

# Explaining the Birthday Problem

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12/5/2017

## Birthday problem

What is the probability that at least two people share the same birthday in this room?

Is is  $\frac{1}{365} \times 4 = 0.0109589$

## Analytical solution

But actually when we compute the math. We get an surprising result:

$$\begin{aligned} 1 - \bar{p}(n) &= 1 \times \left(1 - \frac{1}{365}\right) \times \left(1 - \frac{2}{365}\right) \times \cdots \times \left(1 - \frac{n-1}{365}\right) \\ &= \frac{365 \times 364 \times \cdots \times (365 - n + 1)}{365^n} \\ &= \frac{365!}{365^n (365 - n)!} = \frac{n! \cdot \binom{365}{n}}{365^n} \\ p(n = 4) &= 0.016 \end{aligned} \tag{1}$$

## Simulations

- 1 - Simulate 10,000 rooms with  $n = 4$  random birthdays, and store the results in matrix where each row represents a room.
- 2 - For each room (row) compute the number of unique birthdays.
- 3 - Compute the average number of times a room has 4 unique birthdays, across 10,000 simulations, and report the complement.

```
birthday.prob = function(n.pers, n.sims) {  
  # simulate birthdays  
  birthdays = matrix(round(runif(n.pers * n.sims, 1,  
                                365)), nrow = n.sims,  
                      ncol = n.pers)  
  # for each room (row) get unique birthdays  
  unique.birthdays = apply(birthdays, 1, unique)  
  # Indicator with 1 if all are unique birthdays  
  all.different = (lapply(unique.birthdays, length) == n.pers) # Compute average time all have differen  
  result = 1 - mean(all.different)  
  return(result)  
}  
n.pers.param = n.pers  
n.sims.param = 1e4  
birthday.prob(n.pers.param, n.sims.param)
```

```
## [1] 0.0172
```

Ok, I am convinced that the probability of at least two people in this room share the same birthday is 0.0185