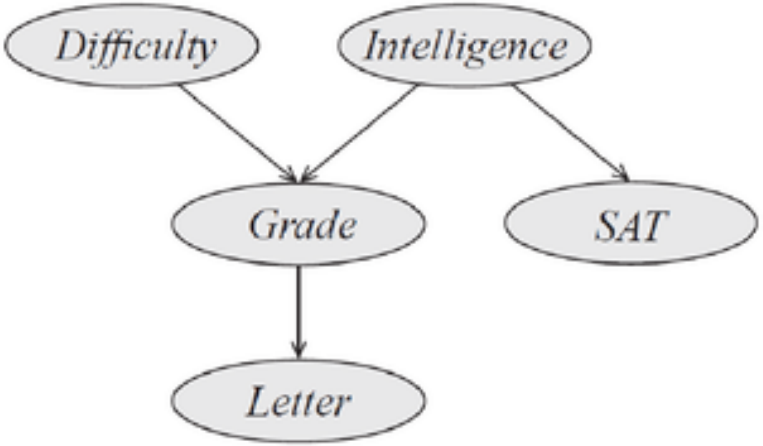
Probabilistic Graphical Models

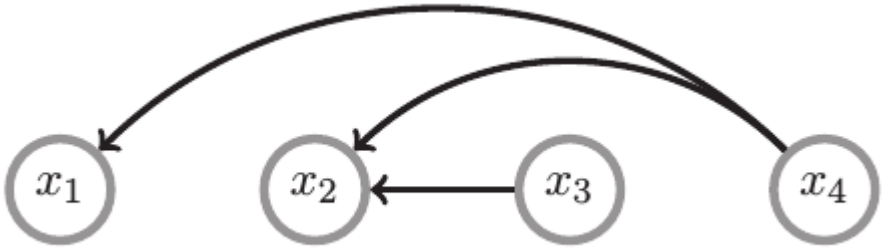
- **Graph based models which express the conditional dependence structure between random variables.**
- **Each node represents a random variable, edges represent dependence**

- **Directed graphs – Bayes nets**
- **Undirected graphs – Markov Fields**



Definition

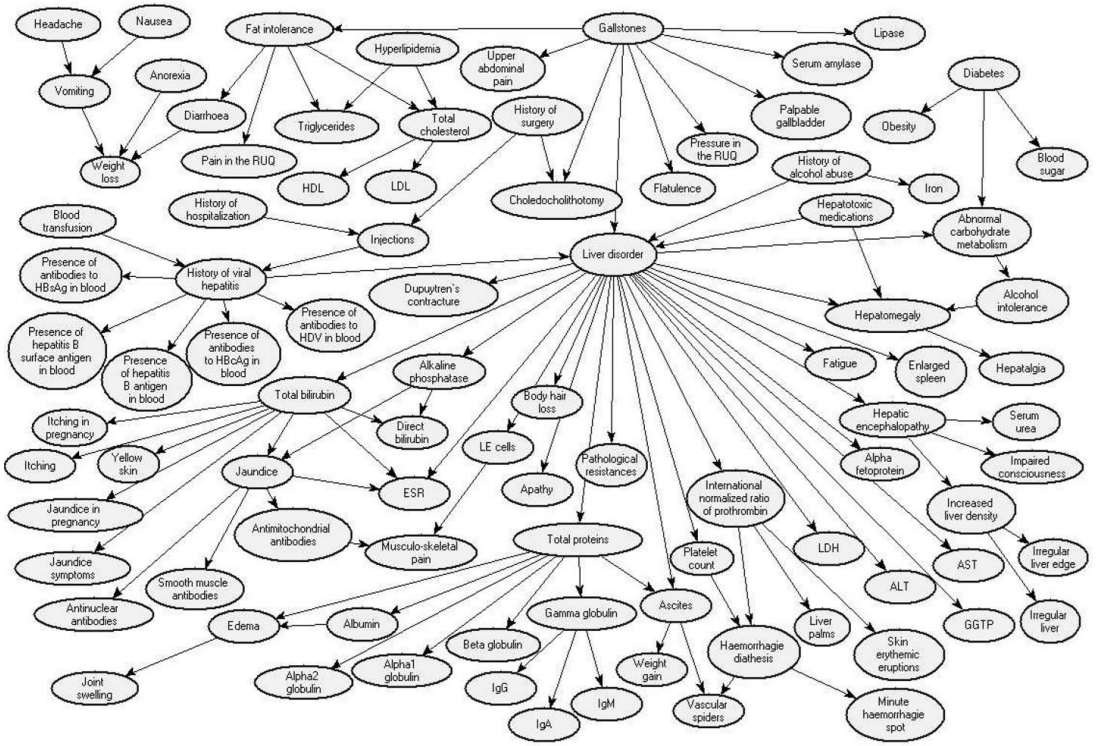
- **A Bayesian network is a Directed Acyclic Graph or DAG. A DAG is a graph with directed links and has no cycles.**
- **A link between two nodes indicates that the nodes directly influence each other.**
- **For each node xi, we have a corresponding conditional probablity distribution P(xi | parents(xi))**
- **Given a bayes-net with the following structure, find the joint probablity distribution p(x1, x2, x3, x4)**



