

第二章 电磁波的传播和传输

吴比翼

北京理工大学集成电路与电子学院
射频技术与软件研究所

biyi.wu@bit.edu.cn



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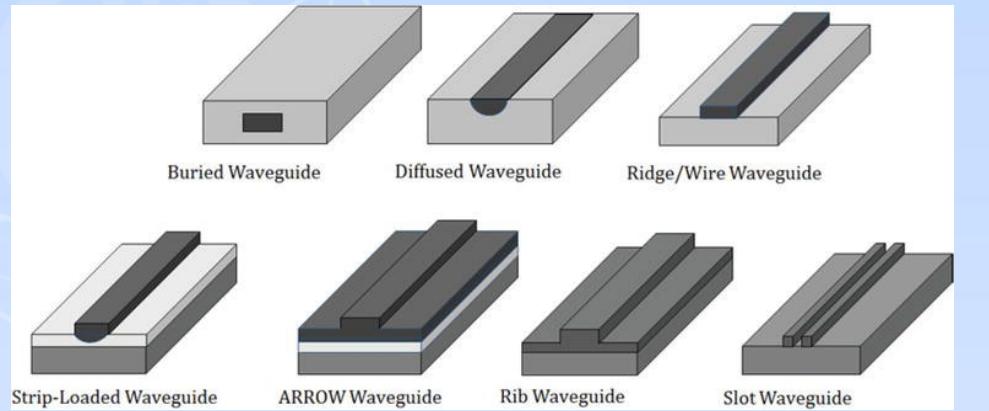
Beijing Institute of Technology

