Towards Systematic Design and Control of Soft Robotics

PROEFSCHRIFT

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Dynamics of Soft Robots

1.1 Lie Group Theory for Robotics

The analytical tools used in this work are derived from Lie group theory. Here, we give a brief preliminary on the basics of Lie groups and their associated Lie algebras whose properties will be used later for deriving the kinematic and dynamic model applicable to a set of soft robotic systems.

The Lie group encompasses the concepts of 'group' and 'smooth manifold' in a unique embodiment: a Lie group $\mathcal{G}$ is a smooth manifold whose elements satisfy the group axioms. Within the perspective of robotics, the Lie group is viewed as a smooth surface on which the states of the system evolve, that is, the manifold describes or is defined by constraints imposed on the state. The smoothness of the manifold implies there exists a unique tangent space for each point on the manifold. The tangent space of the Lie group at the identity is called the Lie algebra, and it allows us to perform algebra computation concerning the Lie group.

1.2 Configuration spaces

In contrast to a rigid robot, whose mechanical structure consists of static links and joints, a soft robot lacks the physical notion of joints and therefore cannot be viewed as a single- or multi-body. From a mechanical perspective, a soft robotic system is more closely related to a continuous deformable medium with infinite degrees-of-freedom rather than a hyper-redundant system. Given this notion, the hyper-flexible soft robot is modeled as a one-dimensional Cosserat beam together with the geometrically exact beam theories proposed by [2]. The main idea is to regard the material solid as a series of infinitesimally thin (semi)-rigid body that co-align with a spatial curve passing through the geometrical center of each cross-section. To characterized the spatial dimension along this curve, let us introduce a spatial parameter $\sigma \in \mathcal{X}$ with $\mathcal{X} \in [0, L] \subset \mathbb{R}$ (where $L \in \mathbb{R}_{<0}$ is the undeformed length of the soft robot). To further elaborate its physical notation, the spatial parameter σ represents a material point inside the hyper-flexible body of soft robot, or more precisely, located at the center of the cross section of the continuous elastic body at σ.

1.3 Dynamics through Hamilton's variational principle

In this section, we derive the dynamical model of the soft robot through Hamilton's variational principle. The principle states that the time evolution of a state vector $q(t)$ between two state instances $q_1 = q(t_1)$ and $q_2 = q(t_2)$ over a fixed time interval $[t_1, t_2]$ is a stationary point regarding an action functional, $\mathcal{S} = \int_{t_1}^{t_2} \mathcal{L}(q, \dot{q}, t)\, dt$ in which $\mathcal{L}(q, \dot{q}) := \mathcal{T}(q, \dot{q}) - \mathcal{V}(q)$ is the Lagrangian. The extension of Hamilton's principle [1] also considers external potential contributions, and can be formally written as

$$\delta(\mathcal{S}) = \int_{t_0}^{t_1} \left[\delta(\mathcal{T}) - \delta(\mathcal{V}) + \delta(\mathcal{W}_{ex})\right]\, dt = 0, \tag{1.1}$$

where the operator $\delta(\cdot)$ denotes a variation applied along the trajectory of the system that are fixed at the boundaries of $[t_0, t_1]$, and $\mathcal{W}_{ex}$ is the external virtual work produced by nonconservative external forces to the dynamical system. First, the kinetic energy of the soft robot is defined as

$$\mathcal{T} := \frac{1}{2} \int_{\mathcal{X}} \eta^\top \mathcal{M} \eta \, d\sigma, \tag{1.2}$$

where $\mathcal{M} \in \mathbb{R}^{6\times 6}$ is a the inertia tensors whose components correspond to the mass and moment of inertias. Then, the variation of the kinetic energy function is given by

$$\begin{aligned}
\delta(\mathcal{T}) &= \frac{\partial}{\partial a} \mathcal{T}(\eta + a\delta(\eta))\bigg|_{a=0}, \\
&= \frac{1}{2} \int_{\mathcal{X}} \delta(\eta)^\top \mathcal{M} \eta + \eta^\top \mathcal{M} \delta(\eta)\, d\sigma, \\
&= \int_{\mathcal{X}} \delta(\eta)^\top \mathcal{M} \eta \, d\sigma,
\end{aligned} \tag{1.3}$$

Recalling the commutativity of the Lie algebra, we can express the variation as $\delta(\eta) = \dot{\epsilon} + \mathrm{ad}_\eta \epsilon$. Therefore, substitution into (1.3) and followed by integration by parts leads to

$$\int_{t_1}^{t_2} \delta(\mathcal{T})\, dt = \left[\int_{\mathcal{X}} \epsilon^\top M \eta\right]_{t_0}^{t_1} + \int_{t_1}^{t_2} \int_{\mathcal{X}} \epsilon^\top \left(M\dot{\eta} - \mathrm{ad}_\eta^\top M \eta\right)\, d\sigma dt. \tag{1.4}$$

Since the variations are fixed at the boundaries of $[t_0, t_1]$, the first right hand part in (1.4) vanishes.

The internal strain energy of the soft robot is defined as

$$\mathcal{V}_{in} := \frac{1}{2} \int_{\mathcal{X}} \xi^\top \Lambda \, d\sigma. \tag{1.5}$$

where $\Lambda \in \mathbb{R}^6$ is the vector field representing the internal forces of the stress resultants over the continuum body. These internal force vector field and the strains vector field are related through a material constitutive law. In general concerning soft robotic applications, the use of linear constitutive relations for an isotropic elastic material are not sufficient, since large deformations introduce nonlinear material behavior. However, for the sake of simplicity, we consider the simplest viscoelastic constitutive model - the Kelvin-Voigt model. The Kelvin-Voigt model is a linear elasticity model with a linear viscous contribution that is proportional to the rate of strain ξ,

$$\Lambda = K\xi + \Gamma\dot{\xi} \tag{1.6}$$

where K and Γ are the elasticity and viscosity material tensor, respectively.

Bibliography

[1] Frederic Boyer, Mathieu Porez, and Alban Leroyer. *Poincaré-cosserat equations for the lighthill three-dimensional large amplitude elongated body theory: Application to robotics*, volume 20. 2010.

[2] J. C. Simo and L. Vu-Quoc. A three-dimensional finite-strain rod model. part II: Computational aspects. *Computer Methods in Applied Mechanics and Engineering*, 58(1):79–116, 1986.