

第二章 电磁波的传播和传输

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北京理工大学

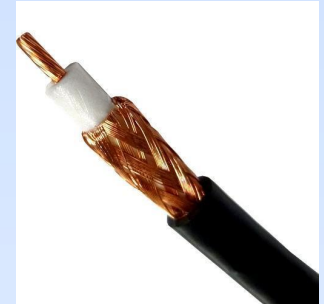
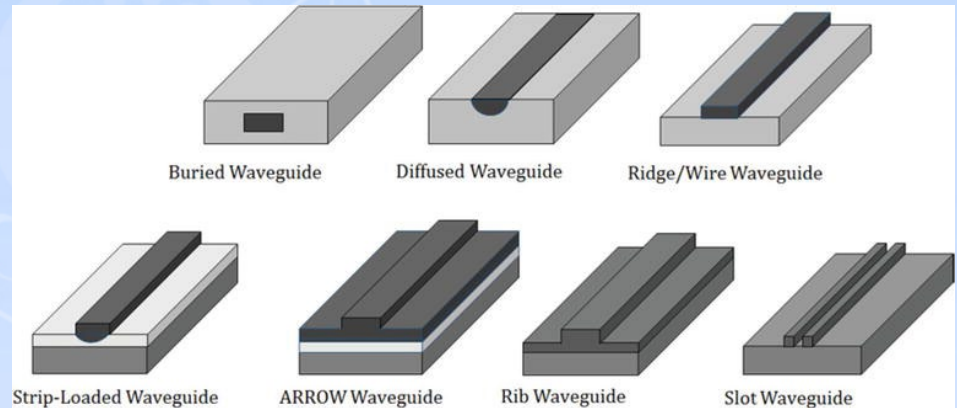
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第2章 电磁波的传播和传输

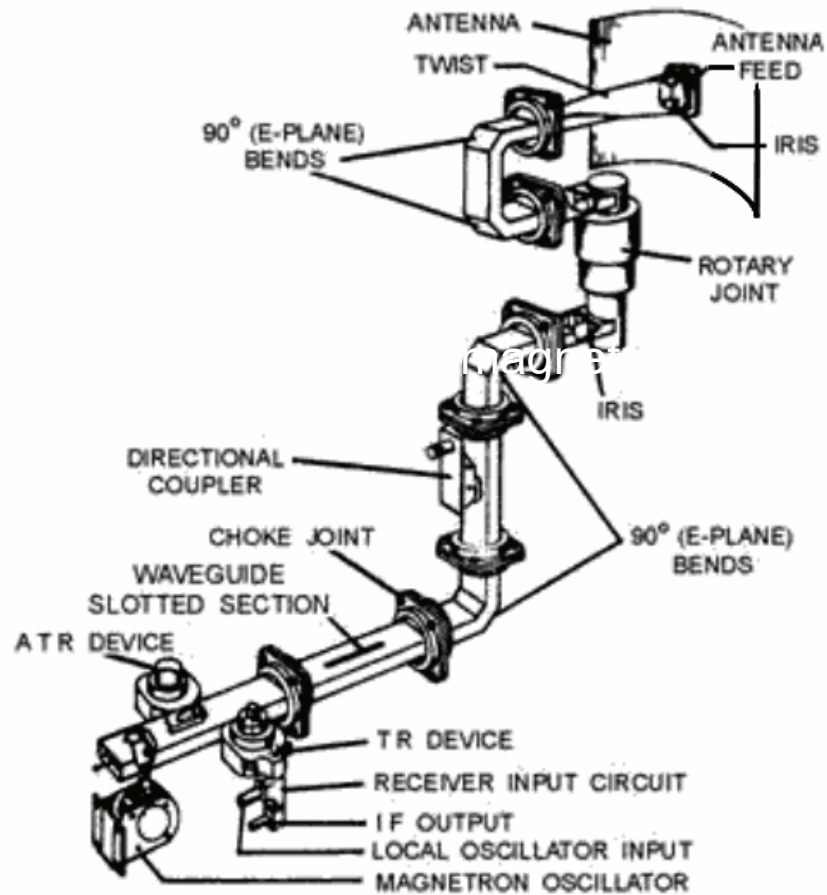
- 电磁波传播
- 波导中的传输

什么是波导

A **waveguide** is a structure that guides waves, such as electromagnetic waves or sound, with minimal loss of energy by restricting the transmission of energy to **one direction**. Without the physical constraint of a waveguide, wave intensities decrease according to the inverse square law as they expand into three dimensional space.

