A Systematic Approach for Design, Modeling, and Control of Soft Robotic Systems

Brandon Jonathan Caasenbrood

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A Systematic Approach for Design, Modeling, and Control of Soft Robotic Systems

Proefschrift

ter verkrijging van de graad van doctor aan de
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rector magnificus prof.dr.ir. F.P.T. Baaijens, voor een
commissie aangewezen door het College voor
Promoties, in het openbaar te verdedigen
op maandag 30 december 2022 om 16:00 uur

door

Brandon Jonathan Caasenbrood

geboren te Jupiter

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Het onderzoek dat in dit proefschrift wordt beschreven is uitgevoerd in overeenstemming met de TU/e Gedragscode Wetenschapsbeoefening.

Abstract

A Systematic Approach for Design, Modeling, and Control of Soft Robotic Systems

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Societal summary

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Contents

Nomenclature

Vector and matrix notation

x	Scalar notation
$\boldsymbol{x}$	Vector notation
$\boldsymbol{X}$	Matrix notation
$\boldsymbol{\mathcal{X}}$	Tensor notation
$\boldsymbol{\mathcal{Q}}$	Manifold

Compact sets

$\emptyset$	Empty set
$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}_{>0}$	Strictly positive reals
$\mathbb{R}_{\geq 0}$	Positive reals
$\mathbb{N}$	Set of natural numbers
$\mathbb{T}$	Finite time horizon
$\mathbb{X}$	1-dimensional spatial set or domain (*i.e.*, line)
$\mathbb{V}$	3-dimensional spatial set or domain (*i.e.*, volume)

Groups

id	Identity
$\mathrm{SO}(n)$	Lie group of rotations on $\mathbb{R}^n$ (*i.e.*, special orthonormal matrices)
$\mathrm{SE}(n)$	Lie group of homogeneous transformations on $\mathbb{R}^n$
$\mathrm{so}(n)$	Lie algebra of $\mathrm{SO}(n)$
$\mathrm{se}(n)$	Lie algebra of $\mathrm{SE}(n)$

Vector- and matrix operations

$\dot{(\cdot)}$	First time derivative
$\ddot{(\cdot)}$	Second time derivative
$\hat{(\cdot)}, (\cdot)^{\wedge}$	Isomorphism from $\mathbb{R}^6 \to \mathrm{se}(3)$
$\check{(\cdot)}, (\cdot)^{\vee}$	Isomorphism from $\mathrm{se}(3) \to \mathbb{R}^6$
$(\cdot)_0$	Reference configuration
$(\cdot)^\top$	Transpose
$(\cdot)^{-1}$	Square matrix inverse
$(\cdot)^\dagger$	Moore-Penrose pseudo inverse
$(\cdot)^+$	Generalized matrix inverse
$(\cdot)^d$	Generalized matrix inverse

Operators and letter-like symbols

δ	Variation of a field
∂	Boundary of a set
int	Interior of a set
$\sup_t$	Supremum over continuous time t
dim	Dimension of vector
trace	Trace of matrix
$\\|\cdot\\|_{\mathrm{ma}}$	Mean absolute norm
$\\|\cdot\\|_{\mathrm{rms}}$	Root-mean-square norm

Acronyms

CoM	Center of mass
CoR	Coefficient of restitution

1
Introduction

1.1 Name of first section

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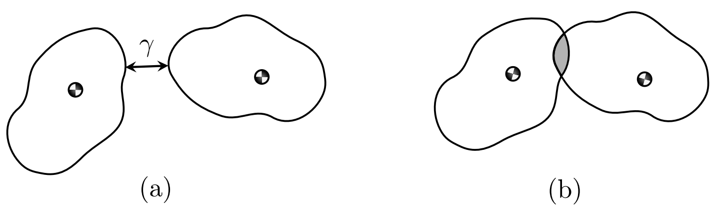

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1.3 Problem statement

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State the objective of your PhD research here.

1.4 Research challenges and contributions

In this section, the research problem is divided in ... research challenges with the topics: topic 1, topic 2, topic 3, ... Per research challenge, a contribution of this thesis addressing the subproblem is presented.

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1.5 Outline of the thesis

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A note for the reader. Chapters ...-... are all based on submitted/published articles and consequently are self-contained and can be read independently. A reference to the corresponding research paper is included at the beginning of each chapter. An overview of how the chapters of this thesis relate to the contributions presented in Section 1.4 is given in Table ...

I

Design Optimization

II

Modeling of Soft Robots

2

Dynamic modeling – The Piece-wise Constant Approach

Abstract - *The motion complexity and use of exotic materials in soft robotics call for accurate and computationally efficient models intended for control. To reduce the gap between material and control-oriented research, we build upon the existing Piecewise-Constant Curvature framework by incorporating hyper-elastic and visco-elastic material behavior. In this work, the continuum dynamics of the soft robot are derived through the differential geometry of spatial curves, which are then related to Finite-Element data to capture the intrinsic geometric and material nonlinearities. To enable fast simulations, a reduced-order integration scheme is introduced to compute the dynamic Lagrangian matrices efficiently, which in turn allows for real-time (multi-link) models with sufficient numerical precision. By exploring the passivity and using the parametrization of the hyper-elastic model, we propose a passivity-based adaptive controller that enhances robustness towards material uncertainty and unmodeled dynamics – slowly improving their estimates online. As a study case, a fully 3D-printed soft robot manipulator is developed, which shows good correspondence with the dynamic model under various conditions, e.g., natural oscillations, forced inputs, and under tip-loads. The solidity of the approach is demonstrated through extensive simulations, numerical benchmarks, and experimental validations.*

This chapter is based on:

.

