

1 Define Set: A set is an unordered collection of objects known as elements or members of the set.

2 Explain the roster notation and set builder notation of set with example.

Roster notation: List element of a set inside braces $\{ \}$, separated by commas.

Ex. $A = \{ 1, 2, 3, 4, 5 \}$.

Set builder notation: A set builder notation describes or defines the elements of a set instead of listing the elements.

Ex. $A = \{ x / x \text{ is counting natural numbers less than } 5 \}$

3 Define null set and singleton set.

Null set: A set which contains no element at all is called Null set / Empty set / Void set.

Singleton set: A set which has only one element is called a singleton set.

4 What is Cardinality of a set?

If there are exactly n distinct elements in set A where n is a non-negative integer, we can say that A is finite set and that n is the cardinality of set A .

Size of the set A is known as cardinality number, denoted as $|A|$.

5. When are two sets said to be equal?
Two sets are equal when they have exactly the same elements, and sets are equivalent.
Two sets A and B are said to be equal if every element of A is an element of B.

6. What is a Powerset?
Powerset is the set of all possible subsets of the given set, denoted as $P(S)$.

7. Define the Cartesian product of two sets and give example.

The Cartesian product of two sets A and B is denoted as $A \times B$, is the set of all possible ordered pairs where the elements of A are first and the elements of B are second.

Example: $A = \{1, 2\}$ $B = \{p, q, r\}$

$$A \times B = \{(1, p), (1, q), (1, r), (2, p), (2, q), (2, r)\}$$

8. Define the Complement and Relative Complement of a set. Give example.

denoted by A^c and A^c is the set of all the elements except A.
 A^c is $U - A$

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8. Define the Complement and Relative Complement of a set. Give example.

denoted by A^c and A' is the set of all the elements except A.
 A' is $U - A$

$$\text{Ex} \Rightarrow U = \{1, 3, 4, 6, 7, 9, 10\} \quad A = \{4, 6, 7\}$$
$$A' \text{ or } A^c = \{1, 3, 9, 10\}$$

Relative Complement: The relative complement of A with respects to a set B , also termed the set difference of B and A , written as $B \setminus A$.

Relative Complement ($B \setminus A$) is the set of element in B but not in A .

Ex.

9 Define union and intersection of two set. give Venn diagram representation.

union: The union of two set is a new set that contains all of the elements that are in at least one of the two sets.

intersection: The intersection of two set is a new set that contains all of the elements that are in both sets (common elements).

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A-10 Define the symmetric difference of two sets.
 Difference between set B denoted as $A \Delta B$.
 $A \Delta B$ is the set containing elements of A
 except the element of B.

$$\text{Ex. } A = \{1, 3, 5, 6\}$$

$$B = \{0, 5, 6, 7\} \text{ find } A \Delta B$$

$$\begin{aligned} A \setminus B &= \{1, 3, 5, 6\} \setminus \{0, 5, 6, 7\} \\ &= \{1, 3\} \end{aligned}$$

$$\begin{aligned} B \setminus A &= \{0, 5, 6, 7\} \setminus \{1, 3, 5, 6\} \\ &= \{0, 7\} \end{aligned}$$

$$\begin{aligned} \text{Formula} = A \Delta B &= (A \setminus B) \cup (B \setminus A) \\ &= \{1, 3\} \cup \{0, 7\} \end{aligned}$$

$$\therefore A \Delta B = \{1, 3, 0, 7\}$$

A-11 Given that $D = \{1, 2, \dots, 10\}$, $A = \{1, 2, 3, 4, 5\}$,
 $B = \{1, 2, 4, 8\}$, $C = \{1, 2, 3, 5, 7\}$, $D = \{2, 4, 6, 8\}$
 find each of the following.

$$\begin{aligned} * (A \cup B) \cap C &= (\{1, 2, 3, 4, 5\} \cup \{1, 2, 4, 8\}) \cap \{1, 2, 3, 5, 7\} \\ &= \{1, 2, 3, 4, 5, 8\} \cap \{1, 2, 3, 5, 7\} \\ &= \{1, 2, 3, 5\} \end{aligned}$$

$$\begin{aligned} * (A \cup (B \cap C)) &= \{1, 2, 3, 4, 5\} \cup (\{1, 2, 4, 8\} \cap \{1, 2, 3, 5, 7\}) \\ &= \{1, 2, 3, 4, 5\} \cup \{1, 2\} \\ &= \{1, 2, 3, 4, 5\} \end{aligned}$$

$$\begin{aligned} * C' \cup D' &= \{4, 6, 8, 9, 10\} \cup \{1, 3, 5, 7, 9, 10\} \\ &= \{1, 3, 4, 5, 6, 7, 8, 9, 10\} \end{aligned}$$