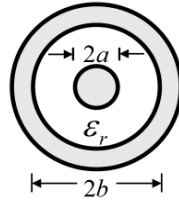


雷达系统中波导

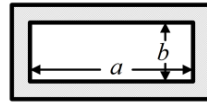


波导中的传输

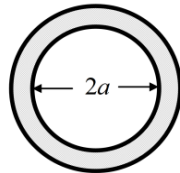
同轴线



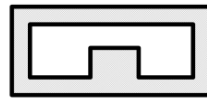
矩形波导



圆波导



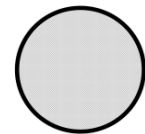
脊波导



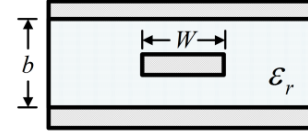
矩形介质波导



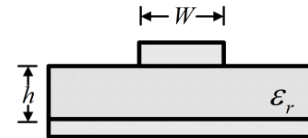
圆形介质波导



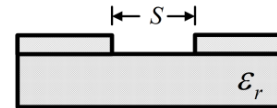
带状线



微带线



槽形线



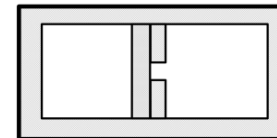
共面波导



悬置微带线



鳍线



镜像线



波导中的传输

- 波导传输问题的求解途径
- 矩形波导中电磁波的传输特性
- 波导正规模的特性
- 任意截面空波导电磁波传输模式的有限元分析
- 波导激励分析

波导传输问题的求解途径

矢量亥姆霍茨方程(2.1.7)的z分量可简化成标量亥姆霍茨方程

$$(\nabla^2 + k^2)E_z = 0 \quad (2.2.1)$$

其中 $k = \omega\sqrt{\mu\varepsilon}$.

下面证明(2.2.1)式

(2.1.6)式中 $\nabla \times \nabla \times \mathbf{E}$ 的z分量是由 $\nabla \times \mathbf{E}$ 的横向分量决定。

$\nabla = \nabla_t + \hat{e}_z \frac{\partial}{\partial z}$, 展开计算可得

$$\begin{aligned} \left(\nabla_t + \hat{\mathbf{z}} \frac{\partial}{\partial z} \right) \times \mathbf{E} \Big|_t &= \nabla_t \times \mathbf{E} + \hat{\mathbf{z}} \times \frac{\partial}{\partial z} \mathbf{E} \Big|_t \\ &= \nabla_t \times (\hat{\mathbf{z}} E_z) + \hat{\mathbf{z}} \times \frac{\partial E_t}{\partial z} = \nabla_t E_z \times \hat{\mathbf{z}} + \hat{\mathbf{z}} \times \frac{\partial E_t}{\partial z} \end{aligned} \quad (2.2.2a)$$

波导传输问题的求解途径

整理得

$$(\nabla \times \mathbf{E})_t = \left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \times \hat{\mathbf{z}} \quad (2.2.2b)$$

$$(\nabla \times \nabla \times \mathbf{E})_z \hat{\mathbf{z}} = \nabla \times (\nabla \times \mathbf{E})_t = \nabla_t \times \left[\left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \times \hat{\mathbf{z}} \right] \quad (2.2.3)$$

利用 $\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A} \nabla \cdot \mathbf{B} - \mathbf{B} \nabla \cdot \mathbf{A} + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B}$ (2.2.4a)

得

$$\begin{aligned} \nabla_t \times \left[\left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \times \hat{\mathbf{z}} \right] &= \left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \nabla_t \cdot \hat{\mathbf{z}} - \hat{\mathbf{z}} \nabla_t \cdot \left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \\ &\quad + (\hat{\mathbf{z}} \cdot \nabla_t) \left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) - \left[\left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) \cdot \nabla_t \right] \hat{\mathbf{z}} \end{aligned} \quad (2.2.4b)$$

波导传输问题的求解途径

又由于 $\hat{\mathbf{z}} \cdot \nabla_t = 0, \nabla_t \cdot \hat{\mathbf{z}} = 0, (\mathbf{A} \cdot \nabla_t) \hat{\mathbf{z}} = 0$ (2.2.5a)

则 (2.2.3) 式可化成

$$(\nabla \times \nabla \times \mathbf{E})_z \hat{\mathbf{z}} = -\hat{\mathbf{z}} \nabla_t \cdot \left(\nabla_t E_z - \frac{\partial \mathbf{E}_t}{\partial z} \right) = \hat{\mathbf{z}} \left(\nabla_t \cdot \frac{\partial \mathbf{E}_t}{\partial z} - \nabla_t^2 E_z \right) \quad (2.2.5b)$$

即
$$(\nabla \times \nabla \times \mathbf{E})_z = \nabla_t \cdot \left(\frac{\partial \mathbf{E}_t}{\partial z} \right) - \nabla_t^2 E_z \quad (2.2.6)$$

$$[\nabla(\nabla \cdot \mathbf{E})]_z = \frac{\partial}{\partial z} \left[\left(\nabla_t + \hat{\mathbf{z}} \frac{\partial}{\partial z} \right) \cdot \mathbf{E} \right] \quad (2.2.7)$$

$$= \frac{\partial}{\partial z} \left(\nabla_t \cdot \mathbf{E} + \frac{\partial E_z}{\partial z} \right) = \frac{\partial (\nabla_t \cdot \mathbf{E})}{\partial z} + \frac{\partial^2 E_z}{\partial z^2}$$

