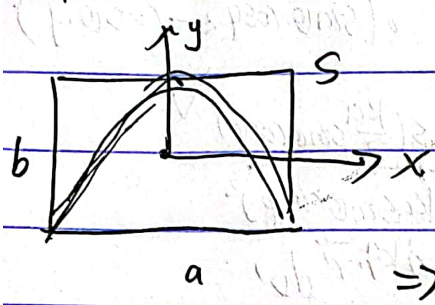
# 第二次作业. 王子菡 1120210446

解3.2: 对于 $TE_{10}$ 波：有 $k_x=\frac{\pi}{a}$ $k_y=0$ , $k_c=k_x$




$k^2=k_x^2+k_y^2+k_z^2 \Rightarrow k_z^2=\sqrt{\omega^2\varepsilon\mu-(\frac{\pi}{a})^2}$

$\vec{E}_s=-\frac{jk_x\omega\mu}{k_c^2}\cos\frac{\pi}{a}x' e^{-jk_z z'}$

由于在S上, $z'=0$

$\Rightarrow \vec{E}_s=-\frac{jk_x\omega\mu}{k_c^2}\cos\frac{\pi}{a}x'\hat{y}=-\frac{ja\omega\mu}{\pi}\cos\frac{\pi}{a}x'\hat{y}$

由等效原理, $\vec{m}=\vec{E}_s\times\hat{n}=\vec{E}_s\times\hat{z}=-\frac{ja\omega\mu}{\pi}\cos\frac{\pi}{a}x'\hat{y}\times\hat{z}$

$=-\frac{j\omega\mu a}{\pi}\cos\frac{\pi}{a}x'\hat{x}$

由镜像原理 $2\vec{m}$ 产生的场与原场一致.

$\Rightarrow \vec{E}=D\hat{r}\times\int_v 2\vec{m}\, e^{jk\vec{r}'\cdot\hat{r}}dv'$

$=2D\int_s j\frac{\omega\mu a}{\pi}\cos\frac{\pi}{a}x'\,\hat{x}\times\hat{r}\cdot e^{jk\vec{r}'\cdot\hat{r}}ds'$

$=2D\frac{j\omega\mu a}{\pi}\int_s\cos\frac{\pi}{a}x'\, e^{jk(x'\sin\theta\cos\varphi+y'\sin\theta\sin\varphi)}\,\hat{x}\times\hat{r}\, ds'$

而 $\hat{x}\times\hat{r}=\hat{x}\times(\hat{x}\sin\theta\cos\varphi+\hat{y}\sin\theta\sin\varphi+\hat{z}\cos\theta)$

$=\hat{z}\sin\theta\sin\varphi-\hat{y}\cos\theta$

$\Rightarrow\vec{E}=2D\frac{ja\omega\mu}{\pi}(\hat{z}\sin\theta\sin\varphi-\hat{y}\cos\theta)\int_s\cos\frac{\pi}{a}x'e^{jk(x'\sin\theta\cos\varphi+y'\sin\theta\sin\varphi)}ds'$

而 $\int_s\cos\frac{\pi}{a}x'e^{j(x'\sin\theta\cos\varphi+y'\sin\theta\sin\varphi)k}ds'$

$=\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\cos\frac{\pi}{a}x'e^{j(x'\sin\theta\cos\varphi)k}\cdot e^{j(y'\sin\theta\sin\varphi)k}dy'dx'$

$=\int_{-\frac{b}{2}}^{\frac{b}{2}}e^{j(y'\sin\theta\sin\varphi)k}dy'\cdot\int_{-\frac{a}{2}}^{\frac{a}{2}}\cos\frac{\pi}{a}x'e^{j(x'\sin\theta\cos\varphi)k}dx'$

$=\int_{-\frac{b}{2}}^{\frac{b}{2}}\cos(ky'\sin\theta\sin\varphi)+j\sin(y'\sin\theta\sin\varphi k)dy'\times$

$\int_{-\frac{a}{2}}^{\frac{a}{2}}\cos\frac{\pi}{a}x'[\cos(x'k\sin\theta\cos\varphi)+j\sin(x'\sin\theta\cos\varphi k)]dx'$

