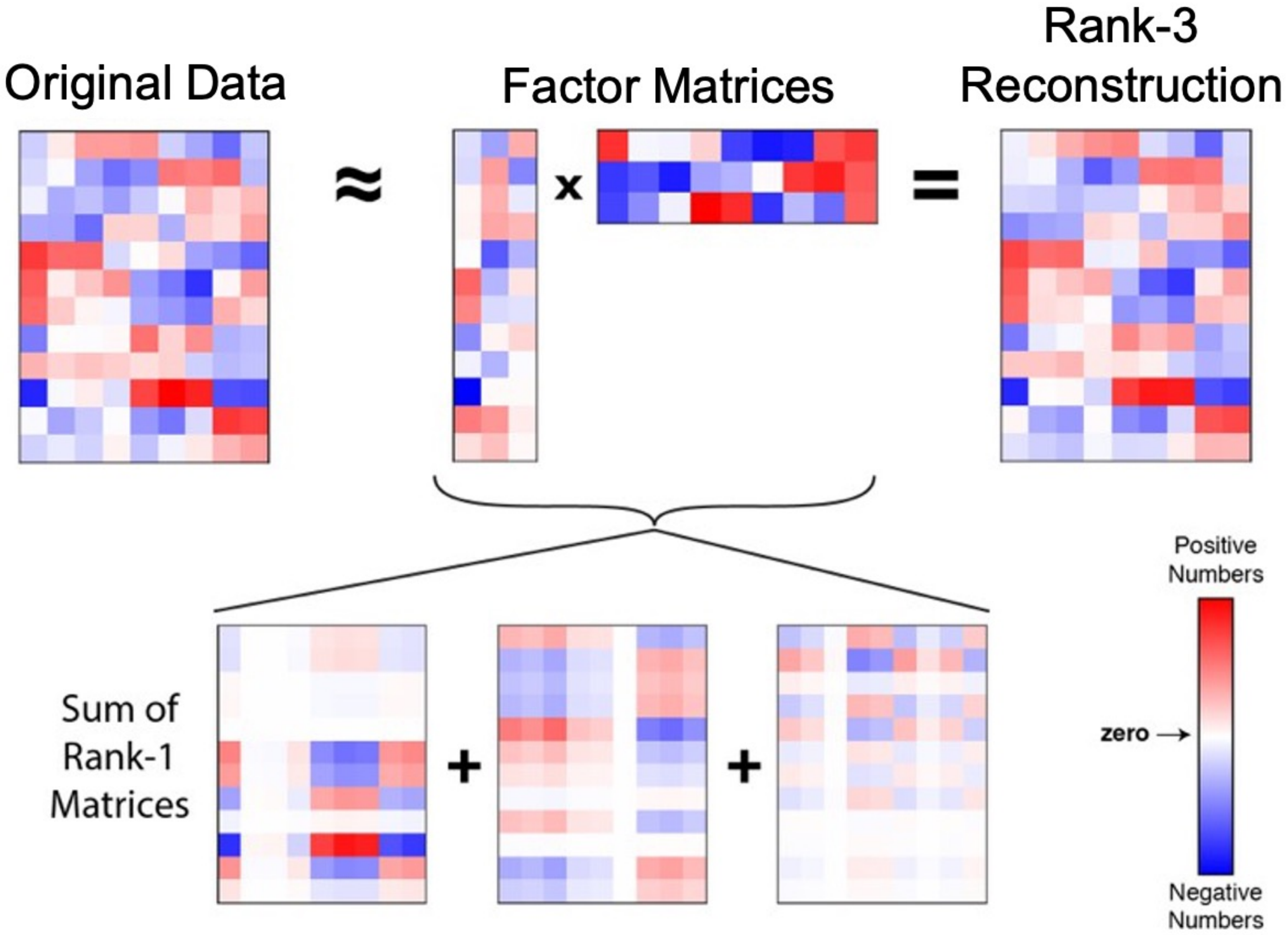


Interlude: Matrix Factorization

- A simple and general framework for extracting correlations and low-dimensional structure from matrix-coded datasets



Interlude: Visualization of Matrix Factorization



Adapted from Alex Williams

Interlude: Matrix Factorization

- List of some matrix decomposition models

Interlude: Matrix Factorization

- Matrix decomposition model, stated formally

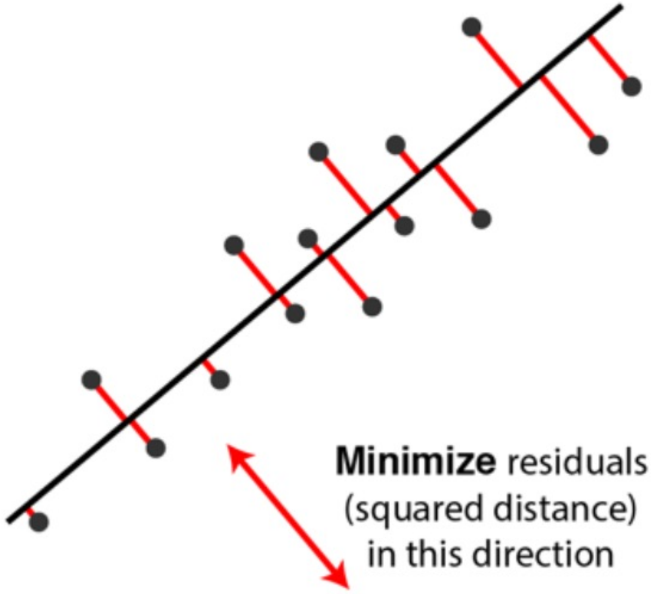
$$
\begin{aligned}
\underset{\mathbf{U},\mathbf{V}}{\text{minimize}} \quad & \underbrace{\|\mathbf{X} - \mathbf{U}\mathbf{V}^T\|_F^2}_{\textit{loss}} \underbrace{+ \lambda_u f_u(\mathbf{U}) + \lambda_v f_v(\mathbf{V})}_{\textit{regularization}} \\
\text{subject to} \quad & \underbrace{\mathbf{U} \in \Omega_u, \; \mathbf{V} \in \Omega_v}_{\textit{constraints}}
\end{aligned}
$$

Interlude: Matrix Factorization

- Principal Components Analysis

$$\underset{\mathbf{U},\mathbf{V}}{\text{minimize}} \quad \|\mathbf{X} - \mathbf{U}\mathbf{V}^T\|_F^2$$

(subject to $V^T V = I_k$)



Interlude: Matrix Factorization

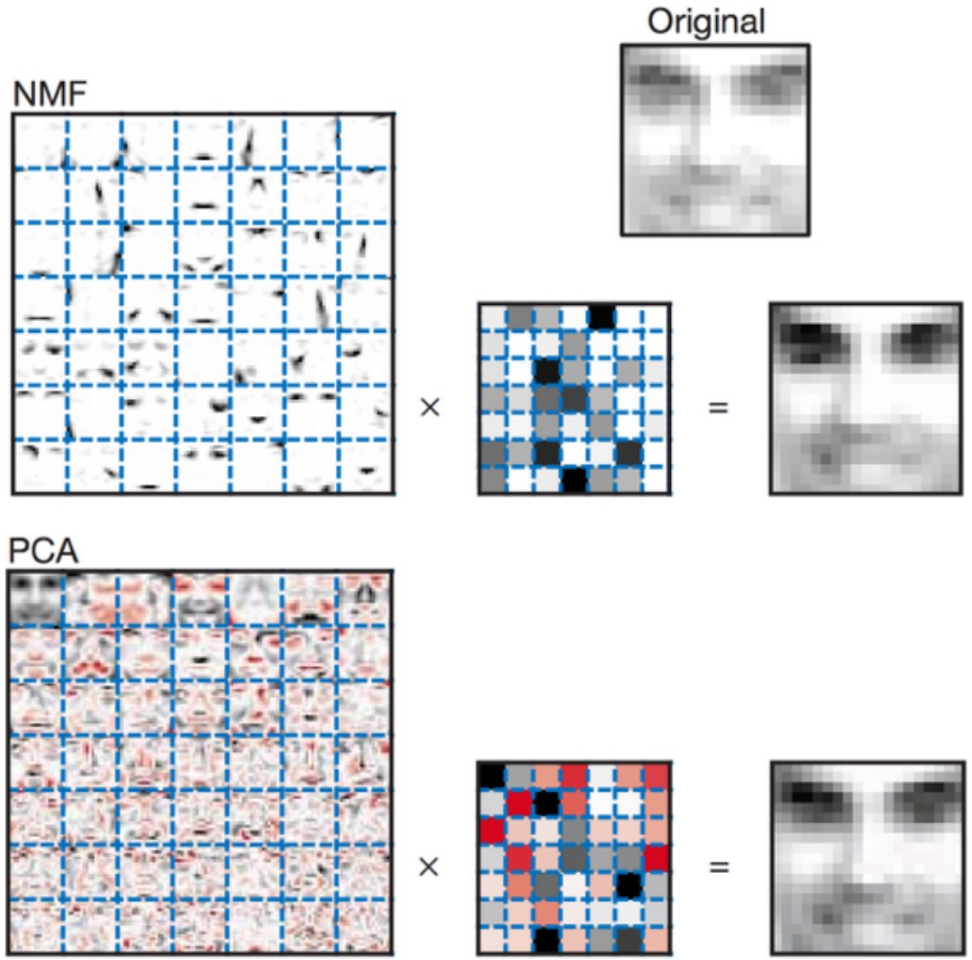
- Non-Negative Matrix Factorization

$$\begin{aligned} \underset{\mathbf{U},\mathbf{V}}{\text{minimize}} \quad & \|\mathbf{X} - \mathbf{U}\mathbf{V}^T\|_F^2 \\ \text{subject to} \quad & \mathbf{U} \geq 0,\ \mathbf{V} \geq 0 \end{aligned}$$

Interlude: Matrix Factorization

- Non-Negative Matrix Factorization

$$\begin{aligned} \underset{\mathbf{U},\mathbf{V}}{\text{minimize}} \quad & \|\mathbf{X} - \mathbf{U}\mathbf{V}^T\|_F^2 \\ \text{subject to} \quad & \mathbf{U} \geq 0,\ \mathbf{V} \geq 0 \end{aligned}$$

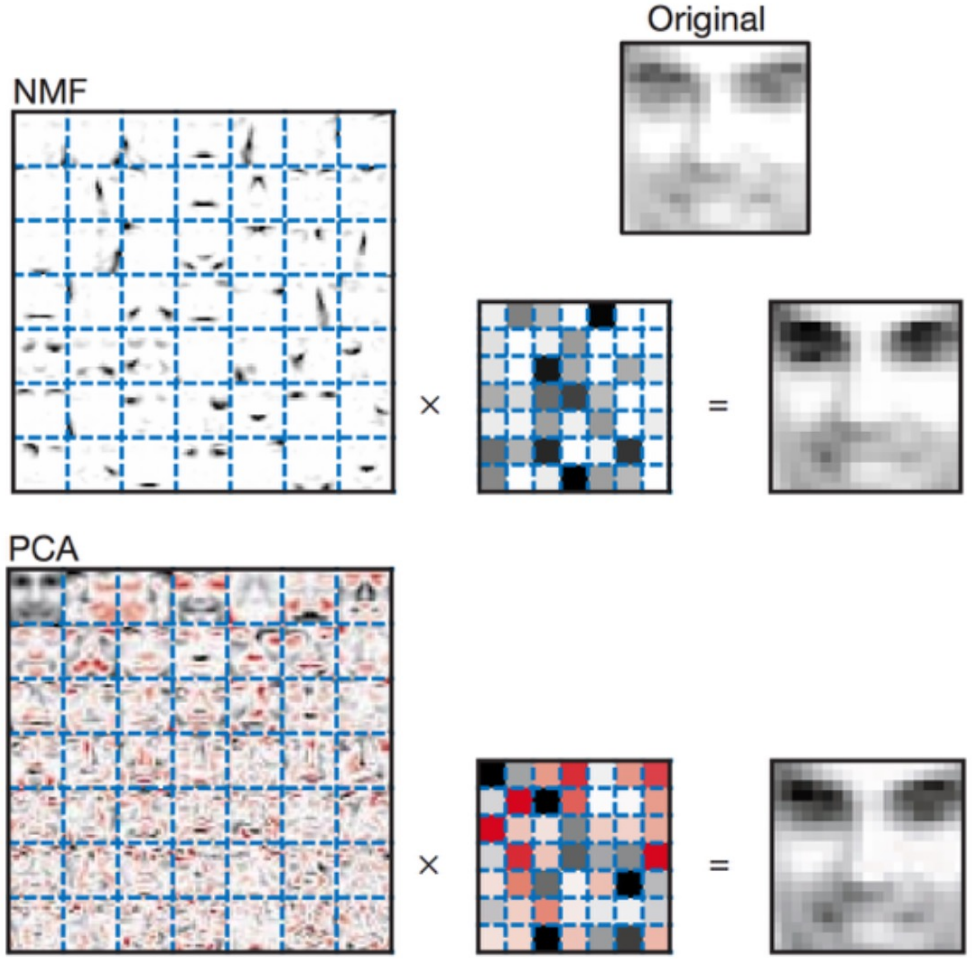


(Lee & Seung, 1999)

Interlude: Matrix Factorization

- Non-Negative Matrix Factorization

$$\underset{\mathbf{U},\mathbf{V}}{\text{minimize}} \quad \|\mathbf{X} - \mathbf{U}\mathbf{V}^T\|_F^2$$
$$\text{subject to} \quad \mathbf{U} \geq 0,\ \mathbf{V} \geq 0$$



(Lee & Seung, 1999)

NMF advantages:

