电磁波的极化

电磁波的极化:

在电磁波的传播方向上任意一点，电场瞬时矢量尾端随时间的运动轨迹。

极化的形式:

①电场矢量尾端轨迹是**直线**，称为线极化波；

②电场矢量尾端轨迹是**圆**， 称为圆极化波；

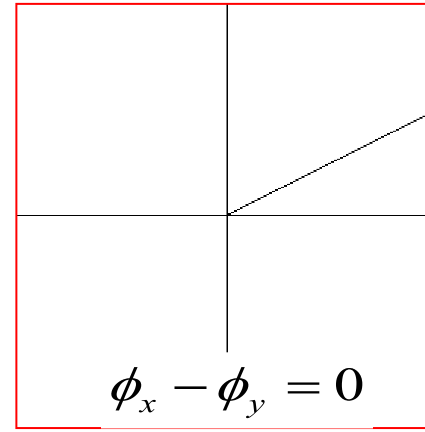
③电场矢量尾端轨迹是**椭圆**，称为椭圆极化波；

以 $\hat{z}$ 方向的电磁波为例，将其电场分解为x， y两个方向的分量。

$$E_x(z,t) = E_{xm}\cos(\omega t - kz + \phi_x) \qquad E_y(z,t) = E_{ym}\cos(\omega t - kz + \varphi_y)$$

1-线极化波

- 条件：$\varphi_x-\varphi_y=0$ **或** $\pm\pi$



- **合成波电场的模**

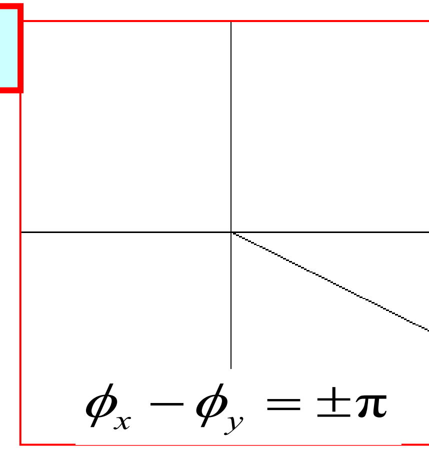
随时间变化

$$E=\sqrt{E_x^2(0,t)+E_y^2(0,t)}=\sqrt{E_{xm}^2+E_{ym}^2}\cos(\omega t+\phi_x)$$

- **合成波电场与$+x$轴的夹角**

常数

$$\alpha=\tan^{-1}\frac{E_y(0,t)}{E_x(0,t)}=\tan^{-1}\frac{\mathrm{E}_{ym}\cos(\omega t-\phi_x)}{\mathrm{E}_{xm}\cos(\omega t-\phi_y)}=\tan^{-1}\left(\pm\frac{\mathrm{E}_{ym}}{\mathrm{E}_{xm}}\right)$$



- 特点：**合成波电场的大小随时间变化但其矢端，轨迹与x轴的夹角始终保持不变。**
- 结论：**任何两个同频率、同传播方向且极化方向互相垂直的线极化波，当它们的相位相同或相差为±π时，其合成波为线极化波。**

2- 圆极化波

- 条件：$E_{xm} = E_{ym} = E_m; \phi_x - \phi_y = \pm \pi/2$

$$E_x(0,t) = E_m \cos(\omega t + \phi_x)$$

常数

$$E_y(0,t) = E_m \cos(\omega t + \phi_x \mp \frac{\pi}{2}) = \pm E_m \sin(\omega t + \phi_x)$$