第2章 电磁波的传播和传输

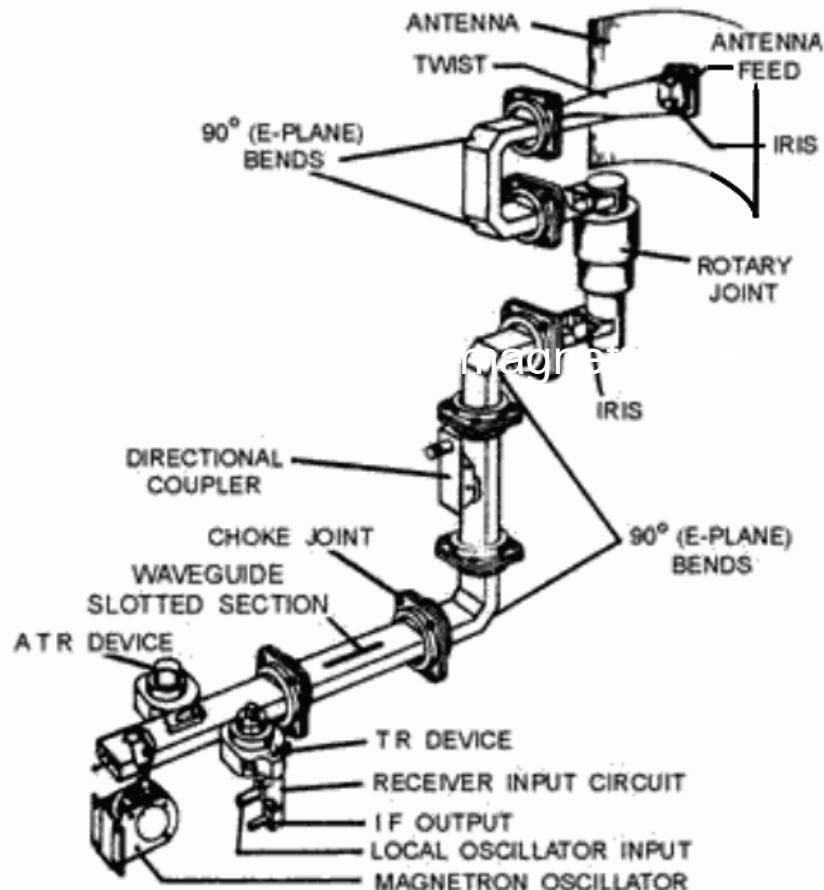
- 电磁波传播
- 波导中的传输

什么是波导

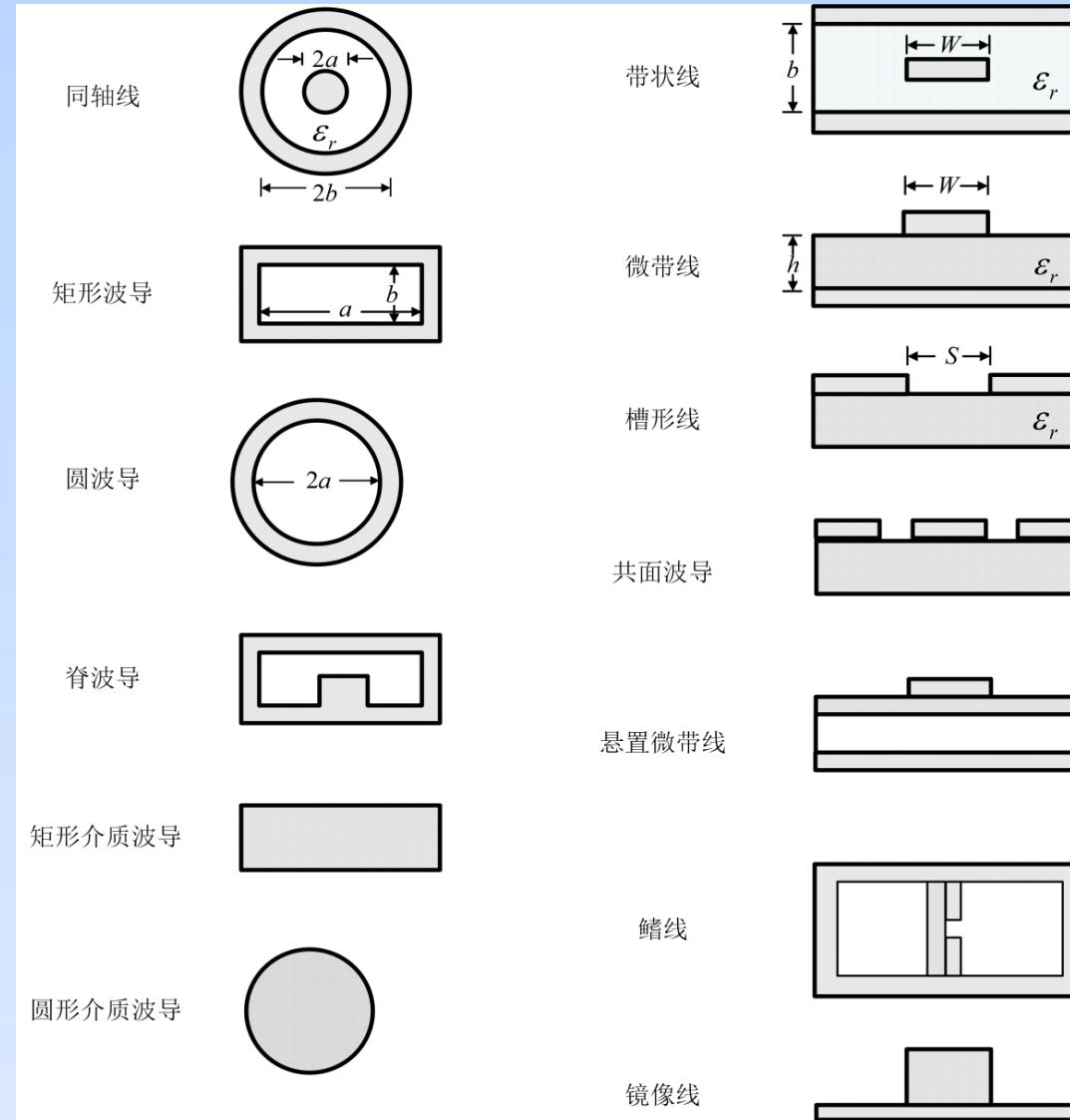
A **waveguide** is a structure that guides waves, such as electromagnetic waves or sound, with minimal loss of energy by restricting the transmission of energy to **one direction**. Without the physical constraint of a waveguide, wave intensities decrease according to the inverse square law as they expand into three dimensional space.



雷达系统中波导



波导中的传输



波导中的传输

- 波导传输问题的求解途径
- 矩形波导中电磁波的传输特性
- 波导正规模的特性
- 任意截面空波导电磁波传输模式的有限元分析
- 波导激励分析

波导传输问题的求解途径

矢量亥姆霍茨方程(2.1.7)的z分量可简化成标量亥姆霍茨方程

$$\left(\nabla^2 + k^2\right)E_z = 0 \quad (2.2.1)$$

其中 $k = \omega\sqrt{\mu\varepsilon}$.

下面证明(2.2.1)式

(2.1.6)式中 $\nabla \times \nabla \times \mathbf{E}$ 的z分量是由 $\nabla \times \mathbf{E}$ 的横向分量决定。

$\nabla = \nabla_t + \hat{e}_z \frac{\partial}{\partial z}$, 展开计算可得

$$\begin{aligned} \left. \left(\nabla_t + \hat{z} \frac{\partial}{\partial z} \right) \times \mathbf{E} \right|_t &= \left. \nabla_t \times \mathbf{E} + \hat{z} \times \frac{\partial}{\partial z} \mathbf{E} \right|_t \\ &= \nabla_t \times (\hat{z} E_z) + \hat{z} \times \frac{\partial E_t}{\partial z} = \nabla_t E_z \times \hat{z} + \hat{z} \times \frac{\partial E_t}{\partial z} \end{aligned} \quad (2.2.2a)$$

波导传输问题的求解途径

整理得

$$(\nabla \times \mathbf{E})_t = \left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \times \hat{\mathbf{z}} \quad (2.2.2b)$$

$$(\nabla \times \nabla \times \mathbf{E})_z \hat{\mathbf{z}} = \nabla \times (\nabla \times \mathbf{E})_t = \nabla_t \times \left[\left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \times \hat{\mathbf{z}} \right] \quad (2.2.3)$$

利用 $\nabla \times (A \times B) = A \nabla \cdot B - B \nabla \cdot A + (B \cdot \nabla) A - (A \cdot \nabla) B$ (2.2.4a)