- sparse factors
- additively combined
- can be “parts-based”
- can be unique

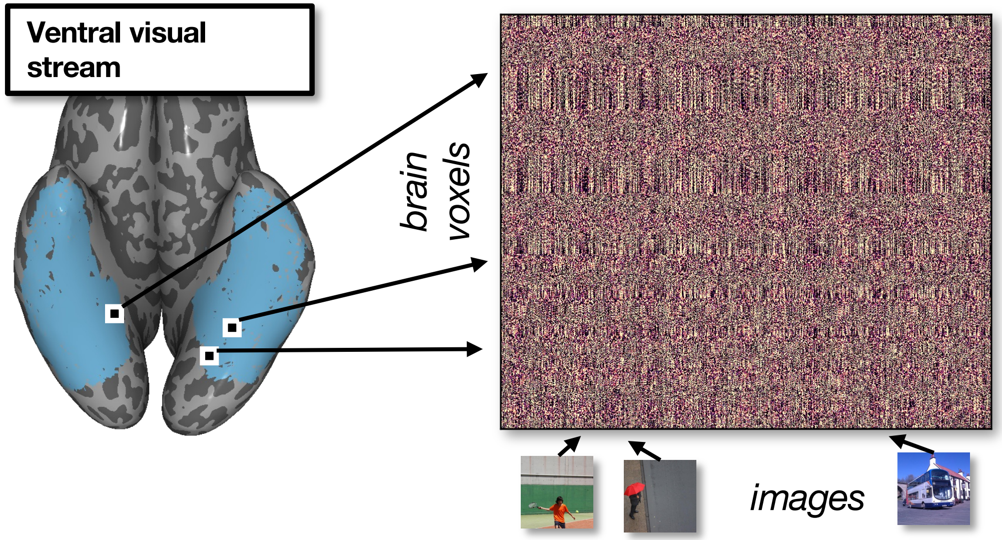
Interlude: Matrix Factorization

List of some other matrix decomposition models

- Sparse Principal Components Analysis:
  sparsity penalties on U and V
- Independent Components Analysis:
  statistical independence of columns of V
- Sparse NMF
  sparsity penalties on U and/or V
- Semi-NMF
  Only U or V constrained to have non-negative entries

Large-scale datasets + Data-driven modeling

Goal: Discover the components ('neural populations') that produce the data

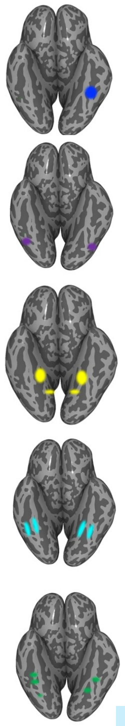


D

Non-Negative Matrix Factorization

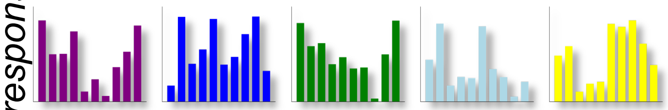
$$D \approx WR$$

W



X

data-driven algorithm



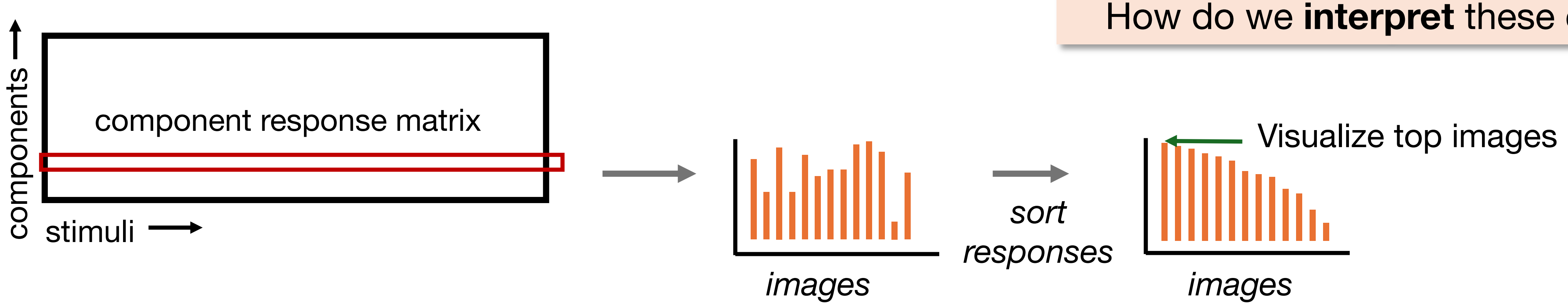
images

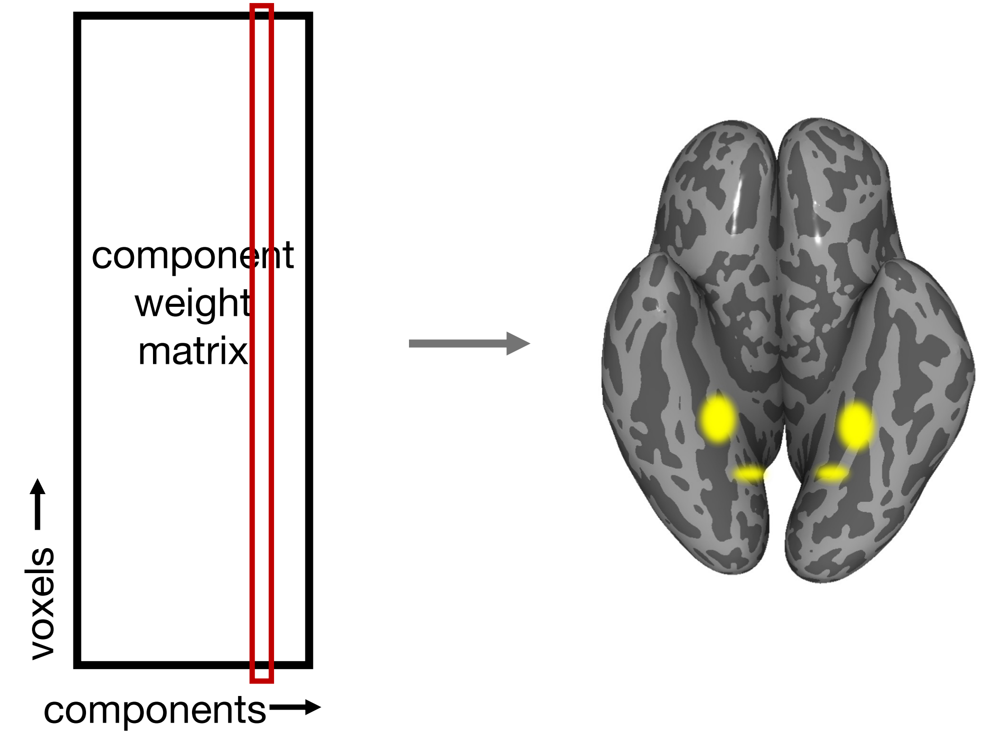
R

5 components present in every subject

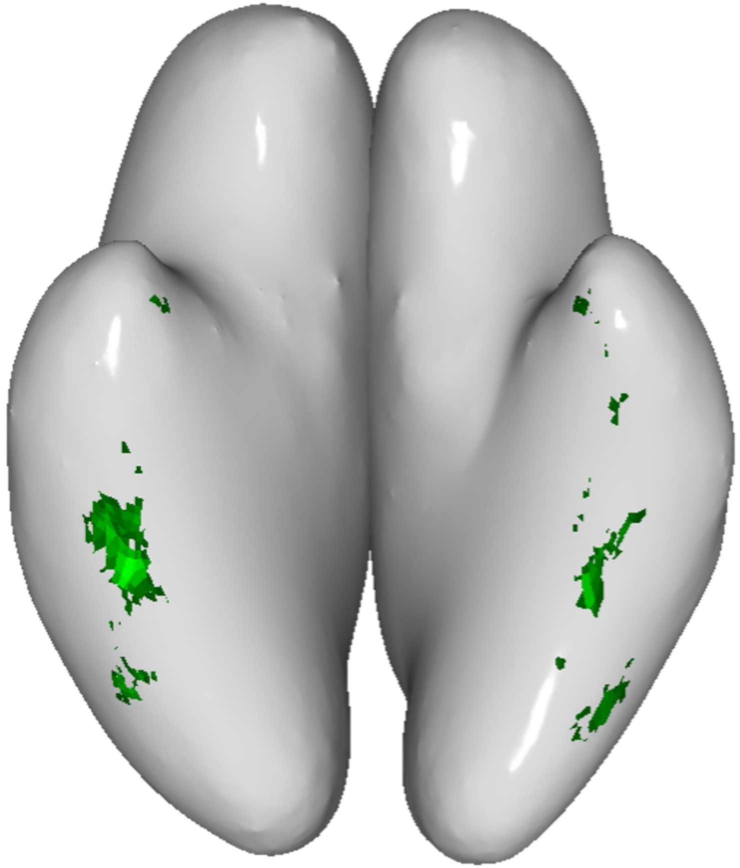
Large-scale datasets + Data-driven modeling

How do we **interpret** these components?

