

$\phi = -150^\circ$ 时 $-\frac{z}{2} = \arctan 3w + \arctan w/w = \frac{\pi}{2}$
 $\arctan 3w = a$
 $\arctan w = b$
 $a+b = \frac{\pi}{2}$ $a = \frac{\pi}{2} - b$
 $\tan a = 3w$
 $\tan b = w$ $w = \sqrt{\frac{1}{9}}$
 $\tan(\frac{\pi}{2} - b) = 3w$
 $\frac{1}{\tan b} = 3w$
 $\tan b = 0.1$ $w = \sqrt{\frac{1}{9}}$
 $\Rightarrow I(s) = \frac{2s^2+1}{R} \cdot U(s) + U(s)$
 $\therefore G(s) = \frac{U(s)}{I(s)} = \frac{R}{2s^2+R(s+1)}$



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一解：设通过L电流为 $i_L(t)$ ，通过C电流为 $i_C(t)$ 则有

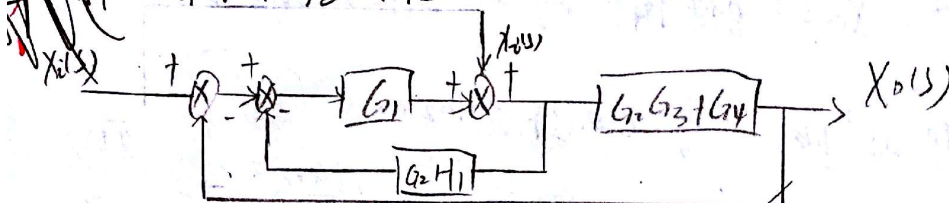
$$\begin{aligned}
 i(t) &= i_L(t) + i_C(t) \\
 u(t) &= \frac{1}{C} \int i_C(t) dt \\
 u(t)R &= L \frac{di_L(t)}{dt} + \frac{1}{C} \int i_C(t) dt
 \end{aligned}$$

两边同拉变 \Rightarrow

$$\begin{aligned}
 I(s) &= I_L(s) + I_C(s) \\
 U(s) &= \frac{I_C(s)}{Cs} \\
 I_L(s)R &= sLI(s) + \frac{I_C(s)}{Cs}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow I(s) &= \frac{2s^2+1}{R} \cdot U(s) + U(s) \\
 \therefore G(s) &= \frac{U(s)}{I(s)} = \frac{R}{2s^2+R(s+1)}
 \end{aligned}$$

解：将A端左移，并得等效电路有



$$\begin{aligned}
 K_p &= \lim_{s \rightarrow 0} G(s)H(s) = \lim_{s \rightarrow 0} \frac{10}{s(s+1)(s+2)} \\
 &= \infty
 \end{aligned}$$

② 当输入 $X_1(s)$ 单独作用时有 $Y(s) = \frac{Y(s)}{X_1(s)} = \frac{G_1(G_3G_4+G_4)}{1+G_1G_2H_1}$

$$K_v = \frac{10}{1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1} = 10$$

③ 当干扰 $X_2(s)$ 单独作用时有 $Y(s) = \frac{Y(s)}{X_2(s)} = \frac{G_2G_3+G_4}{1+G_1G_2H_1}$

$$K_p = \lim_{s \rightarrow 0} \frac{1 \times 5}{s \times 1 \times 1} = \infty$$

则 $Y(s) = Y_1(s) + Y_2(s) = \frac{G_1G_2G_3+G_1G_4}{G_1G_2G_3+G_1G_4+G_2G_3+G_4} \cdot \frac{1}{1+G_1G_2H_1}$

三. 解：由闭环特征方程式 $s^3 + 7s^2 + 17s + K = 0$ 则系统条件 $\begin{cases} K > 0 \\ 7 \times 17 > K \end{cases}$

$\therefore 0 < K < 119$

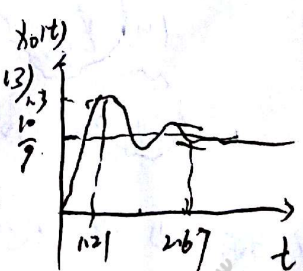
② 令 $s = z - 2$ 代入得 $(z-2)^3 + 7(z-2)^2 + 17(z-2) + K = 0 \Rightarrow z^3 + z^2 + z + K - 14 = 0$

则 $\begin{cases} K - 14 > 0 \\ 1 > K - 14 \end{cases} \Rightarrow 14 < K < 15$ $Y(s) = X(s) \cdot \frac{1}{1+G(s)}$

$$\begin{aligned}
 0 & \frac{1}{K+1} & 0 & K \\
 I & \frac{1}{K} & 0 & K \\
 II & 0 & 0 & K
 \end{aligned}$$

四. 解：由 $G(s) \cdot \frac{1}{s} = \frac{1}{s} - \frac{1}{s+2}$ 有 $G(s) = \frac{2}{s+2}$ $z = \frac{1}{1+K_p} + \frac{1}{K_v} + \frac{1}{K_a}$

(2) $M_p = e^{-\frac{\sqrt{1-\zeta^2}}{1-\zeta^2}} = e^{-\frac{\sqrt{1-\frac{1}{4}}}{1-\frac{1}{4}}} = 16.3\%$ $t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{3 \sqrt{1-\frac{1}{4}}} = 1.21s$



$\begin{cases} \text{有 } G(s) = \frac{1}{s+2} & X(s) = \frac{10}{s} \\ \text{则有 } Y(s) = \frac{1}{9} \cdot \frac{9}{s^2+3s+2} \end{cases} \Rightarrow \begin{cases} \omega_n = 3 \\ \zeta = \frac{1}{2} \end{cases}$