A detailed list of the differences between this chapter and the article on which it is based is provided in the *Modifications* chapter of this thesis.

2.1 Introduction

Traditional robots are made from rigid and dense materials that ensure accurate and repeatable motions. While rigid robotics excel at fast and precise motion, their structural rigidity lacks the compliance and mechanical robustness needed for safe and passive interaction in an unknown environment. Soft robotics is a field of robotics that aims to improve the motion complexity and environmental robustness that is generally lacking its rigid counterpart. To further promote these topics in robotics, researchers aim to mimic living creatures by developing bio-inspired robots with similar morphologies and mechanical properties [? ? ? ? ? ?]. In soft robotics, the hyper-flexible and continuum-bodied structure provide them with a rich family of motion primitives. Besides bio-mimicry, soft robotics has proven to be a prominent alternative for rigid robotics with a variety of applications, e.g., manipulation and adaptive grasping [?], untethered locomotion and exploration through uncertain environments [? ? ?], rehabilitation [?], and even minimal-invasive surgery [? ?]. Although popularity of the field has increased exponentially in recent years, one of the first soft robots date back already to the early 1990's, e.g., the work of Suzumori et al. [?]. Yet, despite years of soft robotics research, their intrinsic hyper-flexible nature still possesses numerous challenges on modeling and control.

One major challenge in modeling is that the soft robot's elastic body undergoes large, continuous deformation. Since its inception, numerous works have addressed the kinematics for soft continuum robots[? ?]; yet, its original framework stems from to hyper-redundant robotics nearly a decade earlier[? ?]. Similar to soft robots, hyper-redundant robots exploit their high joint redundancy to achieve a wider ranges of tasks (e.g., shape control and collision avoidance) besides end-effector manipulation. To some extent, soft robots can be seen as the successor to hyper-redundant robots in which rigid mechanical joints or links are substituted with hyper-flexible soft elements. As a result, their dynamics involve a continuously deformable inertial body rather than the classical notion of rigid bodies. As such, conventional modeling approaches cannot be applied directly to these continuously deformable robots, stressing the importance of novel modeling strategies. In this respect, the dynamics of a continuously deformable soft robot are in theory of an infinite-dimensional nature. This paradigm shift has further emphasized the challenges in control-oriented modeling of soft robots; as their physical description are often more suited for a Partial Differential Equations (PDEs) rather than Ordinary Differential Equations (ODEs).

Recently, some significant steps have been made towards formulating reduced-order ODE models for elastic continuum soft robots, paving a path towards model-based controllers. Perhaps one of the most popular techniques of spatial reduction is the so-called Piece-wise Constant Curvature (PCC) model. The PCC model assumes that the continuum shape can be described using a number of spatially-constant curves which are parametrized using a set of generalized coordinates.

Although PCC models can be seen as a significant oversimplification of true continuum mechanics at hand, these models have proven to be remarkably viable for various control applications. Besides its use in inverse kinematic control [? ? ?], PCC models have also shown to be suitable for feedforward controllers as demonstrated by Falkenhahn et al. [?]; and more recently, closed-loop feedback controllers by Della Santina et al. [? ?]. Although the aforementioned works utilize the lumped-mass description, others have employed PCC models with uniform mass distribution [? ? ? ?] and current models even extend beyond the constant curvature [? ? ?]. However, in the face of significant external loading or (distributed) contact with the environment, the PCC assumption is relatively conservative and leads to undesired kinematic constraints on the continuum deformation. Besides, these models often need additional identification to model the compliance as they do not originate from continuum mechanical framework.

On the other hand, Finite-Element Method (FEM) models do originate from continuum mechanics and due to their PDE description provide a more accurate representation of deformations; and are particularly suited to deal with geometric and material nonlinearities. Duriez et al.[?] and related works[? ?] showed that reduced-order FEM models can play an important role in closed-loop control – allowing accurate volumetric deformation and hyper-elastic behavior. Although such real-time simulations for FEM-based models are possible, a significant state-reduction is required to ensure sufficient computational speed. In the process, FEM-based models often loose desirable control properties, e.g., passivity preservation, which might play an important role in control. An alternative modeling strategy is the recently emerging geometrically-exact Cosserat-beam model. Similar to the PCC models, the Cosserat models have the merit benefit that they can be structured into a standard Lagrangian form – the basis for robotics control theory. Rooted in a geometric method for describing the continuum mechanics using Lie theory[?], Boyer et al. [? ?] proposed a geometrically-exact modeling framework for Cosserat beams using nonlinear parametrization of the strain field. Other examples include the work of Renda et al.[? ?] providing various options for Piecewise-Constant Strain (PCS) and Variable Strain modeling approaches. Although recent variants of the Cosserat models offer good computational performance [? ?], its use in model-based control is slowly upcoming.

In this respect, the topic of reduced-order modeling of soft robots is an active area of research. Yet, a challenge that is frequently overlooked in control-orientated research is the anisotropic material behavior, mechanical saturation, and more importantly, the nonlinear and possibly time-varying nature of the highly hyper-elastic soft materials [? ? ? ?]. This is further amplified by the fact that soft robots are known for their diversity in elastic materials and corresponding morphologies. Mustaza et al.[?] proposed modified nonlinear Kelvin–Voigt material model to embody the complex material behavior of silicone-composite manipulators (so-called STIFF-FLOP actuators). A similar silicone composite actuator was experimentally validated by Sadati et. al [?] who proposed a novel modeling approach with an appendage-dependent Hookean model and viscous power-law to describe nonlinear and time-dependent material effects, respectively. Both nonlinear material models show good correspondence with physical soft robots under various dynamic conditions, yet they lack general transferability to the soft robots with different geometries – intrinsically captured by FEM-driven models. As of today, there are little control-oriented models that both offers geometry and material versatilely similar to FEM-models and the control convenience similar to spatial curve models.