得

$$\begin{aligned} \nabla_t \times \left[\left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \times \hat{\mathbf{z}} \right] &= \left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \nabla_t \cdot \hat{\mathbf{z}} - \hat{\mathbf{z}} \nabla_t \cdot \left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \\ &\quad + (\hat{\mathbf{z}} \cdot \nabla_t) \left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) - \left[\left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \cdot \nabla_t \right] \hat{\mathbf{z}} \end{aligned} \quad (2.2.4b)$$

波导传输问题的求解途径

又由于 $\hat{z} \cdot \nabla_t = 0, \nabla_t \cdot \hat{z} = 0, (\mathbf{A} \cdot \nabla_t) \hat{z} = 0$ (2.2.5a)

则 (2.2.3) 式可化成

$$(\nabla \times \nabla \times \mathbf{E})_z \hat{z} = -\hat{z} \nabla_t \cdot \left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) = \hat{z} \left(\nabla_t \cdot \frac{\partial \mathbf{E}_t}{\partial z} - \nabla_t^2 E_z \right) \quad (2.2.5b)$$

即 $(\nabla \times \nabla \times \mathbf{E})_z = \nabla_t \cdot \left(\frac{\partial \mathbf{E}_t}{\partial z} \right) - \nabla_t^2 E_z \quad (2.2.6)$

$$\begin{aligned} [\nabla(\nabla \cdot \mathbf{E})]_z &= \frac{\partial}{\partial z} \left[\left(\nabla_t + \hat{z} \frac{\partial}{\partial z} \right) \cdot \mathbf{E} \right] \\ &= \frac{\partial}{\partial z} \left(\nabla_t \cdot \mathbf{E} + \frac{\partial E_z}{\partial z} \right) = \frac{\partial(\nabla_t \cdot \mathbf{E})}{\partial z} + \frac{\partial^2 E_z}{\partial z^2} \end{aligned} \quad (2.2.7)$$

将(2.2.6)和(2.2.7)代入(2.1.6)得

$$(\nabla^2 \mathbf{E})_z = \nabla^2 E_z \quad (2.2.8)$$

于是标量亥姆霍茨方程(2.2.1)得证

波导传输问题的求解途径

同样可证得磁场标量亥姆霍茨方程

$$\left(\nabla^2 + k^2\right)H_z = 0 \quad (2.2.9)$$

下面将进一步导出由纵向场求出横向场的表达式

设纵向上的传播因子是 $e^{-\gamma z}$ ($\gamma = \alpha + j\beta$)

电磁波在波导中的空间分布可表述成

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}(t)e^{-\gamma z} \quad (2.2.10)$$

$$\nabla_z \times (\mathbf{E}e^{-\gamma z}) = \hat{\mathbf{z}} \frac{\partial}{\partial z} \times (\mathbf{E}e^{-\gamma z}) = \hat{\mathbf{z}} \times \left(\mathbf{E} \frac{\partial}{\partial z} e^{-\gamma z} \right) = -j\gamma \hat{\mathbf{z}} \times (\mathbf{E}e^{-\gamma z}) \quad (2.2.11a)$$

$$\nabla_z \cdot (\mathbf{E}e^{-\gamma z}) = \hat{\mathbf{z}} \frac{\partial}{\partial z} \cdot (\mathbf{E}e^{-\gamma z}) = \hat{\mathbf{z}} \cdot \left(\mathbf{E} \frac{\partial}{\partial z} e^{-\gamma z} \right) = -j\gamma \hat{\mathbf{z}} \cdot (\mathbf{E}e^{-\gamma z}) \quad (2.2.11b)$$

即有

$$\nabla_z \equiv -j\gamma \hat{\mathbf{z}} \quad (2.2.12b)$$

波导传输问题的求解途径

$$\begin{aligned}(\nabla \times \mathbf{E})_t &= \left[(\nabla_t + \nabla_z) \times (\hat{\mathbf{z}} E_z + \mathbf{E}_t) \right]_t \\&= \nabla_t \times (\hat{\mathbf{z}} E_z) + \nabla_z \times \mathbf{E}_t \\&= (\nabla_t E_z + \gamma \mathbf{E}_t) \times \hat{\mathbf{z}}\end{aligned}\quad (2.2.13)$$

根据法拉第定律(2.1.1)得

$$(\nabla_t E_z + \gamma \mathbf{E}_t) \times \hat{\mathbf{z}} = -j\omega\mu H_t \quad (2.2.14)$$

同样

$$(\nabla_t H_z + \gamma \mathbf{H}_t) \times \hat{\mathbf{z}} = j\omega\epsilon E_t \quad (2.2.15)$$

$$\mathbf{E}_t = j \frac{\omega\mu}{k_c^2} \hat{\mathbf{z}} \times \nabla_t H_z - \frac{\gamma}{k_c^2} \nabla_t E_z \quad (2.2.16)$$

截止波数(**Cutoff Wavenumber**) $k_c^2 = k^2 + \gamma^2$

截止波长(**Cutoff Wavelength**)、截止频率(**Cutoff Frequency**)

同样可推得

$$\mathbf{H}_t = -j \frac{\omega\epsilon}{k_c^2} \hat{\mathbf{z}} \times \nabla_t E_z - \frac{\gamma}{k_c^2} \nabla_t H_z \quad (2.2.17)$$