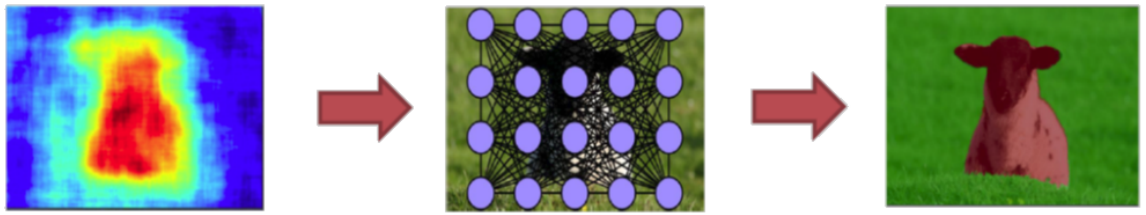
- **Ans** $p(x_1, x_2, x_3, x_4) = p(x_1|x_4)p(x_2|x_3, x_4)p(x_3)p(x_4).$

Applications of PGMs

- **Bayesian nets have been used in medical diagnosis and for statistical learning**



- **Markov Networks are used in Image Segmentation and physics for simulation**



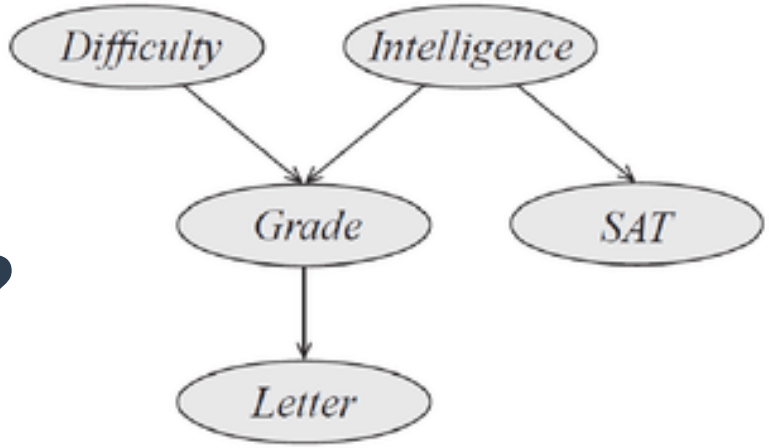
Coarse output from the pixel-wise classifier