将(2.2.6)和(2.2.7)代入(2.1.6)得

$$(\nabla^2 \mathbf{E})_z = \nabla^2 E_z \quad (2.2.8)$$

于是标量亥姆霍茨方程(2.2.1)得证

波导传输问题的求解途径

同样可证得磁场标量亥姆霍茨方程

$$(\nabla^2 + k^2)H_z = 0 \quad (2.2.9)$$

下面将进一步导出由纵向场求出横向场的表达式

设纵向上的传播因子是 $e^{-\gamma z}$ ($\gamma = \alpha + j\beta$)

电磁波在波导中的空间分布可表述成

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}(t)e^{-\gamma z} \quad (2.2.10)$$

$$\nabla_z \times (\mathbf{E}e^{-\gamma z}) = \hat{\mathbf{z}} \frac{\partial}{\partial z} \times (\mathbf{E}e^{-\gamma z}) = \hat{\mathbf{z}} \times \left(\mathbf{E} \frac{\partial}{\partial z} e^{-\gamma z} \right) = -j\gamma \hat{\mathbf{z}} \times (\mathbf{E}e^{-\gamma z}) \quad (2.2.11a)$$

$$\nabla_z \cdot (\mathbf{E}e^{-\gamma z}) = \hat{\mathbf{z}} \frac{\partial}{\partial z} \cdot (\mathbf{E}e^{-\gamma z}) = \hat{\mathbf{z}} \cdot \left(\mathbf{E} \frac{\partial}{\partial z} e^{-\gamma z} \right) = -j\gamma \hat{\mathbf{z}} \cdot (\mathbf{E}e^{-\gamma z}) \quad (2.2.11b)$$

即有

$$\nabla_z \equiv -\gamma \hat{\mathbf{z}} \quad (2.2.12b)$$

波导传输问题的求解途径

$$\begin{aligned}(\nabla \times \mathbf{E})_t &= \left[(\nabla_t + \nabla_z) \times (\hat{\mathbf{z}} E_z + \mathbf{E}_t) \right]_t \\&= \nabla_t \times (\hat{\mathbf{z}} E_z) + \nabla_z \times \mathbf{E}_t \\&= (\nabla_t E_z + \gamma \mathbf{E}_t) \times \hat{\mathbf{z}}\end{aligned} \quad (2.2.13)$$

根据法拉第定律(2.1.1)得

$$(\nabla_t E_z + \gamma \mathbf{E}_t) \times \hat{\mathbf{z}} = -j\omega\mu \mathbf{H}_t \quad (2.2.14)$$

同样

$$(\nabla_t H_z + \gamma \mathbf{H}_t) \times \hat{\mathbf{z}} = j\omega\varepsilon \mathbf{E}_t \quad (2.2.15)$$

$$\mathbf{E}_t = j \frac{\omega\mu}{k_c^2} \hat{\mathbf{z}} \times \nabla_t H_z - \frac{\gamma}{k_c^2} \nabla_t E_z \quad (2.2.16)$$

截止波数(Cutoff Wavenumber) $k_c^2 = k^2 + \gamma^2$

截止波长(Cutoff Wavelength)、截止频率(Cutoff Frequency)

同样可推得

$$\mathbf{H}_t = -j \frac{\omega\varepsilon}{k_c^2} \hat{\mathbf{z}} \times \nabla_t E_z - \frac{\gamma}{k_c^2} \nabla_t H_z \quad (2.2.17)$$

波导传输问题的求解途径

求解波导问题的具体方式：

先从(2.2.1)和(2.2.9)解出纵向场 E_z 和 H_z

再由(2.2.16)和(2.2.17)算出横向场。

方程(2.2.1)和(2.2.9)在(2.2.10)的假设下，可进一步简化为

$$\left(\nabla_t^2 + k_c^2\right)E_z = 0 \quad (2.2.18)$$

$$\left(\nabla_t^2 + k_c^2\right)H_z = 0 \quad (2.2.19)$$

波导中的传输

- 波导传输问题的求解途径
- 矩形波导中电磁波的传输特性
- 波导正规模的特性
- 任意截面空波导电磁波传输模式的有限元分析
- 波导激励分析

矩形波导中电磁波的传输特性

TM模式

纵向场 E_z 除了满足(2.2.18)还满足

$$E_z|_{x=0} = 0, \quad E_z|_{x=a} = 0, \quad E_z|_{y=0} = 0, \quad E_z|_{y=b} = 0$$

(2.2.20)

分离变量法解得

$$E_z = \sin k_x x \sin k_y y e^{-jk_z z} \quad (2.2.21)$$

$$k_x = \frac{m\pi}{a}, \quad m = 1, 2, \dots \quad (2.2.22a) \quad k_y = \frac{n\pi}{b}, \quad n = 1, 2, \dots \quad (2.2.22b)$$

且

$$k_x^2 + k_y^2 + k_z^2 = k_c^2 + k_z^2 = \omega^2 \mu \epsilon = k^2 \quad (2.2.23)$$