波导传输问题的求解途径

求解波导问题的具体方式：

先从(2.2.1)和(2.2.9)解出纵向场 E_z 和 H_z

再由(2.2.16)和(2.2.17)算出横向场。

方程(2.2.1)和(2.2.9)在(2.2.10)的假设下，可进一步简化为

$$\left(\nabla_t^2 + k_c^2\right)E_z = 0 \quad (2.2.18)$$

$$\left(\nabla_t^2 + k_c^2\right)H_z = 0 \quad (2.2.19)$$

波导中的传输

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矩形波导中电磁波的传输特性

TM模式

纵向场 E_z 除了满足(2.2.18)还满足

$$E_z \Big|_{x=0} = 0, \quad E_z \Big|_{x=a} = 0, \quad E_z \Big|_{y=0} = 0, \quad E_z \Big|_{y=b} = 0 \quad (2.2.20)$$

分离变量法解得

$$E_z = \sin k_x x \sin k_y y e^{-jk_z z} \quad (2.2.21)$$

$$k_x = \frac{m\pi}{a}, \quad m = 1, 2, \dots \quad (2.2.22a) \quad k_y = \frac{n\pi}{b}, \quad n = 1, 2, \dots \quad (2.2.22b)$$

且

$$k_x^2 + k_y^2 + k_z^2 = k_c^2 + k_z^2 = \omega^2 \mu \epsilon = k^2 \quad (2.2.23)$$

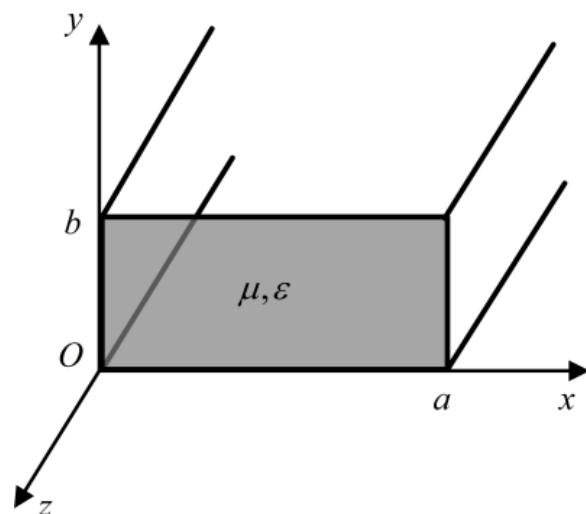


图2-6 矩形波导

矩形波导中电磁波的传输特性

由(2.2.16)和(2.2.17)可算出TM模式的横向分量

$$E_x = -\frac{jk_x k_z}{k_c^2} \cos k_x x \sin k_y y e^{-jk_z z} \quad (2.2.24)$$

$$E_y = -\frac{jk_y k_z}{k_c^2} \sin k_x x \cos k_y y e^{-jk_z z} \quad (2.2.25)$$

$$H_x = \frac{j\omega\epsilon k_y}{k_c^2} \sin k_x x \cos k_y y e^{-jk_z z} \quad (2.2.26)$$

$$H_y = -\frac{j\omega\epsilon k_x}{k_c^2} \cos k_x x \sin k_y y e^{-jk_z z} \quad (2.2.27)$$

式(2.2.22a)和(2.2.22b)中m和n的不同取值，对应于不同的 TM_{mn} 模，其截止波数为

$$k_{cmn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \quad (2.2.28)$$

矩形波导中电磁波的传输特性

$$E_t = -\frac{\gamma}{k_c^2} \nabla_t E_z \quad (2.2.29)$$

$$H_t = -j \frac{\omega \epsilon}{k_c^2} \hat{z} \times \nabla_t E_z \quad (2.2.30)$$

