Description of the Birthday Problem

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# Description of the problem

What is the probability that at least two people this room share the same birthday?

There are 16 in this room.

Is it something like

# The mathematical solution

Define p(n) as the probability that at least one pair has the same birthday, then the 1-p(n) is the probability that all are born in a different day. Which we can compute as:

# The simulated solution

We will simulate the probability:

1 - Simulate 10^{4} rooms with 16 random birthdays, and store the results in matrix where each row represents a room.

2 - For each room (row) compute the number of unique birthdays.

3 - Compute the average number of times a room has 16 unique birthdays, across 10^{4} simulations, and report the complement.

birthday.prob = function(n.pers\_var, n.sims\_var) {  
 # simulate birthdays  
 birthdays = matrix(round(runif(n = n.pers\_var \* n.sims\_var, min = 1, max = 365) ),  
 nrow = n.sims\_var, ncol = n.pers\_var)  
 # for each room (row) get unique birthdays  
 unique.birthdays = apply(birthdays, 1,  
 function(x) length( unique(x) ) )  
 # Indicator with 1 if all are unique birthdays  
 all.different = 1 \* (unique.birthdays==n.pers\_var)  
 # Compute average time all have different birthdays   
 result = 1 - mean(all.different)  
return(result)  
}  
bp\_sim = birthday.prob(n.pers\_var = 21, n.sims\_var = 10000)  
print(bp\_sim)

## [1] 0.4531

# Results

* Many people think the solution is
* The math says: 0.284
* A simulation with 10^{4} rooms with 16 people in each room, says: 0.4531