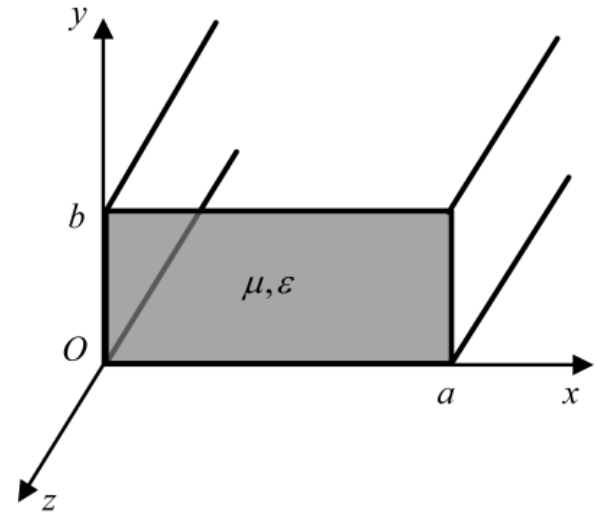


图2-6 矩形波导

矩形波导中电磁波的传输特性

由(2.2.16)和(2.2.17)可算出TM模式的横向分量

$$E_x = -\frac{jk_x k_z}{k_c^2} \cos k_x x \sin k_y y e^{-jk_z z} \quad (2.2.24)$$

$$E_y = -\frac{jk_y k_z}{k_c^2} \sin k_x x \cos k_y y e^{-jk_z z} \quad (2.2.25)$$

$$H_x = \frac{j\omega\epsilon k_y}{k_c^2} \sin k_x x \cos k_y y e^{-jk_z z} \quad (2.2.26)$$

$$H_y = -\frac{j\omega\epsilon k_x}{k_c^2} \cos k_x x \sin k_y y e^{-jk_z z} \quad (2.2.27)$$

式(2.2.22a)和(2.2.22b)中m和n的不同取值，对应于不同的 TM_{mn} 模，其截止波数为

$$k_{cmn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \quad (2.2.28)$$

矩形波导中电磁波的传输特性

$$\mathbf{E}_t = -\frac{\gamma}{k_c^2} \nabla_t E_z \quad (2.2.29)$$

$$\mathbf{H}_t = -j \frac{\omega \epsilon}{k_c^2} \hat{\mathbf{z}} \times \nabla_t E_z \quad (2.2.30)$$

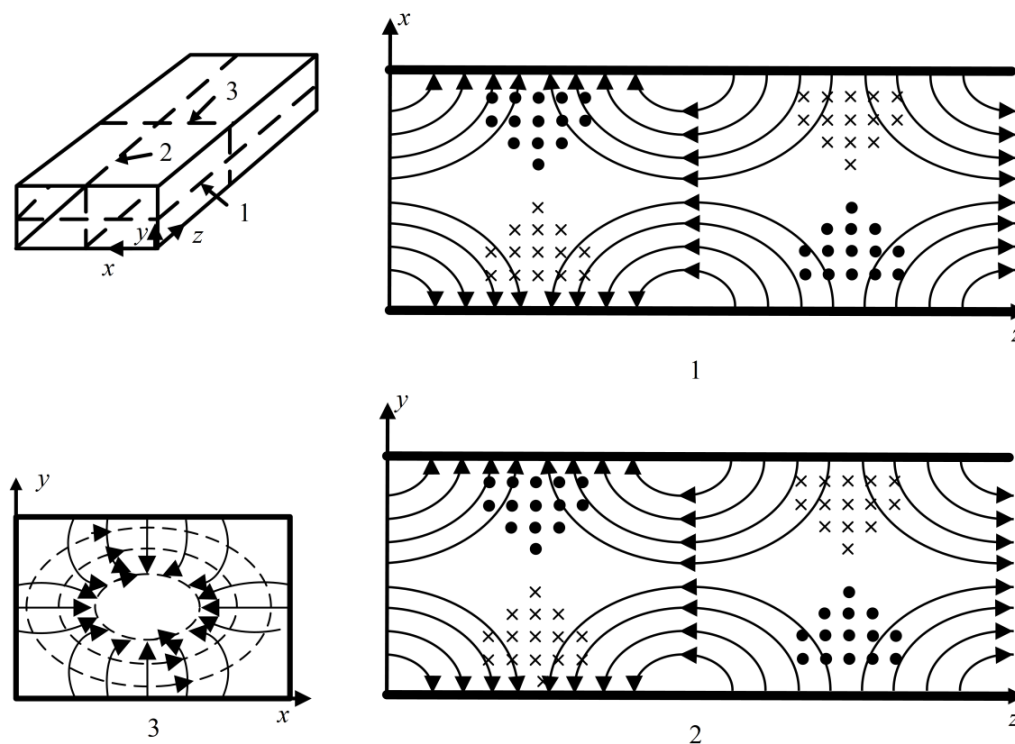


图2-7 矩形波导主模 TM_{11} 的场分布

矩形波导中电磁波的传输特性

TE模式

纵向场 H_z 除了满足(2.2.18)还满足

$$\left. \frac{\partial H_z}{\partial n} \right|_{x=0} = 0, \quad \left. \frac{\partial H_z}{\partial n} \right|_{x=a} = 0, \quad \left. \frac{\partial H_z}{\partial n} \right|_{y=0} = 0, \quad \left. \frac{\partial H_z}{\partial n} \right|_{y=b} = 0 \quad (2.2.31)$$