Ultimately, the strong nonlinearities paired with its continuous nature encourage the use of model-based controllers. Nevertheless, regarding the aforementioned model-based control approaches [? ? ?], the stability and performance of the closed-loop system could be undermined by uncertainties in physical parameters or unmodelled dynamics. Particularly for state-feedback linearization (e.g., inverse dynamic), as the inversion of inaccurately estimated systems could lead to poor performance and even instability. Adaptive control [? ?] or energy-based controllers [?] might offer the needed robustness towards material uncertainties and unmodelled dynamics. Unfortunately, up till now, the applicability of adaptive and energy-based control techniques on soft robotics is scarcely explored. Franco et al. [?] used an adaptive energy-based controller that compensate for external disturbances on the end-effector, yet these controller can be extended to include various slowly-varying material uncertainties, e.g., hyper-elasticity and viscosity.

The contributions of the work are two-fold. First, to derive a finite-dimensional dynamic model of a continuum soft robot, where we briefly recapitulate on existing modeling technique for soft robot manipulators. To address the issue of infinite-dimensionality, we explore the PCC condition that allows for a low-dimensional description of the continuum dynamics. Although such modeling approaches have been thoroughly developed, we will address two issues that will aid the development of model-based controllers. We aim to bridge the gap between the PCC model and the underlying continuum mechanics by matching the quasi-static behavior to a Finite-Element-driven model (FEM); and we propose a reduced-order integration scheme using Matrix-Differential Equations (MDEs) to compute the spatio-temporal dynamics in real-time. Preliminary results of this work were shown in Caasenbrood et al.[?].

Second, in regards to the FEM-based hyper-elastic modeling and the possible presence of unmodelled dynamics (e.g., material uncertainties or external loads on the end-effector), a passivity-based adaptive controller is proposed that enhances robustness towards material uncertainties and unmodelled dynamics in closed-loop, slowly improving their estimates online. All source code is made publicly available at Caasenbrood et al.[?] (see the open software repository)

2.2 Design and fabrication

By using additive manufacturing, we developed a soft and flexible robot manipulator that is suitable for pick-and-place application. The 3-DOF soft robot can be seen in Figure ??. The soft robot manipulator in this work is loosely inspired by the elephant whose trunk-appendage consist mainly of parallel muscles without skeletal support. The anatomy of elephant's trunk provides an excellent study case, as they naturally exhibit continuum-body bending and moderate elongation[? ? ?]. Similar to the earlier soft robotic designs [? ?], the developed soft robot can undergo three-dimensional movement by inflation or deflation of embedded pneumatic bellow network. The soft robot can achieve bending in any preferred direction by differential pressurization of each channel (<0.1 MPa). Whereas, simultaneous pressurization accomplishes moderate elongation.

The soft robot is exclusively composed of a printable, flexible thermoplastic elastomer (Young's modulus $\leq$ 80 MPa), which intrinsically promotes softness and dexterity. The elastomer material is developed explicitly for Selective Laser Sintering (SLS), a 3-Dimensional (3D) printing method that uses a laser to solidify powdered material. The main advantage of SLS printing over other techniques is that the printed parts are fully self-supported, which allows for complex and highly detailed structures. It should be mentioned that the layer-by-layer material deposition will introduce undesired anisotropic mechanical effects. To mitigate anisotropy, the bellows are printed orthogonal to the printing plane, thereby ensuring mechanical symmetry. For the majority of this work, the 3D-printed soft robot in Figure ?? will form the basis of the dynamical model. The 3D-model is made available at the open repository [?].

2

2.3 Continuum dynamic model

As mentioned previously, soft robots are composed of soft bodies that may be regarded as a continuum body with (theoretically) infinitely many degrees-of-freedom (DOF). In this section, we aim to derive a compact and computationally efficient model that envelops the continuous dynamics of a soft robot through a small set of generalized coordinates $\boldsymbol{q} \in \mathcal{Q}$ and their respective generalized velocities $\dot{\boldsymbol{q}}(t) \in T_{\boldsymbol{q}}\mathcal{Q}$ with n the number of active joint variables. We base the modeling framework on the work of Mochiyama et al.[?] who outlined a theoretical foundation for continuum manipulators. Their work is extended upon by including extensibility, serial-chaining of multiple soft-links, pneumatic actuation, and the introduction of nonlinear and time-dependent material behavior. Earlier modeling strategies addressing similar issues can be found in from Godage et al. [? ?], Della Santina et al. [? ? ?], Renda et al. [?], and Boyer et al. [?]. Leveraging from the aforementioned works, the continuous dynamics of a soft robot manipulator can be written in the familiar Lagrangian form:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{Q}^{\text{nc}}, \tag{2.1}$$

where $\boldsymbol{M}(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ denotes the generalized inertia matrix, $\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \mathbb{R}^n$ a vector of nonlinear state-dependent force contributions. In this work, a similar modeling framework is adopted; however, we propose an extension to incorporate FEM-driven data to more accurately reflect the underlying continuum mechanics – in particular hyper-elasticity; and we propose a numerical scheme that allows for fast computation of the continuous dynamics. For completeness, we will recapitulate on the modeling approach here.

