

ACM 模板 *for* 北湖咸鱼

三条咸鱼——zhber、strawberry、miamiao

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一、头文件

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  #define fi first
4  #define se second
5  #define pb push_back
6  #define mp make_pair
7  #define SZ(x) ((int)(x).size())
8  #define All(x) (x).begin(),(x).end()
9  #define rep(i,a,n) for (int i=a;i<n;i++)
10 #define per(i,a,n) for (int i=n-1;i>=a;i--)
11 #define Close() ios::sync_with_stdio(0),cin.tie(0)
12 typedef vector<int> VI;
13 typedef pair<int, int> PII;
14 typedef long long ll;
15 typedef long double ld;
16 typedef unsigned long long ull;
17 const int maxn = 1e5 + 10;
18 const int maxm = 3e3 + 5;
19 const int mod = 1e9 + 7;

```

注：①部分模板可能会重新定义新类型；②不同模板的 maxn、maxm 大小取决于题目要求

二、算法

1.快速读入

①普通快速读

```

1  inline ll read() {
2      ll x=0,f=1;char ch=getchar();
3      while(ch<'0' || ch>'9'){if(ch=='-')f=-1;ch=getchar();}
4      while(ch>='0' && ch<='9'){x=x*10+ch-'0';ch=getchar();}
5      return x*f;
6  }

```

②文件读入流

```

1  namespace fastIO {
2  #define BUF_SIZE 100000
3      //fread -> read
4      bool IOerror = 0;
5      inline char nc() {
6

```

```

7         static char buf[BUF_SIZE], *p1 = buf + BUF_SIZE, *pend = buf
8 + BUF_SIZE;
9         if(p1 == pend) {
10             p1 = buf;
11             pend = buf + fread(buf, 1, BUF_SIZE, stdin);
12             if(pend == p1) {
13                 IOError = 1;
14                 return -1;
15             }
16         } return *p1++;
17     }
18     inline bool blank(char ch) {
19         return ch == ' ' || ch == '\n' || ch == '\r' || ch == '\t';
20     }
21     inline void read(ll &x) {
22         char ch;
23         ll f = 1;
24         while(blank(ch = nc()));
25         while (ch == '-')f = -f, ch = nc();
26         if(IOerror)return;
27         for(x = ch - '0'; (ch = nc()) >= '0' && ch <= '9'; x = x * 10
28 + ch - '0');
29         x *= f;
30     }
31     inline void read(int &x) {
32         char ch;
33         int f = 1;
34         while(blank(ch = nc()));
35         while (ch == '-')f = -f, ch = nc();
36         if(IOerror)return;
37         for(x = ch - '0'; (ch = nc()) >= '0' && ch <= '9'; x = x * 10
38 + ch - '0');
39         x *= f;
40     }
41     inline void read(char *s) {
42         char ch;
43         while(blank(ch = nc()));
44         if(IOerror)return;
45         int i = 0;
46         for (; !blank(ch); s[i++] = ch, ch = nc());
47         s[i] = '\0';
48     }
49 #undef BUF_SIZE
50 };
    using namespace fastIO;

```

2.LCA 问题

①RMQ 在线求 LCA , 可以配合 ST

```

1 void RMQ() {
2     for (int j = 1; (1 << j) <= n; j++)
3         for (int i = 1; i <= n; i++)
4             if (~pa[i][j - 1])
5                 pa[i][j] = pa[pa[i][j - 1]][j - 1];
6 }
7 int LCA(int x, int y) {
8     if (deep[x] < deep[y]) swap(x, y);
9     int i, j;
10    for (i = 0; (1 << i) <= deep[x]; i++); i--;
11    for (j = i; j >= 0; j--) // 把深度统一
12        if (deep[x] - (1 << j) >= deep[y])
13            x = pa[x][j];
14    if (x == y) return x;
15    for (j = i; j >= 0; j--) // 去找 LCA
16        if (pa[x][j] != -1 && pa[x][j] != pa[y][j]) {
17            x = pa[x][j];
18            y = pa[y][j];
19        }
20    return pa[x][0];
21 }

```

②Tarjan 离线求 LCA

3.STL 技巧

①string

```

1 //1.string 类重载运算符 operator>>用于输入，同样重载运算符 operator<<用于输出操
2 作
3 string str1;
4 cin >> str1;//当用 cin>>进行字符串的输入的时候，遇到空格的地方就停止字符串的读取
5 输入
6 cout << str1 << endl;
7 cin.get();//这个的作用就是读取 cin>>输入的结束符，不用对 getline 的输入产生影响！
8 getline(cin, str1);//字符串的行输入
9 cout << str1 << endl;
10
11
12 //2.string 类的构造函数
13 string str2 = "aaaaa";//最简单的字符串初始化
14 cout << str2 << endl;
15
16 char *s = "bbbbbb";
17 string str3(s);//用 c 字符串 s 初始化
18 cout << str3 << endl;
19
20 char ch = 'c';
21 string str4(5, ch);//用 n 个字符 ch 初始化
22 cout << str4 << endl;
23

```

```

24 //3.string 类的字符操作
25 string str5 = "abcde";
26 ch = str5[3];//operator[]返回当前字符串中第 n 个字符的位置
27 cout << ch << endl;
28
29 string str6 = "abcde";
30 ch = str6.at(4);//at()返回当前字符串中第 n 个字符的位置,并且提供范围检查,当越界
31 时会抛出异常!
32 cout << ch << endl;
33
34 //4.string 的特性描述
35 string str7 = "abcdefgh";
36 int size;
37 size = str7.capacity();//返回当前容量
38 cout << size << endl;
39 size = str7.max_size();//返回 string 对象中可存放的最大字符串的长度
40 cout << size << endl;
41 size = str7.size();//返回当前字符串的大小
42 cout << size << endl;
43 size = str7.length();//返回当前字符串的长度
44 cout << size << endl;
45 bool flag;
46 flag = str7.empty();//判断当前字符串是否为空
47 cout << flag << endl;
48 int len = 10;
49 str7.resize(len, ch);//把字符串当前大小置为 len, 并用字符 ch 填充不足的部分
50 cout << str7 << endl;
51
52 //5.string 的赋值
53 string str8;
54 str8 = str7;//把字符串 str7 赋给当前字符串
55 cout << str8 << endl;
56 str8.assign(str7);//把字符串 str7 赋给当前字符串
57 cout << str8 << endl;
58 str8.assign(s);//用 c 类型字符串 s 赋值
59 cout << str8 << endl;
60 str8.assign(s, 2);//用 c 类型字符串 s 开始的 n 个字符赋值
61 cout << str8 << endl;
62 str8.assign(len, ch);//用 len 个字符 ch 赋值给当前字符串
63 cout << str8 << endl;
64 str8.assign(str7, 0, 3);//把字符串 str7 中从 0 开始的 3 个字符赋给当前字符串
65 cout << str8 << endl;
66 string str9 = "0123456789";
67 str8.assign(str9.begin(), str9.end());//把迭代器之间的字符赋给字符串
68 cout << str8 << endl;
69
70 //6.string 的连接
71 string str10;
72 str10 += str9;//把字符串 str9 连接到当前字符串的结尾

```

```

73  cout << str10 << endl;
74  str10.append(s); //把 c 类型字符串 s 连接到当前字符串的结尾
75  cout << str10 << endl;
76  str10.append(s, 2); //把 c 类型字符串 s 的前 2 个字符连接到当前字符串的结尾
77  cout << str10 << endl;
78  str10.append(str9.begin(), str9.end()); //把迭代器之间的一段字符连接到当前字符串的结尾
79
80  cout << str10 << endl;
81  str10.push_back('k'); //把一个字符连接到当前字符串的结尾
82  cout << str10 << endl;
83
84  //7.string 的比较
85  flag = (str9 == str10); //判断两个字符串是否相等
86  cout << flag << endl;
87  flag = (str9 != str10); //判断两个字符串是否不相等
88  cout << flag << endl;
89  flag = (str9 > str10); //判断两个字符串是否大于关系
90  cout << flag << endl;
91  flag = (str9 < str10); //判断两个字符串是否为小于关系
92  cout << flag << endl;
93  flag = (str9 >= str10); //判断两个字符串是否为大于等于关系
94  cout << flag << endl;
95  flag = (str9 <= str10); //判断两个字符串否为小于等于关系
96  cout << flag << endl;
97
98  //以下的 3 个函数同样适用于 c 类型的字符串，在 compare 函数中>时返回 1，<时返回 -1，=
99  时返回 0
100 flag = str10.compare(str9); //比较两个字符串的大小，通过 ASCII 的相减得出！
101 cout << flag << endl;
102 flag = str10.compare(6, 12, str9); //比较 str10 字符串从 6 开始的 12 个字符组成的
103 字符串与 str9 的大小
104 cout << flag << endl;
105 flag = str10.compare(6, 12, str9, 3, 5); //比较 str10 字符串从 6 开始的 12 个字符
106 组成的字符串与 str9 字符串从 3 开始的 5 个字符组成的字符串的大小
107 cout << flag << endl;
108
109 //8.string 的字串
110 string str11;
111 str11 = str10.substr(10, 15); //返回从下标 10 开始的 15 个字符组成的字符串
112 cout << str11 << endl;
113
114 //9.string 的交换
115 str11.swap(str10); //交换 str11 与 str10 的值
116 cout << str11 << endl;
117
118 //10.string 的查找，查找成功时返回所在位置，失败时返回 string::npos 的值，即是 -1
119 string str12 = "abcdefghijklmnopqrstuvwxyz";
120 int pos;
121 pos = str12.find('i', 0); //从位置 0 开始查找字符 i 在当前字符串的位置

```



```

122 cout << pos << endl;
123 pos = str12.find("ghijk", 0); //从位置 0 开始查找字符串“ghijk”在当前字符串的位置
124 cout << pos << endl;
125 pos = str12.find("opqrstuvw", 0, 4); //从位置 0 开始查找字符串“opqrstuvw”前 4 个
126 字符组成的字符串在当前字符串中的位置
127 cout << pos << endl;
128 pos = str12.rfind('s', string::npos); //从字符串 str12 反向开始查找字符 s 在字
129 串中的位置
130 cout << pos << endl;
131 pos = str12.rfind("klmn", string::npos); //从字符串 str12 反向开始查找字符串
132 “klmn”在字符串中的位置
133 cout << pos << endl;
134 pos = str12.rfind("opqrstuvw", string::npos, 3); //从 string::pos 开始从后向前
135 查找字符串 s 中前 n 个字符组成的字符串在当前串中的位置
136 cout << pos << endl;
137
138 string str13 =
139 "aaaabbbbccccdddeeefffggghhhiiijjjkkllmmmandjfaklsdfpopdtwptioczx";
140 pos = str13.find_first_of('d', 0); //从位置 0 开始查找字符 d 在当前字符串第一次出
141 现的位置
142 cout << pos << endl;
143 pos = str13.find_first_of("eefff", 0); //从位置 0 开始查找字符串“eefff”在当前
144 字符串中第一次出现的位置
145 cout << pos << endl;
146 pos = str13.find_first_of("efff", 0, 3); //从位置 0 开始查找当前串中第一个在字
147 符串“efff”的前 3 个字符组成的数组里的字符的位置
148 cout << pos << endl;
149 pos = str13.find_first_not_of('b', 0); //从当前串中查找第一个不在串 s 中的字符出
150 现的位置
151 cout << pos << endl;
152 pos = str13.find_first_not_of("abcdefghij", 0); //从当前串中查找第一个不在串 s
153 中的字符出现的位置
154 cout << pos << endl;
155 pos = str13.find_first_not_of("abcdefghij", 0, 3); //从当前串中查找第一个不在
156 由字符串“abcdefghij”的前 3 个字符所组成的字符串中的字符出现的位置
157 cout << pos << endl;
158 //下面的 last 的格式和 first 的一致，只是它从后面检索！
159 pos = str13.find_last_of('b', string::npos);
160 cout << pos << endl;
161 pos = str13.find_last_of("abcdef", string::npos);
162 cout << pos << endl;
163 pos = str13.find_last_of("abcdef", string::npos, 2);
164 cout << pos << endl;
165 pos = str13.find_last_not_of('a', string::npos);
166 cout << pos << endl;
167 pos = str13.find_last_not_of("abcdef", string::npos);
168 cout << pos << endl;
169 pos = str13.find_last_not_of("abcdef", string::npos, 3);
170 cout << pos << endl;

```