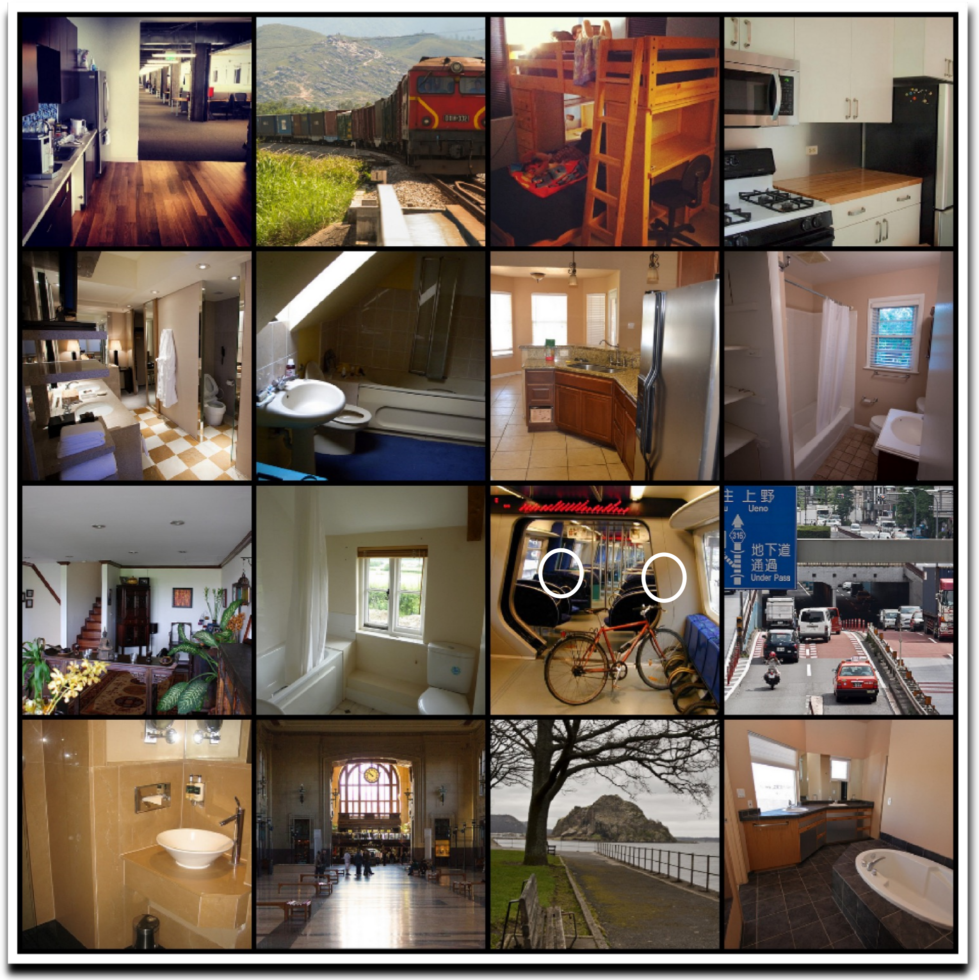


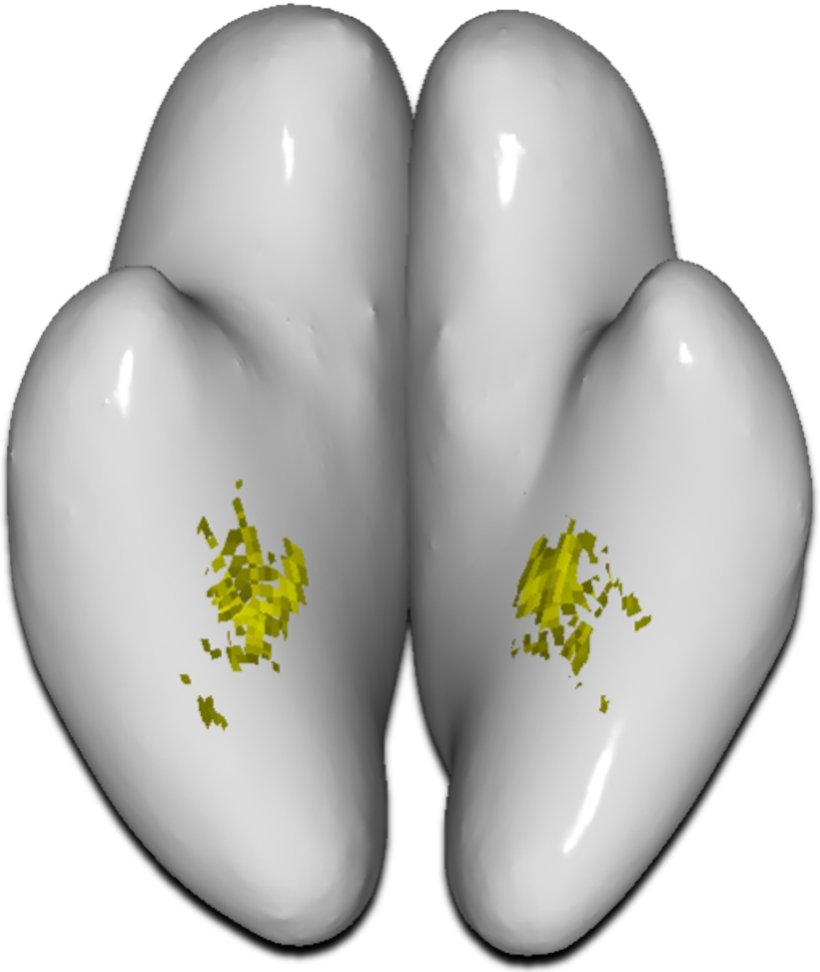


Component 1



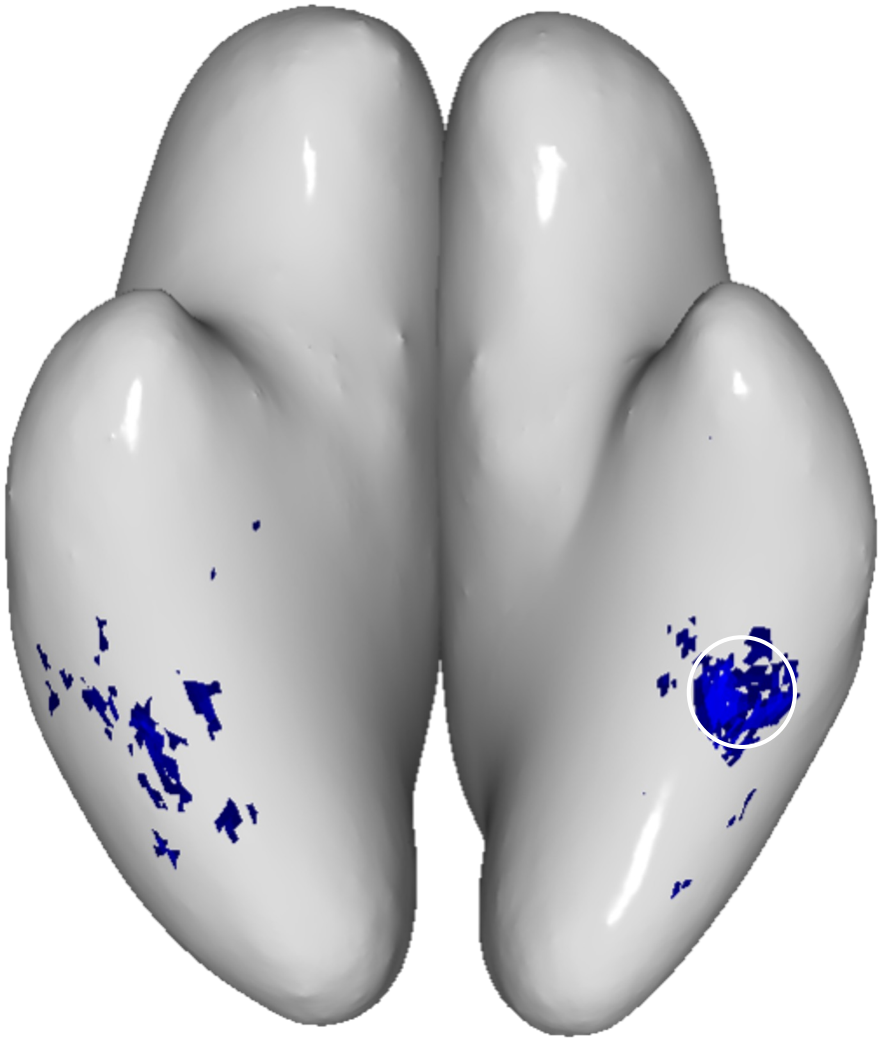
Component 2





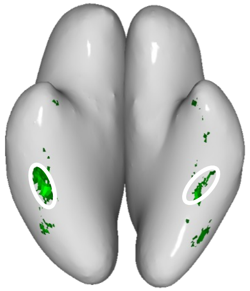
Component 3


TAOS
ARRÊT
ONE WAY
STOP
ALL WAY
AMERICAN
AMOCO
GAS
SKATE ROUTE
TALBOT ST.
SHOPS &
RESTAURANTS
CANYON
HIGH SCHOOL
ようこそ河口湖へ
STOP
HAMMERTIME



