第二章 电磁波的传播和传输

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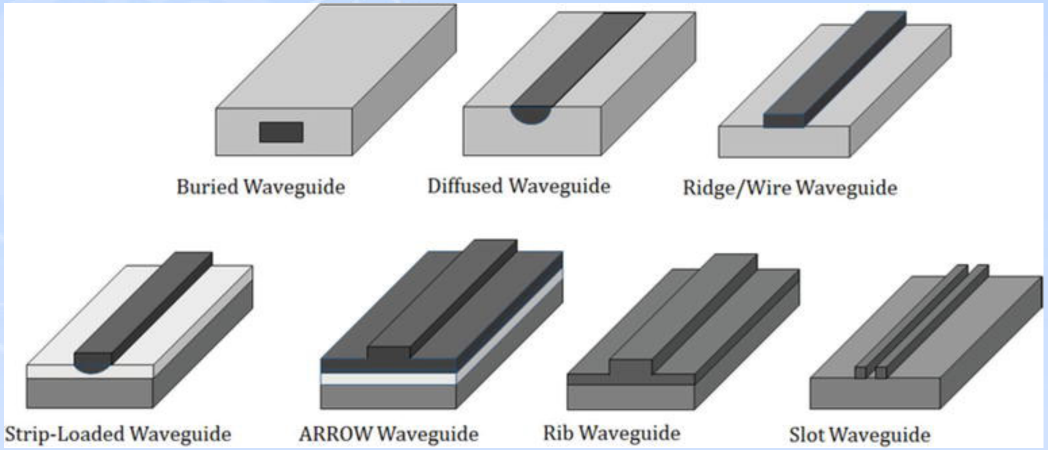
第2章 电磁波的传播和传输

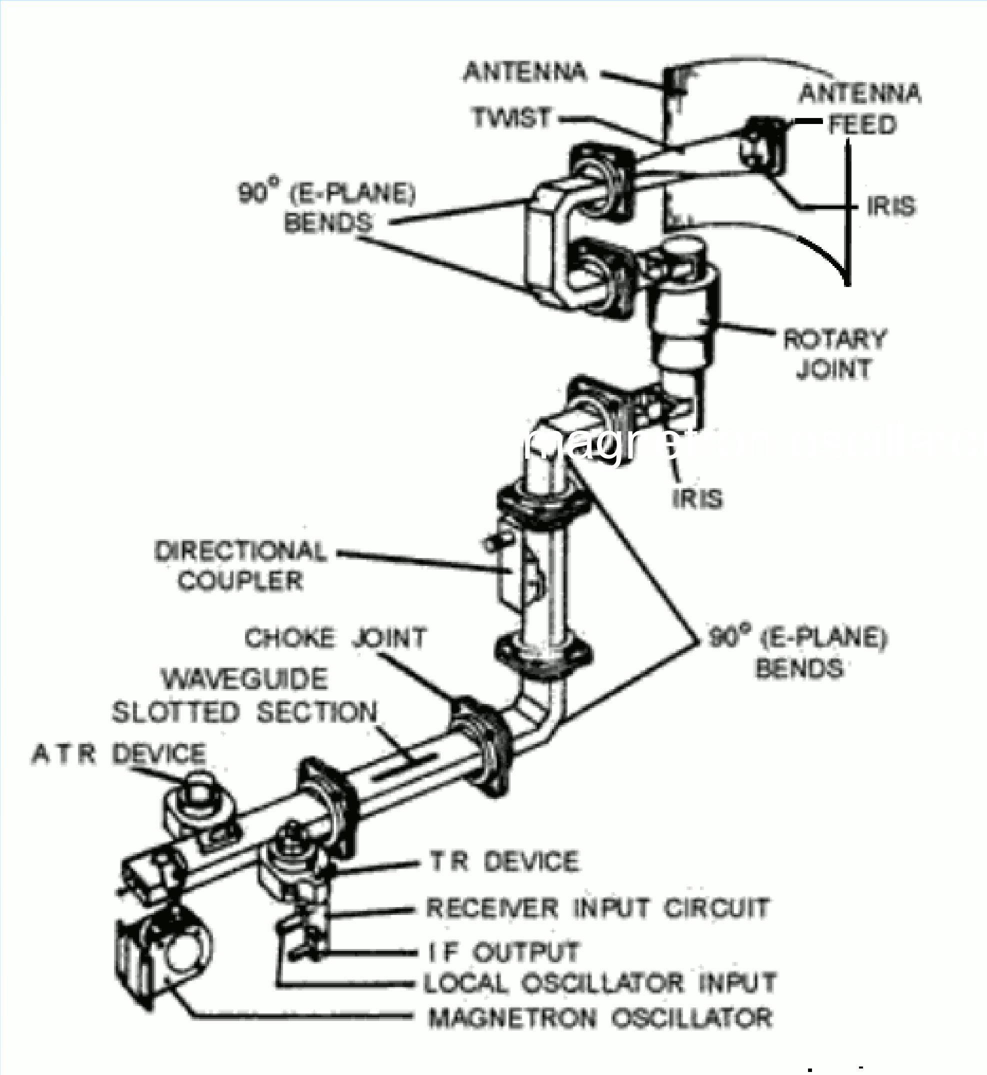
- 电磁波传播
- 波导中的传输

什么是波导

A **waveguide** is a structure that guides waves, such as electromagnetic waves or sound, with minimal loss of energy by restricting the transmission of energy to **one direction**. Without the physical constraint of a waveguide, wave intensities decrease according to the inverse square law as they expand into three dimensional space.