```

171
172 //11.string 的替换
173 string str14 = "abcdefghijlmn";
174 str14.replace(0, 3, "qqqq");//删除从 0 开始的 3 个字符，然后在 0 处插入字符串
175 "qqqq"
176 cout << str14 << endl;
177 str14.replace(0, 3, "vvvv", 2);//删除从 0 开始的 3 个字符，然后在 0 处插入字符串
178 "vvvv"的前 2 个字符
179 cout << str14 << endl;
180 str14.replace(0, 3, "opqrstuvw", 2, 4);//删除从 0 开始的 3 个字符，然后在 0 处插
181 入字符串"opqrstuvw"从位置 2 开始的 4 个字符
182 cout << str14 << endl;
183 str14.replace(0, 3, 8, 'c');//删除从 0 开始的 3 个字符，然后在 0 处插入 8 个字符 c
184 cout << str14 << endl;
185 //上面的位置可以换为迭代器的位置，操作是一样的，在这里就不再重复了！
186
187 //12.string 的插入，下面的位置处亦可以用迭代器的指针表示，操作是一样的
188 string str15 = "abcdefg";
189 str15.insert(0, "mnop");//在字符串的 0 位置开始处，插入字符串"mnop"
190 cout << str15 << endl;
191 str15.insert(0, 2, 'm');//在字符串的 0 位置开始处，插入 2 个字符 m
192 cout << str15 << endl;
193 str15.insert(0, "vwxy", 3);//在字符串的 0 位置开始处，插入字符串"vwxy"中的前
194 3 个字符
195 cout << str15 << endl;
196 str15.insert(0, "vwxy", 1, 2);//在字符串的 0 位置开始处，插入从字符串"vwxy"的
197 1 位置开始的 2 个字符
198 cout << str15 << endl;
199
200 //13.string 的删除
201 string str16 = "gfedcba";
202 string::iterator it;
203 it = str16.begin();
204 it++;
205 str16.erase(it);//删除 it 指向的字符，返回删除后迭代器的位置
206 cout << str16 << endl;
207 str16.erase(it, it + 3); //删除 it 和 it+3 之间的所有字符，返回删除后迭代器的位
208 置
209 cout << str16 << endl;
210 str16.erase(2);//删除从字符串位置 3 以后的所有字符，返回位置 3 前面的字符
211 cout << str16 << endl;
212
213 //14.字符串的流处理
214 string str17("hello,this is a test");
215 istringstream is(str17);
216 string s1, s2, s3, s4;
217 is >> s1 >> s2 >> s3 >> s4; //s1="hello,this",s2="is",s3="a",s4="test"
218 ostringstream os;
219 os << s1 << s2 << s3 << s4;

```

```

220 cout << os.str() << endl;
②bitset
1  //bitset 使用整数初始化 bitset
2  bitset<3> bs(7);
3  //输出 bs 各个位的值
4  cout << "bs[0] is " << bs[0] << endl;
5  cout << "bs[1] is " << bs[1] << endl;
6  cout << "bs[2] is " << bs[2] << endl;
7  //下面的语句会抛出 outofindexexception
8  //cout<<"bs[3] is "<<bs[3]<<endl;
9
10 //使用字符串初始化 bitset
11 //注意：使用 string 初始化时从右向左处理，如下初始化的各个位的值将是 110，而非 011
12 string strVal("011");
13 bitset<3> bs1(strVal);
14 //输出各位
15 cout << "bs1[0] is " << bs1[0] << endl;
16 cout << "bs1[1] is " << bs1[1] << endl;
17 cout << "bs1[2] is " << bs1[2] << endl;
18 //cout 输出时也是从右边向左边输出
19 cout << bs1 << endl;
20
21 //bitset 的方法
22 //any()方法如果有一位为 1，则返回 1
23 cout << "bs1.any() = " << bs1.any() << endl;
24
25 //none()方法，如果有一个为 1none 则返回 0，如果全为 0 则返回 1
26 bitset<3> bsNone;
27 cout << "bsNone.none() = " << bsNone.none() << endl;
28
29 //count()返回几个位为 1
30 cout << "bs1.count() = " << bs1.count() << endl;
31
32 //size()返回位数
33 cout << "bs1.size() = " << bs1.size() << endl;
34
35 //test()返回某一位是否为 1
36 //flip()诸位取反
37 bitset<3> bsFlip = bs1.flip();
38 cout << "bsFlip = " << bsFlip << endl;
39
40 //to_ulong
41 unsigned long val = bs1.to_ulong();
42 cout << val;

```

4.二维 RMQ

```

1  int num[maxn][maxn];
2  int st[maxn][maxn][9][9];
3  void build_st(int n, int m) {

```

```

4         for (int i = 1; i <= n ; ++i)
5             for (int j = 1; j <= m ; ++j)
6                 st[i][j][0][0] = num[i][j];
7         for (int g = 0; g < 10 ; ++g) {
8             for (int t = 0; t < 10 ; ++t) {
9                 if(g == 0 && t == 0)
10                     continue;
11                 for (int i = 1; i <= n; ++i) {
12                     for (int j = 1; j <= m ; ++j) {
13                         if(i + (1 << g) - 1 > n || j + (1 << t)
14 - 1 > m)
15                             break;
16                         if(g == 0)
17                             st[i][j][g][t] =
18 max(st[i][j][g][t - 1], st[i][j + (1 << (t - 1))][g][t - 1]);
19                         else
20                             st[i][j][g][t] = max(st[i][j][g -
21 1][t], st[i + (1 << (g - 1))][j][g - 1][t]);
22                     }
23                 }
24             }
25         }
26     }
27     int query(int a, int b, int c, int d) {
28         int k = (c - a + 1);
29         int t = (d - b + 1);
30         k = log2(1.0 * k) + eps;
31         t = log2(1.0 * t) + eps;
32         return max(max(st[a][b][k][t], st[c - (1 << k) + 1][b][k][t]),
33 max(st[a][d - (1 << t) + 1][k][t], st[c - (1 << k) + 1][d - (1 << t) +
34 1][k][t]));
35     }

```

5.背包

```

1     int dp[maxn];
2     //0-1 背包, 代价为 cost, 获得的价值为 weight
3     void ZeroOnePack(int cost, int weight, int nValue) {
4         for (int i = nValue; i >= cost; i--)
5             dp[i] = max(dp[i], dp[i - cost] + weight);
6     }
7     //完全背包, 代价为 cost, 获得的价值为 weight
8     void CompletePack(int cost, int weight, int nValue) {
9         for (int i = cost; i <= nValue; i++)
10             dp[i] = max(dp[i], dp[i - cost] + weight);
11     }
12     //多重背包
13     void MultiplePack(int cost, int weight, int amount, int nValue) {
14         if (cost * amount >= nValue)
15             CompletePack(cost, weight, nValue);

```

```

16     else {
17         int k = 1;
18         while (k < amount) {
19             ZeroOnePack(k * cost, k * weight, nValue);
20             amount -= k;
21             k <<= 1;
22         }
23         ZeroOnePack(amount * cost, amount * weight, nValue);
24     }
25 }

```

6.莫队

①线性莫队

```

1  int a[maxn], pos[maxn], cnt[maxn];
2  ll ans;
3  struct Query
4  {
5      int l, r, id;
6      ll ans;
7  } Q[maxn];
8  bool cmp1(const Query& a, const Query& b) {
9      return a.id < b.id;
10 }
11 bool cmp2(const Query& a, const Query& b) {
12     return pos[a.l] < pos[b.l] || (pos[a.l] == pos[b.l] && a.r < b.r);
13 }
14 inline void modify(int p, bool op) {
15     if (op == 1) {
16         cnt[a[p]]++;
17         ans += (2LL * cnt[a[p]] - 1) * a[p];
18     } else if (op == 0) {
19         cnt[a[p]]--;
20         ans -= (2LL * cnt[a[p]] + 1) * a[p];
21     }
22 }
23 inline void solve() {
24     int n, t;
25     scanf("%d%d", &n, &t);
26     rep(i, 1, n + 1) scanf("%d", &a[i]);
27     memset(cnt, 0, sizeof cnt);
28     int block = sqrt(n);
29     rep(i, 1, n + 1) pos[i] = (i - 1) / block + 1;
30     rep(i, 0, t) {
31         scanf("%d%d", &Q[i].l, &Q[i].r);
32         Q[i].id = i;
33     }
34     sort(Q, Q + t, cmp2);
35     ans = 0;
36     int L = 1, R = 1;

```

```

37     rep(i, 0, t) {
38         int l = Q[i].l, r = Q[i].r;
39         while (L < l) modify(L, 0), L++;
40         while (L > l) modify(L - 1, 1), L--;
41         while (R <= r) modify(R, 1), R++;
42         while (R > r + 1) modify(R - 1, 0), R--;
43         Q[i].ans = ans;
44     }
45     sort(Q, Q + t, cmp1);
46     rep(i, 0, t) printf("%lld\n", Q[i].ans);
47 }

```

②树上莫队

7.搜索

①A*

②IDA*

③DancingLinks

三、数据结构

1.线段树

注意：这里每个节点存了左和右边界，在极限情况可以省略。

①结构体版本

```

1  //这是一个区间加和 RMQ
2  class SegmentTree {
3  public:
4      #define lson (root << 1)
5      #define rson (root << 1 | 1)
6      #define lent (t[root].r - t[root].l + 1)
7      #define lenl (t[lson].r - t[lson].l + 1)
8      #define lenr (t[rson].r - t[rson].l + 1)
9      struct Tree {
10         int l, r, val, lazy;
11     } t[maxn << 4];
12
13     void pushup(int root) {
14         t[root].val = t[lson].val + t[rson].val;
15     }
16
17     void pushdown(int root) {
18         if (t[root].lazy) {
19             t[lson].lazy += t[root].lazy;
20             t[rson].lazy += t[root].lazy;
21             t[lson].val += lenl * t[root].lazy;
22             t[rson].val += lenr * t[root].lazy;
23             t[root].lazy = 0;
24         }
25     }

```