$=\int_{-\frac{b}{2}}^{\frac{b}{2}}[\cos(ky'\sin\theta\sin\varphi)+0]dy'\cdot\int_{-\frac{a}{2}}^{\frac{a}{2}}[\cos\frac{\pi}{a}x'\cdot\cos(kx'\sin\theta\cos\varphi)]dx'$

$=\frac{2}{k\sin\theta\sin\varphi}\cdot\sin(\frac{bk\sin\theta\sin\varphi}{2})\cdot\frac{2\pi a}{\pi^2-(ka\sin\theta\cos\varphi)^2}\cdot\cos(\frac{ka}{2}\sin\theta\cos\varphi)$

$\Rightarrow\vec{E}=\frac{2Dja\omega\mu}{\pi}(\hat{z}\sin\theta\sin\varphi-\hat{y}\cos\theta)\cdot\frac{2\sin(\frac{bk\sin\theta\sin\varphi}{2})}{k\sin\theta\sin\varphi}\cdot\frac{2\pi a\cos(\frac{ka}{2}\sin\theta\cos\varphi)}{\pi^2-(ka\sin\theta\cos\varphi)^2}$

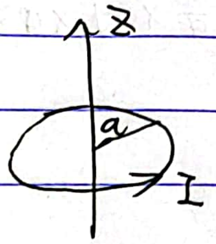
$=2\cdot jk\frac{e^{-jkr}}{4\pi r}\cdot\frac{ja\omega\mu}{\pi}(\hat{z}\sin\theta\sin\varphi-\hat{y}\cos\theta)\frac{2\sin(\frac{bk\sin\theta\sin\varphi}{2})}{k\sin\theta\sin\varphi}\cdot\frac{2\pi a\cos(\frac{ka}{2}\sin\theta\cos\varphi)}{\pi^2-(ka\sin\theta\cos\varphi)^2}$

$\rightarrow$ 故 $\vec{E} = -\dfrac{e^{-jkr}\cdot wu}{2\pi^2 r}\cdot\dfrac{2\sin(\frac{bk}{2}\sin\theta\sin\varphi)}{\sin\theta\sin\varphi}\cdot\dfrac{2\pi a\cos(\frac{ka}{2}\sin\theta\cos\varphi)}{\pi^2-(ka\sin\theta\cos\varphi)^2}(\sin\theta\cos\varphi\hat{z}-\cos\theta\hat{y})$

$\cdot(\sin\theta\cos\varphi\hat{z}-\cos\theta\hat{y})$

$$= -\frac{e^{-jkr}}{2\pi^2 r}\cdot kwu\cdot\frac{2\sin(\frac{bk}{2}\sin\theta\sin\varphi)}{k\sin\theta\sin\varphi}\cdot\frac{2\pi a\cos(\frac{ka}{2}\sin\theta\cos\varphi)}{\pi^2-(ka\sin\theta\cos\varphi)^2}$$

解3.3: 在远场 $\vec{E} = ZD\int_{V'}(\hat\theta\hat\theta+\hat\varphi\hat\varphi)\cdot\vec{J}\cdot e^{jk\hat{r}\cdot\vec{r}'}dv'$




而 $I = I\hat\varphi'$

$\hat\varphi' = -\hat{x}\sin\varphi' + \hat{y}\cos\varphi'$

$\hat\varphi = -\hat{x}\sin\varphi + \hat{y}\cos\varphi$

$\hat\theta = \hat{x}\cos\theta\cos\varphi + \hat{y}\cos\theta\sin\varphi - \hat{z}\sin\theta$

$\Rightarrow \hat\theta\hat\theta\cdot\hat\varphi' = (-\sin\varphi'\cos\theta\cos\varphi + \cos\varphi'\cos\theta\sin\varphi)\hat\theta$

$\hat\varphi\hat\varphi\cdot\hat\varphi' = \qquad = \cos\theta\sin(\varphi-\varphi')\hat\theta$

$= (\sin\varphi\sin\varphi' + \cos\varphi\cos\varphi')\hat\varphi = \cos(\varphi-\varphi')\hat\varphi$

