

一、

1. 无散无旋

2. 零

$$3. \quad \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$4. \quad \vec{E} = -\nabla U - \frac{\partial \vec{A}}{\partial t}$$

5. 等相位面上电场、磁场的大小、方向均相等；等相位面为平面

6. 连续；连续；不连续

7. 零

8. 无；有；有；

$$9. \quad \text{反射系数的模值为 } 1; \text{ 反射系数为零}; \quad \theta_B = \arctan \sqrt{\frac{\mu_1 \varepsilon_1}{\mu_2 \varepsilon_2}}; \quad \theta_c = \arcsin \sqrt{\frac{\mu_2 \varepsilon_2}{\mu_1 \varepsilon_1}}$$

$$10. \quad a < \lambda < 2a; \quad \lambda < a$$

二、

1. 线极化

2. 右旋椭圆极化

3. 右旋椭圆极化

4. 左旋椭圆极化

5. 左旋圆极化

三、

将 $\rho_l dz'$ 看成为一个点电荷，则其镜像电荷位于 $\frac{a^2}{z'}$ ，电荷大小为 $-\frac{a}{z'} \rho_l dz'$ 。

则在导体球外的任意一点处电位为

$$dU = \frac{\rho_l dz'}{4\pi\varepsilon_0 R_1} + \frac{-\frac{a}{z'} \rho_l dz'}{4\pi\varepsilon_0 R_2}, \quad \text{其中 } R_1 = \sqrt{\rho^2 + (z - z')^2};$$

那么整个线段的总电位为

$$U = \int_b^{b+l} \frac{\rho_l dz'}{4\pi\varepsilon_0 R_1} + \frac{-\frac{a}{z'} \rho_l dz'}{4\pi\varepsilon_0 R_2} = \frac{\rho_l}{4\pi\varepsilon_0} \int_b^{b+l} \left(\frac{1}{R_1} + \frac{-\frac{a}{z'}}{R_2} \right) dz';$$

$$\begin{aligned}
& \frac{\rho_l}{4\pi\epsilon_0} \int_b^{b+l} \frac{1}{R_1} dz' = \frac{\rho_l}{4\pi\epsilon_0} \int_b^{b+l} \frac{1}{\sqrt{\rho^2 + (z-z')^2}} dz' \\
& = \frac{\rho_l}{4\pi\epsilon_0} \ln \left[(z'-z) + \sqrt{\rho^2 + (z-z')^2} \right] \Big|_b^{b+l} \\
& = \frac{\rho_l}{4\pi\epsilon_0} \ln \left[\frac{(b-l-z) + \sqrt{\rho^2 + (b-l-z)^2}}{(b-z) + \sqrt{\rho^2 + (b-z)^2}} \right]
\end{aligned}$$

第二部分积分计算：

$$\begin{aligned}
U_m &= \frac{\rho_l}{4\pi\epsilon_0} \int_b^{b+l} \left(\frac{-\frac{a}{z'}}{R_2} \right) dz' = \frac{\rho_l}{4\pi\epsilon_0} \int_b^{b+l} \frac{-\frac{a}{z'}}{\sqrt{\rho^2 + (z-\frac{a^2}{z'})^2}} dz' = \frac{\rho_l}{4\pi\epsilon_0} \int_b^{b+l} \frac{-a}{\sqrt{(z'\rho)^2 + (z'z-a^2)^2}} dz' \\
&= -\frac{a\rho_l}{4\pi\epsilon_0} \int_b^{b+l} \frac{1}{\sqrt{(z')^2(\rho^2 + z^2) - 2(z a^2)z' + a^4}} dz' = -\frac{a\rho_l}{4\pi\epsilon_0} \int_b^{b+l} \frac{1}{\sqrt{(z')^2 R^2 - 2(z a^2)z' + a^4}} dz' \\
&= -\frac{a\rho_l}{4\pi\epsilon_0 R} \int_b^{b+l} \frac{1}{\sqrt{(z')^2 - \frac{2(z a^2)}{R^2} z' + \frac{a^4}{R^2}}} dz' = -\frac{a\rho_l}{4\pi\epsilon_0 R} \int_b^{b+l} \frac{1}{\sqrt{\left(z' - \frac{z a^2}{R^2}\right)^2 - \frac{z^2 a^4}{R^4} + \frac{a^4}{R^2}}} dz'
\end{aligned}$$

$$\text{令 } Z = \frac{z a^2}{R^2}; \quad Q^2 = -\frac{z^2 a^4}{R^4} + \frac{a^4}{R^2};$$

则上式变为

$$U_m = -\frac{a\rho_l}{4\pi\epsilon_0 R} \int_b^{b+l} \frac{1}{\sqrt{(z'-Z)^2 + Q^2}} dz' = -\frac{a\rho_l}{4\pi\epsilon_0 R} \ln \left[\frac{(b-l-Z) + \sqrt{Q^2 + (b-l-Z)^2}}{(b-Z) + \sqrt{Q^2 + (b-Z)^2}} \right]$$