2.3.1 Kinematics of elastic continuum bodies

To represent the hyper-flexible configuration of the soft robot, let us consider a smooth spatial curve that passes through the geometric center of the continuously deformable body, as shown in Figure ??. In literature, this curve is called the '*backbone curve*' as it simplifies the three-dimensional deformation imposed by distributed forces acting on the elastic body. The arc-length of the backbone corresponds to the extensible length of the soft robot denoted by the variable $l(t) \in [l_-, l_+]$ which we assume bounded $l_+ \geq l \geq l_-$, and let L be a constant denoting the total unstressed length of the soft robot. Next, let us introduce a spatial variable $\sigma \in \mathbb{X}$ that belongs to the one-dimensional material domain of the backbone curve, i.e., $\mathbb{X} = [0, L]$. Let it be clear that the spatial variable σ represents the arc-length of a material coordinate along the undeformed material domain of the soft robot manipulator.

Figure here of smooth curve for p and Phi

Given each material coordinate, we wish to find a suitable low-dimensional joint representation $q(t)$ such that the position vector ${}^0p$ anywhere on the continuous backbone can be written as a mapping from generalized coordinates and space into $\mathbb{R}^3$:

$$ {}^0\boldsymbol{\gamma} : \mathbb{X} \times \mathcal{Q}(t) \to \mathbb{R}^3; \tag{2.2} $$

and similarly the rotation matrix ${}^0\boldsymbol{\Phi}(\sigma, \boldsymbol{q})$ by a mapping from the generalized coordinates and space into SO(3):

$$ {}^0\boldsymbol{\Phi} : \mathbb{X} \times \mathcal{Q}(t) \to \mathrm{SO}(3), \tag{2.3} $$

where SO(3) denotes the special orthogonal group for rotations about the origin of $\mathbb{R}^3$, and $n = \dim(\boldsymbol{q})$ the state dimension. Under this notion, the position vectors ${}^0p(q,0)$ and ${}^0\boldsymbol{\gamma}(L,\boldsymbol{q})$ relate to the base and the end-effector of the soft robot, respectively. Please note that left-sided superscript are used to indicate the frame of reference. The set of all points on the backbone $\mathcal{P} = \left\{{}^0\boldsymbol{\gamma} \in \mathbb{R}^3 \,|\, \sigma \in \mathbb{X}\right\}$ draws a possible spatial configuration of the soft robot given a time instance $t \in \mathbb{T}$ on a finite horizon $\mathbb{T} = [0, T]$.

Intermezzo 2.1. *Despite the inherent flexibility in soft robotics, it is sometimes sufficient to express the kinematics according to the Piecewise Constant Curvature (PCC) condition. Mathematically, it implies that the curvature of the continuous body satisfies $\kappa(q,\sigma_1) = \kappa(q,\sigma_2)$ for a neighboring region of points $\sigma_1, \sigma_2 \subseteq \mathbb{X}$. As a result, this condition allows us to describe the full forward kinematics with a significantly reduced set of generalized coordinates, mitigating kinematic complexity in the model. Numerous works employ PCC models [**? ? ? ? ? ?**], and depending on the degrees of elasticity, the PCC condition has been proven to be consistent for various soft robotic systems.* ▲

Following this Piecewise Constant Curvature (PCC) description, let us assign a coordinate frame that twists minimally along the backbone – a Bishop frame [**?**]– parametrized by the following generalized coordinate vector:

$$ \boldsymbol{q} = \begin{pmatrix} \varepsilon & \kappa_x & \kappa_y \end{pmatrix}^\top \in \mathcal{Q}, \tag{2.4} $$

where $\varepsilon \in \mathbb{R}$ is the elongation strain, and κ_x, $\kappa_y \in \mathbb{R}$ are the curvatures or angular strains in x-z and y-z plane, respectively; and $\mathcal{Q} \subset \mathbb{R}^3$ is an admissible space on which q evolves.It is worth mentioning that the joint description above is somewhat related to Renda. et al. [**?**] who proposed a Piece-wise Constant Strain (PCS) parametrization with the exception of including the twist along the tangent.

By exploring the differential geometry of the smooth backbone curve similar to Mochiyama et al.[**?**], we can express the spatial change of the position vector ${}^0\boldsymbol{p}(0,\boldsymbol{q})$ and the orientation matrix ${}^0\boldsymbol{\Phi}(q,\sigma)$ for each material point σ along the smooth backbone by

$$\frac{\partial\, {}^0\boldsymbol{\Phi}}{\partial \sigma}(\sigma, \boldsymbol{q}) = {}^0\boldsymbol{\Phi}(\sigma, \boldsymbol{q})\ [\boldsymbol{\Gamma}(\sigma, \boldsymbol{q})]_\times\,, \tag{2.5}$$

$$\frac{\partial\, {}^0\boldsymbol{p}}{\partial \sigma}(\sigma, \boldsymbol{q}) = {}^0\boldsymbol{\Phi}(\boldsymbol{q}, \sigma)\, \boldsymbol{U}(\sigma, \boldsymbol{q}), \tag{2.6}$$

where $[\boldsymbol{\Gamma}]_\times \in \mathrm{so}(3)$ is a skew-symmetric matrix composed of the entries of the vector $\boldsymbol{\Gamma} \in \mathbb{R}^3$, and $\boldsymbol{U} \in \mathbb{R}^3$ a vector representing the tangent along the extensible backbone. The vectors $\boldsymbol{\Gamma}$ and $\boldsymbol{U}$ are vectors that define the differential geometry of the backbone, which are unique entries that lives in the tangent space of the rigid-body transformation group SE(3). Given the Bishop parametrization as described by (2.4), these geometric entities yield

$$\boldsymbol{\Gamma} = \begin{pmatrix} -\kappa_y \\ \kappa_x \\ 0 \end{pmatrix}; \qquad \boldsymbol{U} = \begin{pmatrix} 0 \\ 0 \\ \varepsilon \end{pmatrix} + \boldsymbol{U}_0, \tag{2.7}$$