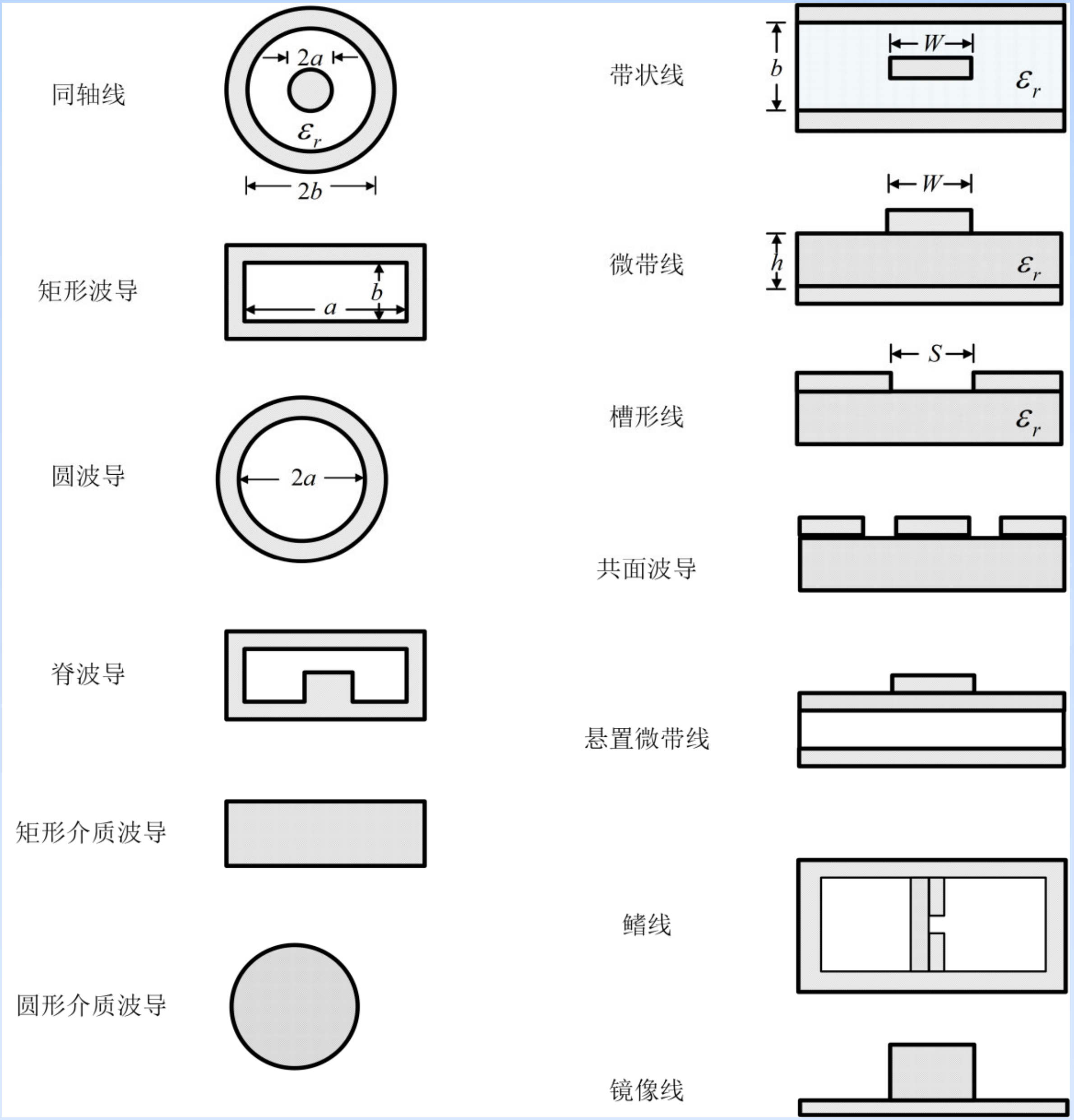


雷达系统中波导


ANTENNA
TWIST
ANTENNA FEED
90° (E-PLANE) BENDS
IRIS
ROTARY JOINT
IRIS
DIRECTIONAL COUPLER
CHOKE JOINT
90° (E-PLANE) BENDS
WAVEGUIDE SLOTTED SECTION
A T R DEVICE
T R DEVICE
RECEIVER INPUT CIRCUIT
I F OUTPUT
LOCAL OSCILLATOR INPUT
MAGNETRON OSCILLATOR


AIR OUTLET COVER MUST BE LATCHED IN OPEN POSITION TO OPERATE EQUIPMENT

波导中的传输


同轴线
2a
ε_r
2b
矩形波导
a
b
圆波导
2a
脊波导
矩形介质波导
圆形介质波导
带状线
W
b
ε_r
微带线
W
h
ε_r
槽形线
S
ε_r
共面波导
悬置微带线
鳍线
镜像线

波导中的传输

- 波导传输问题的求解途径
- 矩形波导中电磁波的传输特性
- 波导正规模的特性
- 任意截面空波导电磁波传输模式的有限元分析
- 波导激励分析

波导传输问题的求解途径

$$\nabla^2\mathbf{E}+k^2\mathbf{E}=0$$

$$\nabla=\nabla_t+\hat{z}\frac{\partial}{\partial z}$$

$$\left(\nabla^2+k^2\right)E_z=0$$

$$e^{-\gamma z}$$

$$\gamma=\alpha+j\beta$$

$$\boldsymbol{E}(\boldsymbol{r})=\boldsymbol{E}(t)e^{-\gamma z}$$

$$\boldsymbol{E}_t=j\frac{\omega\mu}{k_c^2}\hat{\boldsymbol{z}}\times\nabla_t H_z-\frac{\gamma}{k_c^2}\nabla_t E_z$$

$$\boldsymbol{H}_t=-j\frac{\omega\varepsilon}{k_c^2}\hat{\boldsymbol{z}}\times\nabla_t E_z-\frac{\gamma}{k_c^2}\nabla_t H_z$$

$$\left(\nabla_t^2+k_c^2\right)E_z=0$$

$$\left(\nabla_t^2+k_c^2\right)H_z=0$$

波导传输问题的求解途径

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$$\boldsymbol{H}_t=-j\frac{\omega\varepsilon}{k_c^2}\hat{\boldsymbol{z}}\times\nabla_t E_z-\frac{\gamma}{k_c^2}\nabla_t H_z$$