四、

解：求解空间电位满足拉普拉斯方程， $\nabla^2 \varphi = 0$

边界条件： $\varphi(x, 0) = 0$ ， $\varphi(x, d) = 0$ ， $0 < x < \infty$

$\varphi(0, y) = 0$ ， $\varphi(x \rightarrow \infty, y) = 0$ ， $0 < y < d$

通解为：

$$\begin{aligned}
\varphi(\rho, \phi) &= (A_0 + B_0 x)(C_0 + D_0 y) + \\
&\sum_{n=1}^{\infty} (A_n \sinh k_n x + B_n \cosh k_n x) (C_n \sin k_n y + D_n \cos k_n y)
\end{aligned}$$

进行待定系数求解，傅里叶级数展开
得到电位结果为

$$\varphi(x, y) = \sum_{n=1,3,5}^{\infty} \frac{4U_0}{n\pi} \sin \frac{n\pi}{d} y e^{-\frac{n\pi}{d} x}$$

五、

$$1) \quad k = \omega \sqrt{\varepsilon_r \varepsilon_0 \mu_r \mu_0} = \omega \sqrt{\varepsilon_r \mu_r} \sqrt{\varepsilon_0 \mu_0} = \frac{\omega \sqrt{\varepsilon_r \mu_r}}{c} = \frac{10^{10} \pi 9}{3 \cdot 10^8} = 300\pi$$

$$\eta = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_r \varepsilon_0}} = \sqrt{\frac{\mu_r}{\varepsilon_r}} \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{1}{9} 120\pi = \frac{40}{3} \pi$$

$$v_p = \frac{\omega}{k} = \frac{\omega}{\omega \sqrt{\varepsilon_r \mu_r}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = 0.33e10^8$$

$$2) \quad \vec{H}(y, t) = 0.1 \sin(10e10\pi t - \pi/3 - ky) \hat{x} \quad A/m$$

$$\vec{H}(y, t) = 0.1 \cos\left(10e10\pi t - \pi/3 - ky - \frac{\pi}{2}\right) \hat{x}$$

$$\Rightarrow \vec{H}(y) = 0.1 \exp\left(j\left(10e10\pi t - ky - \frac{5\pi}{6}\right)\right) \hat{x}$$

$$\vec{E}(y) = -\eta \hat{k} \times \vec{H}(y) = \frac{4}{3} \pi \hat{z} \exp\left(j\left(10e10\pi t - ky - \frac{5\pi}{6}\right)\right) \quad V/m$$

$$3) \quad \vec{P} = \frac{1}{2} \text{Re}[\vec{E}(y) \times \vec{H}^*(y)] = \frac{\pi}{15} \hat{y} \quad W$$

六、

(1) 左旋圆极化波

$$(2) \quad \vec{E}_r = -\vec{e}_x E_0 \cos(\omega t + kz) - \vec{e}_y E_0 \sin(\omega t + kz)$$

(3) 右旋圆极化波

七、

$$1) \quad TE_{10}: f_c = \frac{\sqrt{\left(\frac{1}{a}\right)^2}}{2\sqrt{\varepsilon_0 \mu_0}} = \frac{c}{2a}$$

$$\beta = k \sqrt{1 - \frac{f_c^2}{f^2}} = \frac{2\pi f}{c} \sqrt{1 - \frac{f_c^2}{f^2}} = 80\pi \sqrt{1 - \left(\frac{1}{80a}\right)^2} = 102.65 \text{ rad/m} \Rightarrow a = 0.0137 \text{ m}$$

2)

$$\lambda_g = \frac{2\pi}{\beta} = 0.0612 \text{ m}; \quad v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = 7.345e8$$

$$\eta_{TE} = \frac{\omega \mu}{\beta} = \frac{\omega \mu}{k \sqrt{1 - \frac{f_c^2}{f^2}}} = \eta \left(1 - \frac{f_c^2}{f^2}\right)^{-\frac{1}{2}} = 120\pi \sqrt{1 - \left(\frac{1}{80a}\right)^2} = 154.295 \Omega$$

3) 已知 TE_{10} 模的纵向磁场为: $H_z(x, y, z) = H_m \cos(\frac{\pi}{a}x)e^{-j\gamma z}$

则横向磁场和电场为:

$$H_x(x, y, z) = -\frac{j\beta}{k_c^2} \nabla_t H_z = \frac{ja\beta}{\pi} H_m \sin(\frac{\pi}{a}x)e^{-j\gamma z}$$

$$E_y(x, y, z) = -Z_{TE} \frac{ja\beta}{\pi} H_m \sin(\frac{\pi}{a}x)e^{-j\gamma z}$$

$$\vec{S}_{av} = \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}^*] = \frac{1}{2} \text{Re}[(\vec{e}_y E_y) \times (\vec{e}_x H_x + \vec{e}_z H_z)^*]$$

$$= -\hat{z} \frac{1}{2} \text{Re}(E_y H_x^*) = \hat{z} \frac{1}{2} \frac{|E_y|^2}{Z_{TE_{10}}} = \hat{z} \frac{1}{2} Z_{TE_{10}} |H_x|^2 = \hat{z} \frac{1}{2} Z_{TE_{10}} \left| \frac{a\beta}{\pi} H_m \sin(\frac{\pi}{a}x) \right|^2$$