分离变量法解得

$$H_z = \cos k_x x \cos k_y y e^{-jk_z z} \quad (2.2.32)$$

$$k_x = \frac{m\pi}{a}, \quad m = 0, 1, 2, \dots \quad (2.2.33a) \quad k_y = \frac{n\pi}{b}, \quad n = 0, 1, 2, \dots \quad (2.2.33b)$$

注意 m 和 n 不能同时为零, k_x, k_y, k_z 同样满足色散关系(2.2.23)

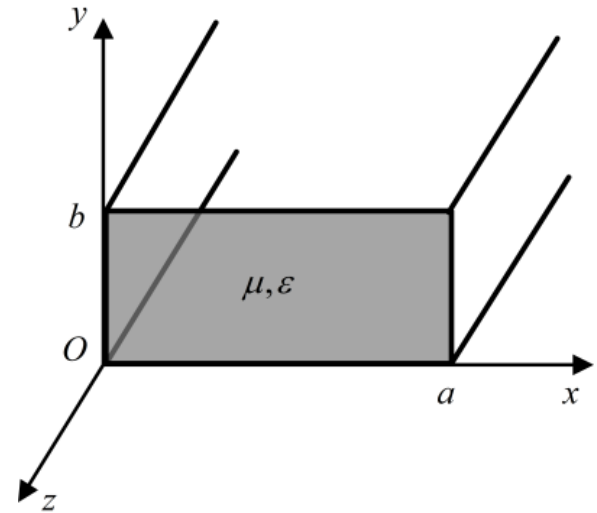


图2-6 矩形波导

矩形波导中电磁波的传输特性

由(2.2.16)和(2.2.17)可算出TE模式的横向分量

$$E_x = \frac{j\omega\mu k_y}{k_c^2} \cos k_x x \sin k_y y e^{-jk_z z} \quad (2.2.34)$$

$$E_y = -\frac{j\omega\mu k_x}{k_c^2} \sin k_x x \cos k_y y e^{-jk_z z} \quad (2.2.35)$$

$$H_x = \frac{jk_x k_z}{k_c^2} \sin k_x x \cos k_y y e^{-jk_z z} \quad (2.2.36)$$

$$H_y = \frac{jk_y k_z}{k_c^2} \cos k_x x \sin k_y y e^{-jk_z z} \quad (2.2.37)$$