with $\boldsymbol{U}_0 = (0, 0, 1)^\top$ the unit-tangent. Now, given an initial configuration of backbone's base, i.e., ${}^0\boldsymbol{\Phi}(0, \boldsymbol{q}) = \boldsymbol{\Phi}_0$ and ${}^0\boldsymbol{p}(0, \boldsymbol{q}) = 0_3$, we can now solve for the position and orientation for each material coordinate σ along the backbone:

$${}^0\boldsymbol{\Phi}(\sigma, \boldsymbol{q}) = \boldsymbol{\Phi}_0 \exp(\sigma[\boldsymbol{\Gamma}(\boldsymbol{q})]_\times), \tag{2.8}$$

$${}^0\boldsymbol{p}(\sigma, \boldsymbol{q}) = \int_0^\sigma {}^0\boldsymbol{\Phi}(\eta, \boldsymbol{q})\, \boldsymbol{U}(\boldsymbol{q})\; d\eta, \tag{2.9}$$

where $\exp : \mathrm{so}(3) \to \mathrm{SO}(3)$ is the exponential map. Let it be clear that the closed-form solutions (2.8) and (2.9) form the forward configuration kinematics of the backbone curve. To express the forward velocity kinematic, let $\boldsymbol{V}(\sigma, \boldsymbol{q}, \dot{\boldsymbol{q}}) = \left({}^\sigma\boldsymbol{\omega}^\top, {}^\sigma\boldsymbol{v}^\top\right)^\top \in \mathbb{R}^6 \cong \mathrm{se}(3)$ be the aggregate of the angular velocity and linear velocity components relative to an inertial frame at σ (the frame of reference is denoted by a left superscript), where the space se(3) denotes the Lie algebra of SE(3). The velocity twist is computed by the following integration procedure:

$$\boldsymbol{V}(\sigma, \boldsymbol{q}, \dot{\boldsymbol{q}}) = \mathrm{Ad}^{-1}_{\boldsymbol{g}(\sigma,\cdot)} \int_0^\sigma \mathrm{Ad}_{\boldsymbol{g}(\eta,\cdot)}\, J^* \dot{q}\; d\eta \;=:\; J(q, \sigma)\dot{q}, \tag{2.10}$$

where $\mathrm{Ad}_g : \mathrm{SE}(3) \to \mathbb{R}^{6\times 6}$ denotes the adjoint transformation matrix regarding the rigid body transformation $g \in \mathrm{SE}(3)$ that maps local velocities (i.e., twist) to a frame located at σ, and J^* a constant joint-axis matrix. The joint-axis matrix for an extensible and bendable PCC segment parametrized by the Bishop parameters is given by

$$J^* := \begin{pmatrix} \dfrac{\partial \Gamma}{\partial q}^\top & \dfrac{\partial U}{\partial q}^\top \end{pmatrix}^\top = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^\top. \tag{2.11}$$

Although we based the forward kinematics on the work of Mochiyama et al.[?], the derived expression for the velocity twist in (2.10) is analogous to the work of Renda et al.[? ?], and Boyer et al. [? ?]. Please also note that (2.10) gives rise to the geometric manipulator Jacobian $J(q, \sigma)$ that defines the mapping from joint velocities to the velocity twist for a particular material point σ on the continuous body. In continuation, let us also introduce the acceleration twist[? ? ?] – obtained through time differentiation of (2.10):

$$\begin{aligned}\dot{V}(q, \dot{q}, \ddot{q}, \sigma) &= J\ddot{q} + \mathrm{Ad}^{-1}_{g(\cdot,\sigma)} \int_0^{\sigma} \mathrm{Ad}_{g(\cdot,\eta)} \mathrm{ad}_{V(\cdot,\eta)}\, J^{*}\dot{q}\; d\eta \\ &:= J(q,\sigma)\ddot{q} + \dot{J}(q,\dot{q},\sigma)\dot{q},\end{aligned} \tag{2.12}$$

where $\mathrm{ad}_V \in \mathbb{R}^{6\times6}$ denotes the adjoint transformation regarding the velocity twist $V \in \mathrm{se}(3)$. The reader is referred to Appendix A for more detailed expressions on the adjoint transformations.

2.3.2 Euler-Lagrange equations

Given the forward kinematics in (2.8), (2.9), (2.10) and (2.12), we can shift our attention to formulating the finite-dimensional dynamics of the soft robot. Our goal here is to write the spatio-temporal dynamics of the hyper-elastic soft robot as a second-order ODE into the Lagrangian form:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = Q^{\mathrm{nc}}, \tag{2.13}$$

where $\mathcal{L}(q, \dot{q}) :=^{\top} (q, \dot{q}) - \mathcal{U}(q)$ is the Lagrangian function, $^{\top} \in \mathbb{R}_{\geq 0}$ and $\mathcal{U} \in \mathbb{R}$ the kinetic and potential energy, respectively; and $Q^{\mathrm{nc}} \in \mathbb{R}^n$ a vector of generalized non-conservative forces. To apply the Lagrangian formalism to a continuum dynamical system, regard an infinitesimal slice of the continuum body for each material coordinate σ along the backbone curve. Given this notion, we embody this infinitesimal slice with an inertia tensor $\mathcal{M} = \mathrm{blkdiag}(\rho I_3, \mathcal{J}_\sigma)$ with $\rho = m/L$ the line-density and J_σ a tensor for the second moment of inertia. The kinetic energy can be obtained through spatial integration of its respective kinetic energy densities[? ? ?], i.e., $\mathfrak{T} = \frac{1}{2} V^{\top} \mathcal{M} V$:

$$\begin{aligned}\mathcal{T}(q, \dot{q}) &= \frac{1}{2} \int_{\mathbb{X}} V(q, \dot{q}, \sigma)^{\top} \mathcal{M}\, V(q, \dot{q}, \sigma)\; d\sigma, \\ &= \frac{1}{2} \dot{q}^{\top} \int_{\mathbb{X}} J(q,\sigma)^{\top} \mathcal{M}\, J(q,\sigma)\; d\sigma\, \dot{q}, \\ &= \frac{1}{2} \dot{q}^{\top} M(q) \dot{q}.\end{aligned} \tag{2.14}$$