- **合成波电场的模** $E = \sqrt{E_x^2(0,t) + E_y^2(0,t)} = E_m$

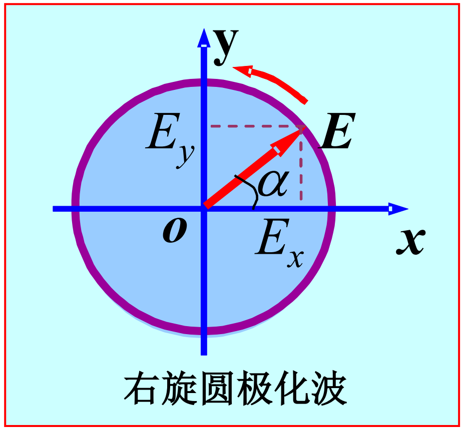
随时间变化

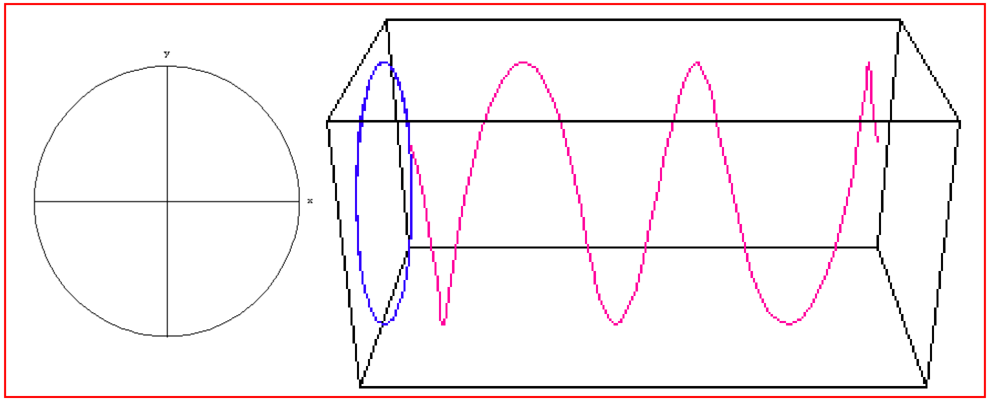
- **合成波电场与**$+x$ **轴的夹角** $\alpha = \arctan[\pm \tan(\omega t + \phi_x)] = \pm(\omega t + \phi_x)$

- 特点：**合成波电场的大小不随时间改变，但方向却随时间变化，电场的矢端在一个圆上并以角速度**ω **旋转。**

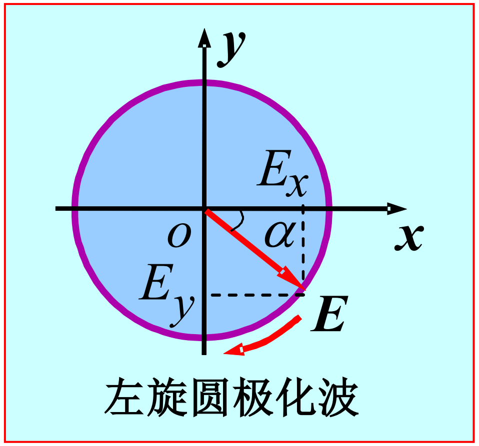
- 结论：**任何两个同频率、同传播方向且极化方向互相垂直的线极化波，当它们的振幅相同、相位差为**$\pm\pi/2$ **时，其合成波为圆极化波。**

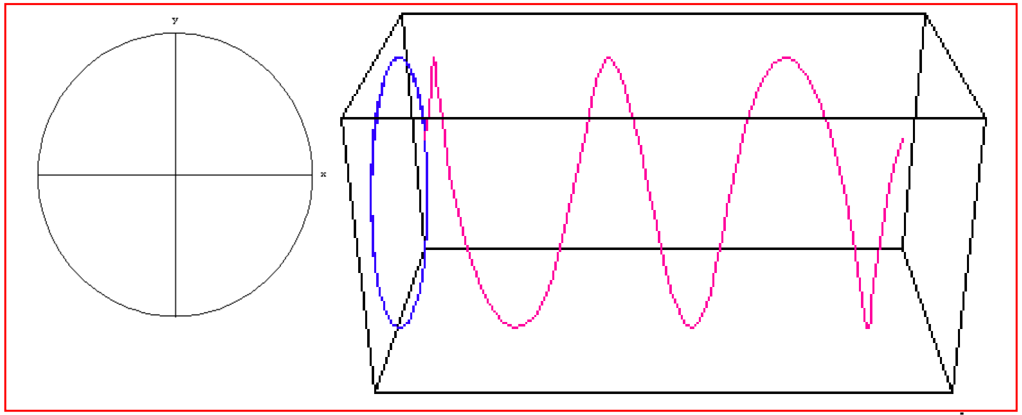
- 右旋圆极化波：**若$\varphi_x - \varphi_y = \pi/2$，则电场矢端的旋转方向与电磁波传播方向成右手螺旋关系，称为右旋圆极化波**