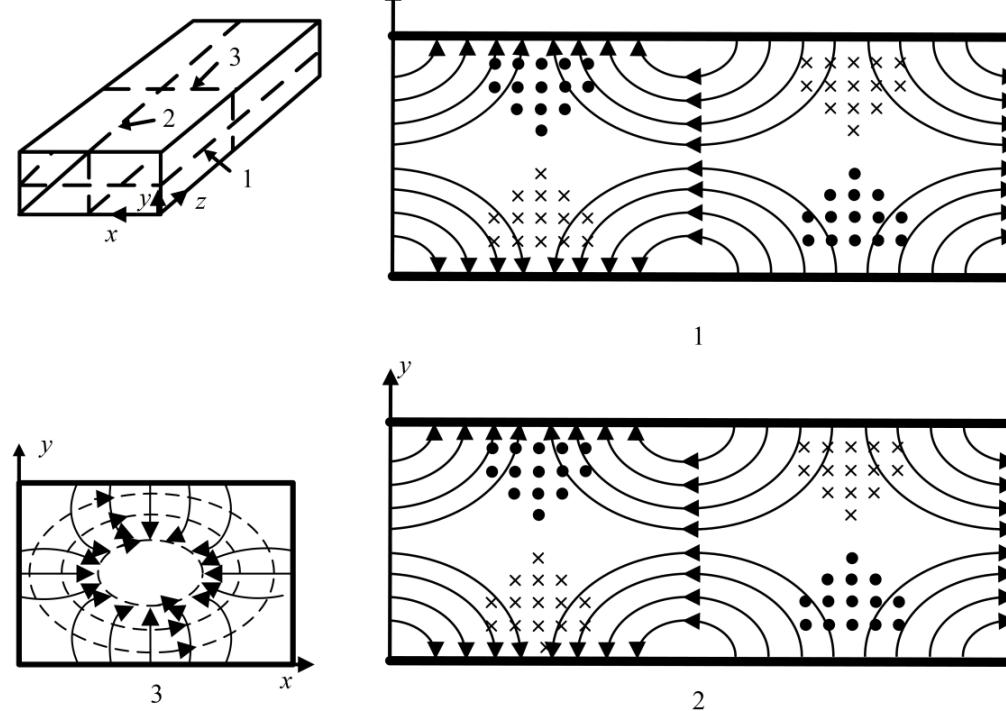


图2-7 矩形波导主模TM₁₁的场分布

矩形波导中电磁波的传输特性

TE模式

纵向场 H_z 除了满足(2.2.18)还满足

$$\frac{\partial H_z}{\partial n} \Big|_{x=0} = 0, \quad \frac{\partial H_z}{\partial n} \Big|_{x=a} = 0, \quad \frac{\partial H_z}{\partial n} \Big|_{y=0} = 0, \quad \frac{\partial H_z}{\partial n} \Big|_{y=b} = 0 \quad (2.2.31)$$

分离变量法解得

$$H_z = \cos k_x x \cos k_y y e^{-jk_z z} \quad (2.2.32)$$

$$k_x = \frac{m\pi}{a}, \quad m = 0, 1, 2 \dots \quad (2.2.33a)$$

$$k_y = \frac{n\pi}{b}, \quad n = 0, 1, 2 \dots \quad (2.2.33b)$$

注意m和n不能同时为零, k_x, k_y, k_z 同样满足色散关系(2.2.23)

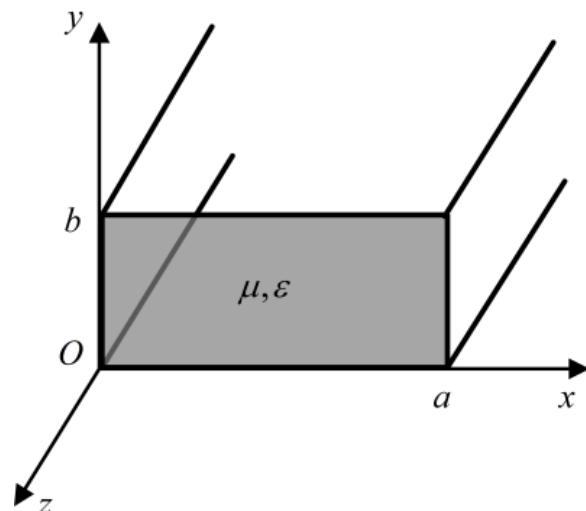


图2-6 矩形波导

矩形波导中电磁波的传输特性

由(2.2.16)和(2.2.17)可算出TE模式的横向分量

$$E_x = \frac{j\omega\mu k_y}{k_c^2} \cos k_x x \sin k_y y e^{-jk_z z} \quad (2.2.34)$$

$$E_y = -\frac{j\omega\mu k_x}{k_c^2} \sin k_x x \cos k_y y e^{-jk_z z} \quad (2.2.35)$$

$$H_x = \frac{jk_x k_z}{k_c^2} \sin k_x x \cos k_y y e^{-jk_z z} \quad (2.2.36)$$

$$H_y = \frac{jk_y k_z}{k_c^2} \cos k_x x \sin k_y y e^{-jk_z z} \quad (2.2.37)$$