TE波

$$\boldsymbol{E}_t=j\frac{\omega\mu}{k_c^2}\hat{\boldsymbol{z}}\times\nabla_t H_z \quad \boldsymbol{H}_t=-\frac{\gamma}{k_c^2}\nabla_t H_z$$

TM波

$$\boldsymbol{E}_t=-\frac{\gamma}{k_c^2}\nabla_t E_z \quad \boldsymbol{H}_t=-j\frac{\omega\varepsilon}{k_c^2}\hat{\boldsymbol{z}}\times\nabla_t E_z$$

$$\left(\nabla_t^2+k_c^2\right)E_z=0$$

$$\left(\nabla_t^2+k_c^2\right)H_z=0$$

矩形波导中电磁波的传输特性

TE波

$$\boldsymbol{E}_t = j\frac{\omega\mu}{k_c^2}\hat{\boldsymbol{z}}\times\nabla_t H_z \quad \boldsymbol{H}_t = -\frac{\gamma}{k_c^2}\nabla_t H_z$$

$$Z_{\mathrm{TE}} = \frac{j\omega\mu}{-\gamma}$$

TM波

$$\boldsymbol{E}_t = -\frac{\gamma}{k_c^2}\nabla_t E_z \quad \boldsymbol{H}_t = -j\frac{\omega\varepsilon}{k_c^2}\hat{\boldsymbol{z}}\times\nabla_t E_z$$

$$Z_{TM} = \frac{\gamma}{j\omega\epsilon}$$

波导中的传输

- 波导传输问题的求解途径
- 矩形波导中电磁波的传输特性
- 波导正规模的特性
- 任意截面空波导电磁波传输模式的有限元分析
- 波导激励分析

矩形波导中电磁波的传输特性

TM模式

纵向场 E_z 除了满足(2.2.18)还满足

$$E_z\big|_{x=0}=0,\quad E_z\big|_{x=a}=0,\quad E_z\big|_{y=0}=0,\quad E_z\big|_{y=b}=0$$

(2.2.20)

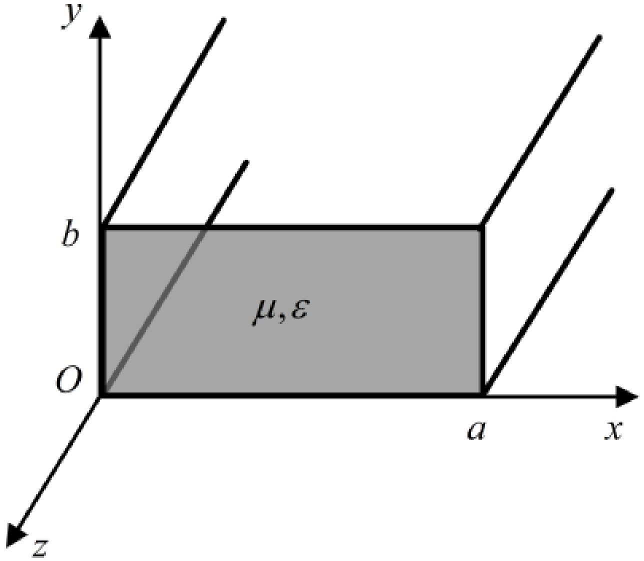

图2-6 矩形波导

分离变量法解得

$$E_z=\sin k_x x\sin k_y y e^{-jk_z z} \qquad (2.2.21)$$

$$k_x=\frac{m\pi}{a},\quad m=1,2\cdots \qquad (2.2.22a)$$

$$k_y=\frac{n\pi}{b},\quad n=1,2\cdots \qquad (2.2.22b)$$

且

$$k_x^2+k_y^2+k_z^2=k_c^2+k_z^2=\omega^2\mu\varepsilon=k^2 \qquad (2.2.23)$$

矩形波导中电磁波的传输特性

由(2.2.16)和(2.2.17)可算出TM模式的横向分量

$$E_x = -\frac{jk_x k_z}{k_c^2}\cos k_x x \sin k_y y e^{-jk_z z} \tag{2.2.24}$$

$$E_y = -\frac{jk_y k_z}{k_c^2}\sin k_x x \cos k_y y e^{-jk_z z} \tag{2.2.25}$$

$$H_x = \frac{j\omega\varepsilon k_y}{k_c^2}\sin k_x x \cos k_y y e^{-jk_z z} \tag{2.2.26}$$

$$H_y = -\frac{j\omega\varepsilon k_x}{k_c^2}\cos k_x x \sin k_y y e^{-jk_z z} \tag{2.2.27}$$

式(2.2.22a)和(2.2.22b)中m和n的不同取值，对应于不同的TM_{mn}模，其截止波数 为

$$k_{cmn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{2.2.28}$$

矩形波导中电磁波的传输特性

$$\boldsymbol{E}_t = -\frac{\gamma}{k_c^2}\nabla_t E_z \qquad (2.2.29)$$

$$\boldsymbol{H}_t = -j\frac{\omega\varepsilon}{k_c^2}\hat{\boldsymbol{z}}\times\nabla_t E_z \qquad (2.2.30)$$

