Description of the Birthday Problem

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Description of the problem

What is the probability that at least two people this room share the same birthday?

There are 16 in this room.

Is it something like $\frac{16}{365} = 0.0438356$

The mathematical solution

Define p(n) as the probability that at least one pair has the same birthday, then the 1-p(n) is the probability that all are born in a different day. Which we can compute as:

$$
\begin{aligned}
1 - p(n) &= 1 \times \left(1 - \frac{1}{365}\right) \times \left(1 - \frac{2}{365}\right) \times \cdots \times \left(1 - \frac{n-1}{365}\right) \\
&= \frac{365 \times 364 \times \cdots \times (365 - n + 1)}{365^n} \\
&= \frac{365!}{365^n (365-n)!} = \frac{n! \cdot \binom{365}{n}}{365^n} \qquad (1) \\
p(n = 16) &= 0.284
\end{aligned}
$$

The simulated solution

We will simulate the probability:

1 - Simulate 10^{4} rooms with 16 random birthdays, and store the results in matrix where each row represents a room.

2 - For each room (row) compute the number of unique birthdays.

3 - Compute the average number of times a room has 16 unique birthdays, across 10^{4} simulations, and report the complement.

```r
birthday.prob = function(n.pers_var, n.sims_var) {
  # simulate birthdays
  birthdays = matrix(round(runif(n = n.pers_var * n.sims_var, min = 1, max = 365) ),
                     nrow = n.sims_var, ncol = n.pers_var)
  # for each room (row) get unique birthdays
  unique.birthdays = apply(birthdays, 1,
                           function(x)  length( unique(x) ) )
  # Indicator with 1 if all are unique birthdays
  all.different = 1 * (unique.birthdays==n.pers_var)
```

```r
  # Compute average time all have different birthdays
  result = 1 - mean(all.different)
return(result)
}
bp_sim = birthday.prob(n.pers_var = 21, n.sims_var = 10000)
print(bp_sim)
```

```
## [1] 0.4531
```

Results

- Many people think the solution is $\frac{1}{365} \times n = 0.0438356$
- The math says: 0.284
- A simulation with 10^4 rooms with 16 people in each room, says: 0.4531