五. 解: (1) 判别系统稳定性. 静态偏差系数等于静态误差系数. 为型系统

设 $k_p = \lim_{s \rightarrow 0} G(s)H(s) = \infty$ $k_v = \lim_{s \rightarrow 0} s G(s)H(s) = 10$ $k_a = \lim_{s \rightarrow 0} s^2 G(s)H(s) = 0$

1) 由输入信号 $x(t) = 1 + 2t + 3t^2$ 则 $A_1 = A_2 = 2, A_3 = 3, x_2 = 6$. $\frac{1}{s^r} \Leftrightarrow \frac{1}{r!} t^r$

由 $e_{ss} = e_{ss} = \frac{A_1}{1+k_p} + \frac{A_2}{k_v} + \frac{A_3}{k_a} = \infty$

13) $G(s) = as^2 + bs$ 时 系统为 II 型系统 则 $k_p = k_v = k_a = \infty$

由此 $E(s) = X(s) - X_0(s)$ 其中 $X_0(s) = \frac{G(s)}{1+G(s)} X(s)$ 注: 将 $\frac{X_0(s)}{X(s)}$ 写出来 即为 $G(s) = \frac{as^2 + bs}{1 + as^2 + bs}$ 此时 $\frac{X_0(s)}{X(s)} = \frac{as^2 + bs}{1 + as^2 + bs}$ 其他都为 0.

则 $E(s) = \frac{1 - G(s)H(s)}{1 + G(s)H(s)} X(s)$ 要使偏差等于 0 有 $E(s) = 0$

则 $G(s)H(s) = \frac{1}{as^2 + bs}$ 即 $as^2 + bs = \frac{s}{10} \cdot (0.1s + 1)(0.02s + 1)$

则 $a = 0.012, b = 0.1$ 则 $\frac{G(s)}{1+G(s)} = \frac{as^2 + bs}{1 + as^2 + bs} = \frac{1 + 0.1s}{1 + 0.012s^2 + 0.1s} = \frac{1 + 0.1s}{1 + 0.1s + 0.012s^2}$

六. 解: 由闭环 $T(s) = \frac{10}{s^2 + s + 10}$ 则 $A(j\omega) = \frac{10}{\sqrt{\omega^2 + (0.5\omega)^2}} = \frac{10}{\sqrt{4 + 100}}$

代入 $j\omega$ 有 $A(j\omega) = \frac{10}{j\omega + 10 + 0.5\omega^2}$ 则 $A = A(j\omega) \cdot A_0 = \frac{10}{\sqrt{100}} \cdot 5 = 4.9$

$\phi(j\omega) = -\arctan \frac{\omega}{10 + 0.5\omega^2}$ 则 $\phi = \phi(j\omega) + \phi_0 = -\arctan \frac{\omega}{10 + 0.5\omega^2} - 60^\circ = -48.69^\circ$

则 $x(t) = 4.9 \cos(2t - 48.69^\circ)$

七. 解: 1) 由 $G(s) = \frac{5}{s(0.5s+1)(\frac{1}{10}s+1)}$ 12)

13) $-40 \lg \frac{\omega_c}{\frac{1}{10}} = 0 - 27.52 \Rightarrow \omega_c = 1.3 \text{ rad/s}$

14) $\phi_{\omega_c} = -\frac{\pi}{2} - \arctan \frac{3\omega_c}{1} - \arctan \frac{1}{10\omega_c} = -\frac{\pi}{2} - \arctan 3.9 - \arctan 0.13 = -\frac{\pi}{2} - 1.107 - 0.13 = -2.837 \text{ rad}$

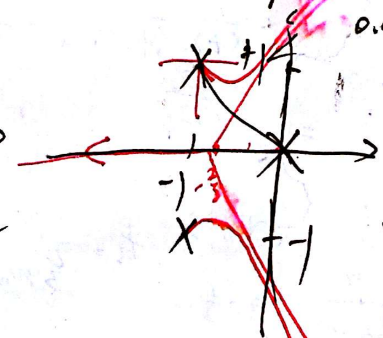
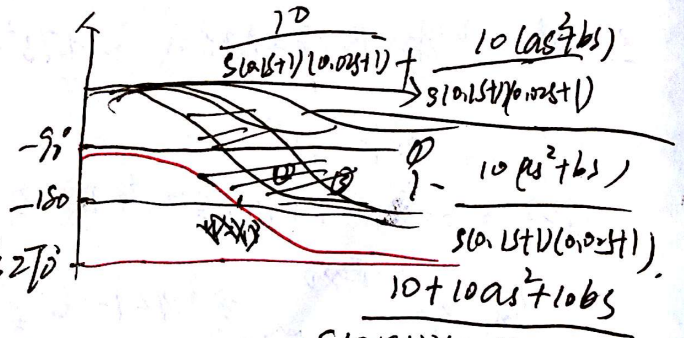
则 $\gamma = 180^\circ + \phi_{\omega_c} = 151.26^\circ$

八. 解: 1) ① 初始相位: $\phi = 180^\circ - 90^\circ - 135^\circ = -45^\circ$

② 无零极点. ③ 3 个极点 $\phi_{\omega_c} = 180^\circ - 180^\circ = 0^\circ$

④ 与 $\omega_c = 2\sqrt{2}$ 此时 $k^* = 4$

12) $0 < k^* < 4$



$\frac{G(s)H(s)}{1 + G(s)H(s)} = \frac{5}{s(0.5s+1)(\frac{1}{10}s+1)} \cdot \frac{1}{1 + \frac{5}{s(0.5s+1)(\frac{1}{10}s+1)}} = \frac{5}{s(0.5s+1)(\frac{1}{10}s+1) + 5}$