TE模的最小截止波数要小于TM模。因此波导主模是TE模

对于 $a > b$ 情形，波导主模便是TE₁₀，其截止波数为 $k_c = \pi / a$

场分布如图2-8

矩形波导中电磁波的传输特性

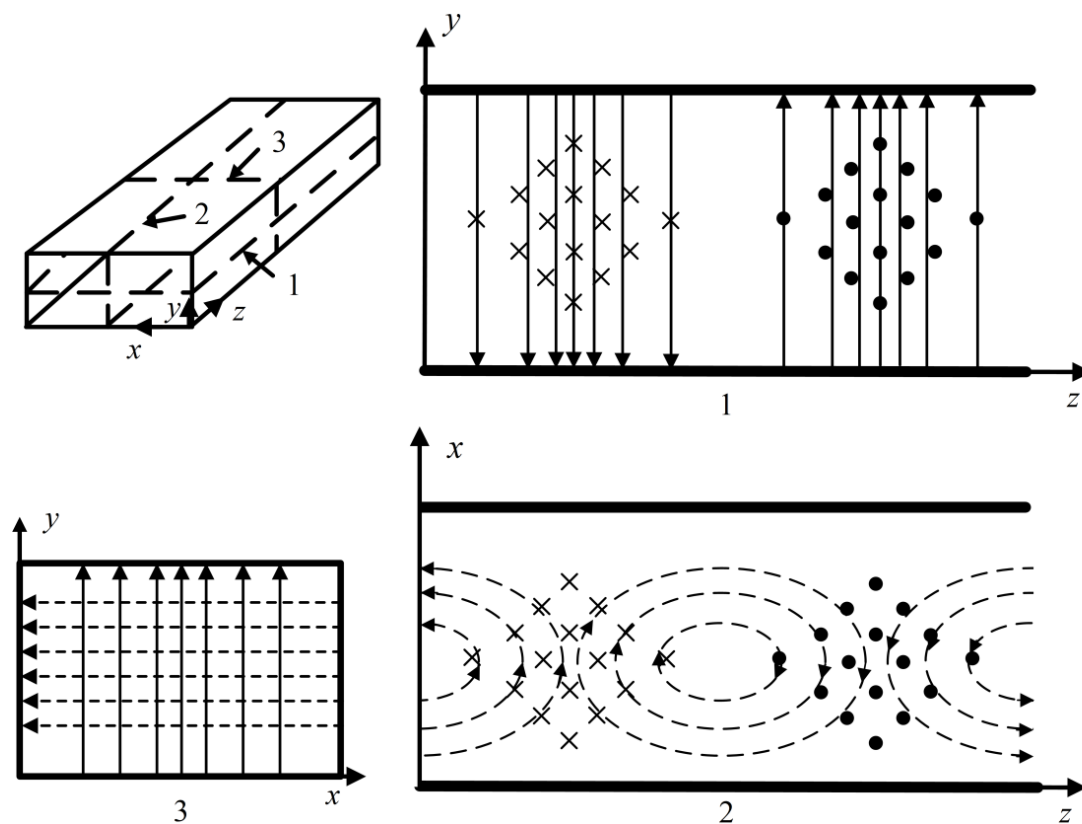


图2-8 矩形波导主模 TE_{10} 的场分布

矩形波导中电磁波的传输特性

单模传输

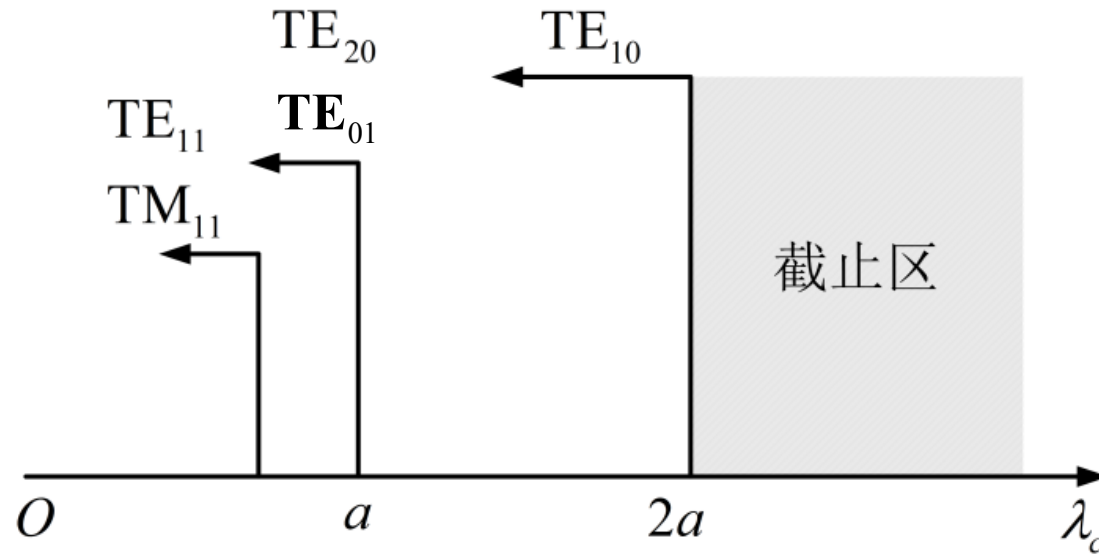


图2-9 $a=2b$ 的矩形波导，其截止波长分布

波导中的传输

- 波导传输问题的求解途径
- 矩形波导中电磁波的传输特性
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波导正规模的特性

假设波导中第m个模的场为 \mathbf{E}_m 、 \mathbf{H}_m ，第n个模的场为 \mathbf{E}_n 、 \mathbf{H}_n

$$\nabla \times \mathbf{H}_m = j\omega\epsilon\mathbf{E}_m \quad (2.2.38)$$

$$\nabla \times \mathbf{E}_m = -j\omega\mu\mathbf{H}_m \quad (2.2.39)$$

$$\nabla \times \mathbf{H}_n = j\omega\epsilon\mathbf{E}_n \quad (2.2.40)$$

$$\nabla \times \mathbf{E}_n = -j\omega\mu\mathbf{H}_n \quad (2.2.41)$$

\mathbf{H}_n 点乘(2.2.39)减去 \mathbf{E}_m 点乘(2.2.40)得

$$\mathbf{H}_n \cdot \nabla \times \mathbf{E}_m - \mathbf{E}_m \cdot \nabla \times \mathbf{H}_n = \nabla \cdot (\mathbf{E}_m \times \mathbf{H}_n) = j\omega(-\mathbf{H}_n \cdot \mu\mathbf{H}_m - \mathbf{E}_m \cdot \epsilon\mathbf{E}_n) \quad (2.2.42)$$

\mathbf{E}_n 点乘式(2.2.38)减去 \mathbf{H}_m 点乘式(2.2.41)得

$$\mathbf{E}_n \cdot \nabla \times \mathbf{H}_m - \mathbf{H}_m \cdot \nabla \times \mathbf{E}_n = \nabla \cdot (\mathbf{H}_m \times \mathbf{E}_n) = j\omega(\mathbf{H}_m \cdot \mu\mathbf{H}_n + \mathbf{E}_n \cdot \epsilon\mathbf{E}_m) \quad (2.2.43)$$

波导正规模的特性

式(2.2.42)加上(2.2.43)得

$$\nabla \cdot (\mathbf{E}_m \times \mathbf{H}_n - \mathbf{E}_n \times \mathbf{H}_m) = 0 \quad (2.2.44)$$