- 左旋圆极化波：**若$\varphi_x - \varphi_y = -\pi/2$，则电场矢端的旋转方向电磁波传播方向成左手螺旋关系，称为左旋圆极化波**

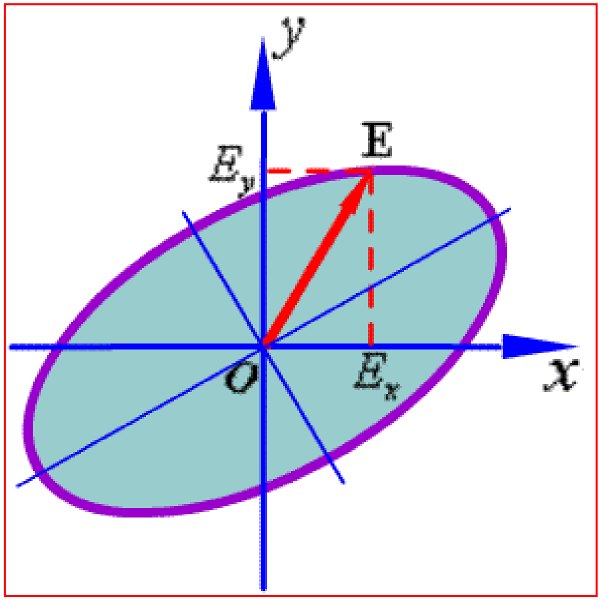




3- 椭圆极化波 $E_{xm} \neq E_{ym} \quad \varphi_x - \varphi_y = \varphi$

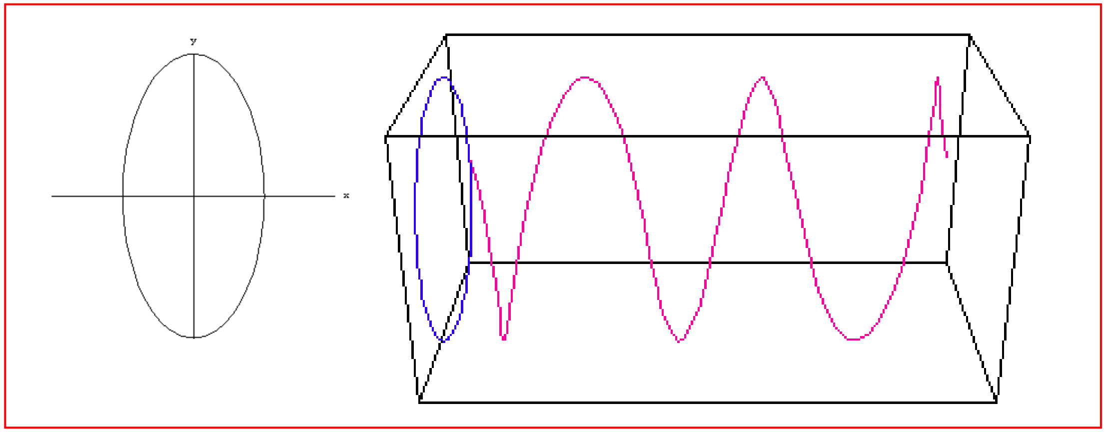
$$E_x(0,t) = E_{xm}\cos(\omega t + \phi_x)$$

$$E_y(0,t) = E_{ym}\cos(\omega t + \phi_x - \phi)$$



可得到 $$\frac{E_x^2}{E_{xm}^2} + \frac{E_y^2}{E_{ym}^2} - \frac{2E_x E_y}{E_{xm}E_{ym}}\cos\phi = \sin^2\phi$$

- 特点：**合成波电场的大小和方向都随时间改变，其端点在一个椭圆上旋转。**



波极化方式小结

- **电磁波的极化状态取决于**E_x**和** E_y **的振幅**E_{xm}、 E_{ym} **和相位差** $\Delta\varphi = \varphi_x - \varphi_y$
- **对于沿**$+z$ **方向传播的均匀平面波：**
  - **线极化：**$\Delta\varphi = 0$、 $\pm\pi$ 。

    $\Delta\varphi = 0$，**在1、3象限；**$\Delta\varphi = \pm\pi$，**在2、4象限。**
  - **圆极化：**$\Delta\varphi = \pm\pi/2$，$E_{xm} = E_{ym}$。

    取"+"，右旋圆极化；取"-"，左旋圆极化。
  - **椭圆极化：其它情况。**

    $0 < \Delta\varphi < \pi$，**右旋；** $-\pi < \Delta\varphi < 0$，**左旋**。

椭圆极化波

x分量，$E_x(0,t) = E_{xm}\cos(\omega t + \varphi_x)$ (1)

y分量，$E_y(0,t) = E_{ym}\cos(\omega t + \varphi_y)$ (2)