MRF/CRF modelling

Output after the CRF inference

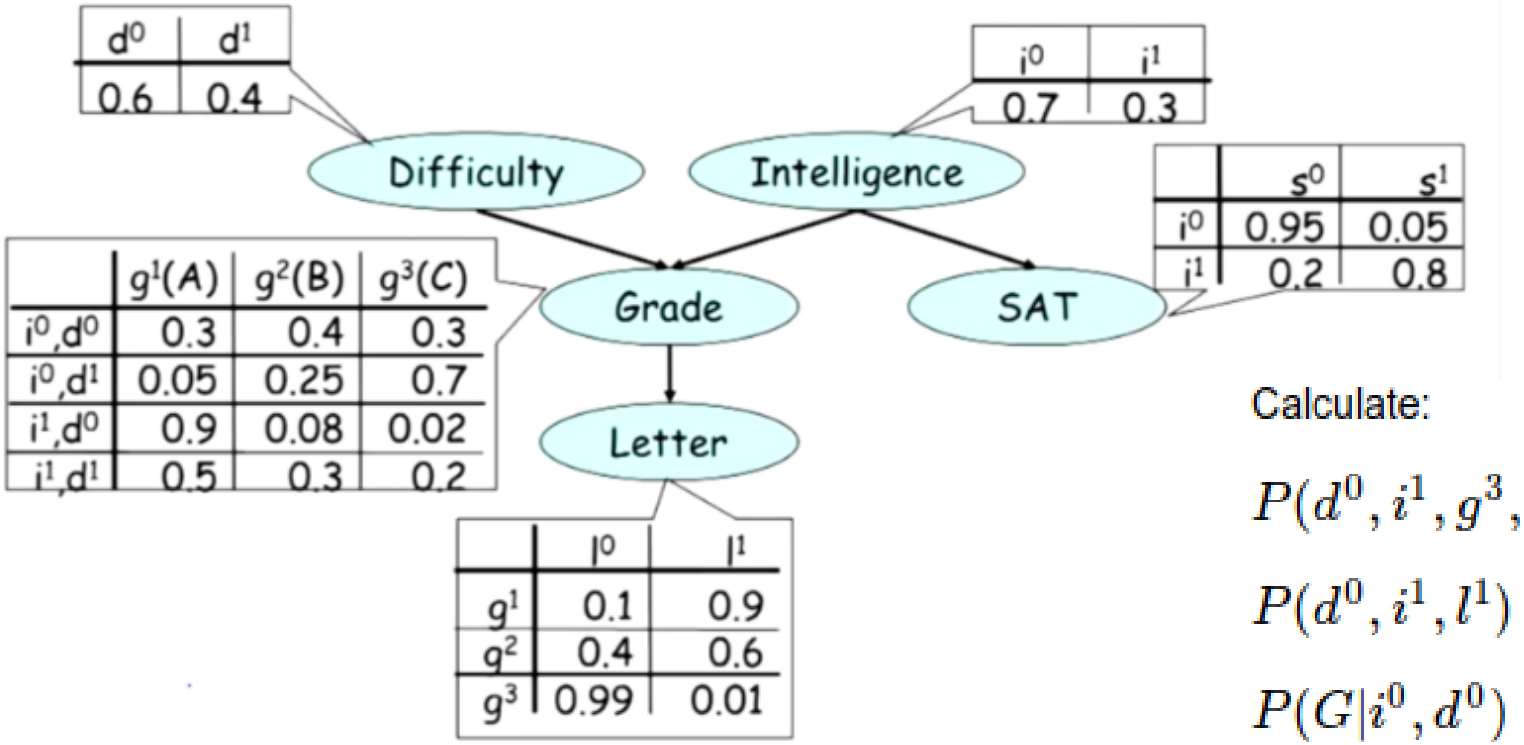
Flow of influence in bayes nets

- Let us go back to the student example
- In a bayes net, when can a random variable (r.v.) influence another r.v.?
- Are Difficulty and Intelligence always independent? What if we know the grade?



Questions

- **Given a bayesian network with the following CPDs**



d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

	$g^1(A)$	$g^2(B)$	$g^3(C)$
i^0,d^0	0.3	0.4	0.3
i^0,d^1	0.05	0.25	0.7
i^1,d^0	0.9	0.08	0.02
i^1,d^1	0.5	0.3	0.2

	l^0	l^1
g^1	0.1	0.9
g^2	0.4	0.6
g^3	0.99	0.01

Calculate:

$P(d^0, i^1, g^3, s^1, l^1)$

$P(d^0, i^1, l^1)$

$P(G|i^0, d^0)$

$P(G|i^0)$