```

26
27     void build(int l, int r, int root) {
28         t[root].l = l;
29         t[root].r = r;
30         t[root].lazy = 0;
31         if (l == r) {
32             t[root].val = 0;
33             return;
34         }
35         int mid = l + r >> 1;
36         build(l, mid, lson);
37         build(mid + 1, r, rson);
38         pushup(root);
39     }
40
41     void update(int l, int r, int val, int root) {
42         if (l <= t[root].l && t[root].r <= r) {
43             t[root].val += lent * val;
44             t[root].lazy += val;
45             return;
46         }
47         pushdown(root);
48         int mid = t[root].l + t[root].r >> 1;
49         if (l <= mid) update(l, r, val, lson);
50         if (r > mid) update(l, r, val, rson);
51         pushup(root);
52     }
53
54     int query(int l, int r, int root) {
55         if (l <= t[root].l && t[root].r <= r)
56             return t[root].val;
57         pushdown(root);
58         int mid = t[root].l + t[root].r >> 1;
59         int ans = 0;
60         if (l <= mid) ans += query(l, r, lson);
61         if (r > mid) ans += query(l, r, rson);
62         return ans;
63     }
64     #undef lenr
65     #undef lenl
66     #undef lent
67     #undef rson
68     #undef lson
69 };

```

②数组版本（占坑）

2.树状数组

①一维树状数组

```

1     int c[maxn];

```

```

2  inline int lowbit(int x) { return x & (-x); }
3  inline void modify(int x, int y, int data) {
4      while (x <= y) {
5          c[x] += data;
6          x += lowbit(x);
7      }
8  }
9  inline int query(int x) {
10     int sum = 0;
11     while (x > 0) {
12         sum += c[x];
13         x -= lowbit(x);
14     }
15     return sum;
16 }

```

②二维树状数组——记得更新四个点

```

1  class BTree {
2      ll a[N][N];
3      int n;
4      void init() {
5          n = 0;
6          memset(a, 0, sizeof a);
7      }
8      void lowbit(int x) {
9          return x & (-x);
10     }
11     void modify(int x, int y, int d) {
12         for (int i = x; i <= n; i += lowbit(i))
13             for (int j = y; j <= n; j += lowbit(j))
14                 a[i][j] += d;
15     }
16     ll query(int x, int y) {
17         ll res = 0;
18         for (int i = x; i >= 0; i -= lowbit(i))
19             for (int j = y; j >= 0; j -= lowbit(j))
20                 res += a[i][j];
21         return res;
22     }
23 };

```

3.树链剖分

①点权式

```

1  // 点权式 - 配合线段树的单点修改+区间查询
2  struct Edge {
3      int to, next;
4      Edge() {}
5      Edge(int a, int b) { to = a; next = b; }
6  } E[maxn << 1];
7  int head[maxn], cnt, tot;

```



```

8  int top[maxn], son[maxn], size[maxn], deep[maxn], pa[maxn], id[maxn];
9  int a[maxn];
10 void init() {
11     memset(head, -1, sizeof head);
12     tot = cnt = 0;
13 }
14 void addedge(int u, int v) {
15     E[cnt].to = v;
16     E[cnt].next = head[u];
17     head[u] = cnt++;
18 }
19 void dfs1(int u, int fa, int d) {
20     size[u] = 1; deep[u] = d; son[u] = 0;
21     for (int i = head[u]; ~i; i = E[i].next) {
22         int v = E[i].to;
23         if (v != fa) {
24             dfs1(v, u, d + 1);
25             pa[v] = u;
26             size[u] += size[v];
27             if (size[v] > size[son[u]]) son[u] = v;
28         }
29     }
30 }
31 void dfs2(int u, int first) {
32     top[u] = first;
33     id[u] = ++tot;
34     if (son[u]) dfs2(son[u], first);
35     for (int i = head[u]; ~i; i = E[i].next) {
36         int v = E[i].to;
37         if (v != pa[u] && v != son[u]) dfs2(v, v);
38     }
39 }
40 void solve(int u, int v) { //注意改的是这里
41     int x = top[u], y = top[v], res = 0;
42     while (x != y) {
43         if (deep[x] < deep[y]) {
44             swap(u, v);
45             swap(x, y);
46         }
47         res += T.query(id[x], id[u], 1);
48         u = pa[x];
49         x = top[u];
50     }
51     if (deep[u] > deep[v]) swap(u, v);
52     res += T.query(id[u], id[v], 1);
53     printf("%d\n", res);
54 }

```

②边权式

1 // 边权式：把边权偏移到深度较大的点，化成点权式

```

2  struct Point {
3      int from, to, val;
4  } p[maxn];
5  struct Edge {
6      int to, next;
7  } E[maxn << 1];
8  int head[maxn], cnt, tot;
9  int top[maxn], son[maxn], size[maxn], deep[maxn], pa[maxn], id[maxn];
10 int a[maxn];
11 void init() {
12     memset(head, -1, sizeof head);
13     tot = cnt = 0;
14 }
15 void addedge(int u, int v) {
16     E[cnt].to = v;
17     E[cnt].next = head[u];
18     head[u] = cnt++;
19 }
20 void dfs1(int u, int fa, int d) {
21     size[u] = 1; deep[u] = d; son[u] = 0;
22     for (int i = head[u]; ~i; i = E[i].next) {
23         int v = E[i].to;
24         if (v != fa) {
25             dfs1(v, u, d + 1);
26             pa[v] = u;
27             size[u] += size[v];
28             if (size[v] > size[son[u]]) son[u] = v;
29         }
30     }
31 }
32 void dfs2(int u, int first) {
33     top[u] = first;
34     id[u] = ++tot;
35     if (son[u]) dfs2(son[u], first);
36     for (int i = head[u]; ~i; i = E[i].next) {
37         int v = E[i].to;
38         if (v != pa[u] && v != son[u]) dfs2(v, v);
39     }
40 }
41 int solve(int u, int v) {
42     int x = top[u], y = top[v];
43     int ans = 0;
44     while (x != y) {
45         if (deep[x] < deep[y]) {
46             swap(u, v);
47             swap(x, y);
48         }
49         ans += T.query(id[x], id[u], 1);
50         u = pa[x];

```

```

51         x = top[u];
52     }
53     //     printf("%d : %d - %d : ", ans, u, v);
54     if (deep[u] > deep[v])
55         swap(u, v);
56     if (u != v) ans += T.query(id[son[u]], id[v], 1);
57     return ans;
58 }

```

5.Treap

```

1  class Treap{
2  public:
3      struct treap_point{
4          int l,r,x,rep,son,rnd;
5      }t[N];
6      int treesz,root;
7      inline void update(int k){t[k].son=t[t[k].l].son+t[t[k].r].son+t[k].rep;}
8      inline void right_rotate(int &k){int
9  tt=t[k].l;t[k].l=t[tt].r;t[tt].r=k;update(k);update(tt);k=tt;}
10     inline void left_rotate(int &k){int
11  tt=t[k].r;t[k].r=t[tt].l;t[tt].l=k;update(k);update(tt);k=tt;}
12     inline void insert_to_tree(int &k,int x)
13     {
14         if
15  (!k){k=++treesz;t[k].x=x;t[k].rnd=rand();t[k].son=t[k].rep=1;t[k].l=t[k].r=0;ret
16         t[k].son++;
17         if (x==t[k].x){t[k].rep++;return;}
18         if (x<t[k].x){insert_to_tree(t[k].l,x);if
19  (t[t[k].l].rnd<t[k].rnd)right_rotate(k);}
20         if (x>t[k].x){insert_to_tree(t[k].r,x);if
21  (t[t[k].r].rnd<t[k].rnd)left_rotate(k);}
22     }
23     inline bool delete_from_tree(int &k,int x)
24     {
25         if (!k)return 0;
26         if (x==t[k].x)
27         {
28             if (t[k].rep>1){t[k].rep--;t[k].son--;return 1;}
29             if (!t[k].l||!t[k].r){k=t[k].l+t[k].r;return 1;}
30             if (t[t[k].l].rnd<t[t[k].r].rnd){right_rotate(k);return
31  delete_from_tree(k,x);}
32             left_rotate(k);return delete_from_tree(k,x);
33         }
34         int res;
35         if (x<t[k].x){res=delete_from_tree(t[k].l,x);if (res)t[k].son--;return res;
36         res=delete_from_tree(t[k].r,x);if (res)t[k].son--;return res;
37     }
38     inline int get_succ_in_tree(int k,int x)
39     {

```

```

40     if (!k)return -1;
41     int sv;
42     if (x<t[k].x){sv=get_succ_in_tree(t[k].l,x);return sv==-1?t[k].x:sv;}
43     return get_succ_in_tree(t[k].r,x);
44 }
45 inline int get_pred_in_tree(int k,int x)
46 {
47     if(!k)return -1;
48     int sv;
49     if (x>t[k].x){sv=get_pred_in_tree(t[k].r,x);return sv==-1?t[k].x:sv;}
50     return get_pred_in_tree(t[k].l,x);
51 }
52 inline int ask_kth_in_tree(int k,int x)
53 {
54     if (!k)return -1;
55     if (x<=t[t[k].l].son)return ask_kth_in_tree(t[k].l,x);
56     if (x>t[t[k].l].son+t[k].rep)return ask_kth_in_tree(t[k].r,x-t[t[k].l].son
57 t[k].rep);
58     return t[k].x;
59 }
60 inline int ask_rank_in_tree(int k,int x)
61 {
62     if (!k)return -1;
63     if (x==t[k].x)return t[t[k].l].son+1;
64     if (x<t[k].x)return ask_rank_in_tree(t[k].l,x);
65     return t[t[k].l].son+t[k].rep+ask_rank_in_tree(t[k].r,x);
66 }
67 /*
68 inline void bianli(int k)//for debug
69 {
70     if (!k)return;
71     bianli(t[k].l);
72     printf("i=%d l=%d r=%d x=%d rep=%d
73 son=%d\n",k,t[k].l,t[k].r,t[k].x,t[k].rep,t[k].son);
74     bianli(t[k].r);
75 }
76 inline void put(){puts("\n-----
77 ");bianli(root);puts("-----\n");};//for debug
78 */
79 inline void init(){treesz=root=0;}
80 inline void insert(int x){insert_to_tree(root,x);}
81 inline int del(int x){return delete_from_tree(root,x);}
82 inline int succ(int x){return get_succ_in_tree(root,x);}
83 inline int pred(int x){return get_pred_in_tree(root,x);}
84 inline int ask_kth(int x){return ask_kth_in_tree(root,x);}
85 inline int ask_rank(int x){return ask_rank_in_tree(root,x);}
86 }T;
87 int main()
88 {

```

