Description of the Birthday Problem

Fernando Hoces de la Guardia

5/9/2018

```
n.pers =  14
set.seed(1234)
```

Description

This is the B Day problem. There are 14 in this room. The 'intuitive' prob is:
$\frac{1}{365} \times 14 = 0.0383562$

Analytical solution

But actually when we compute the math. We get an surprising result:

$$\begin{aligned}
1 - \bar{p}(n) &= 1 \times \left(1 - \frac{1}{365}\right) \times \left(1 - \frac{2}{365}\right) \times \cdots \times \left(1 - \frac{n-1}{365}\right) \\
&= \frac{365 \times 364 \times \cdots \times (365 - n + 1)}{365^n} \\
&= \frac{365!}{365^n (365 - n)!} = \frac{n! \cdot \binom{365}{n}}{365^n} \\
p(n = 14) &= 0.223
\end{aligned} \tag{1}$$

Simulation

1 - Simulate 10,000 rooms with n = 14 random birthdays, and store the results in matrix where each row represents a room.
2 - For each room (row) compute the number of unique birthdays.
3 - Compute the average number of times a room has 14 unique birthdays, across 10,000 simulations, and report the complement.

```
birthday.prob = function(n.pers, n.sims) {
  # simulate birthdays
  birthdays = matrix(round(runif(n.pers * n.sims, 1,
                                 365)), nrow = n.sims,
                          ncol = n.pers)
  # for each room (row) get unique birthdays
  unique.birthdays = apply(birthdays, 1, unique)
  # Indicator with 1 if all are unique birthdays
  all.different = (lapply(unique.birthdays, length) == n.pers) # Compute average time all have differen
  result = 1 - mean(all.different)
return(result)
}
n.pers.param = n.pers
n.sims.param = 1e4
res1 = birthday.prob(n.pers.param,n.sims.param)
```

The simulated probability is 0.229