

# Permutation and combination

---

## Questions:

- **N out of N:**
  - 5 Seven cars and  $x$  vans can be parked in a line in 39 916 800 ways. Find the number of ways in which five cars and  $x + 2$  vans can be parked in a line.
  - 6 A woman has 10 children. She arranges 11 chairs in a row and sits on the chair in the middle. If her youngest child sits on the adjacent chair to her left, in how many ways can the remaining children be seated?
  - 7 A group of  $n$  boys can be arranged in a line in a certain number of ways. By adding two more boys to the group, the number of possible arrangements increases by a factor of 420. Find the value of  $n$ .
- **N with repetition:**
  - 1 Find the number of distinct arrangements of all the letters in these words:
 

|                      |                       |                    |
|----------------------|-----------------------|--------------------|
| <b>a</b> TABLE       | <b>b</b> TABLET       | <b>c</b> COMMITTEE |
| <b>d</b> MISSISSIPPI | <b>e</b> HULLABALLOO. |                    |
  - 6 There are 420 possible arrangements of all the letters in a particular seven-letter word. Give a description of the letters in this word.
  - 7 Find the number of distinct five-letter arrangements that can be made from:
 

|   |  |
|---|--|
| <b>a</b> two As and three Bs                                      | <b>b</b> two identical vowels and three Bs |
| <b>c</b> two identical vowels and any three identical consonants. |  |
- **N distinct with restriction:**

Find how many ways four men and two women can stand in a line if:

  - a** the two women must be at the front
  - b** there must be a woman at the front and a man at the back
  - c** the two women must be separated
  - d** the four men must not be separated
  - e** no two men may stand next to each other.

Five cows and one set of twin calves can be housed separately in a row of seven stalls in  ${}^7P_7 = 5040$  ways. Find in how many of these arrangements:

  - a** the two calves are not in adjacent stalls
  - b** the two calves and their mother, who is one of the 5 cows, are in adjacent stalls
  - c** each calf is in a stall adjacent to its mother.

Find how many of the six-digit numbers that can be made from 1, 2, 2, 3, 3 and 3:

  - a** begin with a 2
  - b** are not divisible by 2.

Find how many of the arrangements of four letters from A, B, C, D, E and F:

- a** begin with the letter A                      **b** contain the letter A.

From a group of 10 boys and seven girls, two are to be chosen to act as the hero and the villain in the school play. Find in how many ways this can be done if these two roles are to be played by:

- a** any of the children                      **b** two girls or two boys                      **c** a boy and a girl.

Using each digit not more than once, how many even four-digit numbers can be made from the digits 1, 2, 3, 4, 5, 6 and 7?

Find how many three-digit numbers can be made from the digits 0, 1, 2, 3 and 4, used at most once each, if the three-digit number:

- a** must be a multiple of 10                      **b** cannot begin with zero.

• **Combination:**

In a classroom there are four lights, each operated by a switch that has an *on* and an *off* position. How many possible lighting arrangements are there in the classroom?

From six boys and seven girls, find how many ways there are to select a group of three children that consists of more girls than boys.

A bag contains six red fuses, five blue fuses and four yellow fuses. Find how many ways there are to select:

- a** three fuses of different colours                      **b** three fuses of the same colour  
**c** 10 fuses in exactly two colours                      **d** nine fuses in exactly two colours.

## End of chapter note:

- $n! = n(n-1)(n-2)\dots \times 3 \times 2 \times 1$ , for any integer  $n > 0$ .

$$0! = 1$$

- A key word that points to a permutation is *arranged*.

A permutation is a way of selecting and arranging objects in a particular order.

- Key words that point to a combination are *chosen* and *selected*.

A combination is a way of selecting objects in no particular order.

- From  $n$  distinct objects, there are:

${}^n P_n = n!$  permutations of all  $n$  objects.

${}^n P_r = \frac{n!}{(n-r)!}$  permutations of  $r$  objects.

$\frac{n!}{p! \times q! \times r! \times \dots}$  permutations in which there are  $p, q, r, \dots$  of each type.

${}^n C_r = \frac{n!}{r!(n-r)!}$  combinations of  $r$  objects.