
PROBABILITY AND STATISTICS 2

MATHEMATICS

Topic: Probability and Statistics 2

Duration: 1 hour 15 mins

INSTRUCTIONS

- Carry out every instruction in each task.
 - Answer **all** questions.
 - Use a black or dark blue pen.
 - You may use an HB pencil for any diagram, graphs or rough working.
 - **Calculator Allowed.**
 - Show your workings if relevant.
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INFORMATION

- The total marks for this paper is **50 marks**.
- The number of marks for each question or part question is shown in brackets [].

1. Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement error, and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is decided by the density function.

$$f(x) = \begin{cases} k(3 - x^2), & -1 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Determine k that renders $f(x)$ a valid density function. [2]
- (b) Find the probability that a random error in measurement is less than $\frac{1}{2}$. [2]
- (c) For this particular measurement, it is undesirable if the magnitude of the error (i.e., $|x|$) exceeds 0.8. What is the probability that this occurs? [2]
2. Service calls come to a maintenance center according to a Poisson process, and on average, 2.7 calls are received per minute.
- (a) Find the probability that no more than 4 calls come in any minute. [3]
- (b) Find the probability that fewer than 2 calls come in any minute. [3]
- (c) Find the probability that more than 10 calls come in a 5-minute period. [4]
3. A pub adds whisky to its sticky toffee pudding recipe. The old recipe was chosen by 3 out of every 8 people who had a dessert. After the recipe change, sticky toffee pudding is chosen by 4 people out of the next 20 who have a dessert.
- (a) Test, at the 10% level, the claim that the popularity of the sticky toffee pudding has changed. [4]
- (b) What is the critical region and actual significance level? [4]
4. A test statistic has a Poisson distribution with parameter λ . Given that $H_0 : \lambda = 9$ and $H_1 : \lambda \neq 9$
- (a) Find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%. [4]
- (b) State the probability of incorrectly rejecting H_0 using this critical region. [2]
5. A survey was conducted to estimate the proportion of people who support a new policy. Out of a sample of 150 people, 90 supported the policy.
- (a) Construct a 95% confidence interval for the proportion of people who support the new policy. [3]
- (b) Interpret the confidence interval in the context of this problem. [1]
6. The heights of a certain population are normally distributed with a mean of 170 cm and a standard deviation of 10 cm.
- (a) What is the probability that a randomly selected individual from this population has a height between 160 cm and 180 cm? [3]
- (b) If a sample of 25 individuals is taken, what is the probability that the sample mean height is greater than 172 cm? [3]

- (c) Find the height that separates the tallest 5% of the population from the rest. [2]
7. Let X and Y be two independent normally distributed random variables with means $\mu_X = 5$ and $\mu_Y = 10$, and standard deviations $\sigma_X = 2$ and $\sigma_Y = 3$, respectively.
- (a) Find the distribution of the linear combination $Z = 3X - 2Y$. [3]
- (b) Calculate the probability that Z is greater than 0. [2]
8. A sample of size $n = 10$ is taken from a population with unknown mean μ and variance σ^2 . The sample data is as follows: 12, 15, 14, 10, 13, 14, 16, 11, 13, 12.
- (a) Estimate the Population mean \bar{X} . [1]
- (b) Estimate the Population variance S^2 . [2]