TE模的最小截止波数要小于TM模。因此波导主模是TE模
对于 $a > b$ 情形，波导主模便是TE10，其截止波数为 $k_c = \pi / a$

场分布如图2-8

矩形波导中电磁波的传输特性

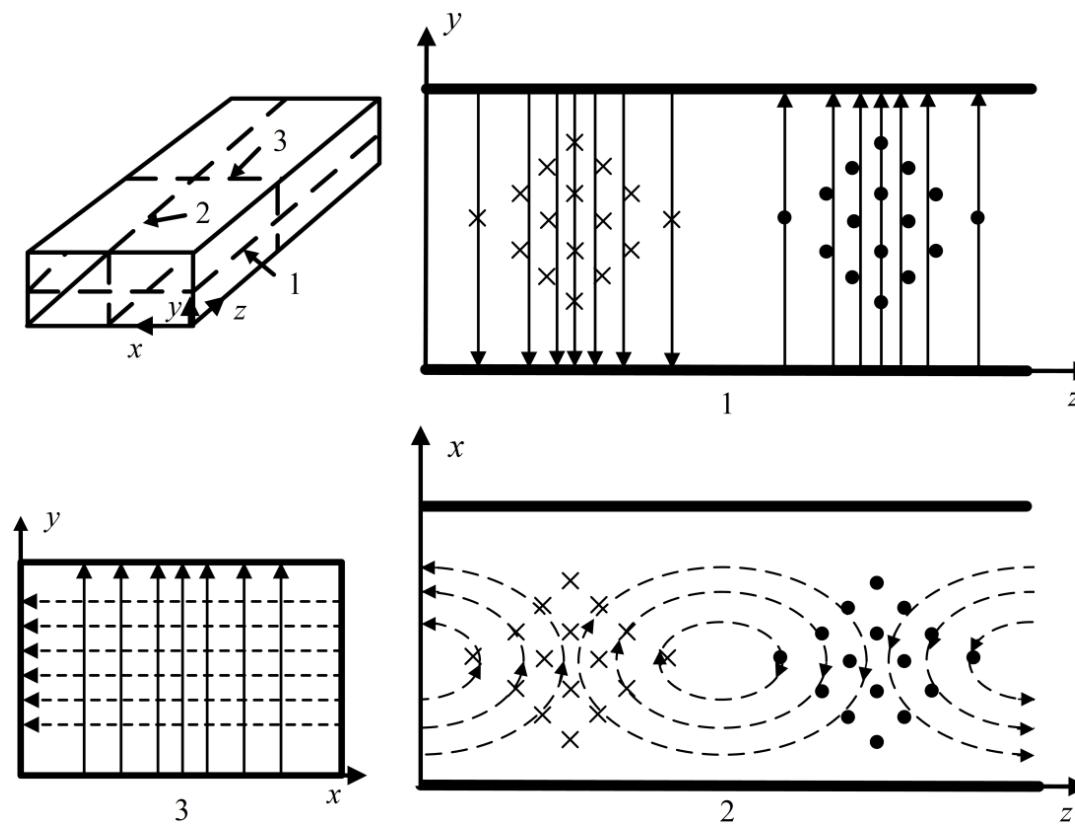


图2-8 矩形波导主模TE₁₀的场分布

例题2.6

矩形波导中电磁波的传输特性

单模传输

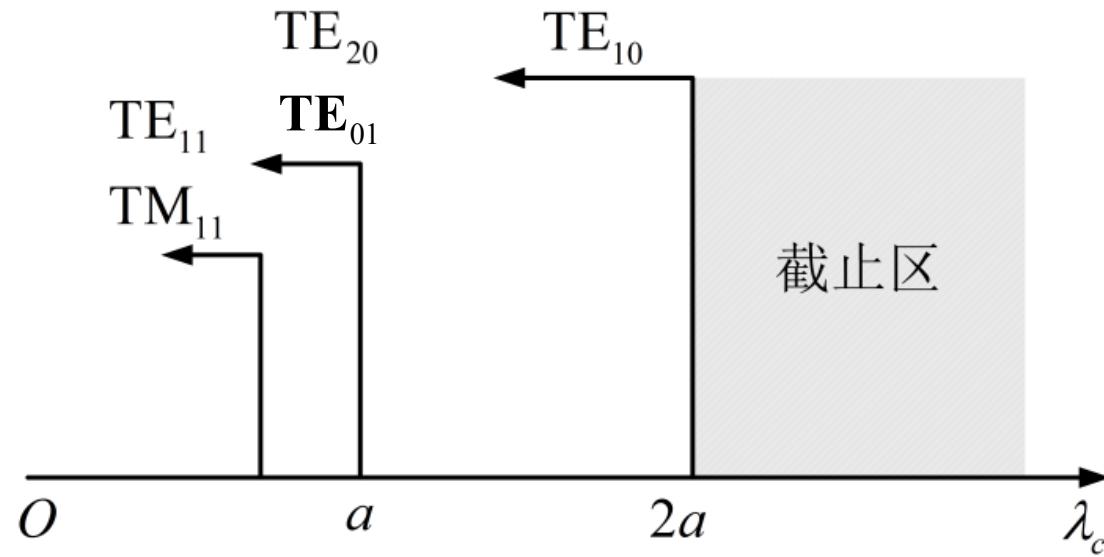


图2-9 $a=2b$ 的矩形波导，其截止波长分布

波导中的传输

- 波导传输问题的求解途径
- 矩形波导中电磁波的传输特性
- 波导正规模的特性
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- 波导激励分析

波导正规模的特性

假设波导中第m个模的场为 \mathbf{E}_m 、 \mathbf{H}_m ， 第n个模的场为 \mathbf{E}_n 、 \mathbf{H}_n

$$\nabla \times \mathbf{H}_m = j\omega \epsilon \mathbf{E}_m \quad (2.2.38)$$

$$\nabla \times \mathbf{E}_m = -j\omega \mu \mathbf{H}_m \quad (2.2.39)$$

$$\nabla \times \mathbf{H}_n = j\omega \epsilon \mathbf{E}_n \quad (2.2.40)$$

$$\nabla \times \mathbf{E}_n = -j\omega \mu \mathbf{H}_n \quad (2.2.41)$$

\mathbf{H}_n 点乘(2.2.39)减去 \mathbf{E}_m 点乘(2.2.40)得

$$\mathbf{H}_n \cdot \nabla \times \mathbf{E}_m - \mathbf{E}_m \cdot \nabla \times \mathbf{H}_n = \nabla \cdot (\mathbf{E}_m \times \mathbf{H}_n) = j\omega (-\mathbf{H}_n \cdot \mu \mathbf{H}_m - \mathbf{E}_m \cdot \epsilon \mathbf{E}_n) \quad (2.2.42)$$

