附录　常用的矢量公式

$$\vec{A}\cdot\vec{B}=A_1B_1+A_2B_2+A_3B_3 \tag{1}$$

$$\vec{A}\times\vec{B}=\vec{e}_1(A_2B_3-A_3B_2)+\vec{e}_2(A_3B_1-A_1B_3)+\vec{e}_3(A_1B_2-A_2B_1) \tag{2}$$

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{C}\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\vec{C}\times\vec{A}) \tag{3}$$

$$\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}(\vec{A}\cdot\vec{C})-\vec{C}(\vec{A}\cdot\vec{B}) \tag{4}$$

$\nabla c=0$ ，$\nabla\cdot\vec{C}=0$ ，$\nabla\times\vec{C}=0$　　c为常数，$\vec{C}$为常矢 (5)

$$\nabla(\varphi\psi)=\varphi\nabla\psi+\psi\nabla\varphi \tag{6}$$

$$\nabla\cdot(\varphi\vec{A})=\varphi\nabla\cdot\vec{A}+\vec{A}\cdot\nabla\varphi \tag{7}$$

$$\nabla\times(\varphi\vec{A})=\varphi\nabla\times\vec{A}+\Delta\varphi\times\vec{A} \tag{8}$$

$$\nabla(\vec{A}\cdot\vec{B})=(\vec{A}\cdot\nabla)\vec{B}+(\vec{B}\cdot\nabla)\vec{A}+\vec{A}\times\nabla\times\vec{B}+\vec{B}\times\nabla\times\vec{A} \tag{9}$$

$$\nabla\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot\nabla\times\vec{A}-\vec{A}\cdot\nabla\times\vec{B} \tag{10}$$

$$\nabla\times(\vec{A}\times\vec{B})=\vec{A}\nabla\cdot\vec{B}-\vec{B}\nabla\cdot\vec{A}+(\vec{B}\cdot\nabla)\vec{A}-(\vec{A}\cdot\nabla)\vec{B} \tag{11}$$

$$\nabla\times\nabla\varphi=0 \tag{12}$$

$$\nabla\cdot\nabla\times\vec{A}=0 \tag{13}$$

$$\nabla\cdot\nabla\varphi=\nabla^2\varphi \tag{14}$$

$$\nabla f(\varphi)=f'(\varphi)\nabla\varphi \tag{15}$$

$$\nabla\times\nabla\times\vec{A}=\nabla\nabla\cdot\vec{A}-\nabla^2\vec{A} \tag{16}$$

$$\int_\tau\nabla\cdot\vec{A}d\tau=\oint_s\vec{A}\cdot d\vec{s} \tag{17}$$

$$\int_\tau\nabla\varphi d\tau=\oint_s\varphi d\vec{s} \tag{18}$$

$$\int_\tau\nabla\times\vec{A}d\tau=-\oint_s\vec{A}\times d\vec{s} \tag{19}$$

$$\int_s(\nabla\times\vec{A})\cdot d\vec{s}=\oint_l\vec{A}\cdot d\vec{l} \tag{20}$$

$$\int_s \nabla\varphi \times d\vec{s} = -\oint_l \varphi\, d\vec{l} \tag{21}$$

若 $\vec{R} = \vec{r} - \vec{r}'$，$R = \left|\vec{R}\right|$，$\hat{R} = \dfrac{\vec{R}}{R}$ 则有：$\nabla R = \hat{R}$ (22)

$$\nabla f(R) = f'(R)\hat{R} = -\nabla' f(R) \tag{23}$$

$$\nabla R^n = nR^{n-1}\hat{R} \tag{24}$$

$$\nabla \cdot [f(R)\hat{R}] = \frac{2}{R} f(R) + f'(R) = -\nabla' \cdot [f(R)\hat{R}] \tag{25}$$

$$\nabla \cdot (R^n \hat{R}) = (n+2)R^{n-1} \tag{26}$$

$$\nabla \cdot \hat{R} = \frac{2}{R} \tag{27}$$

$$\nabla \times [f(R)\hat{R}] = 0 \tag{28}$$

$$\nabla^2 f(R) = \frac{2}{R} f'(R) + f''(R) = \nabla'^2 f(R) \tag{29}$$

$$\nabla^2 R^n = n(n+1)R^{n-2} \tag{30}$$