其中，$E_{xm} \neq E_{ym}, \varphi_y - \varphi_x = \varphi$

对式（1）和（2）整理和三角展开得

$$\frac{E_x}{E_{xm}} = \cos(\omega t + \varphi_x) = \cos(\omega t)\cos\varphi_x - \sin(\omega t)\sin\varphi_x \qquad (3)$$

$$\frac{E_y}{E_{ym}} = \cos(\omega t + \varphi_y) = \cos(\omega t)\cos\varphi_y - \sin(\omega t)\sin\varphi_y \qquad (4)$$

（3）乘 $\sin\varphi_y$ ，以及（4）乘 $\sin\varphi_x$ ，得

$$\frac{E_x}{E_{xm}}\sin\varphi_y = \cos(\omega t)\cos\varphi_x \sin\varphi_y - \sin(\omega t)\sin\varphi_x \sin\varphi_y \qquad (5)$$

$$\frac{E_y}{E_{ym}}\sin\varphi_x = \cos(\omega t)\cos\varphi_y \sin\varphi_x - \sin(\omega t)\sin\varphi_y \sin\varphi_x \qquad (6)$$

（5）和（6）两式相减，并进行三角变换可得，

$$\begin{aligned}\frac{E_x}{E_{xm}}\sin\varphi_y-\frac{E_y}{E_{ym}}\sin\varphi_x&=\cos(\omega t)\cos\varphi_x\sin\varphi_y-\cos(\omega t)\cos\varphi_y\sin\varphi_x\\&=\cos(\omega t)\left(\cos\varphi_x\sin\varphi_y-\cos\varphi_y\sin\varphi_x\right)\\&=\cos(\omega t)\sin\left(\varphi_y-\varphi_x\right)\end{aligned}\tag{7}$$

（3）乘 $\cos\varphi_y$ ，以及（4）乘 $\cos\varphi_x$ ，得

$$\frac{E_x}{E_{xm}}\cos\varphi_y=\cos(\omega t)\cos\varphi_x\cos\varphi_y-\sin(\omega t)\sin\varphi_x\cos\varphi_y\tag{8}$$

$$\frac{E_y}{E_{ym}}\cos\varphi_x=\cos(\omega t)\cos\varphi_y\cos\varphi_x-\sin(\omega t)\sin\varphi_y\cos\varphi_x\tag{9}$$

（8）和（9）两式相减，并进行三角变换可得，

$$\begin{aligned}\frac{E_x}{E_{xm}}\cos\varphi_y-\frac{E_y}{E_{ym}}\cos\varphi_x&=\sin(\omega t)\sin\varphi_y\cos\varphi_x-\sin(\omega t)\sin\varphi_x\cos\varphi_y\\&=\sin(\omega t)\left(\sin\varphi_y\cos\varphi_x-\sin\varphi_x\cos\varphi_y\right)\\&=\sin(\omega t)\sin\left(\varphi_y-\varphi_x\right)\end{aligned}\tag{10}$$

（7）和（10）两式左右分别平方并相加，可得，

$$\begin{aligned}&\left(\frac{E_x}{E_{xm}}\sin\varphi_y-\frac{E_y}{E_{ym}}\sin\varphi_x\right)^2+\left(\frac{E_x}{E_{xm}}\cos\varphi_y-\frac{E_y}{E_{ym}}\cos\varphi_x\right)^2\\&=\cos^2(\omega t)\sin^2(\varphi_y-\varphi_x)+\sin^2(\omega t)\sin^2(\varphi_y-\varphi_x)\\&=\sin^2(\varphi_y-\varphi_x)\left[\cos^2(\omega t)+\sin^2(\omega t)\right]\\&=\sin^2(\varphi_y-\varphi_x)\end{aligned}\tag{11}$$

记，$A=\dfrac{E_x}{E_{xm}}$，$B=\dfrac{E_y}{E_{ym}}$，且将 $\varphi_y-\varphi_x=\varphi$ 代入

则上式可写为，

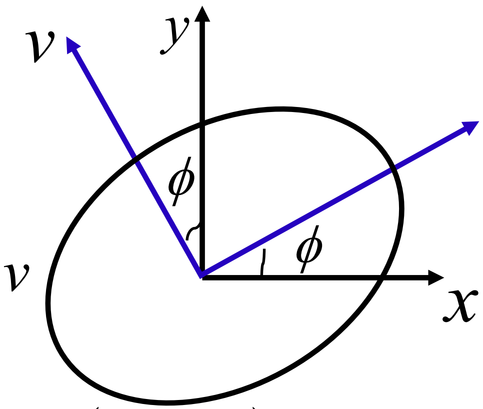
$$\left(A\sin\varphi_y-B\sin\varphi_x\right)^2+\left(A\cos\varphi_y-B\cos\varphi_x\right)^2=\sin^2\varphi\tag{12}$$

