

Toward Design and Control of Soft Robots

PROEFSCHRIFT

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door

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geboren te Roermond

Dit proefschrift is goedgekeurd door de promotoren en de samenstelling van de promotiecommissie is als volgt:

voorzitter: prof. dr. L.P.H. de Goey
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A

Fundamentals on Lie Group Theory

In this chapter, we will discuss the fundamentals on Lie groups and their associated Lie algebras.

A.1 Lie group

A Lie group encompasses the concepts of ‘group’ and ‘smooth manifold’ in a unique embodiment. To be more specific, the Lie Group \mathcal{G} is a smooth manifold whose elements satisfy the group axioms:

1. Closure: if $g_1, g_2 \in \mathcal{G}$, then g_1g_2 is also an element of \mathcal{G} ,
2. Identity: there exists an element e such that $ge = eg = g$ for any $g \in \mathcal{G}$,
3. Inversion: For any $g \in \mathcal{G}$, there exists an element $g^{-1} \in \mathcal{G}$ such that $gg^{-1} = g^{-1}g = e$,
4. Associativity: $(g_1g_2)g_3 = g_1(g_2g_3)$ for any $g_1, g_2, g_3 \in \mathcal{G}$.

The smoothness of the Lie groups intuitively suggests the existence of useful differential geometries. For any elements g on the smooth manifold \mathcal{G} , there exists a tangent linear space denoted by $T_g\mathcal{G}$. The tangent space of the Lie group at the identity element e is referred to as the associative Lie algebra \mathfrak{g} of the group. it allows us to perform algebra computation concerning the Lie group.

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