$\hat{r} = \hat{x}\sin\theta\cos\varphi + \hat{y}\sin\theta\sin\varphi + \hat{z}\cos\theta$

设半径为a: $\vec{r}' = \hat{x}a\cos\varphi' + \hat{y}a\sin\varphi'$

$\Rightarrow \hat{r}\cdot\vec{r}' = a(\sin\theta\cos\varphi\cos\varphi' + \sin\theta\sin\varphi\sin\varphi')$

$= a\sin\theta\cos(\varphi-\varphi')$

$\Rightarrow \vec{E} = ZD\int_l(\hat\theta\hat\theta+\hat\varphi\hat\varphi)\vec{J}e^{jk\hat{r}\cdot\vec{r}'}dv$

$= ZDI\int_0^{2\pi}[\cos\theta\sin(\varphi-\varphi')\hat\theta + \cos(\varphi-\varphi')\hat\varphi]\cdot e^{jka\sin\theta\cos(\varphi-\varphi')}\,ad\varphi'$

$\because$ a很小, $\Rightarrow e^{jka\sin\theta\cos(\varphi-\varphi')} \approx 1 + jka\sin\theta\cos(\varphi-\varphi')$

$\Rightarrow \vec{E} = ZDIa[\int_0^{2\pi}\cos\theta\sin(\varphi-\varphi')\hat\theta + jka\sin\theta\cos(\varphi-\varphi')\cos\theta\sin(\varphi-\varphi')\hat\theta d\varphi'$

$+\int_0^{2\pi}\cos(\varphi-\varphi')\hat\varphi + jka\sin\theta\cos(\varphi-\varphi')\cos(\varphi-\varphi')\hat\varphi\, d\varphi'$

$= ZDIa[0+0+0+\frac{1}{2}jka\sin\theta\cdot 2\pi\,\hat\varphi]$

$= ZDIa\pi\cdot jka\hat\varphi\sin\theta = jkZDI\hat\varphi\pi a^2\sin\theta = jkZDIS\sin\theta\,\hat\varphi$

$= -jk\,ZI\frac{e^{-jkr}}{4\pi r}S\sin\theta\,\hat\varphi\cdot jk = k^2 ZIS\frac{e^{-jkr}}{4\pi r}\hat\varphi\sin\theta$

$= \frac{k^2}{?} z I S \frac{e^{-jkr}}{4\pi r} \hat{\varphi} \sin\theta = \frac{z I \pi S}{\lambda^2 r} e^{-jkr} \sin\theta \ \hat{\varphi}$

故 $E_\varphi = z \frac{\pi I S}{\lambda^2 r} e^{-jkr} \sin\theta$

在远场，$\vec{E}$ 近似只有 $\hat{\varphi}$ 方向，传播方向为 $\hat{r}$ 方向

故 $\vec{H} = \frac{1}{z} \hat{r} \times \vec{E} = \frac{1}{z} E_\varphi \hat{r} \times \hat{\varphi} = -\frac{1}{z} E_\varphi \hat{\theta}$

故 $H_\theta = -E_\varphi / z$

综上所述 $E_\varphi = z \frac{\pi I S}{\lambda^2 r} e^{-jkr} \sin\theta$，$H_\theta = -\frac{E_\varphi}{z}$ 证毕.

3.4：由书上式 3.2.10a 知.

证明：在远场有 $\vec{E}_1 = \frac{j\omega\mu I L}{4\pi r} \sin\theta \, e^{-jkr} \hat{\theta}$ （IL 产生的电场）

由题3.3知：$\vec{E}_2 = z \frac{\pi I S}{\lambda^2 r} e^{-jkr} \sin\theta \, \hat{\varphi}$ （IS产生的电场）

$$\Rightarrow \vec{E} = \vec{E}_1 + \vec{E}_2 = \left( \frac{j\omega\mu I L}{4\pi r} \hat{\theta} + z \frac{\pi I S}{\lambda^2 r} \hat{\varphi} \right) e^{-jkr} \sin\theta$$

$$= \left( \frac{j\omega\mu I L}{4\pi r} \hat{\theta} + z \frac{k^2 I S}{4\pi r} \hat{\varphi} \right) e^{-jkr} \sin\theta .$$