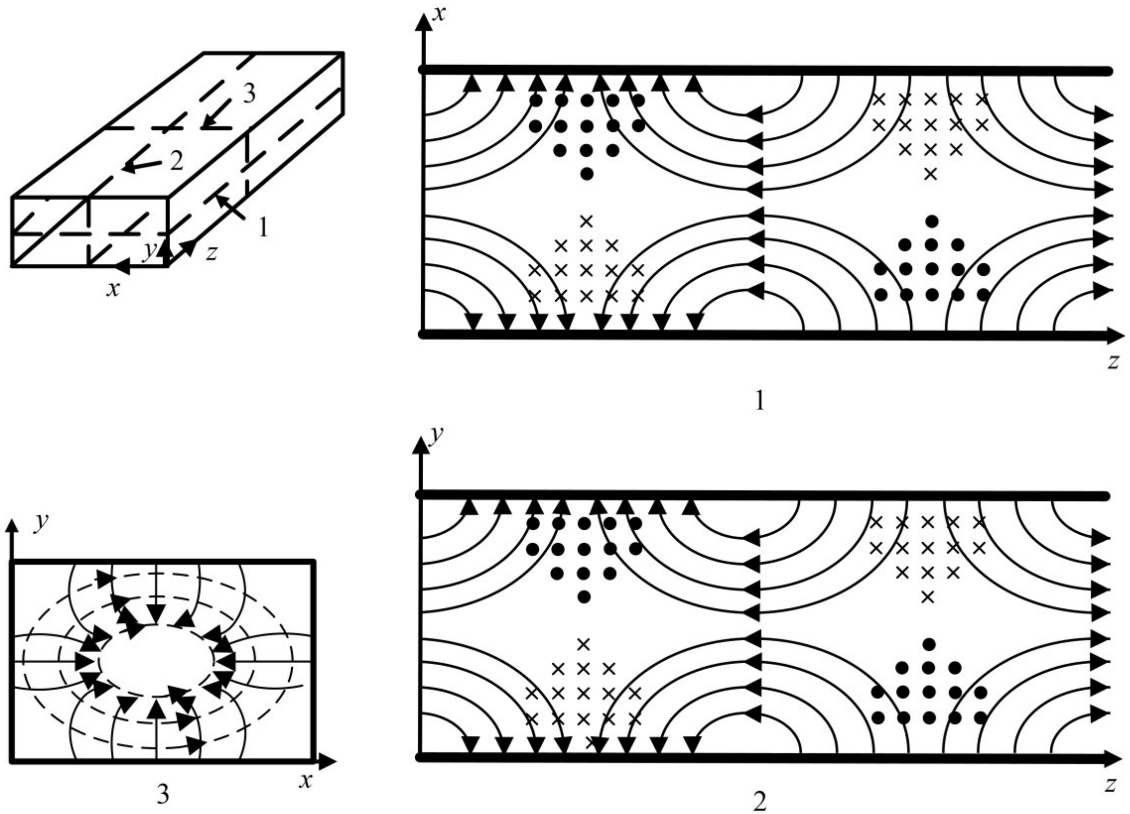

图2-7 矩形波导主模TM_{11}的场分布

矩形波导中电磁波的传输特性

TE模式

纵向场H_z除了满足(2.2.18)还满足

$$\left.\frac{\partial H_z}{\partial n}\right|_{x=0}=0,\quad \left.\frac{\partial H_z}{\partial n}\right|_{x=a}=0,\quad \left.\frac{\partial H_z}{\partial n}\right|_{y=0}=0,\quad \left.\frac{\partial H_z}{\partial n}\right|_{y=b}=0$$

(2.2.31)

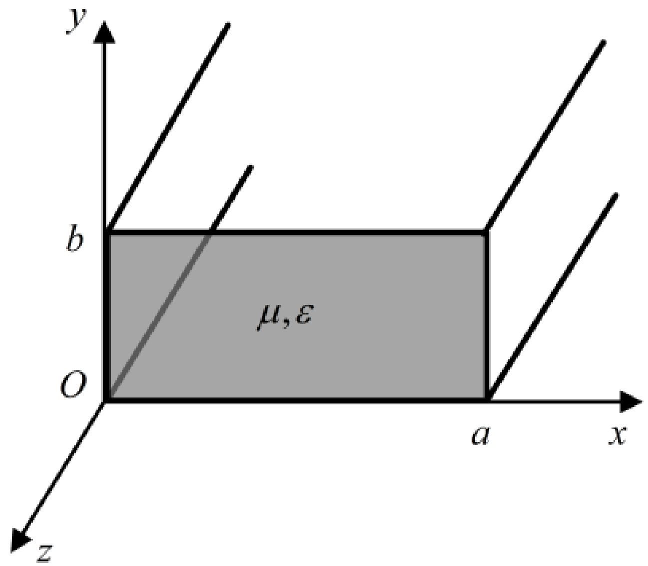

图2-6 矩形波导

分离变量法解得

$$H_z=\cos k_x x\cos k_y y e^{-jk_z z} \qquad (2.2.32)$$

$$k_x=\frac{m\pi}{a},\quad m=0,1,2\cdots \quad (2.2.33a) \qquad k_y=\frac{n\pi}{b},\quad n=0,1,2\cdots \quad (2.2.33b)$$

注意m和n不能同时为零，k_x,k_y,k_z 同样满足色散关系(2.2.23)