将上式展开，整理得，

$$A^2+B^2-2AB\cos\varphi=\sin^2\varphi\tag{13}$$

即，$$\left(\frac{E_x}{E_{xm}}\right)^2+\left(\frac{E_y}{E_{ym}}\right)^2-2\frac{E_x}{E_{xm}}\frac{E_y}{E_{ym}}\cos\varphi=\sin^2\varphi\tag{14}$$

极化倾角



x分量，$E_x(0,t)=E_{xm}\cos(\omega t+\varphi_x)$ (1)

y分量，$E_y(0,t)=E_{ym}\cos(\omega t+\varphi_y)$ (2)

假设标准椭圆沿原点旋转角度 ϕ，并建立一个新的坐标系 $u-v$

那么关系为，

u分量，$E_u(0,t)=E_{um}\cos(\omega t+\gamma)=E_{xm}\cos(\omega t+\varphi_x)\cos\phi+E_{ym}\cos(\omega t+\varphi_y)\sin\phi$ (3)

v分量，$E_v(0,t)=E_{vm}\sin(\omega t+\gamma)=-E_{xm}\cos(\omega t+\varphi_x)\sin\phi+E_{ym}\cos(\omega t+\varphi_y)\cos\phi$ (4)

对上两式（3）和（4）三角展开，并分离时间变量，得

$$\begin{aligned}&\cos(\omega t)\cdot E_{um}\cos\gamma-\sin(\omega t)\cdot E_{um}\sin\gamma\\&=\cos(\omega t)\cdot\left(E_{xm}\cos\phi\cos\varphi_x+E_{ym}\sin\phi\cos\varphi_y\right)-\sin(\omega t)\cdot\left(E_{xm}\cos\phi\sin\varphi_x+E_{ym}\sin\phi\sin\varphi_y\right)\end{aligned} \tag{5}$$

$$\begin{aligned}&\cos(\omega t)\cdot E_{vm}\sin\gamma+\sin(\omega t)\cdot E_{vm}\cos\gamma\\&=\cos(\omega t)\cdot\left(-E_{xm}\sin\phi\cos\varphi_x+E_{ym}\cos\phi\cos\varphi_y\right)+\sin(\omega t)\left(E_{xm}\sin\phi\sin\varphi_x-E_{ym}\cos\phi\sin\varphi_y\right)\end{aligned} \tag{6}$$

由式（5）和（6）可得，

$$\left.\begin{aligned}E_{um}\cos\gamma&=E_{xm}\cos\phi\cos\varphi_x+E_{ym}\sin\phi\cos\varphi_y\\E_{um}\sin\gamma&=E_{xm}\cos\phi\sin\varphi_x+E_{ym}\sin\phi\sin\varphi_y\\E_{vm}\sin\gamma&=-E_{xm}\sin\phi\cos\varphi_x+E_{ym}\cos\phi\cos\varphi_y\\E_{vm}\cos\gamma&=E_{xm}\sin\phi\sin\varphi_x-E_{ym}\cos\phi\sin\varphi_y\end{aligned}\right\} \tag{7}$$

由式（7）可得，

$$\frac{E_{vm}}{E_{um}}=\frac{E_{xm}\sin\phi\sin\varphi_x-E_{ym}\cos\phi\sin\varphi_y}{E_{xm}\cos\phi\cos\varphi_x+E_{ym}\sin\phi\cos\varphi_y}=\frac{-E_{xm}\sin\phi\cos\varphi_x+E_{ym}\cos\phi\cos\varphi_y}{E_{xm}\cos\phi\sin\varphi_x+E_{ym}\sin\phi\sin\varphi_y}\tag{8}$$

上式分子分母交叉相乘，得

$$\left(E_{xm}\sin\phi\sin\varphi_x-E_{ym}\cos\phi\sin\varphi_y\right)\left(E_{xm}\cos\phi\sin\varphi_x+E_{ym}\sin\phi\sin\varphi_y\right)=\left(E_{xm}\cos\phi\cos\varphi_x+E_{ym}\sin\phi\cos\varphi_y\right)\left(-E_{xm}\sin\phi\cos\varphi_x+E_{ym}\cos\phi\cos\varphi_y\right)$$