$$= \left[ \frac{j\omega\mu I L}{4\pi r} \hat{\theta} + \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{\omega \sqrt{\varepsilon\mu} \, k I S}{4\pi r} \hat{\varphi} \right] e^{-jkr} \sin\theta$$

$$= \left[ \frac{j\omega\mu I L}{4\pi r} \hat{\theta} + \frac{\omega\mu I L}{4\pi r} \hat{\varphi} \right] e^{-jkr} \sin\theta$$

$$= \frac{e^{-jkr}}{4\pi r} \cdot \omega\mu I L \sin\theta \ ( j\hat{\theta} + \hat{\varphi} )$$

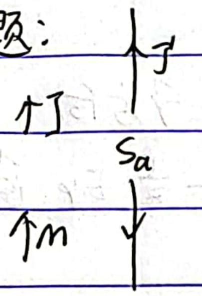
故 $\vec{E}$ 的方向与传播方向 ($\hat{r}$) 垂直，且 $\hat{\theta}$ 和 $\hat{\varphi}$ 方向上 $\vec{E}$ 的相位相差 $\frac{\pi}{2}$，幅度也相同，故 $\vec{E}$ 是圆极化的.

根据对偶定理 可知 $\vec{E}$ 和 $\vec{H}$ 具有相类似的形式，也是圆极化的.

故 辐射场为圆极化，证毕.

证明思考题3.4：

对于a问题：

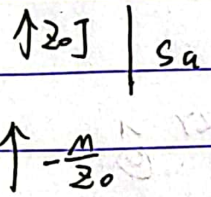



将感应原问题在金属屏上感应出了$\vec{J}'$，导致电场发生了变化。

根据题3.1，$\vec{J}'$ 在Sa不产生切向磁场。

故有 $\begin{cases} \vec{n}\times\vec{E}^a = 0 & r\notin S_a \\ \vec{n}\times\vec{H}^a = \vec{n}\times\vec{H}^i & r\in S_a \end{cases}$ ①

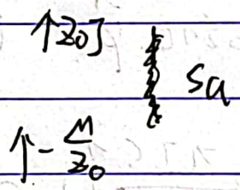
对于c问题：




对其对偶：$\begin{cases} J_c = Z_0 J \\ M_c = -\frac{M}{Z_0} \end{cases} \Rightarrow \begin{cases} M_b = -\frac{M}{Z_0} \\ J_c = Z_0 J \end{cases}$

可见对偶后的电流源与磁流源是一致的，对偶后板变为理想磁导体。

对于b问题：




同a问题，可认为磁导体上产生等效磁流，该磁流在 $r\notin S_a$ 不产生切向电场

故有：$\begin{cases} \vec{n}\times\vec{E}^b = \vec{n}\times\vec{E}^i & r\notin S_a \\ \vec{n}\times H^b = 0 & r\in S_a \end{cases}$ ②

①+②得 $\begin{cases} \vec{n}\times(\vec{E}^b+\vec{E}^a) = \vec{n}\times\vec{E}^i & r\notin S_a & a+b\text{切向电场确定} \\ \vec{n}\times(\vec{H}^b+\vec{H}^a) = \vec{n}\times\vec{H}^i & r\in S_a & a+b\text{切向磁场确定} \end{cases}$

根据唯一性定理：边界场完全一样

则 $a+b=i$ 即场完全一致。$\Rightarrow \begin{cases} \vec{E}^b+\vec{E}^a = \vec{E}^i \\ \vec{H}^b+\vec{H}^a = \vec{H}^i \end{cases}$

根据b与c的对偶性 $\Rightarrow \begin{cases} \vec{H}^b \to -\frac{1}{Z_0}\vec{E}^c \\ \vec{E}^b \to Z_0\vec{H}^c \end{cases}$，此时对偶前后电流源与磁流源一致

$\Rightarrow \begin{cases} \vec{E}^a + Z_0\vec{H}^c = \vec{E}^i \\ \vec{H}^a - \frac{1}{Z_0}\vec{E}^c = \vec{H}^i \end{cases}$

证毕