```

89     int n,x,op;read(n);
90     T.init();
91     for (int i=1;i<=n;i++)
92     {
93         read(op);read(x);
94         if (op==1)T.insert(x);
95         if (op==2)T.del(x);
96         if (op==3)printf("%d\n",T.ask_rank(x));
97         if (op==4)printf("%d\n",T.ask_kth(x));
98         if (op==5)printf("%d\n",T.pred(x));
99         if (op==6)printf("%d\n",T.succ(x));
100    }
101 };

```

6.Splay

7.分块

8.块状链表

9.可持久化数据结构

①可持久化线段树

②可持久化并查集

四、字符串

1.Kmp

①Char 版本

```

1  char n[N], m[N];
2  int f[N];
3  void compute(char *P) {
4      int len = strlen(P);
5      memset(f, 0, sizeof(f));
6      for (int i = 1; i < len; i++) {
7          int j = f[i];
8          while (j && P[i] != P[j])
9              j = f[j];
10         f[i + 1] = (P[i] == P[j]) ? j + 1 : 0;
11     }
12 }
13 bool kmp(char *T, char *P) {
14     int n = strlen(T);
15     int m = strlen(P);
16     compute(P);
17     int q = 0;
18     for (int i = 0; i < n; i++) {
19         while (q && P[q] != T[i])
20             q = f[q];

```

```

21         if (P[q] == T[i])
22             q++;
23         if (q == m)
24             return 1;    //修改这里可以实现很多功能
25     }
26     return 0;
27 }

```

②String 版本

```

1  string n, m;
2  int f[maxn];
3  void compute(string &P) {
4      int len = P.length();
5      memset(f, 0, sizeof(f));
6      for (int i = 1; i < len; i++) {
7          int j = f[i];
8          while (j && P[i] != P[j])
9              j = f[j];
10         f[i + 1] = (P[i] == P[j]) ? j + 1 : 0;
11     }
12 }
13 bool kmp(string &T, string &P) {
14     int n = T.length();
15     int m = P.length();
16     compute(P);
17     int q = 0;
18     for (int i = 0; i < n; i++) {
19         while (q && P[q] != T[i])
20             q = f[q];
21         if (P[q] == T[i])
22             q++;
23         if (q == m)
24             return 1;
25     }
26     return 0;
27 }

```

③扩展 kmp

2.Trie

```

1  struct Trie {
2      #define sigma_size 26
3      int ch[maxn][sigma_size];
4      int val[maxn];
5      int sz;
6
7      void init() {
8          sz = 1;
9          memset(ch[0], 0, sizeof ch[0]);

```

```

10     }
11
12     int idx(char c) {
13         return c - 'a';
14     }
15
16     void insert(char *s, int v) {
17         int u = 0, len = strlen(s);
18         rep(i, 0, len) {
19             int c = idx(s[i]);
20             if (!ch[u][c]) {
21                 memset(ch[sz], 0, sizeof ch[sz]);
22                 val[sz] = 0;
23                 ch[u][c] = sz++;
24             }
25             u = ch[u][c];
26         }
27         val[u] = v;
28     }
29 #undef sigma_size
30 };

```

3.Manacher

```

1  /*
2   *以 char 方式储存字符串，变更字符串为$#*#*#*#*#形式，所以计算应该从 1 开始
3   *每个字符的 p 值大小代表该字符可以扩展的大小也打表回文串长度+1
4   *如果 p 值为奇数，代表此字符左右为中心的字符串为回文串的长度+1
5   *如果 p 值为偶数，代表此字符为中心扩展的字符的长度
6   */
7  int p[maxn * 2 + 10];
8  char re[maxn * 2];
9  char s[maxn];
10 void manacher(char str[], int len) {
11     //memset(p,0,sizeof(p));
12     int mx = 0, id = 0;
13     int ans = 1;
14     for(int i = 1; i <= len; i++) {
15         if(mx > i)
16             p[i] = (p[2 * id - i] < (mx - i)) ? p[2 * id - i] :
17 (mx - i);
18         else p[i] = 1;
19         while(str[i - p[i]] == str[i + p[i]]) p[i]++;
20         if(i + p[i] > mx) {
21             mx = i + p[i];
22             id = i;
23         }
24         ans = max(ans, p[i] - 1);
25     }
26     printf("%d\n", ans);

```

```

27 }
28 // 字符串变更
29 void solve(char str[]) {
30     re[0] = '$'; re[1] = '#';
31     int len = strlen(str + 1);
32     for(int i = 1 ; i <= len; i++) {
33         re[i * 2] = str[i];
34         re[i * 2 + 1] = '#';
35     }
36     manacher(re, len * 2 + 1);
37 }

```

4.AC 自动机

```

1  const int AC_NOD = 28, AC_N = 5e5 + 10;
2  struct Tire {
3      int nxt[AC_N][AC_NOD], fail[AC_N], cnt[AC_N];
4      int root , tot;
5      int id[AC_NOD];
6      int end[AC_N];
7      int newnode() {
8          for(int i = 0; i < AC_NOD ; i++)nxt[tot][i] = -1;
9          fail[tot] = -1;
10         cnt[tot] = 0;
11         return tot++;
12     }
13     void init() {
14         tot = 0;
15         memset(end, false, sizeof end);
16         root = newnode();
17     }
18     int getid(char x) {
19         if(x <= 'z' && x >= 'a') {
20             return x - 'a' + 10;
21         } else if(x <= 'Z' && x >= 'A') {
22             return 36 + x - 'A';
23         } else return x - '0';
24     }
25
26
27     void insert(string s) {
28         int p = root, len = s.length();
29         for(int i = 0; i < len; i++) {
30             int cur = getid(s[i]);
31             if(nxt[p][cur] == -1)nxt[p][cur] = newnode();
32             p = nxt[p][cur];
33         }
34         cnt[p]++; end[p] = true;
35     }
36     void build() {

```



```

37         queue<int> que;
38         que.push(root);
39         int now;
40
41         while(!que.empty()) {
42             now = que.front(); que.pop();
43             if(end[fail[now]])end[now] = true;
44             for(int i = 0; i < AC_NOD; i++) {
45                 if(~nxt[now][i]) {
46                     if(now == root) fail[nxt[now][i]] =
47 root;
48                     else fail[nxt[now][i]] =
49 nxt[fail[now]][i];
50                     que.push(nxt[now][i]);
51                 } else {
52                     if(now == root) nxt[now][i] = root;
53                     else nxt[now][i] = nxt[fail[now]][i];
54                 }
55             }
56         }
57     }
58     int count(string t) {
59         int now = root;
60         int len = t.length();
61         int p;
62         int ans = 0;
63         for(int i = 0; i < len; i++) {
64             p = getid(t[i]);
65             now = nxt[now][p];
66             for(int tmp = now; tmp != root; tmp = fail[tmp]) {
67                 ans += cnt[tmp];
68             }
69         }
70         return ans;
71     }
72 } T;
73 int main() {
74     int t;
75     cin.sync_with_stdio(0);
76     cin >> t;
77     while(t--) {
78         T.init();
79         int n; cin >> n;
80         string q; for(int i = 1; i <= n; i++)cin >> q, T.insert(q);
81         T.build();
82         cin >> q; cout << T.count(q) << endl;
83     }
84     return 0;
85 }

```

5.后缀数组

①Da

```
1  #include <bits/stdc++.h>
2  using namespace std;
3  /*n 为总字符串的长度
4   *r 数组为被操作的字符串，索引从 0 开始，最后一位(r[n-1])一定是'$'(char 数
5  组)||'0'(int 数组)
6   *sa 为生成的后缀数组，从 0 开始到 n-1 结束，保证 sa[0] = n-1
7   *m 代表字符集的大小或者说范围。
8   *height[n-1]的值无效因为其没有前驱字符串
9   *查询两个后缀的 lcp 应该查询 query(sa[l]+1,sa[r]);
10  */
11  const int maxn = 500005;
12  char a[maxn];
13  int wa[maxn],wb[maxn],wv[maxn],wts[maxn];
14  int cmp(int *r,int a,int b,int l)
15  {
16      return r[a] == r[b]&&r[a+l] == r[b+l];
17  }
18  void init_sa(char *r,int *sa,int n,int m)
19  {
20      int i,j,*x = wa,*y = wb,*t;
21      int p;
22      for(i = 0;i < m;i ++) wts[i] = 0;
23      for(i = 0;i < n;i ++) wts[ x[i] = r[i]]++;
24      for(i = 0;i < m;i ++) wts[i] += wts[i-1];
25      for(i = n-1;i >= 0; --i) sa[--wts[x[i]]] = i;
26      for(j = 1,p = 1; p < n ; j <= 1 , m = p )
27      {
28
29          for(p = 0,i = n - j; i < n; i ++) y[p++] = i;
30          for(i = 0;i < n; i ++) if(sa[i] >= j) y[p++] = sa[i] - j;
31          for(i = 0;i < n; i ++) wv[i] = x[y[i]];
32          for(i = 0;i < m; i ++) wts[i] = 0;
33          for(i = 0;i < n; i ++) wts[wv[i]]++;
34          for(i = 1;i < m; i ++) wts[i] += wts[i-1];
35          for(i = n-1;i >= 0;i--) sa[--wts[wv[i]]] = y[i];
36          for(t = x,x = y,y = t,p = 1,x[sa[0]] = 0,i = 1;i < n; i++)
37              x[sa[i]] = cmp(y,sa[i-1],sa[i],j)?p-1:p++;
38      }
39
40      return;
41  }
42  int rnk[maxn],height[maxn];
43  void calheight(char *r,int *sa,int n)
44  {
```

```

45     int i,j,k = 0;
46     for(i = 1;i < n;i++)rnk[sa[i]] = i;
47     for(i = 0;i < n-1;height[rnk[i++]] = k)
48         for(k?k--:0,j = sa[rnk[i]-1];r[i+k] == r[j+k];k++);
49 }
50 int sa[maxn];
51 void solve(int cur)
52 {
53     init_sa(a,sa,len,m);
54     calheight(a,sa,len);
55 }

```

②Dc3

6.后缀自动机

五、图论

1.最短路

①Floyd

```

1  #define inf 0x3fffffff //注意，太大会溢出
2  #define M //最大点数
3  int n, dist[M][M]; //n: 实际点数
4
5  void init () //有时候需要初始化
6  {
7      int i, j;
8      for (i = 1; i <= n; i++)
9          for (j = i + 1; j <= n; j++)
10             dist[i][j] = dist[j][i] = inf;
11 }
12
13 void floyd ()
14 {
15     int i, j, k;
16     for (k = 1; k <= n; k++)
17         for (i = 1; i <= n; i++)
18             for (j = 1; j <= n; j++) //有的题目会溢出就要自己变通了
19                 if (dist[i][k] + dist[k][j] < dist[i][j])
20                     dist[i][j] = dist[i][k] + dist[k][j];
21 }

```

②Dijkstra

● 普通:

```

1  #define inf 0x3fffffff
2  #define M 105
3  int dist[M], map[M][M], n;
4  bool mark[M];

```