\mathbf{E}_n 点乘式(2.2.38)减去 \mathbf{H}_m 点乘式(2.2.41)得

$$\mathbf{E}_n \cdot \nabla \times \mathbf{H}_m - \mathbf{H}_m \cdot \nabla \times \mathbf{E}_n = \nabla \cdot (\mathbf{H}_m \times \mathbf{E}_n) = j\omega (\mathbf{H}_m \cdot \mu \mathbf{H}_n + \mathbf{E}_n \cdot \epsilon \mathbf{E}_m) \quad (2.2.43)$$

波导正规模的特性

式(2.2.42)加上(2.2.43)得

$$\nabla \cdot (E_m \times H_n - E_n \times H_m) = 0 \quad (2.2.44)$$

对式(2.2.44)在波导 z 和 $z+\Delta z$ 两平面及波导内壁所围区域积分得

$$\int_{S_1} (E_m \times H_n - E_n \times H_m) \cdot (-\hat{z}) ds + \int_{S_c} (E_m \times H_n - E_n \times H_m) \cdot \hat{n} ds \quad (2.2.45)$$

$$+ \int_{S_2} (E_m \times H_n - E_n \times H_m) \cdot (\hat{z}) ds = 0$$

$$E_m = E_{tm} + E_{zm} \hat{z}$$

$$\int_{S_1} (E_{tm} \times H_{tn} - E_{tn} \times H_{tm}) \cdot (-\hat{z}) ds + \int_{S_2} (E_{tm} \times H_{tn} - E_{tn} \times H_{tm}) \cdot (\hat{z}) ds = 0 \quad (2.2.46)$$

波导正规模的特性

如果第m个模和第n个模都沿正z方向传输

$$\mathbf{E}_{tm} = e^{-\gamma_m z} \mathbf{e}_m(x, y), \quad \mathbf{H}_{tm} = \frac{1}{Z_m} e^{-\gamma_m z} \mathbf{h}_m(x, y) \quad (2.2.47a)$$

$$\mathbf{E}_{tn} = e^{-\gamma_n z} \mathbf{e}_n(x, y), \quad \mathbf{H}_{tn} = \frac{1}{Z_n} e^{-\gamma_n z} \mathbf{h}_n(x, y) \quad (2.2.47b)$$

$$\mathbf{h}_m(x, y) = \hat{\mathbf{z}} \times \mathbf{e}_m(x, y) \quad (2.2.48a) \quad \int_S \mathbf{e}_m(x, y) \cdot \mathbf{e}_m(x, y) dS = 1 \quad (2.2.48a)$$

$$(\gamma_m + \gamma_n) \int_s \left(\frac{1}{Z_n} \mathbf{e}_m \times \mathbf{h}_n - \frac{1}{Z_m} \mathbf{e}_n \times \mathbf{h}_m \right) \cdot (\hat{\mathbf{z}}) ds = 0 \quad (2.2.49)$$

显然 $\gamma_m + \gamma_n \neq 0$ ，故

$$\int_s \left(\frac{1}{Z_n} \mathbf{e}_m \times \mathbf{h}_n - \frac{1}{Z_m} \mathbf{e}_n \times \mathbf{h}_m \right) \cdot (\hat{\mathbf{z}}) ds = 0 \quad (2.2.50)$$

波导正规模的特性

式(2.2.50)和(2.2.52b)相加

$$\int_S \mathbf{e}_m \times \mathbf{h}_n \cdot \hat{\mathbf{z}} dS = 0, \quad \gamma_m \neq \gamma_n \quad (2.2.53a)$$

利用(2.2.48a)和(2.2.48b)

$$\int_S \mathbf{e}_m \times \mathbf{h}_n \cdot \hat{\mathbf{z}} dS = \delta_{mn} \quad (2.2.53b)$$

$$\int_S \mathbf{e}_m \cdot \mathbf{e}_n dS = \delta_{mn} \quad (2.2.53c)$$

波导正规模最为一般的正交性

$$\mathbf{E} = \sum_{m,n} a_{mn} \mathbf{e}_{TEmn} + \sum_{m,n} b_{mn} \mathbf{e}_{TMmn} \quad (2.2.54)$$