矩形波导中电磁波的传输特性

由(2.2.16)和(2.2.17)可算出TE模式的横向分量

$$E_x = \frac{j\omega\mu k_y}{k_c^2}\cos k_x x \sin k_y y e^{-jk_z z} \qquad (2.2.34)$$

$$E_y = -\frac{j\omega\mu k_x}{k_c^2}\sin k_x x \cos k_y y e^{-jk_z z} \qquad (2.2.35)$$

$$H_x = \frac{jk_x k_z}{k_c^2}\sin k_x x \cos k_y y e^{-jk_z z} \qquad (2.2.36)$$

$$H_y = \frac{jk_y k_z}{k_c^2}\cos k_x x \sin k_y y e^{-jk_z z} \qquad (2.2.37)$$

TE模的最小截止波数要小于TM模。因此波导主模是TE模

对于 $a > b$ 情形，波导主模 便是TE10，其截止波数为 $k_c = \pi / a$

场分布如图2-8

矩形波导中电磁波的传输特性

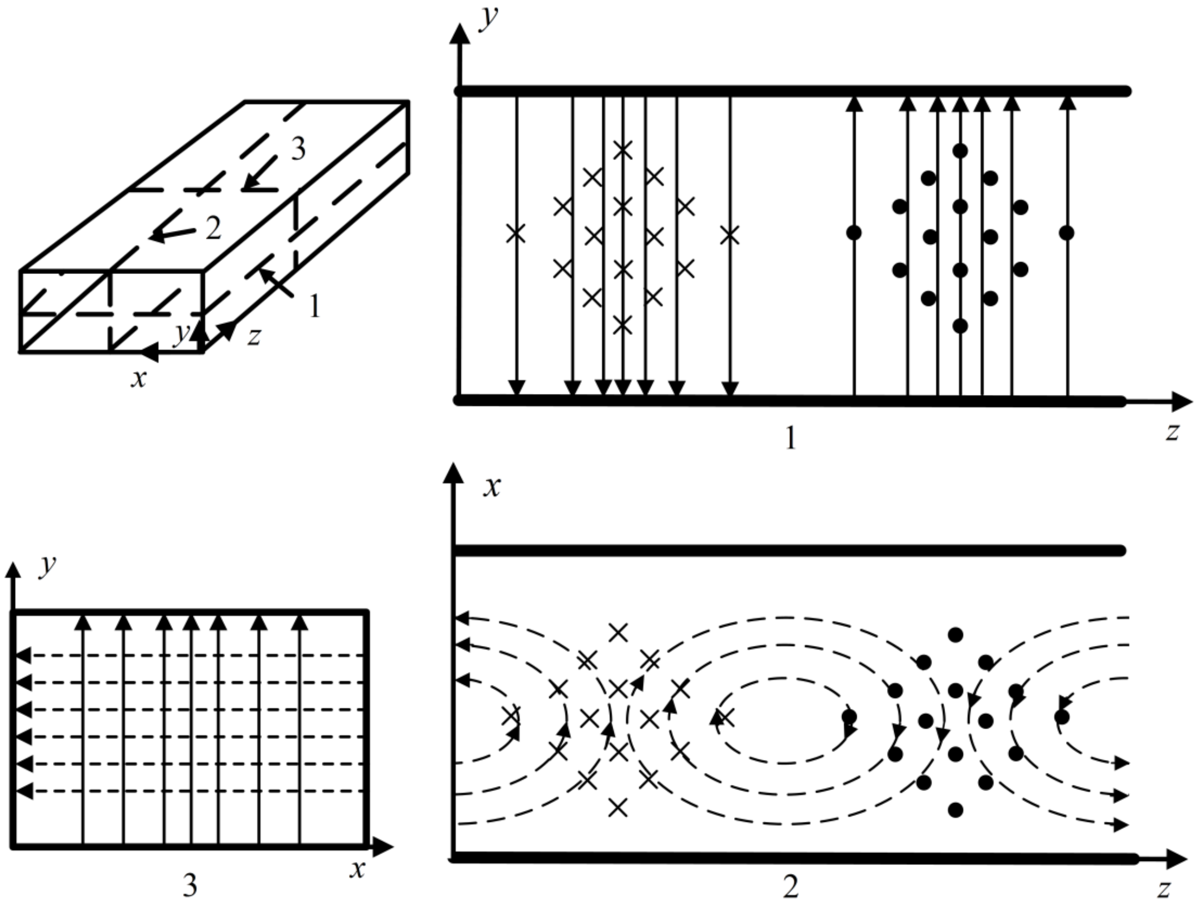

图2-8 矩形波导主模TE_{10}的场分布

例题2.6

矩形波导中电磁波的传输特性

单模传输

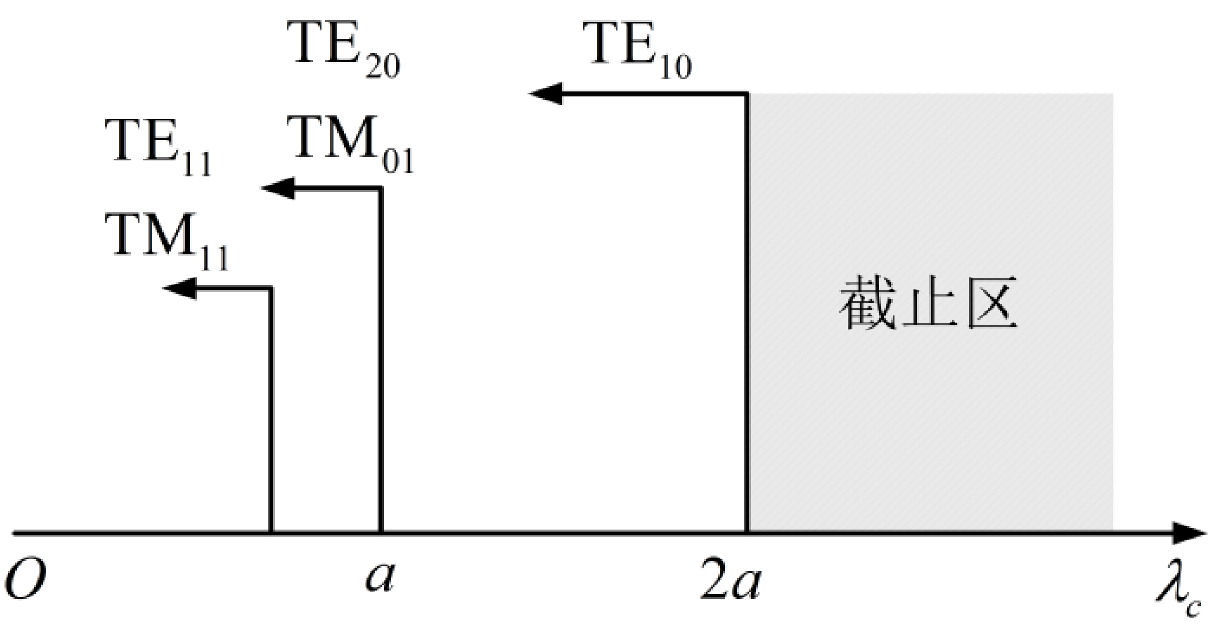

图2-9 a=2b的矩形波导，其截止波长分布

波导中的传输

- 波导传输问题的求解途径
- 矩形波导中电磁波的传输特性
- 波导正规模的特性

波导正规模的特性

假设波导中第m个模的场为$\boldsymbol{E}_\mathrm{m}$、$\boldsymbol{H}_\mathrm{m}$，第n个模的场为$\boldsymbol{E}_n$、$\boldsymbol{H}_n$

$$\nabla\times\boldsymbol{H}_m = j\omega\varepsilon\boldsymbol{E}_m \tag{2.2.38}$$

$$\nabla\times\boldsymbol{E}_m = -j\omega\mu\boldsymbol{H}_m \tag{2.2.39}$$

$$\nabla\times\boldsymbol{H}_n = j\omega\varepsilon\boldsymbol{E}_n \tag{2.2.40}$$

$$\nabla\times\boldsymbol{E}_n = -j\omega\mu\boldsymbol{H}_n \tag{2.2.41}$$

$\boldsymbol{H}_n$点乘(2.2.39)减去$\boldsymbol{E}_m$点乘(2.2.40)得

$$\boldsymbol{H}_n\cdot\nabla\times\boldsymbol{E}_m - \boldsymbol{E}_m\cdot\nabla\times\boldsymbol{H}_n = \nabla\cdot\left(\boldsymbol{E}_m\times\boldsymbol{H}_n\right) = j\omega\left(-\boldsymbol{H}_n\cdot\mu\boldsymbol{H}_m - \boldsymbol{E}_m\cdot\varepsilon\boldsymbol{E}_n\right) \tag{2.2.42}$$

