Towards Design and Control of Soft Robotic Systems

PROEFSCHRIFT

ter verkrijgen van de graad van doctor aan de
Technische Universiteit Eindhoven, op gezag van de
rector magnificus prof. dr. F.P.T. Baaijens, voor een
commissie aangewezen door het College van
Promoties, in het openbaar te verdedigen
op maandag 28 mei 2022 om 16.00uur

door

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geboren te Roermond

Dit proefschrift is goedgekeurd door de promotoren en de samenstelling van de promotiecommissie is als volgt:

voorzitter:	prof. dr. L.P.H. de Goey
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A

Fundamentals on Lie Group Theory

In this chapter, we will discuss the fundamentals on Lie groups and their associated Lie algebras.

A.1 Lie group

A Lie group encompasses the concepts of 'group' and 'smooth manifold' in a unique embodiment. To be more specific, the Lie Group $\mathcal{G}$ is a smooth manifold whose elements satisfy the group axioms:

1. Closure: if $g_1, g_2 \in \mathcal{G}$, then $g_1 g_2$ is also an element of $\mathcal{G}$,
2. Identity: there exists an element e such that $ge = eg = g$ for any $g \in G$,
3. Inversion: For any $g \in \mathcal{G}$, there exists an element $g^{-1} \in \mathcal{G}$ such that $gg^{-1} = g^{-1}g = e$,
4. Associativity: $(g_1 g_2) g_3 = g_1 (g_2 g_3)$ for any $g_1, g_2, g_3 \in \mathcal{G}$.

The smoothness of the Lie groups intuitively suggests the existence of useful differential geometries. For any elements g on the smooth manifold $\mathcal{G}$, there exists a tangent linear space denoted by $T_g\mathcal{G}$. The tangent space of the Lie group at the identity element e is referred to as the associative Lie algebra $\mathfrak{g}$ of the group. it allows us to perform algebra computation concerning the Lie group.

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