Note that expression for the kinetic energy naturally gives rise to the generalized inertia matrix $M(q)$ of the Lagrangian model. By substitution of the kinetic energy into the Euler-Lagrange equation (2.13), we find $M(q)\ddot{q} + C(q,\dot{q})q$ where $C(q,\dot{q})$ denotes the Coriolis matrix. Instead of computing the Coriolis matrix through the conventional Christoffel symbols[?], we adopt a computational scheme by Garofalo et al. [?] used for serial-chain rigid manipulators, in which we replaced the finite summation of N rigid-bodies by a spatial integration over the continuum domain $\mathbb{X}$:

$$C(q,\dot{q}) = \int_{\mathbb{X}} J(q,\sigma)^\top \mathcal{C}_{V(q,\dot{q},\sigma)} J(q,\sigma) \; + J(q,\sigma)^\top \mathcal{M} \dot{J}(q,\dot{q},\sigma)\, d\sigma, \tag{2.15}$$

where $\mathcal{C}_V = -\mathcal{C}_V^\top := \mathcal{M}\mathrm{ad}_V - \mathrm{ad}_V^\top \mathcal{M}$ is a skew-symmetric matrix. The computation above is slight different from existing literature[? ?] to ensure that the matrix $\dot{M} - 2C$ is skew-symmetric; the so-called the passivity condition[?] for Euler-Lagrange systems (see Appendix B for proof). The importance of this property will become apparent later in the energy-based controller design. Lastly, the potential energy is given by sum of gravitational potential energy and internal elastic potential, i.e., $\mathcal{U}(q) = \mathcal{U}_g(q) + \mathcal{U}_e(q)$. Since gravitational potential energy density is given by $\mathfrak{U}_g = -\rho\, {}^0p(q,\sigma)\gamma_g$ with $\gamma_g \in \mathbb{R}^3$ is a vector of body accelerations, the potential energy related to gravity is obtained by spatial integration of their respective energy densities:

$$\mathcal{U}_g(q) = -\rho \int_{\mathbb{X}} {}^0p(q,\sigma)^\top \gamma_g \, d\sigma. \tag{2.16}$$

To model the hyper-elastic nature, lets introduce two nonlinear stiffness functions for both stretching and bending, denoted by $k_e : \mathbb{R} \mapsto \mathbb{R}_{>0}$ and $k_b : \mathbb{R} \mapsto \mathbb{R}_{>0}$, respectively. These functions allow us to describe a collective elastic behavior imposed by the hyper-elastic materials and the continuum-bodied deformation. It shall be clear that these entities are unique to the soft robot's geometry and soft material choice, and thus finding a suitable candidate model requires further analysis. Later, we will sculpt these nonlinear stiffness functions through Finite Element Methods (FEM). For now, we assume that these analytical nonlinear stiffness functions are known, and thus the (hyper)-elastic potential energy takes the form

$$\mathcal{U}_e(q) = \int_0^{\varepsilon} k_e(\eta)\, \eta \; d\eta + \int_0^{\beta(q)} k_b(\eta)\, \eta \; d\eta, \tag{2.17}$$

where ε is the elongation strain, and $\beta(q) = \kappa L(\varepsilon + 1)$ is the bending angle with the total curvature of the soft segment $\kappa = \sqrt{{\kappa_x}^2 + {\kappa_y}^2}$ (see Figure ??).

Overall dynamics

Finally, by combining (2.13), (2.14), (2.15), (2.16), and (2.17), the continuum dynamics of the soft robot can be casted into the familiar closed form [? ? ? ?] similar to aforementioned model (1):

$$M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + P(q,\dot{q}) + G(q) = \tau(u,\delta), \tag{2.18}$$

where $P = d\mathcal{U}_e/dq + R\dot{q}$ is a vector of generalized forces imposed by the deformation of the soft materials with $R \in \mathbb{R}^{n\times n}$ the Rayleigh damping matrix, $G = \partial\mathcal{U}_g/\partial q$ a vector of generalized gravitational forces, and $u \in \mathbb{R}^m$ the control input with the index m the number of pressure inputs. The generalized input vector is chosen of the form: $\tau(u,\delta) = Hu + \delta$ with $H : \mathbb{R}^m \mapsto \mathbb{R}^n$ a mapping from the input space to the joint actuation space, and $\delta(t)$ an external disturbance (e.g., unmodelled material uncertainties).