$\boldsymbol{E}_n$点乘式(2.2.38)减去$\boldsymbol{H}_m$点乘式(2.2.41)得

$$\boldsymbol{E}_n\cdot\nabla\times\boldsymbol{H}_m - \boldsymbol{H}_m\cdot\nabla\times\boldsymbol{E}_n = \nabla\cdot\left(\boldsymbol{H}_m\times\boldsymbol{E}_n\right) = j\omega\left(\boldsymbol{H}_m\cdot\mu\boldsymbol{H}_n + \boldsymbol{E}_n\cdot\varepsilon\boldsymbol{E}_m\right) \tag{2.2.43}$$

波导正规模的特性

式(2.2.42)加上(2.2.43)得

$$\nabla\cdot\left(\boldsymbol{E}_m\times\boldsymbol{H}_n-\boldsymbol{E}_n\times\boldsymbol{H}_m\right)=0 \tag{2.2.44}$$

对式(2.2.44)在波导z和$z+\Delta z$两平面及波导内壁所围区域积分得

$$\int_{s_1}\left(\boldsymbol{E}_m\times\boldsymbol{H}_n-\boldsymbol{E}_n\times\boldsymbol{H}_m\right)\cdot\left(-\hat{z}\right)ds+\int_{s_c}\left(\boldsymbol{E}_m\times\boldsymbol{H}_n-\boldsymbol{E}_n\times\boldsymbol{H}_m\right)\cdot\hat{n}ds \tag{2.2.45}$$

$$+\int_{s_2}\left(\boldsymbol{E}_m\times\boldsymbol{H}_n-\boldsymbol{E}_n\times\boldsymbol{H}_m\right)\cdot\left(\hat{z}\right)ds=0$$

$$\boldsymbol{E}_m=\boldsymbol{E}_{tm}+E_{zm}\hat{\boldsymbol{z}}$$

$$\int_{s_1}\left(\boldsymbol{E}_{tm}\times\boldsymbol{H}_{tn}-\boldsymbol{E}_{tn}\times\boldsymbol{H}_{tm}\right)\cdot\left(-\hat{z}\right)ds+\int_{s_2}\left(\boldsymbol{E}_{tm}\times\boldsymbol{H}_{tn}-\boldsymbol{E}_{tn}\times\boldsymbol{H}_{tm}\right)\cdot\left(\hat{z}\right)ds=0 \tag{2.2.46}$$