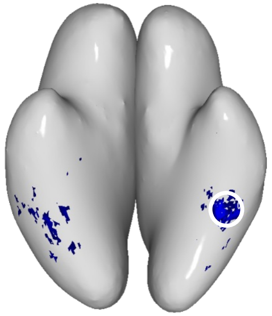
Component 4

Component 1

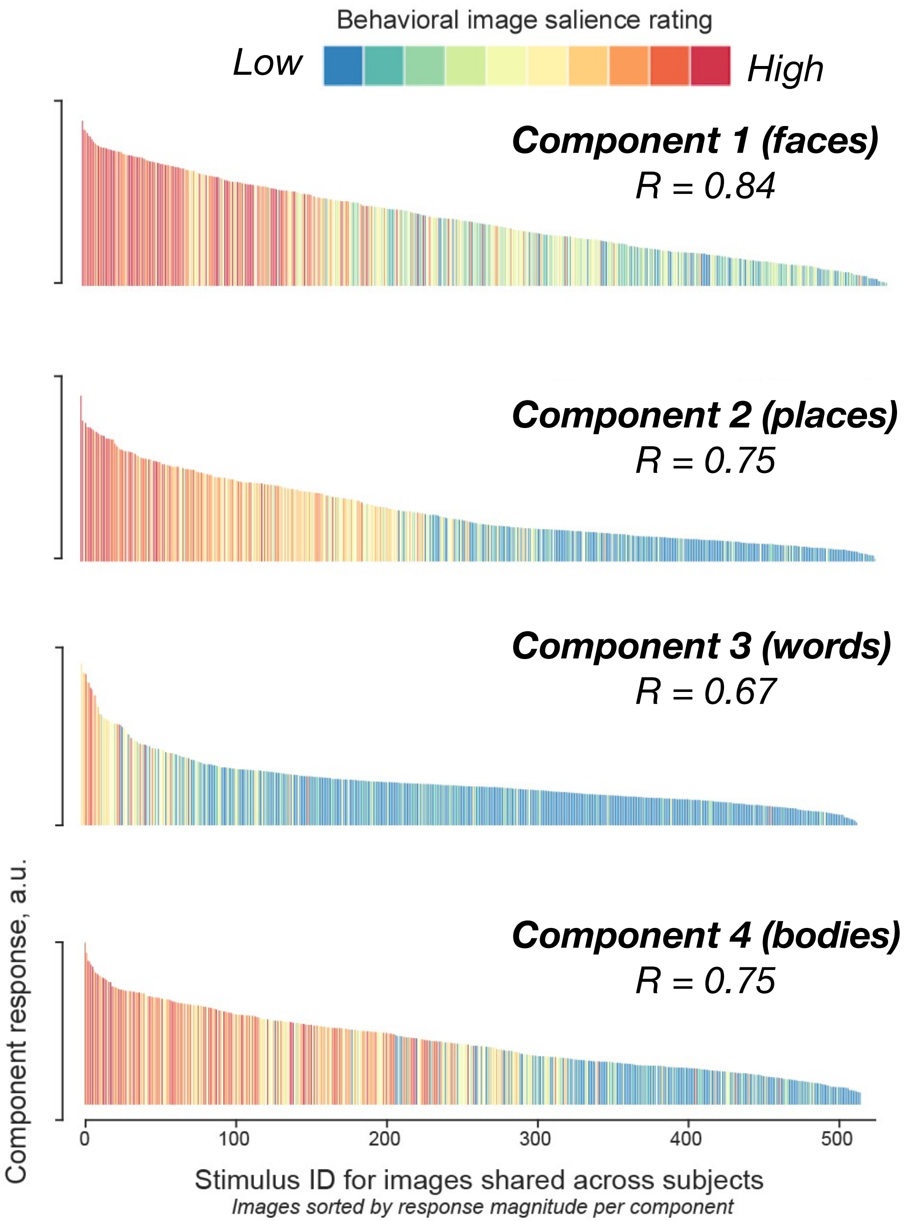


faces

Component 3



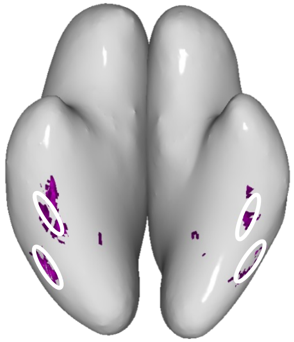
words



Component 2

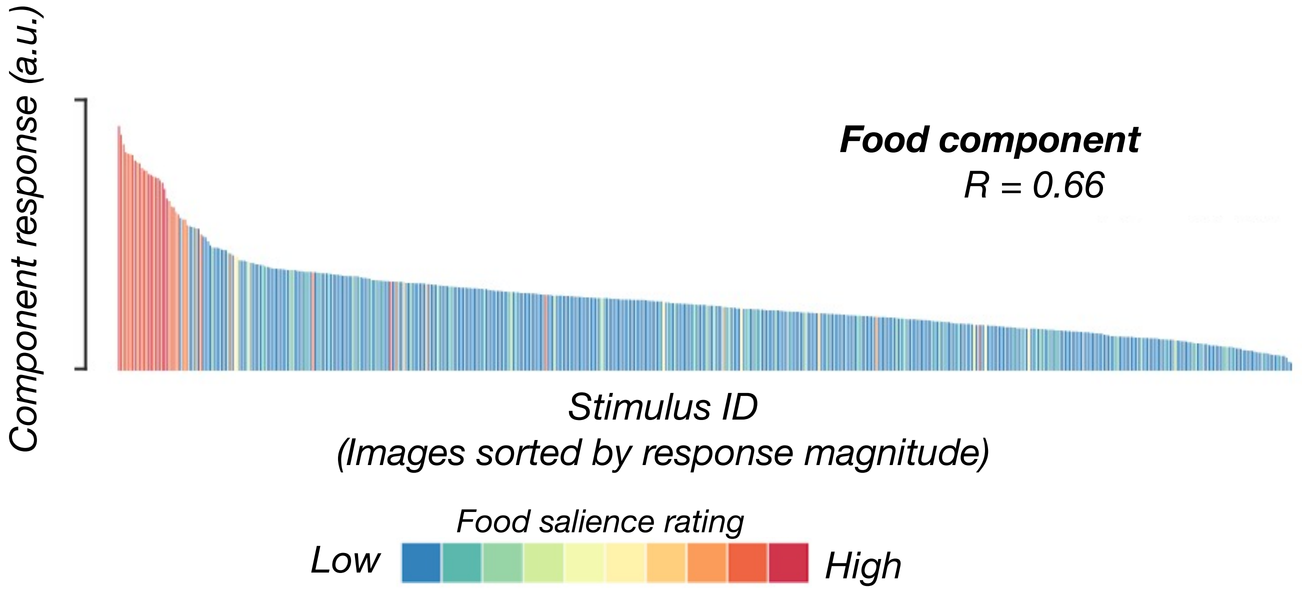
places

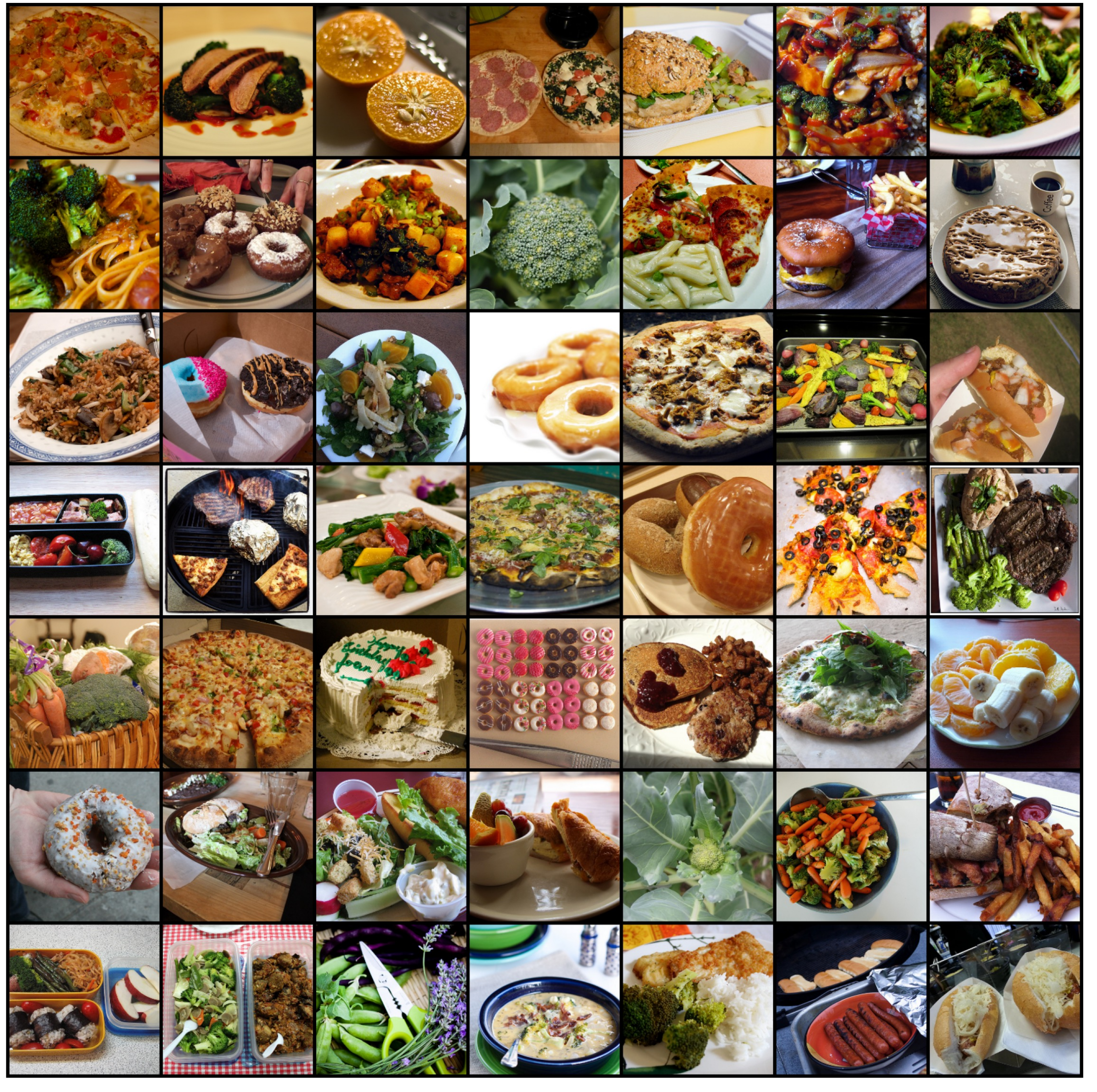
Component 4



bodies

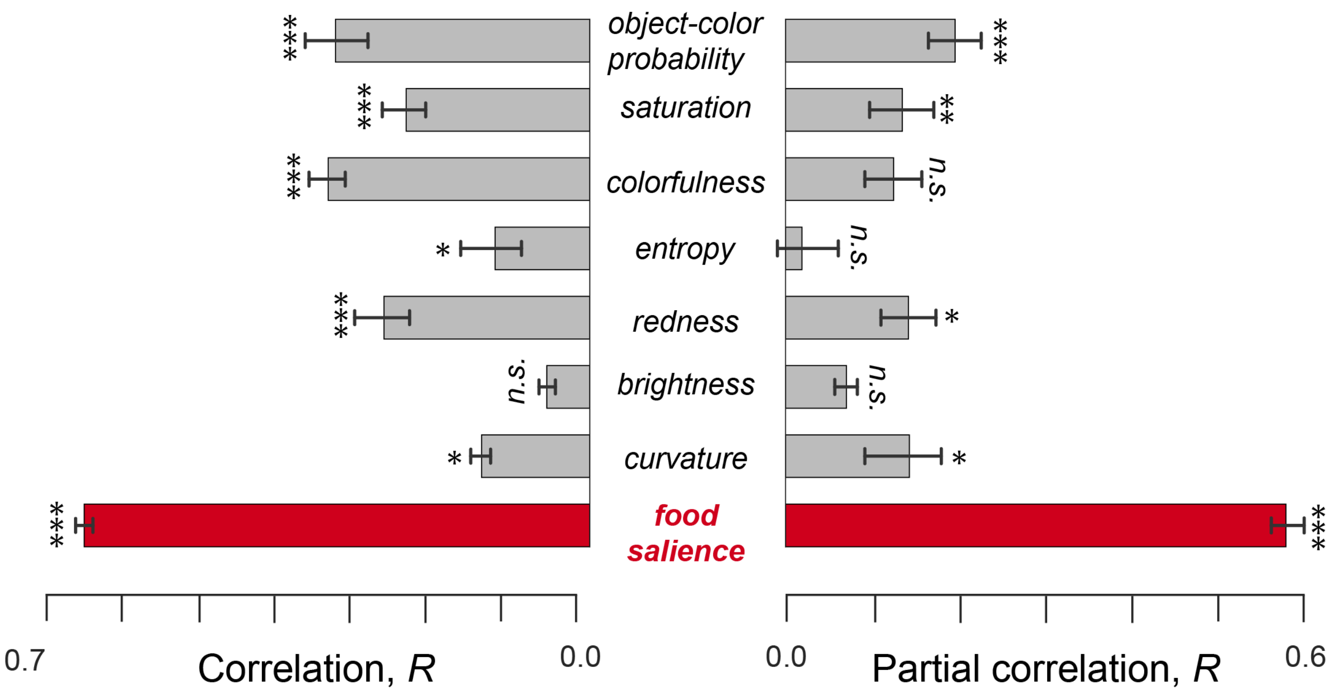
A novel fifth component





...is it really food?

Low-level image properties and component response?

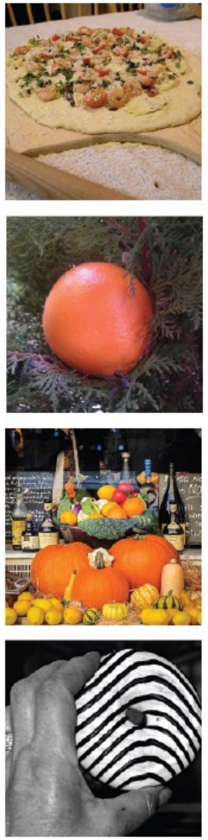


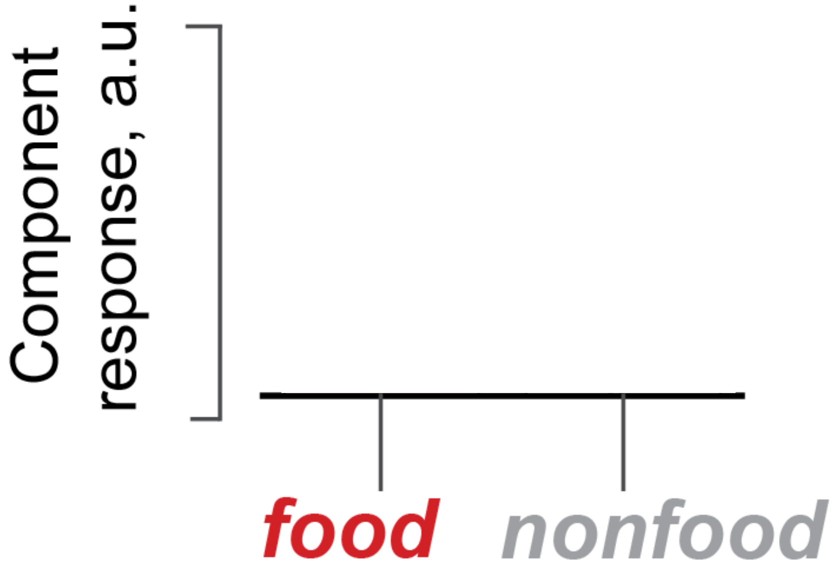
…is it really food?

Big data approaches allow for hypothesis-driven control analyses

food

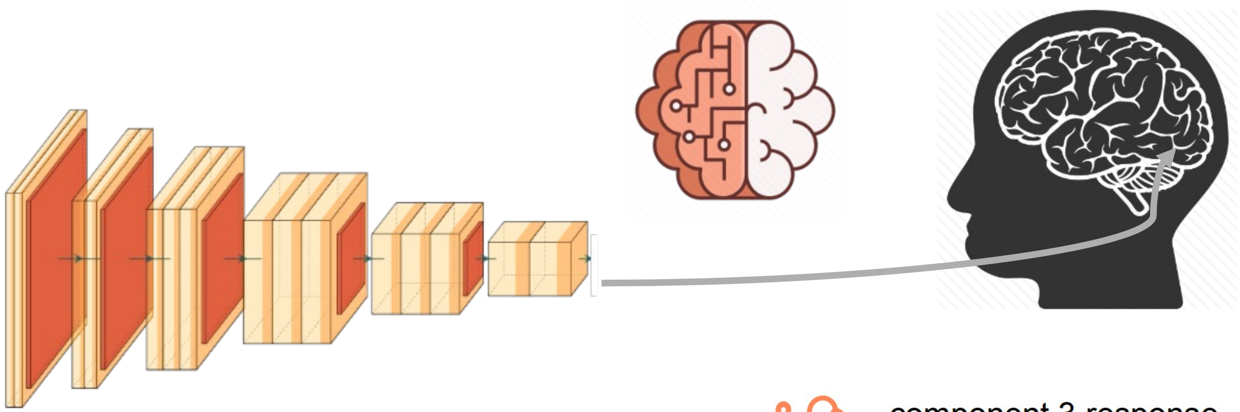
Computational matching algorithm





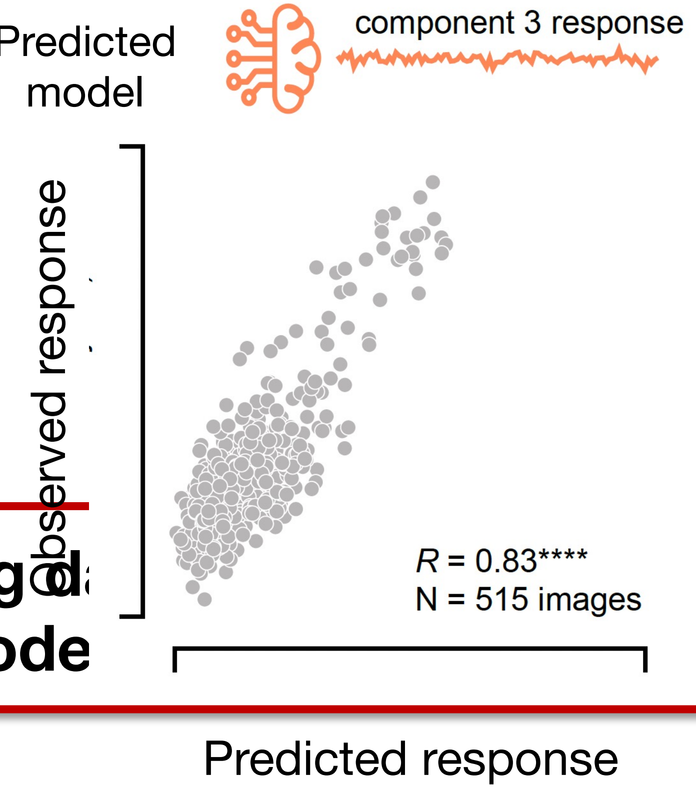
…is it really food?

Simulate the component



Model predictions

Food selectivity persists even for strongly matched controls!



Big d
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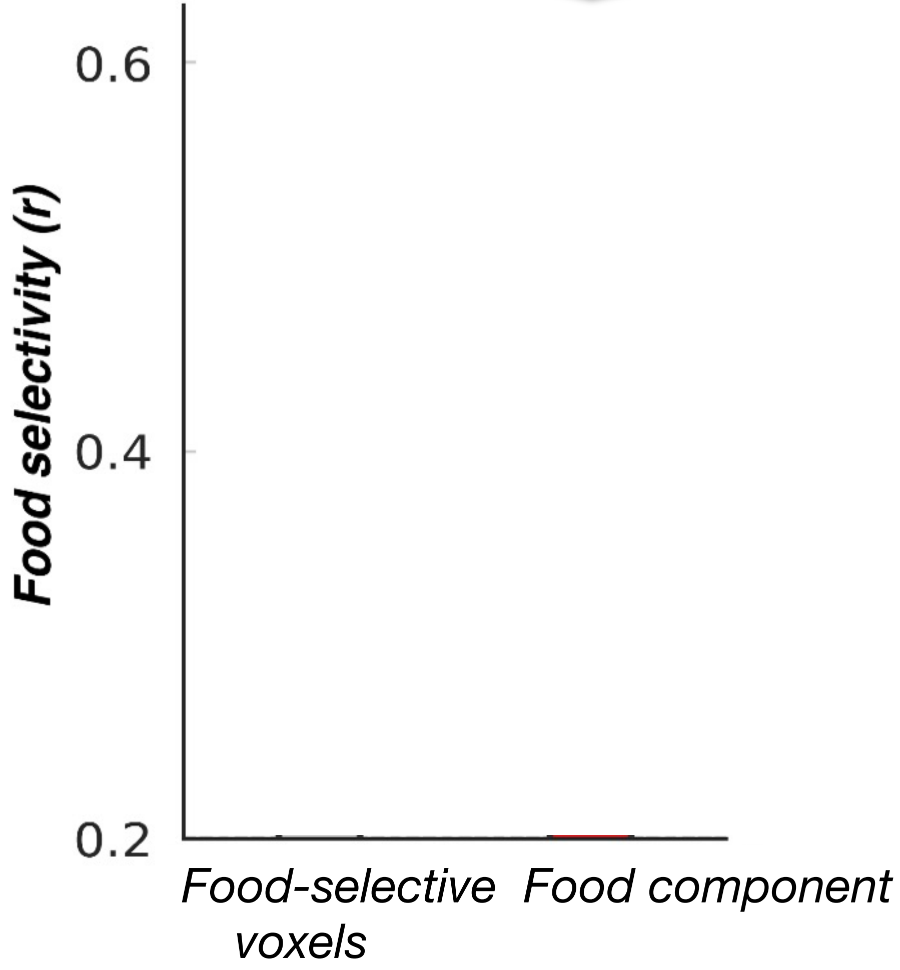
food ***nonfood***

Component has higher selectivity than raw voxels



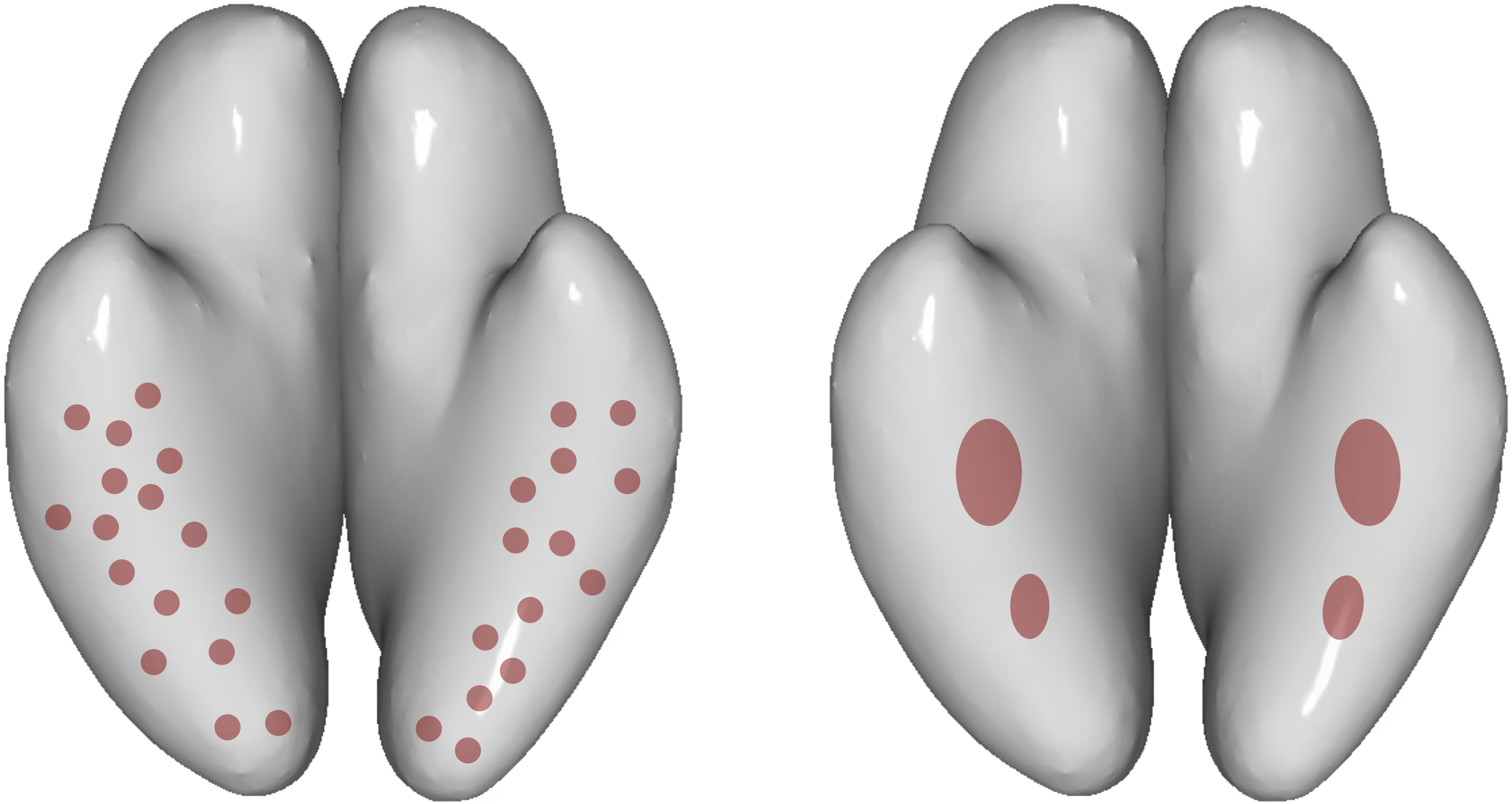
Why was this food component not discovered before?

