

# Description of the Birthday Problem

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9/3/2021

## Description of the problem

What is the probability that at least two people in this room share the same birthday?

There are 30 in this room.

Is it something like  $\frac{30}{365} = 0.0821918$

## The mathematical solution

Define  $p(n)$  as the probability that at least one pair has the same birthday, then the  $1-p(n)$  is the probability that all are born in a different day. Which we can compute as:

$$\begin{aligned} 1 - p(n) &= 1 \times \left(1 - \frac{1}{365}\right) \times \left(1 - \frac{2}{365}\right) \times \cdots \times \left(1 - \frac{n-1}{365}\right) \\ &= \frac{365 \times 364 \times \cdots \times (365 - n + 1)}{365^n} \\ &= \frac{365!}{365^n (365 - n)!} = \frac{n! \cdot \binom{365}{n}}{365^n} \end{aligned} \tag{1}$$
$$p(n = 30) = 0.706$$

## The simulated solution

We will simulate the probability:

1 - Simulate  $10^4$  rooms with 30 random birthdays, and store the results in matrix where each row represents a room.

2 - For each room (row) compute the number of unique birthdays.

3 - Compute the average number of times a room has 30 unique birthdays, across  $10^4$  simulations, and report the complement.

```
birthday.prob = function(n.pers_var, n.sims_var) {  
  # simulate birthdays  
  birthdays = matrix(round(runif(n = n.pers_var * n.sims_var, min = 1, max = 365) ),  
                      nrow = n.sims_var, ncol = n.pers_var)  
  # for each room (row) get unique birthdays  
  unique.birthdays = apply(birthdays, 1,  
                           function(x) length( unique(x) ) )  
  # Indicator with 1 if all are unique birthdays  
  all.different = 1 * (unique.birthdays==n.pers_var)
```

```
# Compute average time all have different birthdays
result = 1 - mean(all.different)
return(result)
}
bp_sim = birthday.prob(n.pers_var = n.pers, n.sims_var = 10000)
print(bp_sim)

## [1] 0.7118
```

## Results

- Many people think the solution is  $\frac{1}{365} \times n = 0.0821918$
- The math says: 0.706
- A simulation with  $10^4$  rooms with 30 people in each room, says: 0.7118