```

5 void init () {
6     int i, j;
7     for (i = 1; i <= n; i++) //i==j 的时候也可以初始化为 0，只是有时候不合适
8         for (j = 1; j <= n; j++)
9             map[i][j] = inf;
10 }
11 void dijkstra (int u) {
12     int i, j, mins, v;
13     for (i = 1; i <= n; i++) {
14         dist[i] = map[u][i];
15         mark[i] = false;
16     }
17     mark[u] = true;
18     dist[u] = 0; //既然上面的 map 当 i==j 时不是 0，就要这句
19     while (1) {
20         mins = inf;
21         for (j = 1; j <= n; j++)
22             if (!mark[j] && dist[j] < mins)
23                 mins = dist[j], v = j;
24         if (mins == inf)
25             break;
26         mark[v] = true;
27         for (j = 1; j <= n; j++)
28             if (!mark[j] && dist[v] + map[v][j] < dist[j])
29                 dist[j] = dist[v] + map[v][j];
30     }
31 }

```

● 堆优化:

```

1 struct node {
2     int to, val;
3     friend bool operator< (const node &a, const node &b) {
4         return a.val > b.val;
5     }
6 };
7 vector<node> g[maxn];
8 int dis[maxn];
9 int dijkstra(int s, int t, int n) {
10     fill(dis, dis + n + 1, INF);
11     priority_queue<node> q;
12     dis[s] = 0;
13     q.push({s, 0});
14     while (!q.empty()) {
15         int u = q.top().to; q.pop();
16         int sz = SZ(g[u]);
17         rep(i, 0, sz) {
18             int v = g[u][i].to, w = g[u][i].val;
19             if (dis[v] > dis[u] + w)
20                 q.push({v, dis[v] = dis[u] + w});

```

```

21         }
22     }
23     return dis[t];
24 }

```

③Spfa

● 普通队列:

```

1  bool vis[maxn];
2  vector<PII> g[maxn];
3  int dis[maxn];
4  inline void spfa(int s, int n) {
5      queue<int> q;
6      memset(vis, 0, sizeof vis);
7      fill(dis, dis + n + 1, INF);
8      dis[s] = 0; vis[s] = 1;
9      q.push(s);
10     while (!q.empty()) {
11         int u = q.front(); q.pop();
12         vis[u] = 0;
13         rep(i, 0, SZ(g[u])) {
14             int v = g[u][i].fi, w = g[u][i].se;
15             if (dis[v] > dis[u] + w) {
16                 dis[v] = dis[u] + w;
17                 if (!vis[v]) {
18                     vis[v] = 1;
19                     q.push(v);
20                 }
21             }
22         }
23     }
24 }

```

● 双端队列:

```

1  bool vis[maxn];
2  vector<PII> g[maxn];
3  int dis[maxn];
4  inline void spfa(int s, int n) {
5      deque<int> q;
6      memset(vis, 0, sizeof vis);
7      fill(dis, dis + n + 1, INF);
8      dis[s] = 0; vis[s] = 1;
9      q.pb(s);
10     while (!q.empty()) {
11         int u = q.front(); q.pop_front();
12         vis[u] = 0;
13         rep(i, 0, SZ(g[u])) {
14             int v = g[u][i].fi, w = g[u][i].se;
15             if (dis[v] > dis[u] + w) {
16                 dis[v] = dis[u] + w;
17                 if (!vis[v]) {

```

```

18         vis[v] = 1;
19         if (!q.empty() && dis[v] <=
20 dis[q.front()]) q.push_front(v);
21         else q.pb(v);
22     }
23     }
24 }
25 }
26 }

```

2.K 短路

3.差分约束

4.次小生成树

5.曼哈顿最小生成树

6.最小树形图

7.2-SAT

8.二分图匹配

①匈牙利算法

● 链表:

```

1  vector<int> g[maxn];
2  int match[maxn];
3  bool vis[maxn];
4  int n;
5  bool dfs(int u) {
6      vis[u] = 1;
7      int sz = g[u].size();
8      rep(i, 0, sz) {
9          int v = g[u][i];
10         if (match[v] == -1 || !vis[v] && dfs(v)) {
11             match[v] = u;
12             match[u] = v;
13             return 1;
14         }
15     }
16     return 0;
17 }
18 int bipartite_matching() {
19     int ans = 0;
20     memset(match, -1, sizeof match);
21     rep(u, 1, n + 1) {
22         if (match[u] == -1) {
23             memset(vis, 0, sizeof vis);
24             if (dfs(u))

```

```

25                                     ans++;
26                                 }
27                             }
28                             return ans;
29     }

    ● 邻接矩阵:
1   int g[N][N], match[N];
2   bool vis[N];
3   int n, m; // n 个点 m 个边
4   bool dfs(int u) {
5       rep(v, 1, n + 1) {
6           if (g[u][v] && !vis[v]) {
7               vis[v] = 1;
8               if (match[v] == -1 || dfs(match[v])) {
9                   match[v] = u;
10                  return 1;
11              }
12          }
13      }
14      return 0;
15  }
16  int hungary() {
17      int ans = 0;
18      memset(match, -1, sizeof match);
19      rep(u, 1, m + 1) {
20          memset(vis, 0, sizeof vis);
21          if (dfs(u))
22              ans++;
23      }
24      return ans;
25  }

```

9.二分图最大权匹配

```

1   int nx, ny; //两边的点数
2   int g[N][N]; //二分图描述
3   int match[N], lx[N], ly[N]; //y 中各点匹配状态, x,y 中的点标号
4   int slack[N];
5   bool visx[N], visy[N];
6
7   bool DFS(int x) {
8       visx[x] = true;
9       for(int y = 0; y < ny; y++) {
10          if(visy[y]) continue;
11          int tmp = lx[x] + ly[y] - g[x][y];
12          if(tmp == 0) {
13              visy[y] = true;
14              if(match[y] == -1 || DFS(match[y])) {
15                  match[y] = x;

```

```

16             return true;
17         }
18     } else if(slack[y] > tmp)
19         slack[y] = tmp;
20 }
21 return false;
22 }
23 int KM() {
24     memset(match, -1, sizeof(match));
25     memset(ly, 0, sizeof(ly));
26     for(int i = 0; i < nx; i++) {
27         lx[i] = -INF;
28         for(int j = 0; j < ny; j++)
29             if(g[i][j] > lx[i])
30                 lx[i] = g[i][j];
31     }
32     for(int x = 0; x < nx; x++) {
33         for(int i = 0; i < ny; i++)
34             slack[i] = INF;
35         while(true) {
36             memset(visx, false, sizeof(visx));
37             memset(visy, false, sizeof(visy));
38             if(DFS(x))break;
39             int d = INF;
40             for(int i = 0; i < ny; i++)
41                 if(!visy[i] && d > slack[i])
42                     d = slack[i];
43             for(int i = 0; i < nx; i++)
44                 if(visx[i])
45                     lx[i] -= d;
46             for(int i = 0; i < ny; i++) {
47                 if(visy[i])ly[i] += d;
48                 else slack[i] -= d;
49             }
50         }
51     }
52     int res = 0;
53     for(int i = 0; i < ny; i++)
54         if(match[i] != -1)
55             res += g[match[i]][i];
56     return res;
57 }

```

10.连通性

①强连通分量

```

1  /*
2  * tarjan:
3  * scnt—缩的点
4  * belong—每个点属于那个块

```



```

5  */
6  struct Edge {
7      int to, next;
8  } E[maxn << 1];
9  int head[maxn], cnt;
10 int dfn[maxn], low[maxn], s[maxn], belong[maxn], sec, top, scnt;
11 bool inS[maxn];
12 void init() {
13     memset(head, -1, sizeof head);
14     memset(dfn, 0, sizeof dfn);
15     memset(low, 0, sizeof low);
16     cnt = scnt = sec = top = 0;
17 }
18 void addedge(int u, int v) {
19     E[cnt].to = v;
20     E[cnt].next = head[u];
21     head[u] = cnt++;
22 }
23 void tarjan(int u) {
24     dfn[u] = low[u] = ++sec;
25     inS[u] = 1;
26     s[top++] = u;
27     for (int i = head[u]; ~i; i = E[i].next) {
28         int v = E[i].to;
29         if (!dfn[v]) {
30             tarjan(v);
31             low[u] = min(low[u], low[v]);
32         } else if (inS[v] && low[u] > dfn[v]) low[u] = dfn[v];
33     }
34     if (dfn[u] == low[u]) {
35         ++scnt;
36         int tmp;
37         do {
38             tmp = s[--top];
39             inS[tmp] = 0;
40             belong[tmp] = scnt;
41         } while (tmp != u);
42     }
43 }

```

②双连通分量

● 边双连通分量

```

1  /*
2  * tarjan
3  * block 边连通分量的数量
4  * belong 标记该点属于哪个连通分量
5  */
6  int low[maxn], dfn[maxn];
7  bool inStack[maxn];
8  int cnt = 0;

```

```

9  stack<int> stk;
10 int bridge = 0;
11 int block = 0;
12 int belong[maxn];
13 struct edge {
14     int to, nxt;
15     bool cut;
16 } e[maxm];
17 int head[maxn], tot = 0;
18 void addEdge(int u, int v) {
19     e[tot].to = v;
20     e[tot].nxt = head[u];
21     e[tot].cut = false;
22     head[u] = tot++;
23 }
24 void tarjan(int u, int pre) {
25     int v;
26     low[u] = dfn[u] = ++cnt;
27     stk.push(u);
28     inStack[u] = true;
29     for(int i = head[u]; i != -1; i = e[i].nxt) {
30         int v = e[i].to;
31         if(v == pre) continue;
32         if(!dfn[v]) {
33             tarjan(v, u);
34             low[u] = min(low[u], low[v]);
35             // bridge
36             if(low[v] > dfn[u]) {
37                 bridge++;
38                 e[i].cut = true;
39                 e[i ^ 1].cut = true;
40             }
41         } else if(inStack[v] && low[u] > dfn[v]) {
42             low[u] = dfn[v];
43         }
44     }
45     if(low[u] == dfn[u]) {
46         block++;
47         do {
48             v = stk.top();
49             inStack[v] = false;
50             stk.pop();
51             belong[v] = block;
52         } while(v != u);
53     }
54 }
55 void gao(int n) {
56     mem(dfn, 0); mem(low, 0);
57     mem(inStack, false);

```

```

58     cnt = 0;
59     bridge = 0;
60     block = 0;
61     rep(i, 1, n + 1) {
62         if(!dfn[i]) {
63             tarjan(i, i);
64         }
65     }
66 }
67 // multi-edge
68 map<int, int> mp;
69 inline bool isHash(int u, int v) {
70     if(mp[u * maxn + v] || mp[v * maxn + u]) return true;
71     mp[u * maxn + v] = mp[v * maxn + u] = 1;
72     return false;
73 }

```

● 点双连通分量