De-mixing is important!

New methodological advances are crucial

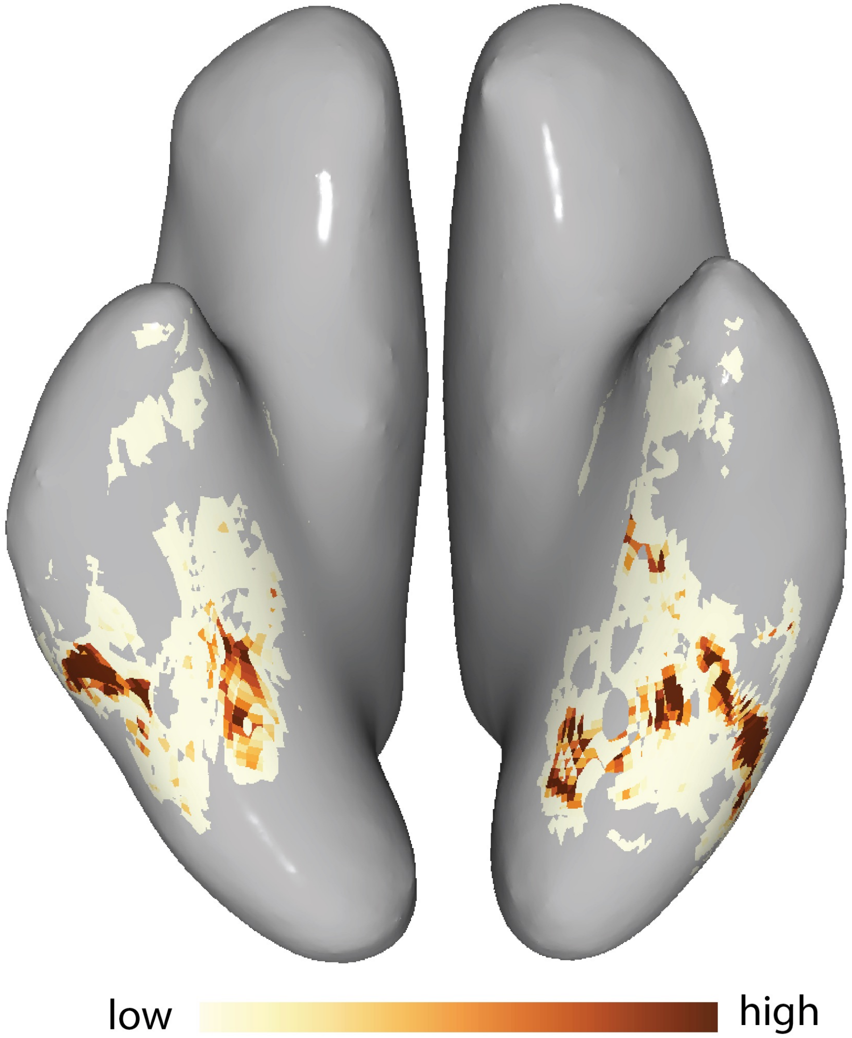
Where does it live in the brain?

Two possibilities



Where does it live in the brain?

Food component voxel weights



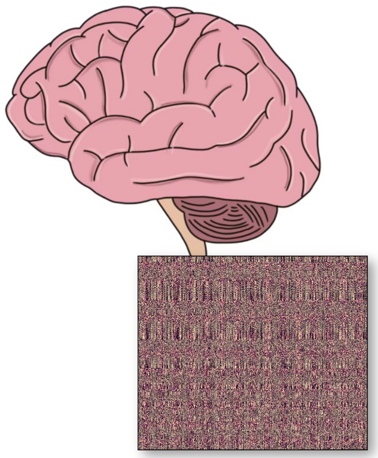
What are the ***fundamental dimensions*** along which the cortex is

What have we missed?

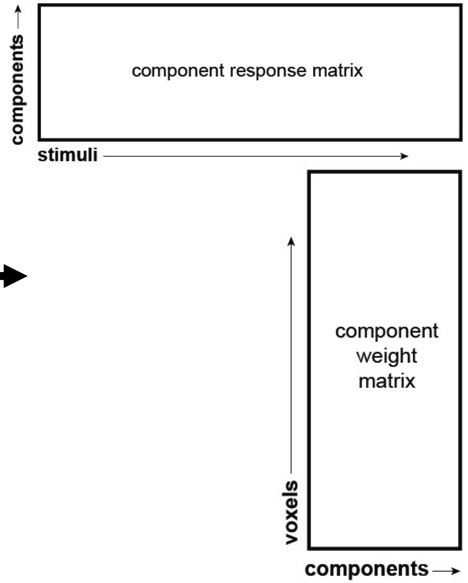
➢ Rediscovered neural selectivities for **faces, places, bodies and words**, suggesting these are **fundamental organizational dimensions.**

These methods could be powerful in other domains, including **social perception** and **language understanding**

➢ Discovered a **novel selectivity for food** in the ventral visual cortex.



Brain activity data

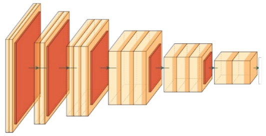


Consensus Bayesian Non-Negative Matrix Factorization

Component visualization (Generate hypothesis)

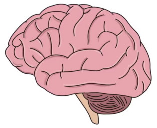
Hypothesis-driven control analysis

Model-driven control analysis



Test hypothesis

*How is the brain **representing**?*

P()

Matrix Factorization

High-level visual cortex is richly populated with areas selective for meaningful categories such as faces, bodies, words, scenes and food

Kanwisher, Khosla, and Dobs. 2022. Trends in Neurosciences (to appear)

***Khosla,** Murty and Kanwisher. 2022. Current Biology*

Lecture Overview

- Example from research: how does our brain represent our visual world?
- High-level introduction to Matrix Factorization Methods
- **Another research example: how does our brain represent sounds?**
- Diverse applications of matrix factorization in neuroscience
- Practical Demonstration: Demixing fMRI data using matrix factorization

Case Study II: How does the brain represent sounds?

Distinct Cortical Pathways for Music and Speech Revealed by Hypothesis-Free Voxel Decomposition

Sam Norman-Haignere [1], Nancy G. Kanwisher [1 2 3], Josh H. McDermott [1 3]

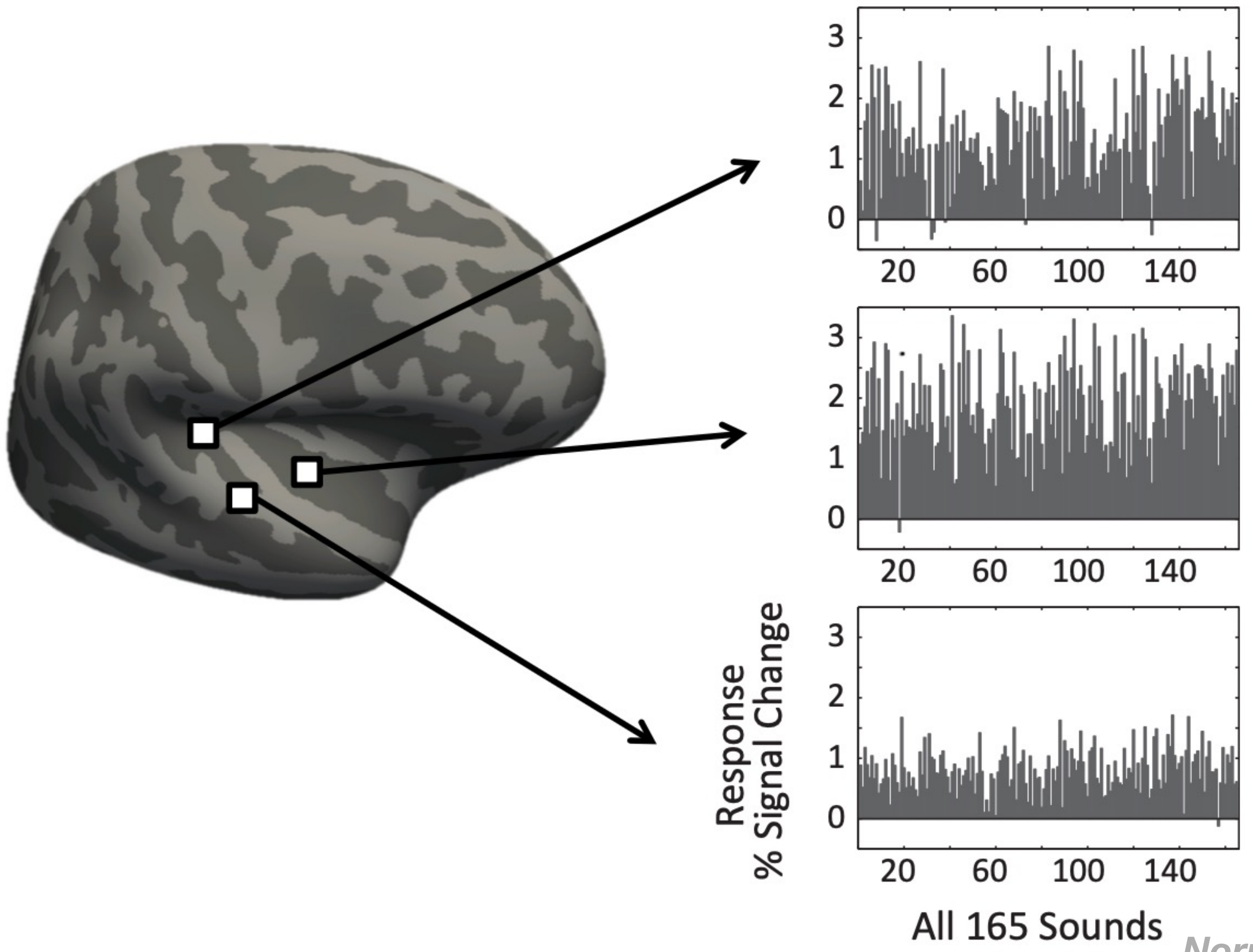
Measured fMRI responses to 165 natural sounds:

1. Man speaking
2. Flushing toilet
3. Pouring liquid
4. Tooth-brushing
5. Woman speaking
6. Car accelerating
7. Biting and chewing
8. Laughing
9. Typing
10. Car engine starting
11. Running water
12. Breathing
13. Keys jangling
14. Dishes clanking
15. Ringtone
16. Microwave
17. Dog barking
18. Walking (hard surface)
19. Road traffic
20. Zipper
21. Cellphone vibrating
22. Water dripping
23. Scratching
24. Car windows
25. Telephone ringing
26. Chopping food
27. Telephone dialing
28. Girl speaking
29. Car horn
30. Writing
31. Computer startup sound
32. Background speech
33. Songbird
34. Pouring water
35. Pop song
36. Water boiling
37. Guitar
38. Coughing
39. Crumpling paper
40. Siren

