1. 解: $D_{min} = \frac{1}{4}\times 0 + \frac{1}{4}\times 0 + \frac{1}{4}\times 0 + \frac{1}{4}\times 0$

$= 0$

$D_1 = D_2 = D_3 = D_4 = \frac{1}{4}\times 0 + 3\times\frac{1}{4}$

$= \frac{3}{4}$

$D_{max} = \min\{D_1, D_2, D_3, D_4\} = \frac{3}{4}$

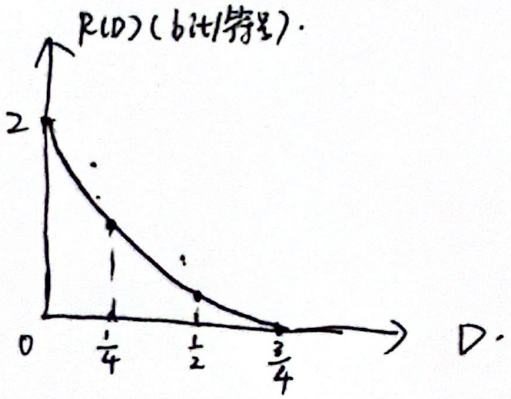
$R(D) = \log 4 - D\log(4-1) - H(D)$

$= 2 - D\log 3 - H(D),\ 0 \le D \le 1-\frac{1}{4} = \frac{3}{4}$.

则 $R(D) = \begin{cases} 2 - D\log 3 - H(D), & 0 \le D \le \frac{3}{4} \\ 0, & D > \frac{3}{4} \end{cases}$

取 $(0,2)\ (\frac{3}{4},0)\ (\frac{1}{2}, \frac{3}{2}-\frac{1}{2}\log 3)\ (\frac{1}{4}, \frac{3}{2}-\frac{1}{4}\log 3)$

得曲线为



2. 解: $D_1 = \frac{1}{3}\times(1+1+2) = \frac{4}{3}$

$D_2 = \frac{1}{3}\times(2+1+1) = \frac{4}{3}$

则 $D_{max} = \min\{D_1, D_2\} = \frac{4}{3}$.

此时 $I(U;V) = 0$

即 $\begin{cases} P(V_1/U_i) = P(V_1) \\ P(V_1) + P(V_2) = 1 \\ P(V_2/U_i) = P(V_2) \end{cases}$

信道为 $\begin{pmatrix} a & 1-a \\ a & 1-a \\ a & 1-a \end{pmatrix}$, $0 \le a \le 1$

$D_{min} = \frac{1}{3}\times 1 + \frac{1}{3}\times 1 + \frac{1}{3}\times 1 = 1$

则 $\begin{cases} P(V_1/U_1) = 1 \\ P(V_1/U_2) + P(V_2/U_2) = 1 \\ P(V_2/U_3) = 1 \end{cases}$

即 $\begin{pmatrix} 1 & 0 \\ a & 1-a \\ 0 & 1 \end{pmatrix}$, $0 \le a \le 1$

3.解：$D_{min}=\frac{1}{2}\times 0+\frac{1}{2}\times 0=0$

$D_1=\frac{a}{2}$，$D_2=\frac{a}{2}$

$D_{max}=\min\{D_1,D_2\}=\frac{a}{2}$

其为二元对称信道

则 $R(D)=\begin{cases}\log 2-D\log(2-1)-H(D), & 0\le D\le\frac{1}{2}\\ 0, & D>\frac{1}{2}\end{cases}$

整理得 $R(D)=\begin{cases}1+D\log D+(1-D)\log(1-D), & 0\le D\le\frac{1}{2}\\ 0, & D>\frac{1}{2}.\end{cases}$