对式(2.2.44)在波导 z 和 $z+\Delta z$ 两平面及波导内壁所围区域积分得

$$\int_{s_1} (\mathbf{E}_m \times \mathbf{H}_n - \mathbf{E}_n \times \mathbf{H}_m) \cdot (-\hat{\mathbf{z}}) ds + \int_{s_c} (\mathbf{E}_m \times \mathbf{H}_n - \mathbf{E}_n \times \mathbf{H}_m) \cdot \hat{\mathbf{n}} ds \quad (2.2.45)$$

$$+ \int_{s_2} (\mathbf{E}_m \times \mathbf{H}_n - \mathbf{E}_n \times \mathbf{H}_m) \cdot (\hat{\mathbf{z}}) ds = 0$$

$$\mathbf{E}_m = \mathbf{E}_{tm} + E_{zm} \hat{\mathbf{z}}$$

$$\int_{s_1} (\mathbf{E}_{tm} \times \mathbf{H}_{tn} - \mathbf{E}_{tn} \times \mathbf{H}_{tm}) \cdot (-\hat{\mathbf{z}}) ds + \int_{s_2} (\mathbf{E}_{tm} \times \mathbf{H}_{tn} - \mathbf{E}_{tn} \times \mathbf{H}_{tm}) \cdot (\hat{\mathbf{z}}) ds = 0 \quad (2.2.46)$$

波导正规模的特性

如果第m个模和第n个模都沿正z方向传输

$$\mathbf{E}_{tm} = e^{-\gamma_m z} \mathbf{e}_m(x, y), \quad \mathbf{H}_{tm} = \frac{1}{Z_m} e^{-\gamma_m z} \mathbf{h}_m(x, y) \quad (2.2.47a)$$

$$\mathbf{E}_{tn} = e^{-\gamma_n z} \mathbf{e}_n(x, y), \quad \mathbf{H}_{tn} = \frac{1}{Z_n} e^{-\gamma_n z} \mathbf{h}_n(x, y) \quad (2.2.47b)$$

$$\mathbf{h}_m(x, y) = \hat{\mathbf{z}} \times \mathbf{e}_m(x, y) \quad (2.2.48a) \quad \int_S \mathbf{e}_m(x, y) \cdot \mathbf{e}_m(x, y) dS = 1 \quad (2.2.48a)$$

$$(\gamma_m + \gamma_n) \int_s \left(\frac{1}{Z_n} \mathbf{e}_m \times \mathbf{h}_n - \frac{1}{Z_m} \mathbf{e}_n \times \mathbf{h}_m \right) \cdot (\hat{\mathbf{z}}) ds = 0 \quad (2.2.49)$$

显然 $\gamma_m + \gamma_n \neq 0$ ，故

$$\int_s \left(\frac{1}{Z_n} \mathbf{e}_m \times \mathbf{h}_n - \frac{1}{Z_m} \mathbf{e}_n \times \mathbf{h}_m \right) \cdot (\hat{\mathbf{z}}) ds = 0 \quad (2.2.50)$$

波导正规模的特性

式(2.2.50)和(2.2.52b)相加

$$\int_S \mathbf{e}_m \times \mathbf{h}_n \cdot \hat{\mathbf{z}} dS = 0, \quad \gamma_m \neq \gamma_n \quad (2.2.53a)$$

利用(2.2.48a)和(2.2.48b)

$$\int_S \mathbf{e}_m \times \mathbf{h}_n \cdot \hat{\mathbf{z}} dS = \delta_{mn} \quad (2.2.53b)$$

$$\int_S \mathbf{e}_m \cdot \mathbf{e}_n dS = \delta_{mn} \quad (2.2.53c)$$

波导正规模最为一般的正交性

$$\mathbf{E} = \sum_{m,n} a_{mn} \mathbf{e}_{TEmn} + \sum_{m,n} b_{mn} \mathbf{e}_{TMmn} \quad (2.2.54)$$

