习题 7.7(P85)

1．求函数 $f(x,y)=2x^2-xy-y^2-6x-3y+5$ 在点 $(1,1)$ 的泰勒公式.

解： $f(1,1)=-4$， $f'_x(x,y)=4x-y-6$， $f'_x(1,1)=-3$， $f'_y(x,y)=-x-2y-3$，

$f'_y(1,1)=-6$， $f''_{xx}(x,y)=4$， $f''_{xy}(x,y)=f''_{yx}(x,y)=-1$， $f''_{yy}(x,y)=-2$，三阶偏导数均为 0.

故 $f(x,y)=f(1,1)+f'_x(1,1)(x-1)+f'_y(1,1)(y-1)+$

$$\frac{1}{2!}[f''_{xx}(1,1)(x-1)^2+2f''_{xy}(1,1)(x-1)(y-1)+f''_{yy}(1,1)(y-1)^2]$$

$$=-4-3(x-1)-6(y-1)+2(x-1)^2-(x-1)(y-1)-(y-1)^2$$

2．将 $f(x,y)=\sin(x^2+y^2)$ 展成二阶麦克劳林公式（皮亚诺余项）.

解： $f'_x(x,y)=2x\cos(x^2+y^2)$， $f''_{xx}(x,y)=2\cos(x^2+y^2)-4x^2\sin(x^2+y^2)$，

$f''_{xy}(x,y)=-4xy\sin(x^2+y^2)$，故 $f(0,0)=0$，

由自变量的对称性得：$f'_x(0,0)=f'_y(0,0)=0$，$f''_{xx}(0,0)=f''_{yy}(0,0)=2$，$f''_{xy}(0,0)=0$

$$f(x,y)=f(0,0)+f'_x(0,0)x+f'_y(0,0)y+$$

$$\frac{1}{2!}[f''_{xx}(0,0)x^2+2f''_{xy}(0,0)xy+f''_{yy}(0,0)y^2]+o(\rho^2)$$

$$=x^2+y^2+o(\rho^2)$$

3．将 $f(x,y)=e^{x+y}$ 展成二阶麦克劳林公式（拉格朗日余项）.

解： $f'_x(x,y)=e^{x+y}$， $f''_{xx}(x,y)=e^{x+y}$， $f''_{xy}(x,y)=e^{x+y}$

$f'''_{xxx}(x,y)=f'''_{xxy}(x,y)=f'''_{xyy}(x,y)=f'''_{yyy}(x,y)=e^{x+y}$，故 $f(0,0)=1$，

由自变量的对称性得：$f'_x(0,0)=f'_y(0,0)=1$，$f''_{xx}(0,0)=f''_{yy}(0,0)=1$，$f''_{xy}(0,0)=1$

$$f'''_{xxx}(\theta x,\theta y)=f'''_{xxy}(\theta x,\theta y)=f'''_{xyy}(\theta x,\theta y)=f'''_{yyy}(\theta x,\theta y)=e^{\theta(x+y)}$$

$$f(x,y)=f(0,0)+f'_x(0,0)x+f'_y(0,0)y+$$

$$\frac{1}{2!}[f''_{xx}(0,0)x^2+2f''_{xy}(0,0)xy+f''_{yy}(0,0)y^2]+$$

$$\frac{1}{3!}[f'''_{xxx}(\theta x,\theta y)x^3+3f'''_{xxy}(\theta x,\theta y)x^2y+f'''_{xyy}(\theta x,\theta y)xy^2+f'''_{yyy}(\theta x,\theta y)y^3]$$

$$f(x,y)=1+x+y+\frac{1}{2!}(x+y)^2+\frac{1}{3!}e^{\theta(x+y)}(x+y)^3 \quad (0<\theta<1)$$