PROBABILITY AND STATISTICS 2

MATHEMATICS

Topic: Probability and Statistics 2
Duration: 1 hour 15 mins

INSTRUCTIONS

- Carry out every instruction in each task.

- Answer **all** questions.

- Use a black or dark blue pen.

- You may use an HB pencil for any diagram, graphs or rough working.

- **Calculator Allowed**.

- Show your workings if relevant.

INFORMATION

- The total marks for this paper is **50 marks**.

- The number of marks for each question or part question is shown in brackets [ ].

1. Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement error, and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is decided by the density function.

$$f(x) = \begin{cases} k(3 - x^2), & -1 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

  (a) Determine k that renders $f(x)$ a valid density function.   [2]

  (b) Find the probability that a random error in measurement is less than $\frac{1}{2}$.   [2]

  (c) For this particular measurement, it is undesirable if the magnitude of the error (i.e., $|x|$) exceeds 0.8. What is the probability that this occurs?   [2]

2. Service calls come to a maintenance center according to a Poisson process, and on average, 2.7 calls are received per minute.

  (a) Find the probability that no more than 4 calls come in any minute.   [3]

  (b) Find the probability that fewer than 2 calls come in any minute.   [3]

  (c) Find the probability that more than 10 calls come in a 5-minute period.   [4]

3. A pub adds whisky to its sticky toffee pudding recipe. The old recipe was chosen by 3 out of every 8 people who had a dessert. After the recipe change, sticky toffee pudding is chosen by 4 people out of the next 20 who have a dessert.

  (a) Test, at the 10% level, the claim that the popularity of the sticky toffee pudding has changed.   [4]

  (b) What is the critical region and actual significance level?   [4]

4. A test statistic has a Poisson distribution with parameter λ. Given that $H_0 : \lambda = 9$ and $H_1 : \lambda \neq 9$

  (a) Find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%.   [4]

  (b) State the probability of incorrectly rejecting H_0 using this critical region.   [2]

5. A survey was conducted to estimate the proportion of people who support a new policy. Out of a sample of 150 people, 90 supported the policy.

  (a) Construct a 95% confidence interval for the proportion of people who support the new policy.   [3]

  (b) Interpret the confidence interval in the context of this problem.   [1]

6. The heights of a certain population are normally distributed with a mean of 170 cm and a standard deviation of 10 cm.

  (a) What is the probability that a randomly selected individual from this population has a height between 160 cm and 180 cm?   [3]

  (b) If a sample of 25 individuals is taken, what is the probability that the sample mean height is greater than 172 cm?   [3]

(c) Find the height that separates the tallest 5% of the population from the rest.   [2]

7. Let X and Y be two independent normally distributed random variables with means $\mu_X = 5$ and $\mu_Y = 10$, and standard deviations $\sigma_X = 2$ and $\sigma_Y = 3$, respectively.

(a) Find the distribution of the linear combination $Z = 3X - 2Y$.   [3]

(b) Calculate the probability that Z is greater than 0.   [2]

8. A sample of size $n = 10$ is taken from a population with unknown mean μ and variance σ^2. The sample data is as follows: 12, 15, 14, 10, 13, 14, 16, 11, 13, 12.

(a) Calculate the sample mean $\bar{X}$.   [1]

(b) Calculate the sample variance S^2.   [2]