波导正规模的特性

如果第m个模和第n个模都沿正z方向传输

$$\boldsymbol{E}_{tm} = e^{-\gamma_m z}\boldsymbol{e}_m(x,y), \qquad \boldsymbol{H}_{tm} = \frac{1}{Z_m}e^{-\gamma_m z}\boldsymbol{h}_m(x,y) \tag{2.2.47a}$$

$$\boldsymbol{E}_{tn} = e^{-\gamma_n z}\boldsymbol{e}_n(x,y), \qquad \boldsymbol{H}_{tn} = \frac{1}{Z_n}e^{-\gamma_n z}\boldsymbol{h}_n(x,y) \tag{2.2.47b}$$

$$\boldsymbol{h}_m(x,y) = \hat{\boldsymbol{z}}\times\boldsymbol{e}_m(x,y) \quad \text{(2.2.48a)} \quad \int_S \boldsymbol{e}_m(x,y)\cdot\boldsymbol{e}_m(x,y)dS = 1 \quad \text{(2.2.48a)}$$

$$(\gamma_m+\gamma_n)\int_s\left(\frac{1}{Z_n}\boldsymbol{e}_m\times\boldsymbol{h}_n - \frac{1}{Z_m}\boldsymbol{e}_n\times\boldsymbol{h}_m\right)\cdot(\hat{\boldsymbol{z}})ds = 0 \tag{2.2.49}$$

显然 $\gamma_m+\gamma_n \neq 0$，故

$$\int_s\left(\frac{1}{Z_n}\boldsymbol{e}_m\times\boldsymbol{h}_n - \frac{1}{Z_m}\boldsymbol{e}_n\times\boldsymbol{h}_m\right)\cdot(\hat{\boldsymbol{z}})ds = 0 \tag{2.2.50}$$

波导正规模的特性

如果第m个模沿正z方向传输，而第n个模都沿负z方向传输

$$\boldsymbol{E}_{tn} = e^{\gamma_n z}\boldsymbol{e}_n(x,y), \qquad \boldsymbol{H}_{tn} = -\frac{1}{Z_n}e^{\gamma_n z}\boldsymbol{h}_n(x,y) \qquad (2.2.51)$$

代入(2.2.46)得

$$(\gamma_m - \gamma_n)\int_s \left(\frac{1}{Z_n}\boldsymbol{e}_m \times \boldsymbol{h}_n + \frac{1}{Z_m}\boldsymbol{e}_n \times \boldsymbol{h}_m\right)\cdot(\hat{z})ds = 0 \qquad (2.2.52a)$$

如果 $\gamma_m \neq \gamma_n$ ，那么

$$\int_s \left(\frac{1}{Z_n}\boldsymbol{e}_m \times \boldsymbol{h}_n + \frac{1}{Z_m}\boldsymbol{e}_n \times \boldsymbol{h}_m\right)\cdot(\hat{z})ds = 0 \qquad (2.2.52b)$$

$$\int_s \boldsymbol{e}_m \times \boldsymbol{h}_n \cdot \hat{z}ds = 0, \qquad \gamma_m \neq \gamma_n \qquad (2.2.53)$$

波导正规模最为一般的正交性

波导正规模的特性

式(2.2.50)和(2.2.52b)相加

$$\int_s \boldsymbol{e}_m \times \boldsymbol{h}_n \cdot \hat{z} ds = 0, \qquad \gamma_m \neq \gamma_n \tag{2.2.53a}$$

利用(2.2.48a)和(2.2.48b)

$$\int_S \boldsymbol{e}_m \times \boldsymbol{h}_n \cdot \hat{z} dS = \delta_{mn} \tag{2.2.53b}$$

$$\int_S \boldsymbol{e}_m \cdot \boldsymbol{e}_n dS = \delta_{mn} \tag{2.2.53c}$$

波导正规模最为一般的**正交性**

$$\boldsymbol{E} = \sum_{m,n} a_{mn} \boldsymbol{e}_{TEmn} + \sum_{m,n} b_{mn} \boldsymbol{e}_{TMmn} \tag{2.2.54}$$