Remark 2.1. *Given the context of manipulators, a possible disturbance $\delta(t)$ could be an external mass applied to the tip of the soft robot. Given the kinematic relations in* (2.10) *and* (2.12)*, one can describe the disturbance (modeled here as a point-mass located at L) by a state-dependent vector:*

$$\delta_m = m_\delta \lfloor J(\cdot, L)\rfloor_3^\top \left(\mathrm{Ad}^{-1}_{g(\cdot,L)}\gamma_g + \lfloor \dot{V}(\cdot, L)\rfloor_3\right), \tag{2.19}$$

where $\lfloor\cdot\rfloor_3$ extracts the last three rows of a matrix or vector, and $m_\delta > 0$ the applied mass to the end-effector. It is worth recalling that the acceleration twist can be computed through the geometric Jacobian and its time derivative, i.e., $\dot{V} = J\ddot{q} + \dot{J}\dot{q}$. Indeed, the PCC condition for a soft body can only accurately describe the true dynamics if external forces produced by mass m_δ do not excessively exceed the intrinsic elastic balancing forces $P(q)$. Alternatively, a soft body can be modeled using multiple PCC curves of smaller size, similar to standard Finite Element discretization. $\triangle$

The actuation mapping H depends on the geometry, placement, and orientation of the (pneumatic) soft actuators. Since the pneumatic chambers are aligned parallel to the backbone curve and are equally spaced along the circumference, we propose the following ansatz:

$$H := \begin{pmatrix} \alpha_\varepsilon & \dots & \alpha_\varepsilon \\ -\alpha_\kappa\cos(\phi_1) & \dots & -\alpha_\kappa\cos(\phi_m) \\ \alpha_\kappa\sin(\phi_1) & \dots & \alpha_\kappa\sin(\phi_m) \end{pmatrix}, \tag{2.20}$$

where $\alpha_\varepsilon, \alpha_\kappa > 0$ are system parameters representing the effective transferal of differential pressure to joint forces, and $\phi_i = (i-1)\cdot\frac{2\pi}{m}$ the angular inter-distance between the m-number of pneumatic bellows. Please note that the parameters α_ε and α_κ are dependent on the bellow area and radius from the bellow to the backbone curve.

2.4 Extension to multi-link dynamics

We previously expressed the position and velocity kinematics as explicit functions of the generalized coordinates (i.e., Bishop parameters) and their time-derivatives. This explicit dependency stems from the PCC conditions inferring the curvature is non-varying along the spatial domain $\mathbb{X}$, i.e., $\kappa(q,\sigma) = \kappa(q)$. Although sufficient for some cases, the condition is generally restrictive, and to some extent inconvenient, since the inclusion of multiple links demands piece-wise integration of the kinematics (2.9), (2.8), (2.10), and (2.12). Rather than separation of integration, we can extend this PCC description by using piece-wise continuous spatial function to distinguishes multiple soft-bodied links along the continuous body of the soft robot. The idea of parametrization through shapes functions has been explored earlier by Chirikjian et al.[? ?], and later by Boyer et al. [?], Della Santina et al. [?]. A similar discontinuous shape function series was used by Berthet-Rayne et al. [?] to pursue multi-body dynamics for growing continuum robots; and proposed by Chirikjian [?] for hyper-redundant robots earlier.

Following the aforementioned works, let us parameterize the the geometric vectors Γ and U for a N-link soft robot through the product of a basis of orthonormal functions $\{s_i\}_{i\in\mathbb{N}}$ and the Bishop parametrization as follows

$$\Gamma(q,\sigma) = \sum_{i=1}^{N} s_i(\sigma) \lceil J^* \rceil_3 \, \tilde{q}_i, \tag{2.21}$$

$$U(q,\sigma) = \sum_{i=1}^{N} s_i(\sigma) \lfloor J^* \rfloor_3 \, \tilde{q}_i + U_0, \tag{2.22}$$

where J^* is the joint-axis matrix as in (**??**), the mathematical operators $\lceil\cdot\rceil_3$ and $\lfloor\cdot\rfloor_3$ extract the first or last three rows of a matrix, respectively; $\tilde{q}_i$ the joint variables of the i-th link, and $s_i : \mathbb{X} \mapsto \{0,1\}$ is a piece-wise continuous shape function, whose purpose is to be non-zero for a given interval on $\mathbb{X}$. The new generalized coordinate vector becomes the aggregate of all joint variables of the multi-body soft robotic system $q = \left(\tilde{q}_1^\top, \tilde{q}_2^\top, ..., \tilde{q}_N^\top\right)^\top$ with the vector $\tilde{q}_i = (\varepsilon_i, \kappa_{x,i}, \kappa_{y,i})^\top$ relating to the Bishop parametrization of the ith-link. Given (2.21) and (2.22), we may now rewrite the velocity-twist as

$$V(q,\dot{q},\sigma) = \mathrm{Ad}_g^{-1} \int_0^\sigma \mathrm{Ad}_g J^* S(\sigma) \, d\sigma \dot{q} := J(q,\sigma)\dot{q} \tag{2.23}$$

where $S = (s_1,\, s_2,\, ...,\, s_N) \otimes I_n$ is an unitary selection matrix derived from the basis of piece-wise continuous shape functions $\{s_i\}_{i=1}^N$. To be less ambiguous about this selection matrix S, lets consider a spatial coordinate $\sigma_2 \in [L_1, L_1 + L_2]$ that lies on the spatial interval of the second link. Consequently, the operation $S(\sigma_2)q = \tilde{q}_2$ returns the corresponding joint variable of the second link. This selection of generalized coordinates follows analogously for other links along the serial-chain of the soft manipulator. We provided a small library of piece-wise continuous shape functions upto $1 \leq N \leq 8$ links under `./src/pwf` on the open repository[?]. Now, substitution of the discontinuous variation of the geometric Jacobian in (2.23) into (2.14) leads to the dynamic model of a N-link soft robot manipulator in the Lagrangian form similar to (2.18).