波导模式的完备性

例题2.6

试求等腰直角三角形波导(图2-9)中最低阶E波和H波的截止波长。

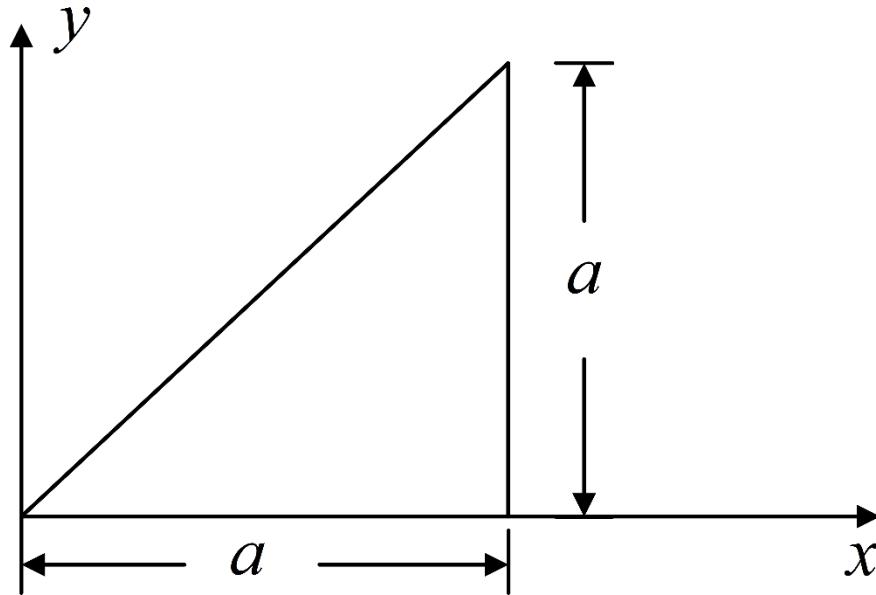


图2-9 等腰直角三角形波导

例题2.6

解：在方形波导的对角线处加一导体平面即得到等腰直角三角形波导。故求解时可利用方形波导中的场解。

(1) E波：设等腰直角三角形波导中的纵向电场分量 E_z 为方形波导中两个不同的E型波的叠加：

$$E_z = A \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a} + B \sin \frac{p\pi x}{a} \sin \frac{q\pi y}{a} \quad (e2.6.1)$$

为了满足 $(\nabla_t^2 + k_t^2)E_z = 0$ ，横向波数 k_t 应为

$$k_t^2 = \left(\frac{\pi}{a}\right)^2 (m^2 + n^2) = \left(\frac{\pi}{a}\right)^2 (p^2 + q^2) \quad (e2.6.2)$$

在 $y=0$ 和 $x=a$ 上 $E_z = 0$ ，满足边界条件；在弦 $y=x$ 上，

$$E_z|_{y=x} = A \sin m\xi \sin n\xi + B \sin p\xi \sin q\xi \quad (e2.6.3)$$

$$\begin{aligned} &= \frac{A}{2} [\cos(m-n)\xi - \cos(m+n)\xi] + \frac{B}{2} [\cos(p-q)\xi - \cos(p+q)\xi] \\ &= 0 \end{aligned}$$

例题2.6

若取正整数 r ,令 $m - p = q - n = r$ (e2.6.4a)

由式(e2.6.2)可得 $m + p = q + n$ (e2.6.4b)

上二式又可写为 $m + n = p + q, m - n = -(p - q)$ (e2.6.5)

$A = -B = 1$ 时式(e2.6.3)成立。又由上式可知, $m = q, n = p$

因此式(e2.6.1)和式(e2.6.2)变为

$$E_z = \sin \frac{(n+r)\pi x}{a} \sin \frac{n\pi y}{a} - \sin \frac{n\pi x}{a} \sin \frac{(n+r)\pi y}{a} \quad (e2.6.1)$$

$$k_t = \frac{\pi}{a} \sqrt{(n+r)^2 + n^2}, \quad n, r = 1, 2, \dots \quad (e2.6.2)$$

可见, 最低E波的截止波长为

$$\lambda_c = 2\pi/k_t = 2a/\sqrt{5}$$

例题2.6

返回

(2) H波: 考虑方形波导 TE_{11} 波型的电场分布(图2-10)

根据场型分割原理

由 $y=0, y=x$ 和 $x=l$ 平面围成的等腰直角三角形波导和由 $y=0, y=x$ 和 $y=l-x$ 平面围成的等腰直角三角形波导中，纵向磁场分量均为

$$H_z = \cos \frac{\pi x}{l} \cos \frac{\pi y}{l} \quad (\text{e2.6.3})$$

对于前者，当直角边为 a 时，

$$l=a, \text{ 截止波长 } \lambda_c = \sqrt{2}l = \sqrt{2}a$$

对于后者，当直角边为 a 时，

$$l=\sqrt{2}a, \text{ 截止波长 } \lambda_c = \sqrt{2}l = 2a$$

因此，最低H波的 $\lambda_c = 2a$

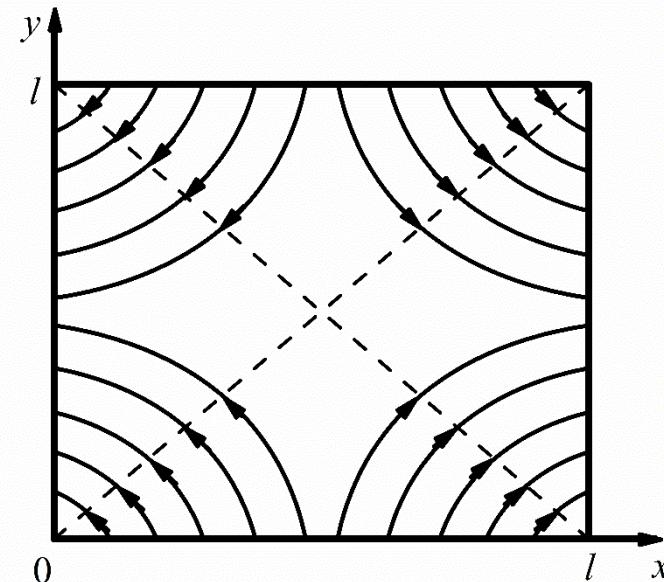


图2-10 方形波导 TE_{11} 波型的电场分布