…

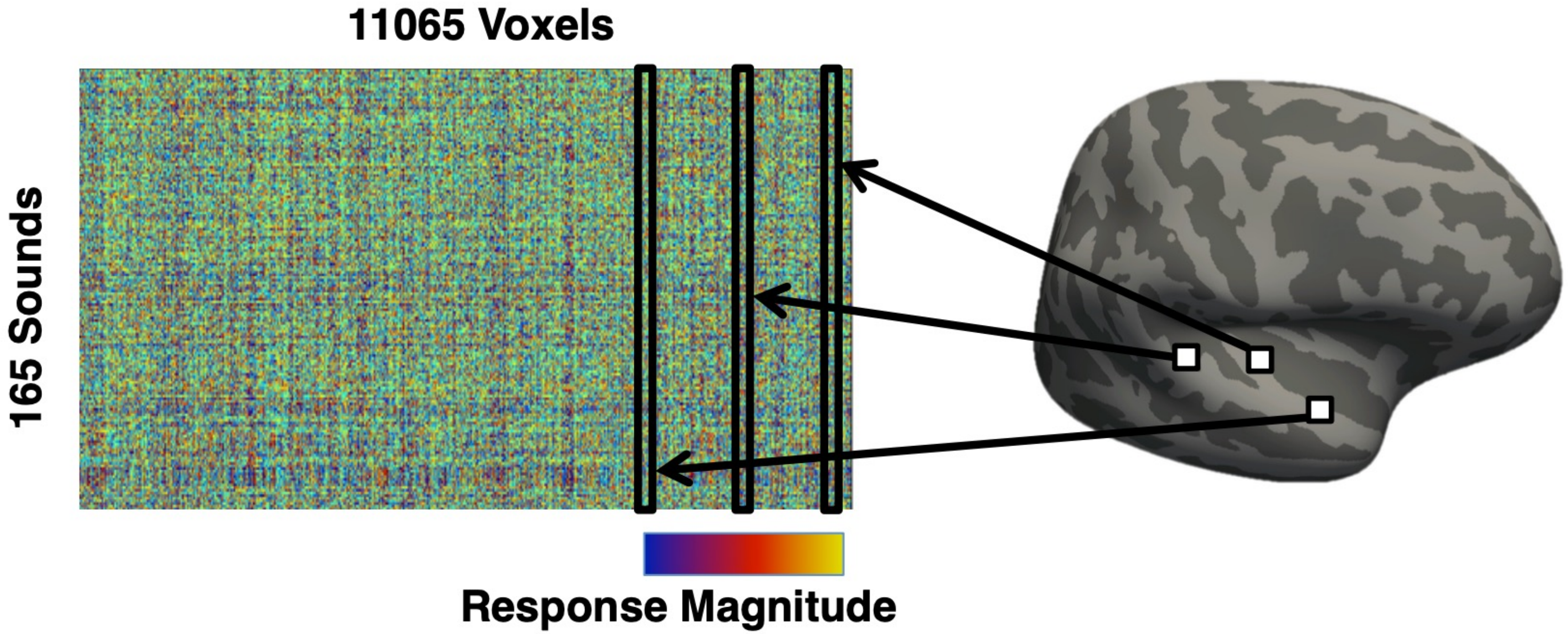
Norman-Haignere et al. Neuron

- Measured fMRI responses to 165 natural sounds:
- For each voxel, measure average response to each sound



Norman-Haignere et al. Neuron

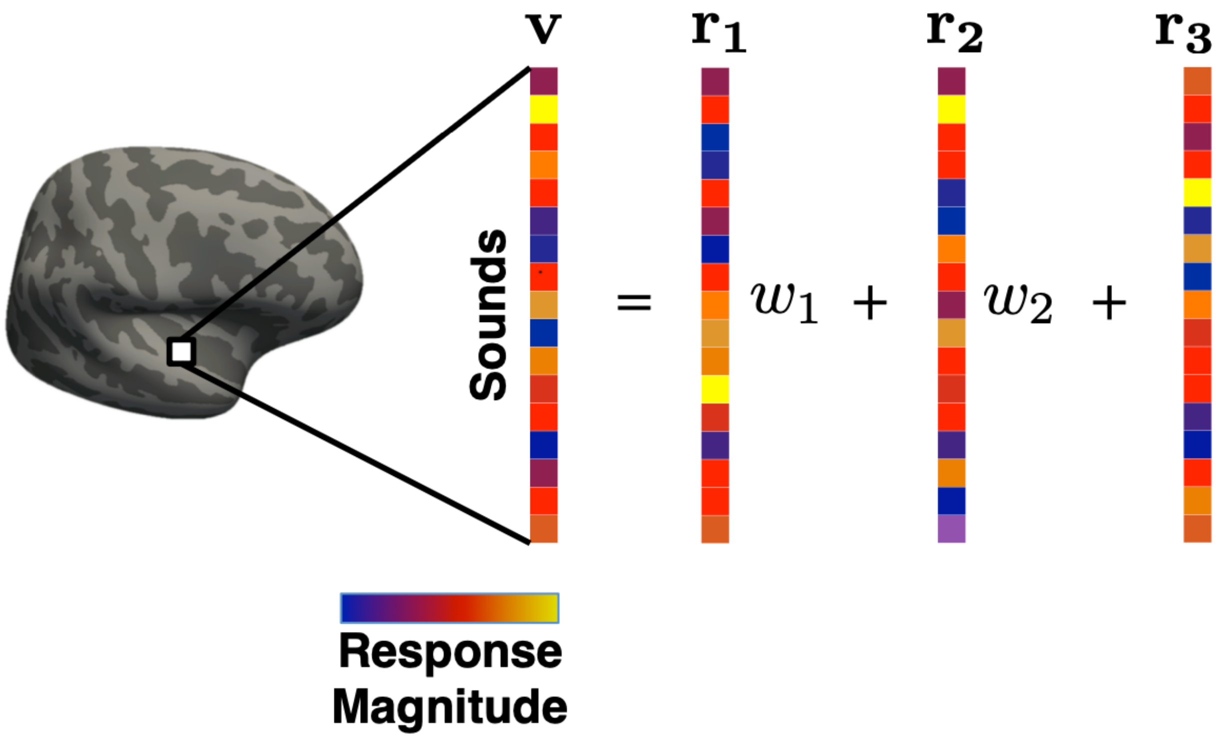
- Measured fMRI responses to 165 natural sounds:
- For each voxel, measure average response to each sound
- Compile all voxel responses into a matrix



Hypothesis: Perhaps a small number of neural populations – each with a canonical response to the sound set – explain the response of thousands of voxels?

Norman-Haignere et al. Neuron

Linear Model of Voxel Responses

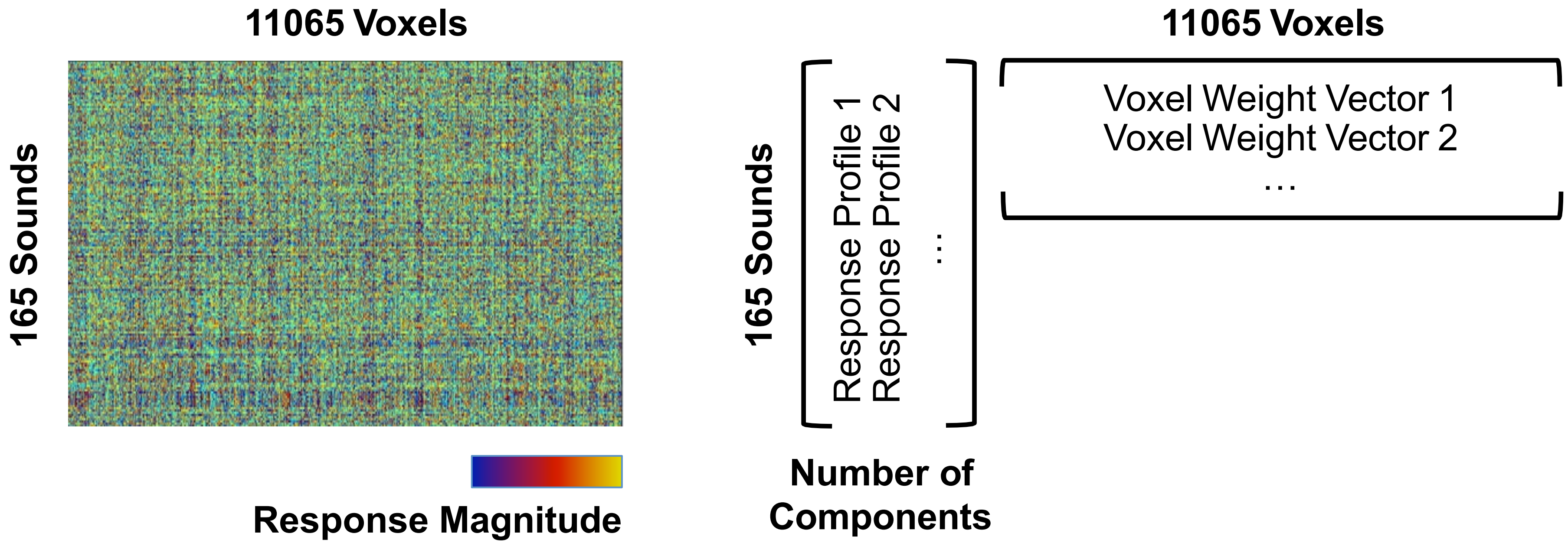


$$\mathbf{v} = \mathbf{r_1} w_1 + \mathbf{r_2} w_2 + \mathbf{r_3} w_3 + \ldots \qquad \mathbf{v} = \sum_{i=1}^{N} \mathbf{r_i} w_i$$

Voxel responses modeled as weighted sum of response profiles

Norman-Haignere et al. Neuron

Matrix Factorization

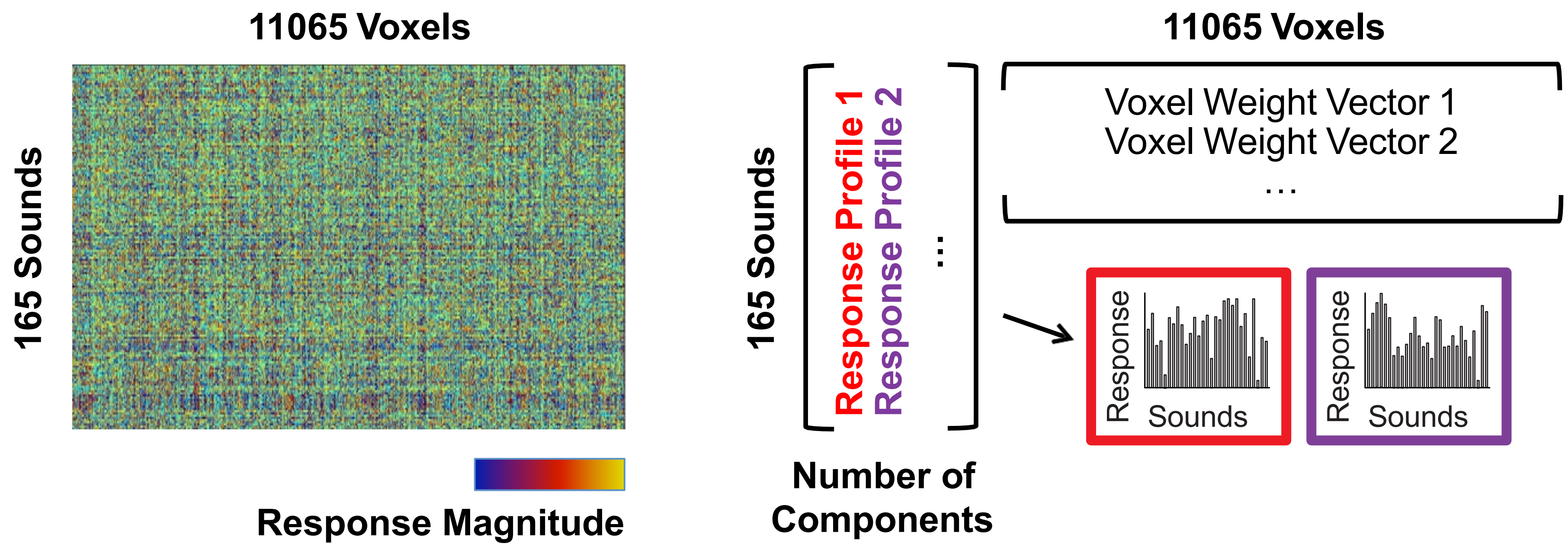


Factor response matrix into set of components, each with:

1. Response profile to all 165 sounds
2. Voxel weights specifying contribution of each component to each voxel

Norman-Haignere et al. Neuron

Matrix Factorization



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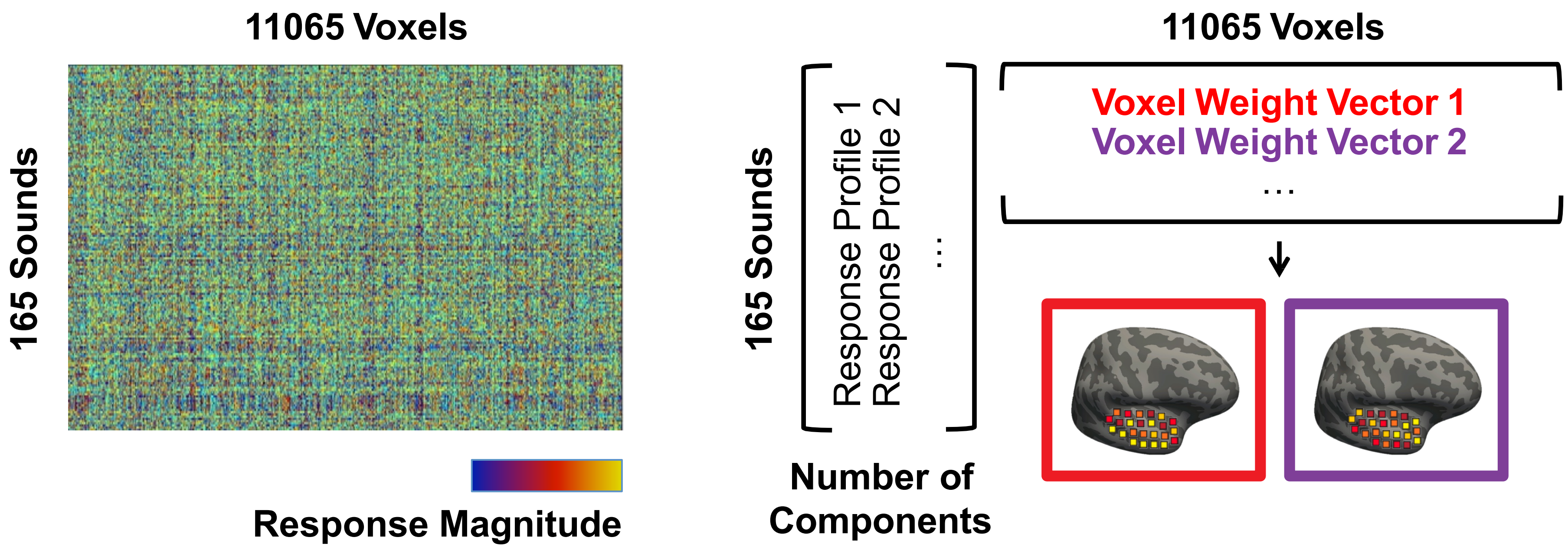
Norman-Haignere et al. Neuron