```

1  /*
2  * tarjan
3  * block 点双连通分量数目
4  * belong 属于哪个分量 割点可能属于多个连通分量 所以割点要特殊判断
5  */
6  int low[maxn], dfn[maxn];
7  bool inStack[maxn];
8  int cnt = 0;
9  stack<int> stk;
10 int bridge = 0;
11 int block = 0;
12 int belong[maxn];
13 struct edge {
14     int to, nxt;
15     bool cut;
16 } e[maxn];
17 int head[maxn], tot = 0;
18 void addEdge(int u, int v) {
19     e[tot].to = v;
20     e[tot].nxt = head[u];
21     e[tot].cut = false;
22     head[u] = tot++;
23 }
24 void tarjan(int u, int pre) {
25     int v;
26     low[u] = dfn[u] = ++cnt;
27     stk.push(u);
28     inStack[u] = true;
29     for(int i = head[u]; i != -1; i = e[i].nxt) {
30         int v = e[i].to;
31         if(v == pre) continue;
32         if(!dfn[v]) {

```

```

33         tarjan(v, u);
34         low[u] = min(low[u], low[v]);
35         // bridge
36         if(low[v] >= dfn[u]) {
37             block++;
38             int vv;
39             do {
40                 vv = stk.top();
41                 belong[vv] = block;
42                 inStack[vv] = false;
43             } while(vv != v);
44         }
45     } else if(inStack[v] && low[u] > dfn[v]) {
46         low[u] = dfn[v];
47     }
48 }
49 }
50 void gao(int n) {
51     mem(dfn, 0); mem(low, 0);
52     mem(inStack, false);
53     cnt = 0;
54     bridge = 0;
55     block = 0;
56     rep(i, 1, n + 1) {
57         if(!dfn[i]) {
58             tarjan(i, i);
59         }
60     }
61 }
62 // multi-edge
63 map<int, int> mp;
64 inline bool isHash(int u, int v) {
65     if(mp[u * maxn + v] || mp[v * maxn + u]) return true;
66     mp[u * maxn + v] = mp[v * maxn + u] = 1;
67     return false;
68 }

```

③割点和桥: low[v]>dfn[u]

```

1  /*
2  * Tarjan O(n + m)
3  * bridge 桥的数量
4  * addBlock 删除该点后增加的连通块
5  */
6  int low[maxn], dfn[maxn];
7  bool inStack[maxn];
8  int cnt = 0;
9  stack<int> stk;
10 int bridge = 0;
11 bool cut[maxn];
12 int addBlock[maxn];

```

```

13 struct edge {
14     int to, nxt;
15     bool cut;
16 } e[maxm];
17 int head[maxn], tot = 0;
18 void addEdge(int u, int v) {
19     e[tot].to = v;
20     e[tot].nxt = head[u];
21     e[tot].cut = false;
22     head[u] = tot++;
23 }
24 void tarjan(int u, int pre) {
25     int v;
26     low[u] = dfn[u] = ++cnt;
27     stk.push(u);
28     inStack[u] = true;
29     int son = 0;
30     for(int i = head[u]; i != -1; i = e[i].nxt) {
31         int v = e[i].to;
32         if(v == pre) continue;
33         if(!dfn[v]) {
34             son++;
35             tarjan(v, u);
36             low[u] = min(low[u], low[v]);
37             // bridge
38             if(low[v] > dfn[u]) {
39                 bridge++;
40                 e[i].cut = true;
41                 e[i ^ 1].cut = true;
42             }
43             // cut point
44             if(u != pre && low[v] >= dfn[u]) {
45                 cut[u] = true;
46                 addBlock[u]++;
47             }
48             } else if(low[u] > dfn[v]) {
49                 low[u] = dfn[v];
50             }
51     }
52     // root
53     if(u == pre && son > 1) cut[u] = true;
54     if(u == pre) addBlock[u] = son - 1;
55     inStack[u] = false;
56     stk.pop();
57 }
58 void gao(int n) {
59     mem(dfn, 0); mem(low, 0);
60     mem(inStack, false);
61     mem(addBlock, 0);

```

```

62     mem(cut, false);
63     cnt = 0;
64     bridge = 0;
65     rep(i, 1, n + 1) {
66         if(!dfn[i]) {
67             tarjan(i, i);
68         }
69     }
70 }
71 // multi-edge
72 map<int, int> mp;
73 inline bool isHash(int u, int v) {
74     if(mp[u * maxn + v] || mp[v * maxn + u]) return true;
75     mp[u * maxn + v] = mp[v * maxn + u] = 1;
76     return false;
77 }

```

11.网络流

①Dinic

```

1  struct Edge {
2      int to, cap, next;
3  } E[maxn];
4  int head[N], pa[N], vis[N], cnt;
5  void init() {
6      memset(head, -1, sizeof head);
7      cnt = 0;
8  }
9  void addedge(int u, int v, int w) {
10     E[cnt].to = v; E[cnt].cap = w; E[cnt].next = head[u]; head[u] =
11     cnt++;
12     E[cnt].to = u; E[cnt].cap = 0; E[cnt].next = head[v]; head[v] =
13     cnt++; // 0 or w
14 }
15 bool bfs(int s, int t) {
16     memset(vis, -1, sizeof vis);
17     queue<int> q;
18     vis[s] = 0;
19     q.push(s);
20     while (!q.empty()) {
21         int u = q.front();
22         q.pop();
23         for (int i = head[u]; i != -1; i = E[i].next) {
24             int v = E[i].to;
25             if (E[i].cap && vis[v] == -1) {
26                 vis[v] = vis[u] + 1;
27                 q.push(v);
28             }
29         }
30     }

```

```

31         return vis[t] != -1;
32     }
33     int dfs(int u, int t, int flow) {
34         if (u == t) return flow;
35         for (int &i = pa[u]; i != -1; i = E[i].next) {
36             int v = E[i].to;
37             if (E[i].cap && vis[v] == vis[u] + 1) {
38                 int res = dfs(v, t, min(flow, E[i].cap));
39                 if (res) {
40                     E[i].cap -= res;
41                     E[i ^ 1].cap += res;
42                     return res;
43                 }
44             }
45         }
46         return 0;
47     }
48     int Dinic(int s, int t) {
49         int max_flow = 0;
50         while (bfs(s, t)) {
51             memcpy(pa, head, sizeof head);
52             int res;
53             do {
54                 res = dfs(s, t, INF);
55                 max_flow += res;
56             } while (res);
57         }
58         return max_flow;
59     }
60 }

```

②ISAP

```

1     struct Edge {
2         int from, to, cap, flow;
3     };
4     class ISAP {
5     public:
6         #define max_V 202
7         int n, m, s, t;
8         vector<Edge> edges;
9         vector<int> g[max_V];
10        bool vis[max_V];
11        int d[max_V], cur[max_V], p[max_V], num[max_V];
12
13        void Addedge(int u, int v, int w) {
14            edges.pb((Edge) {
15                u, v, w, 0
16            });
17            edges.pb((Edge) {
18                v, u, 0, 0

```

```

19         });
20         m = SZ(edges);
21         g[u].pb(m - 2);
22         g[v].pb(m - 1);
23     }
24
25     bool Bfs() {
26         memset(vis, 0, sizeof vis);
27         queue<int> q;
28         q.push(t);
29         vis[t] = 1;
30         d[t] = 0;
31         while (!q.empty()) {
32             int u = q.front(); q.pop();
33             int sz = SZ(g[u]);
34             rep(i, 0, sz) {
35                 Edge &E = edges[g[u][i] ^ 1];
36                 if (!vis[E.from] && E.cap > E.flow) {
37                     vis[E.from] = 1;
38                     d[E.from] = d[u] + 1;
39                     q.push(E.from);
40                 }
41             }
42         }
43         return vis[s];
44     }
45
46     void ClearAll(int n) {
47         this->n = n;
48         rep(i, 0, n) g[i].clear();
49         edges.clear();
50     }
51
52     void ClearFlow() {
53         int sz = SZ(edges);
54         rep(i, 0, sz) edges[i].flow = 0;
55     }
56
57     int Augment() {
58         int x = t, a = INF;
59         while (x != s) {
60             Edge &E = edges[p[x]];
61             a = min(a, E.cap - E.flow);
62             x = edges[p[x]].from;
63         }
64         x = t;
65         while (x != s) {
66             edges[p[x]].flow += a;
67             edges[p[x] ^ 1].flow -= a;

```



```

68         x = edges[p[x]].from;
69     }
70     return a;
71 }
72
73 int MaxFlow(int s, int t) {
74     this->s = s; this->t = t;
75     int flow = 0;
76     Bfs();
77     memset(num, 0, sizeof num);
78     rep(i, 0, n) num[d[i]]++;
79     int x = s;
80     memset(cur, 0, sizeof cur);
81     while (d[s] < n) {
82         if (x == t) {
83             flow += Augment();
84             x = s;
85         }
86         bool ok = 0;
87         int sz = SZ(g[x]);
88         rep(i, cur[x], sz) {
89             Edge &E = edges[g[x][i]];
90             if (E.cap > E.flow && d[x] == d[E.to] + 1) {
91                 ok = 1;
92                 p[E.to] = g[x][i];
93                 cur[x] = i;
94                 x = E.to;
95                 break;
96             }
97         }
98         if (!ok) {
99             int m = n - 1;
100             rep(i, 0, sz) {
101                 Edge &E = edges[g[x][i]];
102                 if (E.cap > E.flow)
103                     m = min(m, d[E.to]);
104             }
105             if (--num[d[x]] == 0) break; // gap
106             num[d[x] = m + 1]++;
107             cur[x] = 0;
108             if (x != s) x = edges[p[x]].from;
109         }
110     }
111     return flow;
112 }
113
114 vector<int> Mincut() {
115     Bfs();
116     vector<int> ans;

```

```

117         int sz = SZ(edges);
118         rep(i, 0, sz) {
119             Edge &E = edges[i];
120             if (!vis[E.from] && vis[E.to] && E.cap > 0)
121                 ans.pb(i);
122         }
123         return ans;
124     }
125
126     void Reduce() {
127         int sz = SZ(edges);
128         rep(i, 0, sz) {
129             edges[i].cap -= edges[i].flow;
130         }
131     }
132
133     void Print() {
134         puts("Graph:");
135         int sz = SZ(edges);
136         rep(i, 0, sz) {
137             printf("%d->%d, %d, %d\n", edges[i].from,
138 edges[i].to, edges[i].cap, edges[i].flow);
139         }
140     }
141     #undef max_V
142 };

```

12.费用流

```

1  struct Edge {
2      int from, to, cap, flow, cost;
3      Edge(int a, int b, int c, int d, int E) { from = a; to = b; cap = c;
4  flow = d; cost = E; }
5  };
6  class MCMF {
7  public:
8      #define max_V 202
9      int n, m, s, t;
10     vector<Edge> edges;
11     vector<int> g[max_V];
12     bool vis[max_V];
13     int d[max_V], p[max_V], a[max_V];
14
15     void Init(int n) {
16         this->n = n;
17         rep(i, 0, n) g[i].clear();
18         edges.clear();
19     }
20
21     void Addedge(int u, int v, int c, int w) {

```