波导模式的**完备性**

例题2.6

试求等腰直角三角形波导(图2-9)中最低阶E波和H波的截止波长。

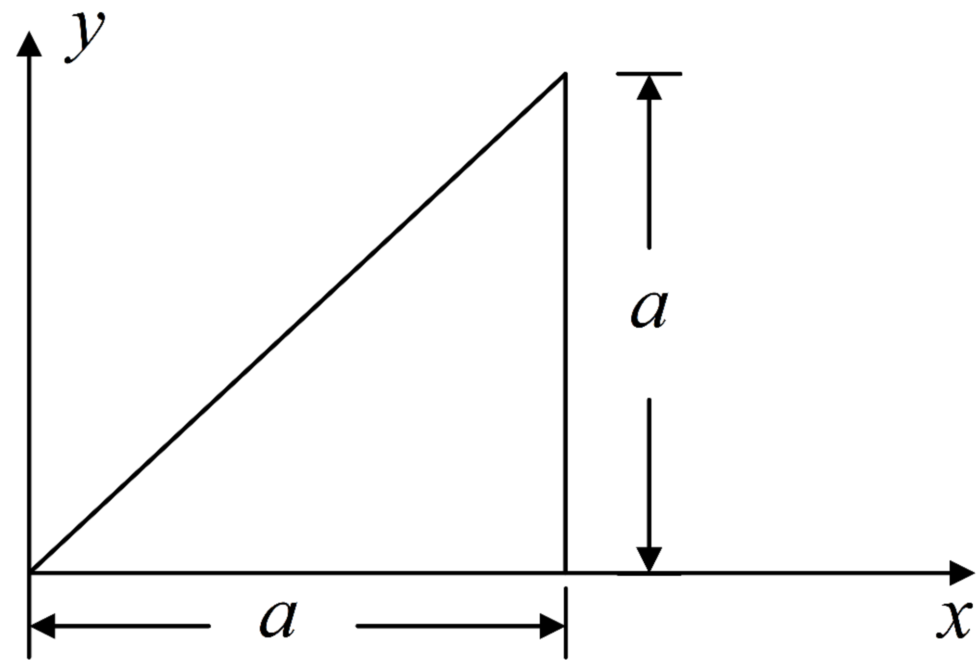

图2-9 等腰直角三角形波导

例题2.6

解：在方形波导的对角线处加一导体平面即得到等腰直角三角形波导。故求解时可利用方形波导中的场解。

1. TM波：设等腰直角三角形波导中的纵向电场分量 E_z 为方形波导中两个不同的E型波的叠加：

$$E_z = A\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{a} + B\sin\frac{p\pi x}{a}\sin\frac{q\pi y}{a} \qquad \text{(e2.6.1)}$$

为了满足 $\left(\nabla_t^2 + k_t^2\right)E_z = 0$ ，横向波数 k_t 应为

$$k_t^2 = \left(\frac{\pi}{a}\right)^2\left(m^2+n^2\right) = \left(\frac{\pi}{a}\right)^2\left(p^2+q^2\right) \qquad \text{(e2.6.2)}$$

在 $y=0$ 和 $x=a$ 上 $E_z=0$ ，满足边界条件；在弦 $y=x$ 上，

$$\begin{aligned} E_z\big|_{y=x} &= A\sin m\xi\sin n\xi + B\sin p\xi\sin q\xi \\ &= \frac{A}{2}\left[\cos(m-n)\xi - \cos(m+n)\xi\right] + \frac{B}{2}\left[\cos(p-q)\xi - \cos(p+q)\xi\right] \\ &= 0 \end{aligned} \qquad \text{(e2.6.3)}$$

例题2.6

若取正整数r,令 $m-p=q-n=r$ (e2.6.4a)

由式(e2.6.2)可得 $m+p=q+n$ (e2.6.4b)

上二式又可写为 $m+n=p+q, m-n=-(p-q)$ (e2.6.5)

$A=-B=1$ 时式(e2.6.3)成立。又由上式可知, $m=q, n=p$

因此式(e2.6.1)和式(e2.6.2)变为

$$E_z=\sin\frac{(n+r)\pi x}{a}\sin\frac{n\pi y}{a}-\sin\frac{n\pi x}{a}\sin\frac{(n+r)\pi y}{a} \quad \text{(e2.6.1)}$$

$$k_t=\frac{\pi}{a}\sqrt{(n+r)^2+n^2}, \qquad n,r=1,2,\cdots \quad \text{(e2.6.2)}$$

可见，最低E波的截止波长为

$$\lambda_c=2\pi/k_t=2a/\sqrt{5}$$

例题2.6

返回

(1)TE波：考虑方形波导 TE_{11} 波型的电场分布(图2-10)

根据场型分割原理

由 $y=0, y=x$ 和 $x=l$ 平面围成的等腰直角三角形波导和由 $y=0, y=x$ 和 $y=l-x$ 平面围成的等腰直角三角形波导中，纵向磁场分量均为

$$H_z=\cos\frac{\pi x}{l}\cos\frac{\pi y}{l} \qquad \text{(e2.6.3)}$$

对于前者，当直角边为a时，

$l=a$，截止波长 $\lambda_c=\sqrt{2}l=\sqrt{2}a$

对于后者，当直角边为a时，

$l=\sqrt{2}a$，截止波长 $\lambda_c=\sqrt{2}l=2a$

因此，最低H波的 $\lambda_c=2a$

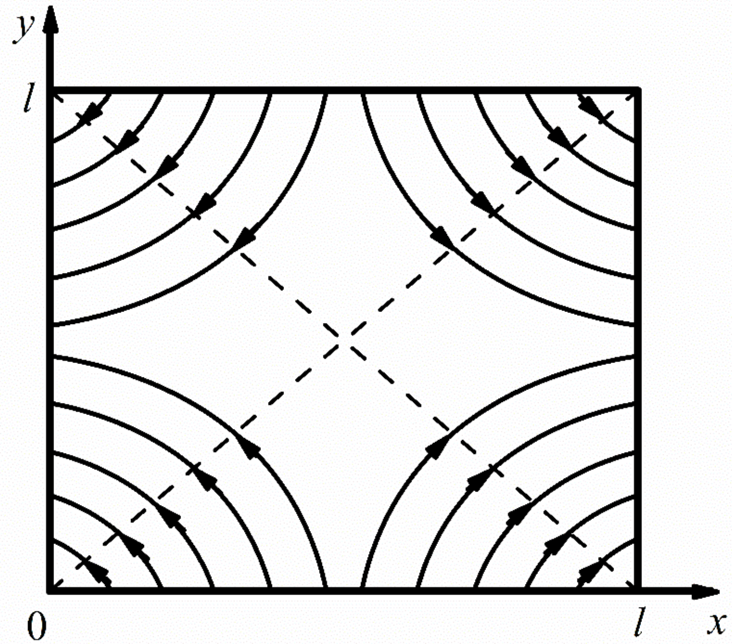

图2-10 方形波导TE_{11} 波型的电场分布