即，

$$\begin{aligned}&E^2{}_{xm}\sin^2\varphi_x\sin\phi\cos\phi+E_{xm}E_{ym}\sin^2\phi\sin\varphi_x\sin\varphi_y-E_{xm}E_{ym}\cos^2\phi\sin\varphi_y\sin\varphi_x-E^2{}_{ym}\sin\phi\cos\phi\sin^2\varphi_y\\&=-E^2{}_{xm}\cos^2\varphi_x\sin\phi\cos\phi+E_{xm}E_{ym}\cos^2\phi\cos\varphi_x\cos\varphi_y-E_{xm}E_{ym}\sin^2\phi\cos\varphi_y\cos\varphi_x+E^2{}_{ym}\sin\phi\cos\phi\cos^2\varphi_y\end{aligned}$$

移项整理得，

$$\begin{aligned}&\sin\phi\cos\phi\left(E^2{}_{xm}\sin^2\varphi_x-E^2{}_{ym}\sin^2\varphi_y+E^2{}_{xm}\cos^2\varphi_x-E^2{}_{ym}\cos^2\varphi_y\right)\\&=E_{xm}E_{ym}\cos\varphi_x\cos\varphi_y\left(\cos^2\phi-\sin^2\phi\right)-E_{xm}E_{ym}\sin\varphi_x\sin\varphi_y\left(\sin^2\phi-\cos^2\phi\right)\end{aligned}$$

即，

$$\sin\left(2\phi\right)\left(E^2{}_{xm}-E^2{}_{ym}\right)=2E_{xm}E_{ym}\cos\left(2\phi\right)\cos\left(\varphi_x-\varphi_y\right)\tag{9}$$

可得，

$$\tan\left(2\phi\right)=\frac{2E_{xm}E_{ym}\cos\left(\varphi_x-\varphi_y\right)}{E^2{}_{xm}-E^2{}_{ym}}\tag{10}$$

由式（10）可得，

$$\phi=\frac{1}{2}\arctan\left(\frac{2E_{xm}E_{ym}\cos\left(\varphi_x-\varphi_y\right)}{E^2{}_{xm}-E^2{}_{ym}}\right)$$

由式（7）以及三角函数关系 $\cos^2\gamma+\sin^2\gamma=1$，可得，

$$\begin{aligned}E^2{}_{um}&=\left(E_{xm}\cos\phi\cos\varphi_x+E_{ym}\sin\phi\cos\varphi_y\right)^2+\left(E_{xm}\cos\phi\sin\varphi_x+E_{ym}\sin\phi\sin\varphi_y\right)^2\\&=E^2{}_{xm}\cos^2\phi+E^2{}_{ym}\sin^2\phi+E_{xm}E_{ym}\sin\left(2\phi\right)\left(\cos\varphi_x\cos\varphi_y+\sin\varphi_x\sin\varphi_y\right)\\&=E^2{}_{xm}\cos^2\phi+E^2{}_{ym}\sin^2\phi+E_{xm}E_{ym}\sin\left(2\phi\right)\cos\left(\varphi_x-\varphi_y\right)\end{aligned}$$

$$\begin{aligned}E^2{}_{vm}&=\left(-E_{xm}\sin\phi\cos\varphi_x+E_{ym}\cos\phi\cos\varphi_y\right)^2+\left(E_{xm}\sin\phi\sin\varphi_x-E_{ym}\cos\phi\sin\varphi_y\right)^2\\&=E^2{}_{xm}\sin^2\phi+E^2{}_{ym}\cos^2\phi-E_{xm}E_{ym}\sin\left(2\phi\right)\left(\cos\varphi_y\cos\varphi_x+\sin\varphi_x\sin\varphi_y\right)\\&=E^2{}_{xm}\sin^2\phi+E^2{}_{ym}\cos^2\phi-E_{xm}E_{ym}\sin\left(2\phi\right)\cos\left(\varphi_x-\varphi_y\right)\end{aligned}$$

标准椭圆沿原点旋转 ϕ ，方程为

$$\frac{\left(E_x\cos\phi+E_y\sin\phi\right)^2}{E^2{}_{um}}+\frac{\left(-E_x\sin\phi+E_y\cos\phi\right)^2}{E^2{}_{vm}}=1$$