```

22         edges.pb(Edge(u, v, c, 0, w));
23         edges.pb(Edge(v, u, 0, 0, -w));
24         int m = SZ(edges);
25         g[u].pb(m - 2);
26         g[v].pb(m - 1);
27     }
28
29     bool Spfa(int s, int t, int &flow, int &cost) {
30         fill(d, d + n, INF);
31         memset(vis, 0, sizeof vis);
32         d[s] = 0; vis[s] = 1; p[s] = 0; a[s] = INF;
33         queue<int> q;
34         q.push(s);
35         while (!q.empty()) {
36             int u = q.front(); q.pop();
37             vis[u] = 0;
38             int sz = SZ(g[u]);
39             rep(i, 0, sz) {
40                 Edge& E = edges[g[u][i]];
41                 if (E.cap > E.flow && d[E.to] > d[u] + E.cost)
42 {
43                     d[E.to] = d[u] + E.cost;
44                     p[E.to] = g[u][i];
45                     a[E.to] = min(a[u], E.cap - E.flow);
46                     if (!vis[E.to]) {
47                         vis[E.to] = 1;
48                         q.push(E.to);
49                     }
50                 }
51             }
52         }
53         if (d[t] == INF) return 0;
54         flow += a[t];
55         cost += d[t] * a[t];
56         int u = t;
57         while (u != s) {
58             edges[p[u]].flow += a[t];
59             edges[p[u] ^ 1].flow -= a[t];
60             u = edges[p[u]].from;
61         }
62         return 1;
63     }
64
65     int MincostMaxflow(int s, int t) {
66         int flow = 0, cost = 0;
67         while (Spfa(s, t, flow, cost));
68         return cost;
69     }
70     #undef max_V

```

71 };

六、数论、组合数学

1.基础数论

①extend gcd

注意：解只是一组， $(x+k*b/g, y-k*a/g)$ 也是一组

```
1  int extend_gcd(int a, int b, int& x, int& y) {
2      int d = a;
3      if (b != 0) {
4          d = extend_gcd(b, a % b, y, x);
5          y -= (a / b) * x;
6      } else {
7          x = 1;
8          y = 0;
9      }
10     return d;
11 } // (x+k*b/g, y-k*a/g)
```

②欧拉函数

③一元线性同余方程

④中国剩余定理

⑤逆元

2.质因子分解

①线性筛法

②Miller Rabin and Pollard Rho

```
1  //质因子分解，小数据用筛法直接判，大数据用 pollard_rho
2  //map[i]是含有多少个质因子 i
3  //map<LL, int>::iterator c, c->first 表示质因子, c->second 表示次方
4
5  #define maxn_for_division 1000001
6  bool is_prime[maxn_for_division];
7  vector<int>prime;
8  map<LL, int>factor;
9  inline void get_prime() {
10     for(int i = 0; i < maxn_for_division; i++)is_prime[i] = 1;
11     is_prime[0] = is_prime[1] = 0;
12     for(int i = 2; i < maxn_for_division; i++)
13         if (is_prime[i]) {
14             prime.push_back(i);
15             for (int j = i; j < maxn_for_division; j +=
16 i)is_prime[j] = 0;
17         }
18 }
19 inline LL gcd(LL a, LL b) {
20     if (!b)return a;
21     return gcd(b, a % b);
```

```

22 }
23 inline LL mod_mul(LL a, LL b, LL p) {
24     LL ans = 0ll;
25     a %= p, b %= p;
26     if (a > b) swap(a, b);
27     while (b) {
28         if (b & 1) ans = (ans + a) % p;
29         a = (a + a) % p;
30         b >>= 1;
31     }
32     return ans;
33 }
34 LL mod_pow(LL a, LL b, LL p) {
35     LL ans = 1ll;
36     a %= p;
37     while (b) {
38         if (b & 1) ans = mod_mul(ans, a, p);
39         a = mod_mul(a, a, p);
40         b >>= 1;
41     }
42     return ans;
43 }
44 bool witness(LL a, LL n) {
45     LL m = n - 1;
46     int j = 0;
47     while(!(m & 1)) j++, m >>= 1;
48     LL x = mod_pow(a, m, n);
49     if (x == 1 || x == n - 1) return 0;
50     while(j--) {
51         x = mod_mul(x, x, n);
52         if(x == n - 1) return 0;
53     }
54     return 1;
55 }
56 #define rep_times 20
57 bool Miller_Rabin(LL n) { //判断 n 是否为素数
58     srand(time(0));
59     if(n < 2) return 0;
60     if(n == 2) return 1;
61     if (!(n & 1)) return 0;
62     for(int i = 0; i < rep_times; i++) {
63         LL a = rand() % (n - 1) + 1;
64         if (witness(a, n)) return 0;
65     }
66     return 1;
67 }
68 #undef rep_times
69 LL Pollard_Rho(LL n, int c) {
70     LL x = 2, y = 2, d = 1;

```

```

71     while (d == 1) {
72         x = mod_mul(x, x, n) + c;
73         y = mod_mul(y, y, n) + c;
74         y = mod_mul(y, y, n) + c;
75         d = gcd((x - y >= 0 ? x - y : y - x), n);
76     }
77     if (d == n) return Pollard_Rho(n, c + 1);
78     return d;
79 }
80 bool Is_Prime(LL n) {
81     return n < maxn_for_division && is_prime[n] || n >=
82 maxn_for_division && Miller_Rabin(n);
83 }
84 void Find_Factor(LL n) {
85     if (Is_Prime(n)) {
86         factor[n]++;
87         return;
88     }
89     for (int i = 0; i < prime.size() && prime[i] <= n; i++)
90         if (n % prime[i] == 0) {
91             while (n % prime[i] == 0) {
92                 factor[prime[i]]++;
93                 n /= prime[i];
94             }
95         }
96     if (n != 1) {
97         if (Is_Prime(n)) factor[n]++;
98         else {
99             LL p = Pollard_Rho(n, 1);
100             Find_Factor(p);
101             Find_Factor(n / p);
102         }
103     }
104 }
105 int main() {
106     LL n;
107     get_prime();
108     while(read(n), !IOerror) {
109         factor.clear();
110         Find_Factor(n);
111         //-----output-----
112 -----
113         for(map<LL, int>::iterator c = factor.begin(); c !=
114 factor.end();) {
115             printf("%lld^%d", c->first, c->second);
116             if(++c != factor.end()) printf("*");
117         }
118         //-----
119 -----

```

```

120             puts("");
121         }
122     }

```

3.卷积

①FFT

```

1  class Fft {
2  public:
3  #define MAX_LEN 120000
4      const double pi = acos(-1.0);
5      struct cp {
6          double a, b;
7          cp operator+ (const cp &o) const { return (cp){a + o.a, b +
8  o.b}; }
9          cp operator- (const cp &o) const { return (cp){a - o.a, b -
10 o.b}; }
11         cp operator* (const cp &o) const { return (cp){a * o.a - b *
12 o.b, b * o.a + a * o.b}; }
13         cp operator* (const double &o) const { return (cp){a * o, b *
14 o}; }
15         cp operator!() const {return (cp){a, -b}; }
16     } w[MAX_LEN];
17     int pos[MAX_LEN];
18     cp x[MAX_LEN], y[MAX_LEN], z[MAX_LEN];
19
20     void fft_init(int len) {
21         int j = 0;
22         while ((1 << j) < len) ++j;
23         --j;
24         rep(i, 0, len) pos[i] = pos[i >> 1] >> 1 | ((i & 1) << j);
25     }
26
27     void fft(cp *x, int len, int sta) {
28         rep(i, 0, len) if (i < pos[i]) swap(x[i], x[pos[i]]);
29         w[0] = {1, 0};
30         for (int i = 2; i <= len; i <= 1) {
31             cp g = {cos(2 * pi / i), sin(2 * pi / i) * sta};
32             for (int j = i >> 1; j >= 0; j -= 2) w[j] = w[j >> 1];
33             for (int j = 1; j < i >> 1; j += 2) w[j] = w[j - 1] *
34 g;
35             for (int j = 0; j < len; j += i) {
36                 cp *a = x + j, *b = a + (i >> 1);
37                 for (int k = 0; k < i >> 1; k++) {
38                     cp o = b[k] * w[k];
39                     b[k] = a[k] - o; a[k] = a[k] + o;
40                 }
41             }
42         }
43         if (sta == -1) rep(i, 0, len) x[i].a /= len, x[i].b /= len;

```

```

44     }
45
46     void FFT(int *a, int *b, int n, int m, int *c) {
47         int len = 1;
48         while (len <= (n + m) >> 1) len <<= 1;
49         fft_init(len);
50         rep(i, (n >> 1), len) x[i].a = x[i].b = 0;
51         rep(i, (m >> 1), len) y[i].a = y[i].b = 0;
52         rep(i, 0, n) (i & 1 ? x[i >> 1].b : x[i >> 1].a) = a[i];
53         rep(i, 0, m) (i & 1 ? y[i >> 1].b : y[i >> 1].a) = b[i];
54         fft(x, len, 1); fft(y, len, 1);
55         rep(i, 0, (len >> 1)) {
56             int j = len - 1 & len - i;
57             z[i] = x[i] * y[i] - (x[i] - !x[j]) * (y[i] - !y[j]) *
58 (w[i] + (cp){1, 0}) * 0.25;
59         }
60         rep(i, (len >> 1), len) {
61             int j = len - 1 & len - i;
62             z[i] = x[i] * y[i] - (x[i] - !x[j]) * (y[i] - !y[j]) *
63 ((cp){1, 0} - w[i ^ len >> 1]) * 0.25;
64         }
65         fft(z, len, -1);
66         rep(i, 0, n + m) {
67             if (i & 1) c[i] = (int)(z[i >> 1].b + 0.5);
68             else c[i] = (int)(z[i >> 1].a + 0.5);
69         }
70     }
71 #undef MAX_LEN
72 } f;
73 char s1[maxm], s2[maxm];
74 int a[maxm], b[maxm], c[maxm << 1];
75 inline void solve() {
76     int n;
77     while (~scanf("%s%s", s1, s2)) {
78         int len1 = strlen(s1);
79         int len2 = strlen(s2);
80         memset(c, 0, sizeof c);
81         rep(i, 0, len1) a[i] = s1[len1 - i - 1] - '0';
82         rep(i, 0, len2) b[i] = s2[len2 - i - 1] - '0';
83         f.FFT(a, b, len1, len2, c);
84         rep(i, 0, len1 + len2 - 1) {
85             c[i + 1] += c[i] / 10;
86             c[i] %= 10;
87         }
88         int len = len1 + len2 - 1;
89         while (c[len] == 0 && len > 0) --len;
90         per(i, 0, len + 1) putchar(c[i] + '0');
91         puts("");
92     }

```



```
93 }
    ②FWT
    ③NTT
```

4.积性函数

①莫比乌斯反演

5.组合数

①线性递推

```
1  C[1][0] = C[1][1] = 1;
2  for (int i = 2; i < maxn; i++) {
3      C[i][0] = 1;
4      for (int j = 1; j < maxn; j++) {
5          C[i][j] = (C[i - 1][j] + C[i - 1][j - 1]) % mod;
6      }
7  }
```

②错排列

```
1  rev[1] = 0; rev[2] = 1;
2  for (int i = 3; i <= maxn; ++i)
3      rev[i] = (i-1)*(rev[i-1]+rev[i-2]);
```

③第二类斯特林数