2.5 Efficient solver of the soft robotic dynamics through Matrix-Differential Equations

Due to the partial differential nature of soft robots, obtaining a closed-form expression for the projected Lagrangian model in (2.18) can become notoriously long and complex (especially for multi-link systems). The origin of this problem stems from the integrands of inertia matrix $M(q)$ in (2.14) and Coriolis forces $C(q, \dot{q})$ in (2.15); which become highly nonlinear and therefore difficult to calculate a-priori. As a result, solving the forward dynamics using traditional solvers often deteriorates the real-time performance, and in turn its usability for closed-loop control. Inspired by Boyer et al. [?] and Godage et al [?], instead of finding an exact solution to the dynamic entries $M(q)$, $C(q, \dot{q})$ and $G(q)$, let us introduce a similar reduced-order integration scheme that produces an approximate of the dynamic model (2.18). Yet, instead of using an inverse Newton-Euler algorithm (i.e., Featherstone or Hollerbach scheme) in which the Lagrangian entries are built column-wise, we propose an explicit integration scheme that efficiently computes all Lagrangian entities in parallel through a so-called Matrix-Differential Equation (MDE).

The idea here is to replace all necessary spatial integrations for the computation of the Lagrangian entities by an equivalent Matrix-Differential Equation of the form:

$$\frac{\partial Z}{\partial \sigma} = F(Z, \sigma), \tag{2.24}$$

where $Z(\cdot, \sigma)$ is a matrix-valued function composed of the necessary elements for the forward kinematics and forward dynamics, and $F(Z, \sigma)$ a matrix-valued flow function that describes the spatial evolution of Z. Then, by choosing the appropriate initial condition for $Z(\cdot, 0) = Z_0$ and numerically solving (2.24) over a finite horizon $\mathbb{X}$, we can retrieve an approximate of the Lagrangian model in (2.18) by extracting the necessary elements from the solution $Z(\cdot, L)$.

Before describing the MDE, let us first introduce two intermediate matrices related to the computation of the manipulator Jacobian and its time-derivative, namely:

$$\frac{\partial B_1}{\partial \sigma} = \mathrm{Ad}_{g(\cdot,\sigma)}\, J^* S(\sigma), \tag{2.25}$$

$$\frac{\partial B_2}{\partial \sigma} = \mathrm{Ad}_{g(\cdot,\sigma)} \mathrm{ad}_{V(\cdot,\sigma)}\, J^* S(\sigma) \tag{2.26}$$

such that they satisfy $J\dot{q} = \mathrm{Ad}_g^{-1} B_1 \dot{q}$ and $\dot{J}\dot{q} = \mathrm{Ad}_g^{-1} B_2 \dot{q}$. Given the expressions above, we can now include a partial computation Jacobians into the MDE. By collecting all the differential relation for the forward kinematics (5), (6) and forward dynamics (14), (15), and (16), we can assign a flow function $F := \mathrm{blkdiag}\,(F_1, F_2)$ composed of two matrices:

$$F_1 = \left(\begin{matrix} {}^0\Phi[\Gamma]_\times & {}^0\Phi U \\ 0_{3\times 3} & 0_3 \end{matrix} \;\middle|\; \mathrm{Ad}_g J^* S \;\middle|\; \mathrm{Ad}_g \mathrm{ad}_V J^* S \right), \tag{2.27}$$

$$F_2 = \begin{pmatrix} \frac{\partial M}{\partial \sigma} & \frac{\partial C}{\partial \sigma} & \frac{\partial G}{\partial \sigma} \end{pmatrix}, \tag{2.28}$$

in which the differential form of the dynamic entities $M(q)$, $C(q, \dot{q})$, and $G(q)$ of the Lagrangian model are given by

$$\frac{\partial M}{\partial \sigma} = (\mathrm{Ad}_g^{-1} B_1)^\top \mathcal{M} (\mathrm{Ad}_g^{-1} B_1), \tag{2.29}$$

$$\frac{\partial C}{\partial \sigma} = (\mathrm{Ad}_g^{-1} B_1)^\top \left[\mathcal{C}_V (\mathrm{Ad}_g^{-1} B_1) + \mathcal{M}(\mathrm{Ad}_g^{-1} B_2)\right], \tag{2.30}$$

$$\frac{\partial G}{\partial \sigma} = (\lfloor B_1 \rfloor_3)^\top \rho \gamma_g, \tag{2.31}$$

We wish to stress that F_1 collects all elements related to the forward kinematics, whereas F_2 contains the dynamic entities related to the Lagrangian model. Following the spatial Matrix-Differential equation in (2.24) above, its solution will be a matrix $Z := \mathrm{blkdiag}\,(Z_1, Z_2)$ composed of two smaller state matrices Z_1 and Z_2:

$$Z_1 := \left(\begin{matrix} {}^0\Phi & {}^0 p \\ 0_{3\times 3} & 0_3 \end{matrix} \;\middle|\; B_1 \;\middle|\; B_2 \right), \tag{2.32}$$

$$Z_2 := \begin{pmatrix} M & C & G \end{pmatrix}, \tag{2.33}$$

Such a Matrix-Differential equation as in (2.24) are not supported natively by standard ODE solvers. Therefore, an explicit second-order Runge-Kutta solver for MDEs is developed such that efficiently computes the evolution of the state matrix Z along $\mathbb{X}$. The solver is written in `MATLAB` and can be found under `./src/Model.m` at Caasenbrood [?].

As for state trajectories along the temporal regime $\mathbb{T} = [0, T]$, an implicit trapezoidal integration scheme is proposed to solve the approximated continuum dynamics, which are generally less conservative on discretization to preserve numerical stability. Here implicit schemes are favored over explicit scheme, since a coarser time integration can significantly increase real-time performance. In addition, to further boost performance of the temporal integration, a cost-effective approximation of the Hessian is introduced. For more detail, see Appendix C for more detail.

III

Control and Sensing Strategies

IV

Appendices

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Appendix name

A.1 Section header

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Peer-reviewed journal articles

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Peer-reviewed articles in conference proceedings

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