$$\begin{aligned}
 * (C \cup D)' &= (C \cup D)^c \\
 &= \{1, 2, 3, 4, 5, 6, 7, 8\}' \\
 &= \{9, 10\}
 \end{aligned}$$

$$\begin{aligned}
 * (A \cup B) - C &= \{1, 2, 3, 4, 5, 8\} - \{1, 2, 3, 5, 7\} \\
 &= \{4, 8\}
 \end{aligned}$$

$$\begin{aligned}
 * A \cup (B - C) &= \{1, 2, 3, 4, 5\} \cup \{4, 8\} \\
 &= \{1, 2, 3, 4, 5, 8\}
 \end{aligned}$$

$$\begin{aligned}
 * (B - C) - D &= \{4, 8\} - \{2, 4, 6, 8\} \\
 &= \{ \} \\
 &= \emptyset
 \end{aligned}$$

$$\begin{aligned}
 * B - (C - D) &= \{1, 2, 4, 8\} - \{1, 3, 5, 7\} \\
 &= \{2, 4, 8\}
 \end{aligned}$$

$$\begin{aligned}
 * (A \cup B) - (C \cap D) &= \{1, 2, 3, 4, 5, 8\} - \\
 (C \cap D) &= \{2\} \\
 (A \cup B) - (C \cap D) &= \{1, 2, 3, 4, 5, 8\} - \{2\} \\
 &= \{1, 3, 4, 5, 8\}
 \end{aligned}$$

12. which of these sets is finite?

- a. $\{x | x \text{ is even}\}$ infinite
 b. $\{x | x < 5\}$ infinite
 c. $\{1, 2, 3, \dots\}$ infinite
 d. $\{1, 2, 3, \dots, 999, 1000\}$ finite

Hence, $\{1, 2, 3, \dots, 999, 1000\}$ {d} is a finite set.

13. which of these set is not null set.

$$A = \{x | 6x = 24 \text{ and } 3x = 1\}$$

$$B = \{x | x + 1 = 10\}$$

$$C = \{x | x \text{ is a man older than 200 years}\}$$

$$D = \{x | x < x\}$$

$$\Rightarrow A = \{x | 6x = 24 \text{ and } 3x = 1\}$$

$$6x = 24$$

$$x = \frac{24}{6}$$

$$x = \frac{1}{3}$$

$$\therefore A = \{4, 1/3\}$$

$$\Rightarrow B = \{x | x + 1 = 10\}$$

$$\therefore B = \{9\}$$

$$\Rightarrow C = \{x | x \text{ is a man older than 200 years}\}$$

possible that there is some person have age 200.

so it is not null set.

$$\Rightarrow D = \{x | x < x\}$$

x is not less than itself.

So, here A , B and C are not a null set / empty set / void set.

14. let consider $U = \{1, 2, 3, \dots, 8, 9\}$, $A = \{1, 3, 5, 7\}$ then find A' , $(A')'$, $A' \cap ((A') \cup A')$.

$$A' = U - A = \{2, 4, 6, 8\}$$

$$(A')' = A = \{1, 3, 5, 7\}$$

$$\begin{aligned} A' \cap ((A') \cup A') &= A' \cap (A' \cup A') \\ &= A' \cap A' \\ &= A' \\ &= \{2, 4, 6, 8\} \end{aligned}$$

15. if $U = \{1, 2, \dots, 15\}$, $A = \{1, 2, 3, \dots, 8, 9\}$ and $B = \{3, 5, 7, 9\}$ then find $A \cup B$, $A \cap B$, A' , B' , $(A \cup B)'$, $(A \cap B)'$, $A' \cup B'$, $A' \cap B'$, $A - B$, $A' - B$, $A \Delta B$, $A \cup B \cap A'$, $(A \Delta B)'$

$$A \cup B = A = \{1, 2, 3, 4, \dots, 9\}$$

$$A \cap B = B = \{3, 5, 7, 9\}$$

$$A' = \{10, 11, 12, 13, 14, 15\}$$

$$B' = \{1, 2, 4, 6, 8, 10, \dots, 15\}$$

$$(A \cup B)' = (A)' = \{10, 11, 12, 13, 14, 15\}$$

($\because A \cup B = A$)

$$(A \cap B)' = (B)' = \{1, 2, 4, 6, \dots, 15\}$$

($\because A \cap B = B$)

$$A - B = \{1, 2, 4, 6, 8\}$$

$$A' - B = A' = \{10, 11, 12, 13, 14, 15\}$$

$$\begin{aligned} A \Delta B &= (A \cup B) - (A \cap B) \\ &= A - B \\ &= \{1, 2, 4, 6, 8\} \end{aligned}$$