```
1  for (int i = 0; i < MX ; ++i) {
2      S[i][0] = 0; S[i][i] = 1;
3      for (int j = 1; j < i ; ++j) {
4          S[i][j] = (S[i - 1][j - 1] + ((i - 1) * S[i - 1][j]));
5      }
6  }
```

④卡特兰数

```
1  catalan[0] = 1;
2  for (int i = 1; i <= 100 ; i++ ) {
3      catalan[i] = 0;
4      for (int j = 0; j < i; j++ ) {
5          catalan[i] = catalan[i] + catalan[j] * catalan[i - j - 1];
6      }
7  }
```

6.Polya

7.组合游戏

8.数值分析

七、计算几何

1.基本函数

2.二维、三维最近点对

3.凸包

4.半平面交

5.旋转卡壳

八、代数

1.矩阵快速幂

```
1  class Matrix {
2  #define N 2
3  public:
4      int a[N][N];
5      int n;
6
7      void Init(int key) {
8          memset(a, 0, sizeof a);
9          if (key)
10             rep(i, 0, n) a[i][i] = 1;
11     }
12
13     Matrix operator+ (const Matrix &b) {
14         Matrix c;
15         c.n = n;
16         rep(i, 0, n) rep(j, 0, n) c.a[i][j] = (a[i][j] + b.a[i][j]) %
17 mod;
18         return c;
19     }
20
21     Matrix operator+ (int x) {
22         Matrix p = *this;
23         rep(i, 0, n) p.a[i][i] = (p.a[i][i] + x % mod) % mod;
24         return p;
25     }
```

```

26
27     Matrix operator- (const Matrix &b) {
28         Matrix c;
29         c.n = n;
30         rep(i, 0, n) rep(j, 0, n) c.a[i][j] = (a[i][j] - b.a[i][j] +
31 mod) % mod;
32         return c;
33     }
34
35     Matrix operator* (const Matrix &b) {
36         Matrix c;
37         c.n = n;
38         c.Init(0);
39         rep(i, 0, n) rep(j, 0, n) rep(k, 0, n)
40             c.a[i][j] = (c.a[i][j] + a[i][k] * b.a[k][j] % mod) %
41 mod;
42         return c;
43     }
44
45     Matrix Power(int t) {
46         Matrix ans, p = *this;
47         ans.n = p.n;
48         ans.Init(1);
49         while (t) {
50             if (t & 1) ans = ans * p;
51             p = p * p;
52             t >>= 1;
53         }
54         return ans;
55     }
56
57     void Print() {
58         rep(i, 0, n) {
59             rep(j, 0, n) {
60                 if (j == 0) printf("%d", a[i][j]);
61                 else printf(" %d", a[i][j]);
62             }
63             puts("");
64         }
65     }
66     #undef N
67 };

```

2.高斯消元

①普通消元法

```

1  bool gauss(int n) {
2      rep(i, 1, n + 1) {
3          int r = i;
4          rep(j, i + 1, n + 1)

```

```

5         if (fabs(a[j][i]) > fabs(a[r][i])) r = j;
6         if (r != i) rep(j, 1, n + 2) swap(a[r][j], a[i][j]);
7         rep(k, i + 1, n + 1) {
8             double y = a[k][i] / a[i][i];
9             rep(j, i, n + 2) a[k][j] -= y * a[i][j];
10        }
11    }
12    per(i, 1, n + 1) {
13        rep(j, i + 1, n + 1) a[i][n + 1] -= a[j][n + 1] * a[i][j];
14        if (fabs(a[i][i]) <= eps && fabs(a[i][n + 1]) > eps)
15            return 0;
16        a[i][n + 1] /= a[i][i];
17    }
18    return 1;
19 }

```

Process —— 改一改高斯消元

```

1  per(j, i, n + 2) {
2      rep(k, i + 1, n + 1) {
3          a[k][j] -= a[k][i] / a[i][i] * a[i][j];
4      }
5  }

```

②异或消元法

```

1  ll V[maxn << 2];
2  int cir, tot;
3  void XORgauss() {
4      for(ll i = b[61]; i; i >>= 1) {
5          int j = tot + 1;
6          while(j <= cir && !(V[j]&i))j++;
7          if(j == cir + 1) continue;
8          tot++;
9          swap(V[tot], V[j]);
10         rep(k, 1, cir + 1)
11             if(k != tot && (V[k]&i))
12                 V[k] ^= V[tot];
13     }
14 }

```

九、JAVA 类

1.基础语法

头文件:

```

import java.io.*;
import java.util.*;
import java.math.*;

```

读入:

```

Scanner cin = Scanner (System.in);
while(cin.hasNext())//等价于!=EOF
n = cin.nextInt();//读入一个int型的数

```

```
n = cin.nextBigInteger();//读入一个大整数
```

输出:

```
System.out.print(n);//打印n
```

```
System.out.println();//换行
```

```
System.out.printf("%d\n",n);//也可以类似 c++里的输出方式
```

定义变量:

```
int i,j,k,a[];
```

```
a = new int[100];
```

```
BigInteger n,m;
```

```
String s;
```

数据类型:

布尔型 **boolean** 1 true,false false

字节型 **byte** 8 -128-127 0

字符型 **char** 16 '\u000' - \uffff '\u0000'

短整型 **short** 16 -32768-32767 0

整型 **int** 32 -2147483648,2147483647 0

长整型 **long** 64 -9.22E18,9.22E18 0

浮点型 **float** 32 1.4E-45-3.4028E+38 0.0

双精度型 **double** 64 4.9E-324,1.7977E+308 0.0

2.BigInteger

基本函数:

valueOf(parameter); 将参数转换为制定的类型

```
BigInteger b = BigInteger.valueOf(a);
```

```
String s = "12345";
```

```
BigInteger c = BigInteger.valueOf(s);
```

add(); 大整数相加

```
BigInteger a = new BigInteger("23");
```

```
BigInteger b = new BigInteger("34");
```

```
a.add(b);
```

subtract(); 相减

multiply(); 相乘

divide(); 相除取整

remainder(); 取余

pow();

```
a.pow(b) = a ^ b
```

gcd(); 最大公约数

abs(); 绝对值

negate(); 取反数

mod();

```
a.mod(b) = a % b = a.remainder(b);
```

max(); **min()**;

```
int compareTo();
```

```
boolean equals(); 是否相等
```

构造函数:

BigInteger(String val); 将指定字符串转换为十进制表示形式;

BigInteger(String val, int radix); 将指定基数的 BigInteger 的字符串表示形式转换为 BigInteger

基本常量:

```
A = BigInteger.ONE 1
```

```

B = BigInteger.TEN 10
C = BigInteger.ZERO 0
    基本操作：
Scanner cin = new Scanner(System.in); // 读入
while(cin.hasNext()) { //等同于!=EOF
    int n;
    BigInteger m;
    n = cin.nextInt(); //读入一个int;
    m = cin.BigInteger(); //读入一个BigInteger;
    System.out.print(m.toString());
}
if( a.compareTo(b) == 0 ) System.out.println("a == b"); //大整数a==b
else if( a.compareTo(b) > 0 ) System.out.println("a > b"); //大整数a>b
else if( a.compareTo(b) < 0 ) System.out.println("a < b"); //大整数a<b
//大整数绝对值
System.out.println(a.abs()); //大整数a的绝对值
//大整数的幂
int exponent = 10;
System.out.println(a.pow(exponent)); //大整数a的exponent次幂
//返回大整数十进制的字符串表示
System.out.println(a.toString());
//返回大整数p进制的字符串表示
int p = 8;
System.out.println(a.toString(p));

```

3.BigDecimal

基本介绍：

```

BigDecimal BigDecimal(double d); //不允许使用
BigDecimal BigDecimal(String s); //常用,推荐使用
static BigDecimal valueOf(double d); //常用,推荐使用

```

常用函数介绍：

```

BigDecimal remainder(BigDecimal divisor); //求余数
BigDecimal negate(); //求相反数
int compareTo(BigDecimal val);
/* compareTo:
 * 将此 BigDecimal 与指定的 BigDecimal 比较
 * 根据此方法,值相等但具有不同标度的两个 BigDecimal 对象 (如, 2.0 和 2.00) 被认为是相等的;
 * 相对六个 boolean 比较运算符 (<, ==, >, >=, !=, <=) 中每一个运算符的各个方法,优先提供此方法;
 * 建议使用以下语句执行上述比较 : (x.compareTo(y) <op> 0), 其中 <op> 是六个比较运算符之一;
 *
 * 指定者 : 接口 Comparable<BigDecimal> 中的 compareTo
 * 返回 : 当此 BigDecimal 在数字上小于、等于或大于 val 时, 返回 -1、0 或 1
 */

```

样例：

```

1 package com.util;
2 import java.math.BigDecimal;
3 /**
4  * 提供精确的浮点数运算(包括加、减、乘、除、四舍五入)工具类
5  */

```

```

6  public class ArithUtil {
7      // 除法运算默认精度
8      private static final int DEF_DIV_SCALE = 10;
9      private ArithUtil() {}
10
11     /**
12      * 精确加法
13      */
14     public static double add(double value1, double value2) {
15         BigDecimal b1 = BigDecimal.valueOf(value1);
16         BigDecimal b2 = BigDecimal.valueOf(value2);
17         return b1.add(b2).doubleValue();
18     }
19
20     /**
21      * 精确减法
22      */
23     public static double sub(double value1, double value2) {
24         BigDecimal b1 = BigDecimal.valueOf(value1);
25         BigDecimal b2 = BigDecimal.valueOf(value2);
26         return b1.subtract(b2).doubleValue();
27     }
28
29     /**
30      * 精确乘法
31      */
32     public static double mul(double value1, double value2) {
33         BigDecimal b1 = BigDecimal.valueOf(value1);
34         BigDecimal b2 = BigDecimal.valueOf(value2);
35         return b1.multiply(b2).doubleValue();
36     }
37
38     /**
39      * 精确除法 使用默认精度
40      */
41     public static double div(double value1, double value2) throws IllegalAccessException {
42         return div(value1, value2, DEF_DIV_SCALE);
43     }
44
45     /**
46      * 精确除法
47      * @param scale 精度
48      */
49     public static double div(double value1, double value2, int scale) throws
50     IllegalAccessException {
51         if(scale < 0) {
52             throw new IllegalAccessException("精确度不能小于0");
53         }
54         BigDecimal b1 = BigDecimal.valueOf(value1);

```

```

55         BigDecimal b2 = BigDecimal.valueOf(value2);
56         // return b1.divide(b2, scale).doubleValue();
57         return b1.divide(b2, scale, BigDecimal.ROUND_HALF_UP).doubleValue();
58     }
59
60     /**
61      * 四舍五入
62      * @param scale 小数点后保留几位
63      */
64     public static double round(double v, int scale) throws IllegalAccessException {
65         return div(v, 1, scale);
66     }
67
68     /**
69      * 比较大小
70      */
71     public static boolean equalTo(BigDecimal b1, BigDecimal b2) {
72         if(b1 == null || b2 == null) {
73             return false;
74         }
75         return 0 == b1.compareTo(b2);
76     }
77 }

```