Matrix Factorization



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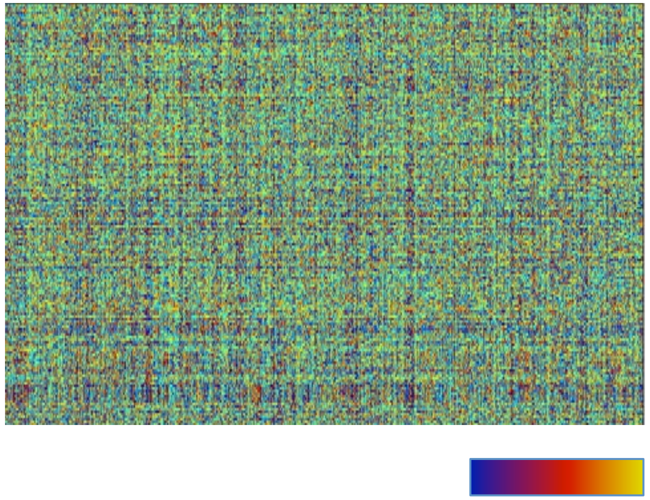
Norman-Haignere et al. Neuron

Matrix Factorization



11065 Voxels

165 Sounds

Response Profile 1
Response Profile 2
…

Voxel Weight Vector 1
Voxel Weight Vector 2
…

Number of Components

Matrix approximation ill-posed (many equally good solutions)

- Must be constrained with additional assumptions
- Different techniques make different assumptions

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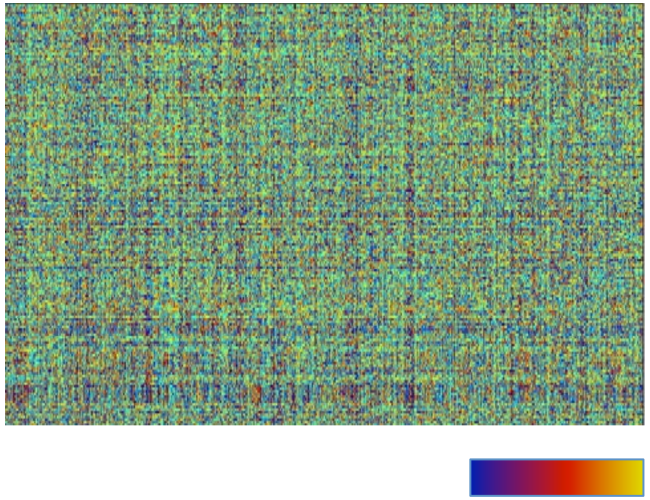
Principal Components Analysis (PCA)

11065 Voxels

165 Sounds



Response Magnitude

165 Sounds

Response Profile 1
Response Profile 2
…

Number of Components

11065 Voxels

Voxel Weight Vector 1
Voxel Weight Vector 2
…

For PCA to infer underlying components, they must have uncorrelated response profiles and voxel weights

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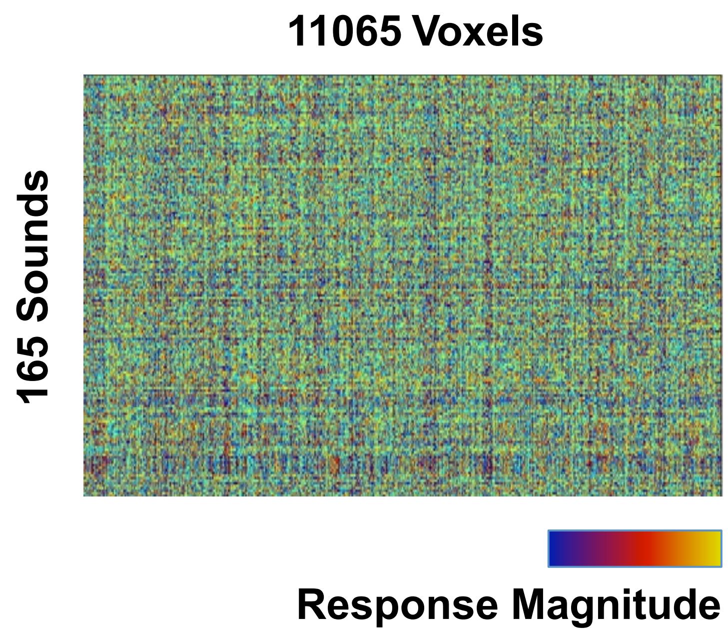
Independent Components Analysis (ICA)



165 Sounds

Response Profile 1
Response Profile 2
...

Number of Components

11065 Voxels

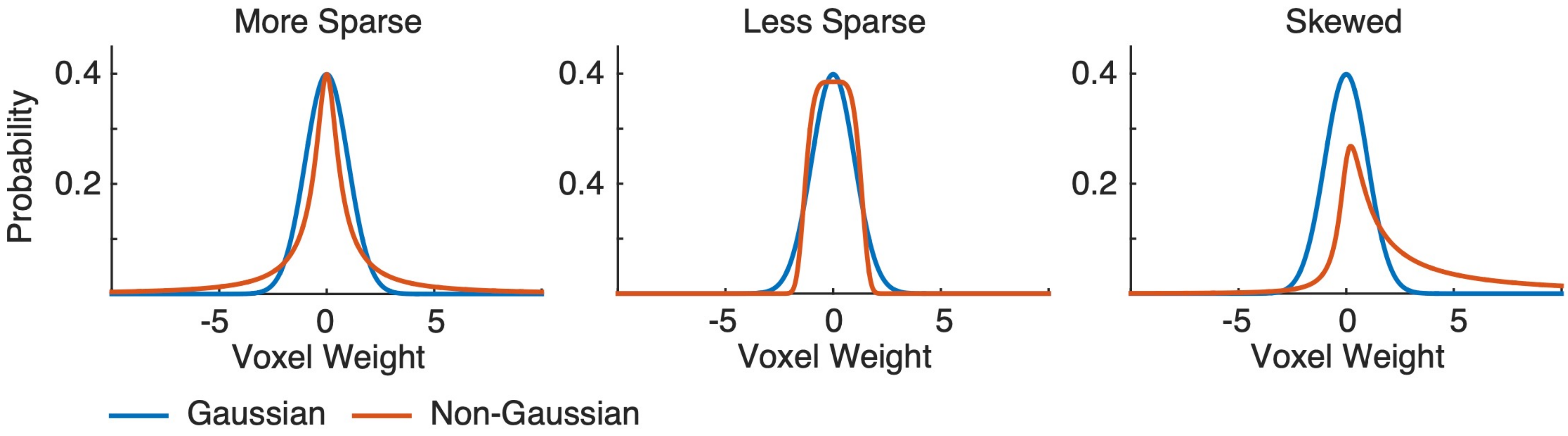
Voxel Weight Vector 1
Voxel Weight Vector 2
...

For ICA to infer underlying components, they must have non-gaussian statistically independent voxel weights

Norman-Haignere et al. Neuron

Independent Components Analysis (ICA): an aside on non-gaussianity

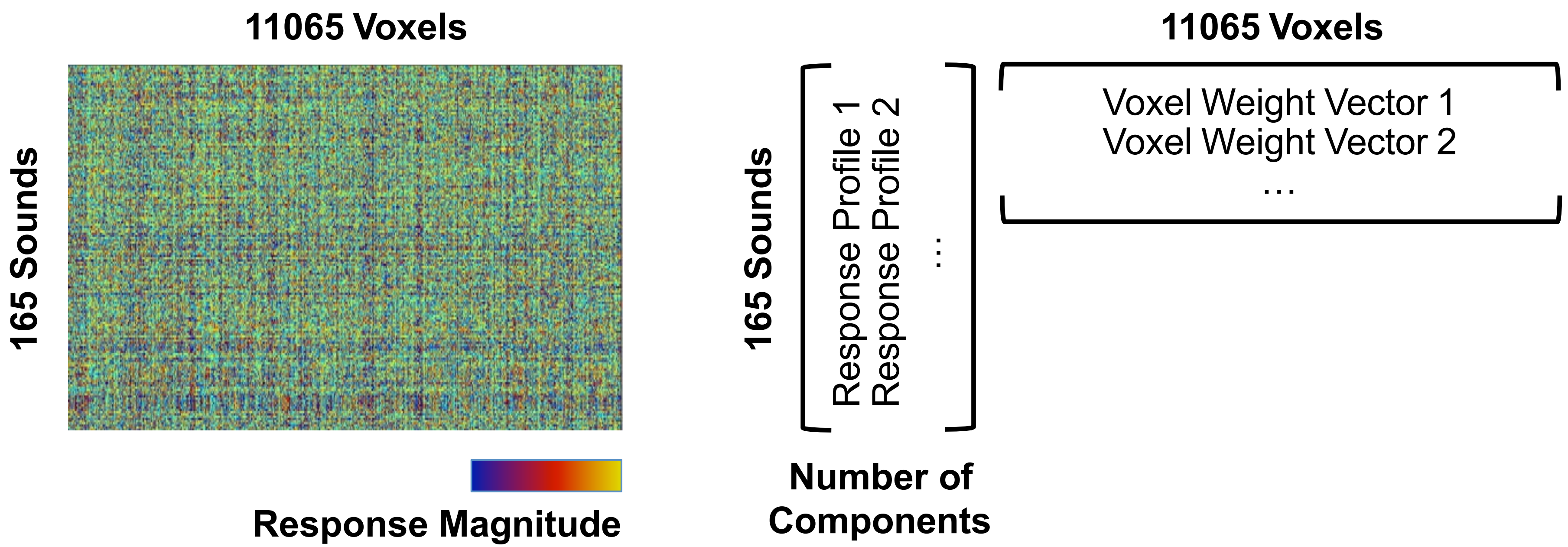
Many ways for a distribution to be non-Gaussian:



For ICA to infer underlying components, they must have non-gaussian statistically independent voxel weights

Norman-Haignere et al. Neuron

Matrix Factorization



Applied Independent Components Analysis to the data matrix

- finding U and V such that $X \approx UV^T$, with the constraint that the components (columns of V) are statistically independent

Found 6 components that could approximate the original data matrix reasonably well

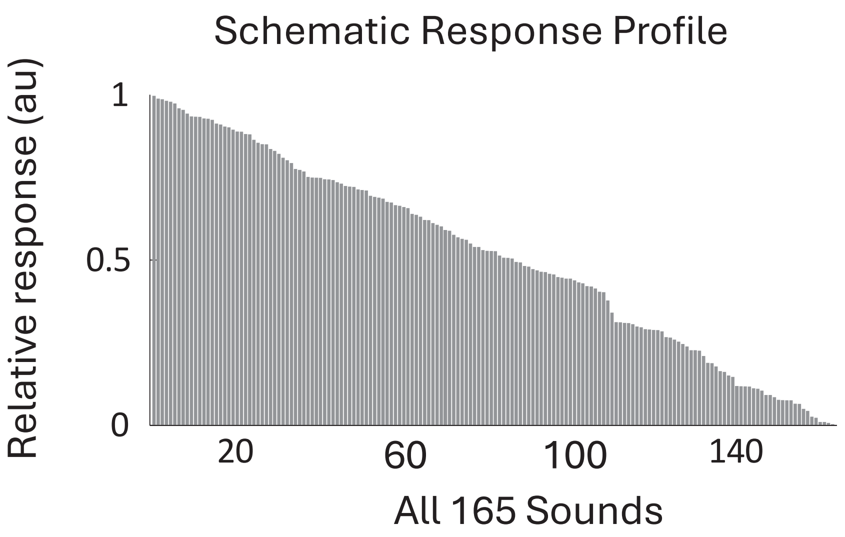
Norman-Haignere et al. Neuron

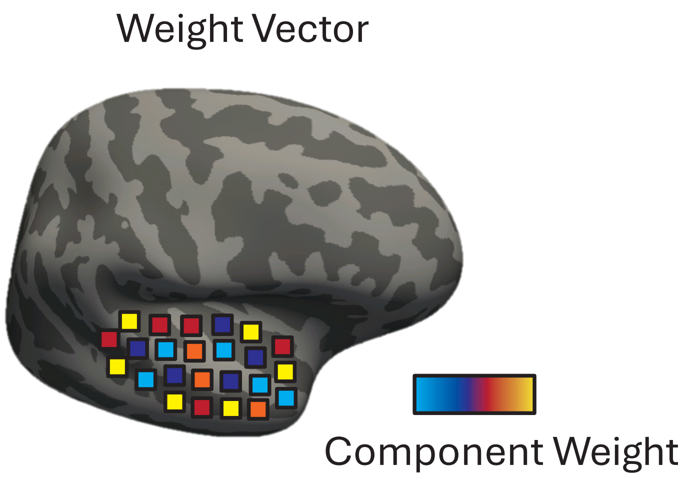
Probing the Inferred Components

Have 6 dimensions, each with:

1. A response profile (165-dimensional vector)
2. A weight vector, specifying its contribution to each voxel

Both response profile and anatomy unconstrained

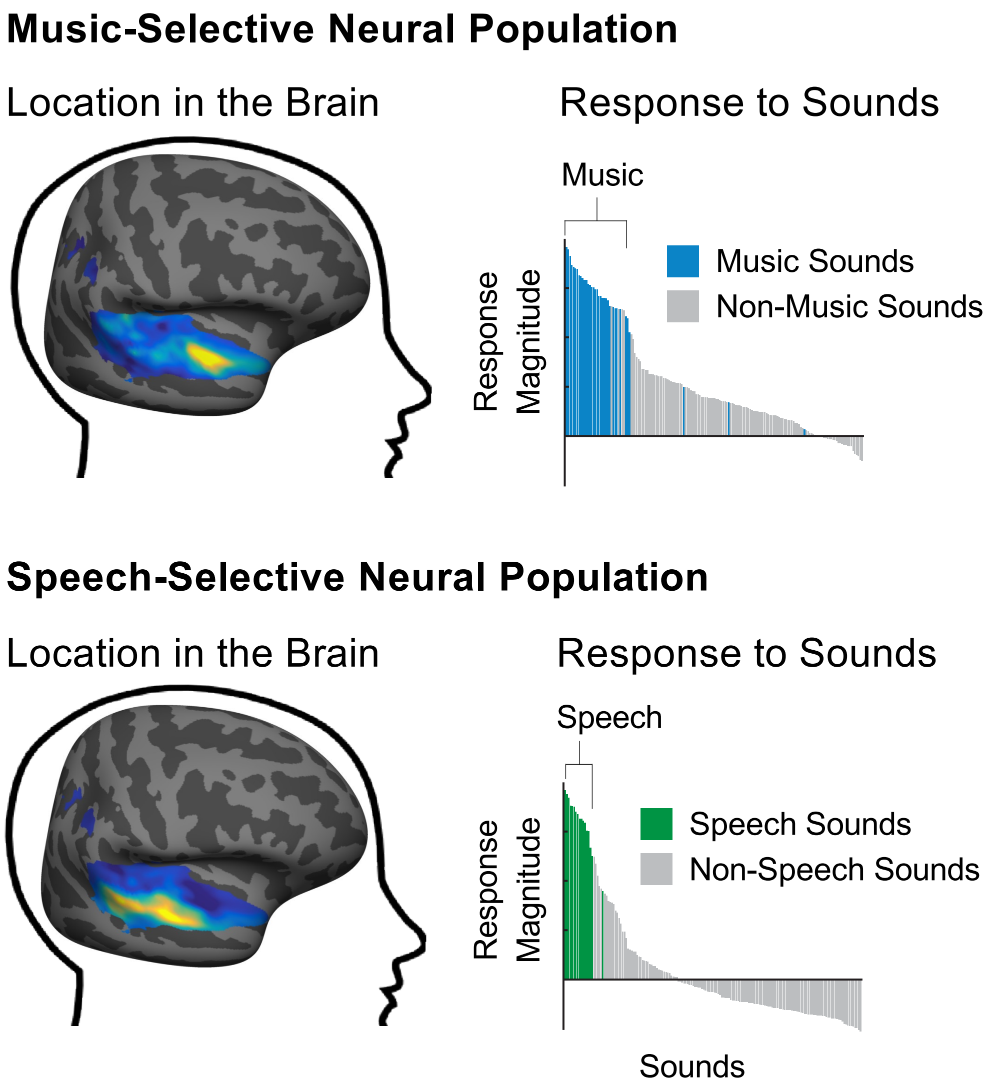




Norman-Haignere et al. Neuron

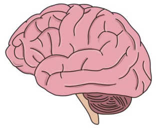
All 6 inferred components have interpretable properties

⇒ 2 components highly selective for speech and music, respectively



Norman-Haignere et al. Neuron

*How is the brain **representing**?*

P()

Matrix Factorization

Auditory cortex is richly populated with areas selective for speech and music

*Kanwisher, **Khosla** and Dobs. 2022. Trends in Neuroscience (to appear)*

Norman-Haignere, Kanwisher and McDermott, Neuron

Lecture Overview

- Example from research: how does our brain represent our visual world?
- High-level introduction to Matrix Factorization Methods
- Another research example: how does our brain represent sounds?
- **Diverse applications of matrix factorization in neuroscience**
- Practical Demonstration: Demixing fMRI data using matrix factorization

Matrix factorization applied to fMRI data

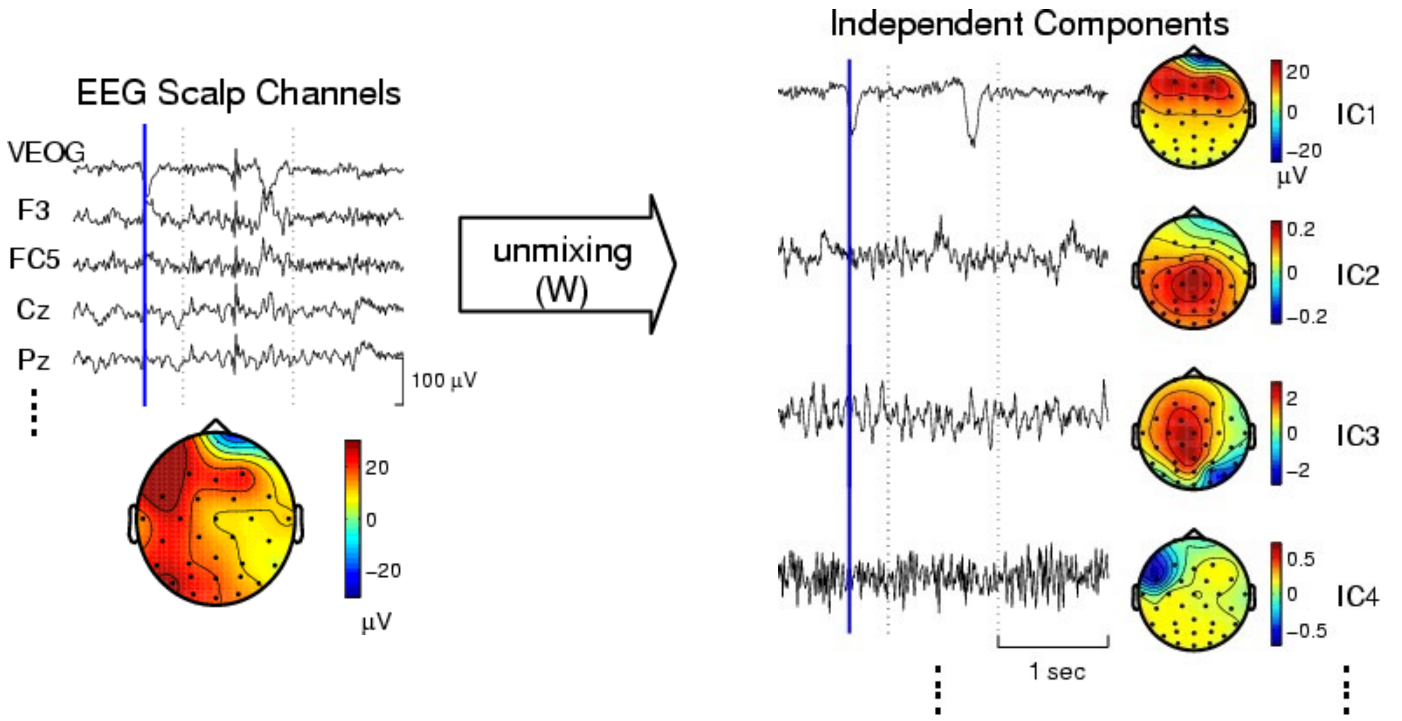
- **Demixing mixed signals**: fMRI ‘voxels’ contain hundreds of thousands of neurons; plausibly contain neural populations with distinct selectivity
  - Matrix factorization helps unmix responses from different neural populations
- **Dimensionality Reduction**: fMRI data is high-dimensional;
  - Matrix factorization can identify the most informative patterns of activity across voxels
- **Noise Reduction**: fMRI/MEG contain noise from various sources (e.g. motion artifacts).
  - Matrix factorization helps separate the signal of interest from noise components

Matrix factorization applied to EEG

Frequently used to denoise EEG timecourses

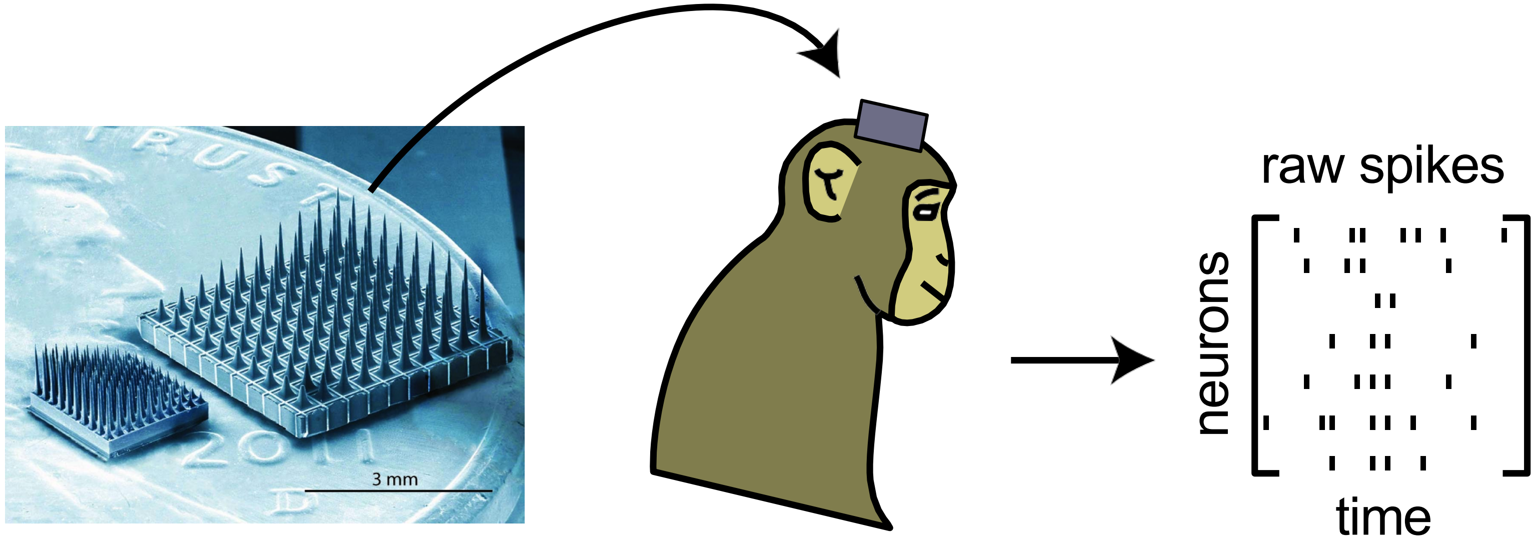
⇒ Artifacts (e.g. eye blinks) mostly independent of neural activity and have non- Gaussian amplitude distribution

⇒ EEG channels modeled as linear mixture of artifacts and neural activity



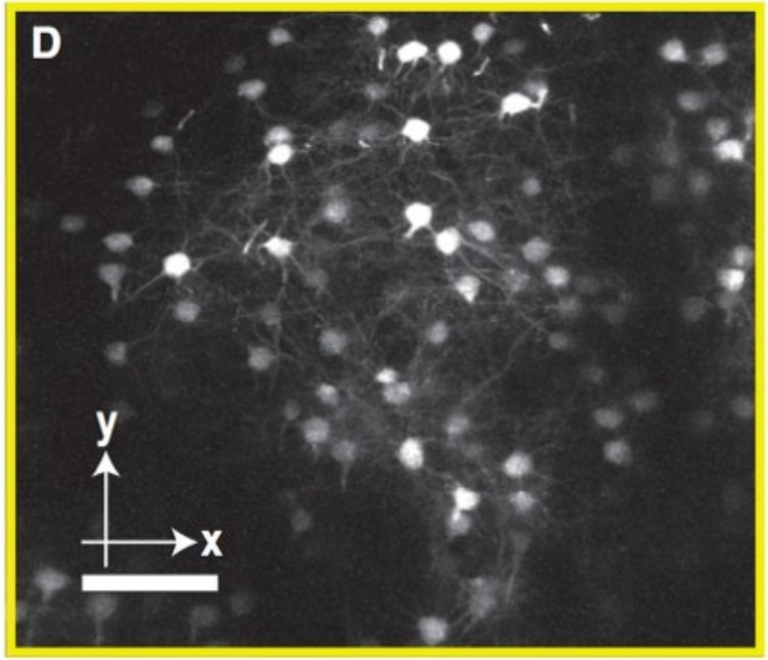
Other types of matrix data in neuroscience

Spiking Activity



Other types of matrix data in neuroscience

Fluorescence Images

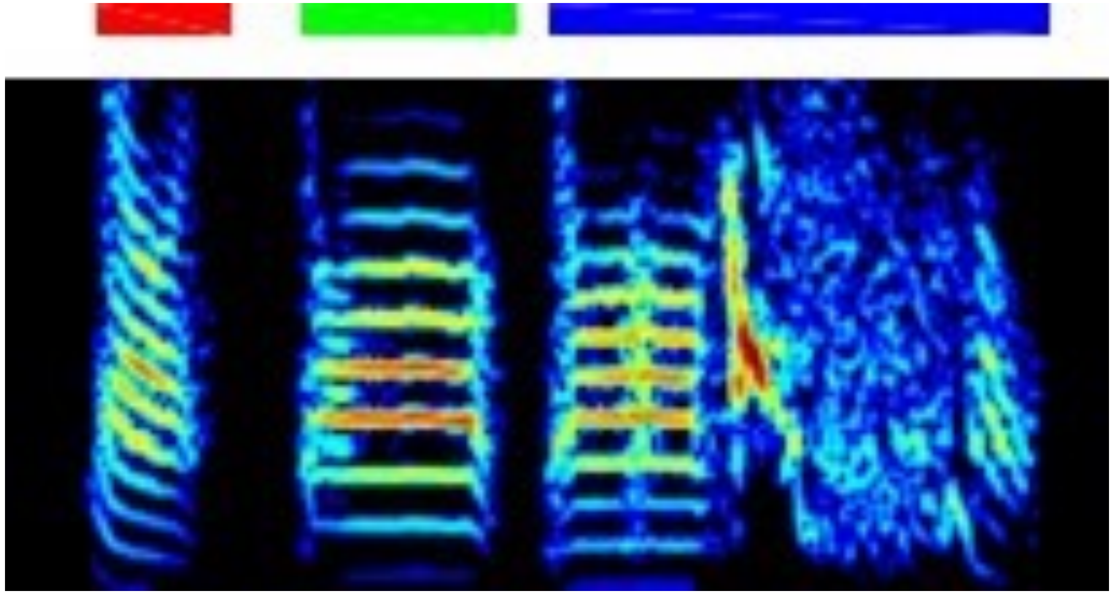


Cortical neurons expressing YFP
(Kim & Zhang et al., 2016)

Other types of matrix data in neuroscience

Spectrograms

frequency



time

Zebra Finch courtship song
(Provided by Emily Mackevicius)

Adapted from Alex Williams

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Lecture Recap

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- High-level introduction to Matrix Factorization Methods
- Another research example: how does our brain represent sounds?
- Diverse applications of matrix factorization in neuroscience
- Practical Demonstration: Demixing fMRI data using matrix factorization