$$\begin{aligned} A \cup B \cap A' &= A \cap A' \\ &= \phi \end{aligned}$$

$$(A \Delta B)' = \{3, 5, 7, 9, 10, 11, \dots, 15\}$$

16 Verify with example.

$$U = \{1, 2, 3, \dots, 10\}$$

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{1, 2, 4, 8\}$$

$$C = \{1, 2, 3, 5, 7, 9\}$$

$$D = \{2, 4, 6, 8\}$$

$$* (A \cup B)' = A' \cap B' \quad (\text{De Morgan's law})$$

$$\begin{aligned} \text{L.H.S.} &= (\{1, 2, 3, 4, 5\} \cup \{1, 2, 4, 8\})' \\ &= \{1, 2, 3, 4, 5, 8\}' \\ &= \{6, 7, 9, 10\} \end{aligned}$$

$$\begin{aligned} \text{R.H.S.} &= \{6, 7, 8, 9, 10\} \cap \{3, 5, 6, 7, 9, 10\} \\ &= \{6, 7, 9, 10\} \end{aligned}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$* (A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

$$L.H.S = (A \cup B) \cap C$$

$$= (\{1, 2, 3, 4, 5\} \cup \{1, 2, 4, 8\}) \cap \{1, 2, 3, 5, 7\}$$

$$= (\{1, 2, 3, 4, 5, 8\}) \cap \{1, 2, 3, 5, 7\}$$

$$= \underline{\underline{\{1, 2, 3, 5\}}}$$

$$R.H.S = (A \cap C) \cup (B \cap C)$$

$$= (\{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 5, 7\}) \cup (\{1, 2, 4, 8\} \cap \{1, 2, 3, 5, 7\})$$

$$= \{1, 2, 3, 5\} \cup \{1, 2\}$$

$$= \underline{\underline{\{1, 2, 3, 5\}}}$$

$$\therefore L.H.S = R.H.S$$

$$* (A')' = A$$

$$L.H.S = (A')'$$

$$= (\{6, 7, 8, 9, 10\})'$$

$$= \{1, 2, 3, 4, 5\}$$

$$= A$$

$$* A \cup \phi = A$$

$$= \{1, 2, 3, 4, 5\} \cup \{\}$$

$$= \{1, 2, 3, 4, 5\}$$

$$* A \cap \phi = \phi$$

$$= \{1, 2, 3, 4, 5\} \cap \{\}$$

$$= \phi$$

$$* A \cup U = U \quad \text{and} \quad A \cap U = A$$

$$= \{1, 2, 3, 4, 5\} \cup \{1, 2, \dots, 10\}$$

$$= \{1, 2, 3, 4, \dots, 10\}$$

$$A \cap U = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, \dots, 10\} \\ = \{1, 2, 3, 4, 5\}$$

$$* A \cup A' = U$$

$$\begin{aligned} \text{L.H.S} &= A \cup A' \\ &= \{1, 2, 3, 4, 5\} \cup \{6, 7, 8, 9, 10\} \\ &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \\ &= U \\ &= \text{R.H.S} \end{aligned}$$

$$* A \cap A' = \phi$$

$$\begin{aligned} \text{L.H.S} &= A \cap A' \\ &= \{1, 2, 3, 4, 5\} \cap \{6, 7, 8, 9, 10\} \\ &= \{\} \\ &= \phi \\ &= \text{R.H.S} \end{aligned}$$

$$* (A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

$$\begin{aligned} \text{L.H.S} &= (A \cap B) \cup C \\ &= (\{1, 2, 3, 4, 5\} \cap \{1, 2, 4, 8\}) \cup \{1, 2, 3, 5, 7\} \\ &= \{1, 2, 4\} \cup \{1, 2, 3, 5, 7\} \\ &= \{1, 2, 3, 4, 5, 7\} \end{aligned}$$

$$\text{R.H.S} = (A \cup C) \cap (B \cup C)$$

$$\begin{aligned} &= \{1, 2, 3, 4, 5\} \cup \{1, 2, 3, 5, 7\} \cap \\ &\quad \{1, 2, 4, 8\} \cup \{1, 2, 3, 5, 7\} \\ &= \{1, 2, 3, 4, 5, 7\} \cap \{1, 2, 3, 4, 5, 7, 8\} \\ &= \{1, 2, 3, 4, 5, 7\} \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$* A \cup U' = U$$

$$= A \cup U'$$

$$= \{1, 2, 3, 4, 5\} \cup \{1, 2, 3, \dots, 10\}'$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$* A \cap \phi' = \phi$$

$$= A \cap \phi'$$

$$= A \cap U \quad (\because \phi' = U)$$

$$= \phi$$