波导模式的完备性

例题2.6

试求等腰直角三角形波导(图2-9)中最低阶E波和H波的截止波长。

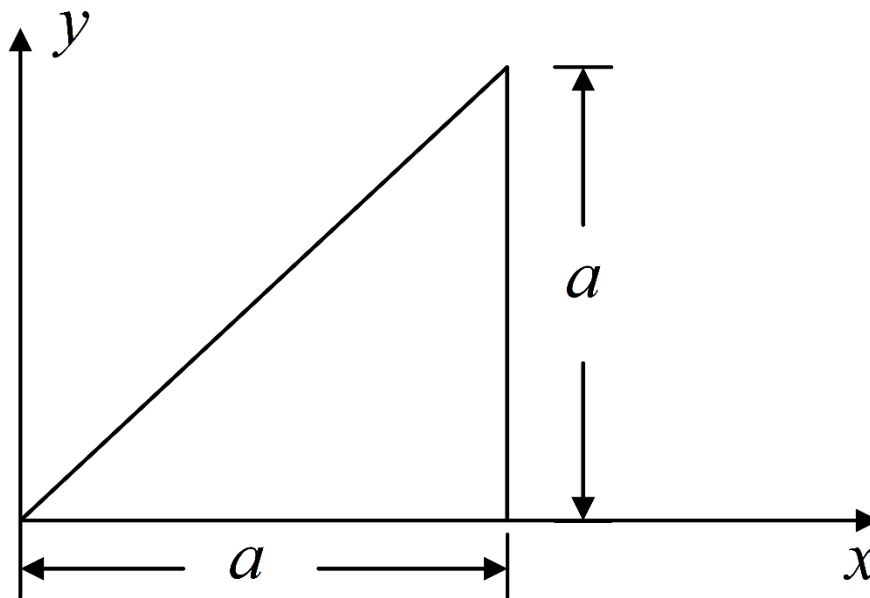


图2-9 等腰直角三角形波导

例题2.6

解：在方形波导的对角线处加一导体平面即得到等腰直角三角形波导。故求解时可利用方形波导中的场解。

(1) E波：设等腰直角三角形波导中的纵向电场分量 E_z 为方形波导中两个不同的E型波的叠加：

$$E_z = A \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a} + B \sin \frac{p\pi x}{a} \sin \frac{q\pi y}{a} \quad (\text{e2.6.1})$$

为了满足 $(\nabla_t^2 + k_t^2)E_z = 0$ ，横向波数 k_t 应为

$$k_t^2 = \left(\frac{\pi}{a}\right)^2 (m^2 + n^2) = \left(\frac{\pi}{a}\right)^2 (p^2 + q^2) \quad (\text{e2.6.2})$$

在 $y=0$ 和 $x=a$ 上 $E_z=0$ ，满足边界条件；在弦 $y=x$ 上，

$$E_z|_{y=x} = A \sin m\xi \sin n\xi + B \sin p\xi \sin q\xi \quad (\text{e2.6.3})$$

$$\begin{aligned} &= \frac{A}{2} [\cos(m-n)\xi - \cos(m+n)\xi] + \frac{B}{2} [\cos(p-q)\xi - \cos(p+q)\xi] \\ &= 0 \end{aligned}$$

例题2.6

若取正整数 r ,令 $m - p = q - n = r$ (e2.6.4a)

由式(e2.6.2)可得 $m + p = q + n$ (e2.6.4b)

上二式又可写为 $m + n = p + q, m - n = -(p - q)$ (e2.6.5)

$A = -B = 1$ 时式(e2.6.3)成立。又由上式可知, $m = q, n = p$

因此式(e2.6.1)和式(e2.6.2)变为

$$E_z = \sin \frac{(n+r)\pi x}{a} \sin \frac{n\pi y}{a} - \sin \frac{n\pi x}{a} \sin \frac{(n+r)\pi y}{a} \quad (\text{e2.6.1})$$

$$k_t = \frac{\pi}{a} \sqrt{(n+r)^2 + n^2}, \quad n, r = 1, 2, \dots \quad (\text{e2.6.2})$$

可见, 最低E波的截止波长为

$$\lambda_c = 2\pi/k_t = 2a/\sqrt{5}$$

(2) H波：考虑方形波导 TE_{11} 波型的电场分布(图2-10)

根据场型分割原理

由 $y=0, y=x$ 和 $x=l$ 平面围成的等腰直角三角形波导和由 $y=0, y=x$ 和 $y=l-x$ 平面围成的等腰直角三角形波导中，纵向磁场分量均为

$$H_z = \cos \frac{\pi x}{l} \cos \frac{\pi y}{l} \quad (\text{e2.6.3})$$

对于前者，当直角边为 a 时，

$l = a$ ，截止波长 $\lambda_c = \sqrt{2}l = \sqrt{2}a$

对于后者，当直角边为 a 时，

$l = \sqrt{2}a$ ，截止波长 $\lambda_c = \sqrt{2}l = 2a$

因此，最低H波的 $\lambda_c = 2a$

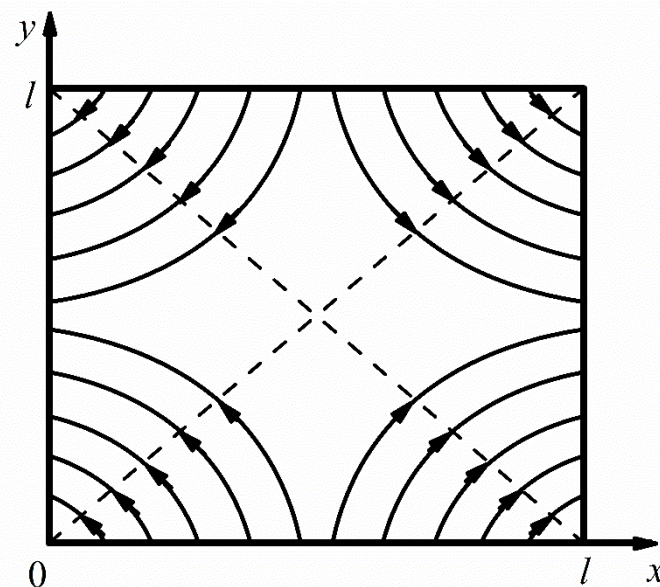


图2-10 方形波导 TE_{11} 波型的电场分布