Towards Design and Control of Soft Robotic Systems

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Towards Design and Control of Soft Robotic Systems

PROEFSCHRIFT

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Epigraph

Summary

Towards Design and Control of Soft Robotic Systems

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Contents

Chapter 1

Dynamic modeling of hyper-elastic soft robots – the PCC approach

The motion complexity and use of exotic materials in soft robotics call for accurate and computationally efficient models intended for control. To reduce the gap between material and control-oriented research,we build upon the existing Piecewise-Constant Curvature framework by incorporating hyper-elastic and visco-elastic material behavior. In this work, the continuum dynamics of the soft robot are derived through the differential geometry of spatial curves, which are then related to Finite-Element data to capture the intrinsic geometric and material nonlinearities. To enable fast simulations, a reduced-order integration scheme is introduced to compute the dynamic Lagrangian matrices efficiently, which in turn allows for real-time (multi-link) models with sufficient numerical precision. By exploring the passivity and using the parametrization of the hyper-elastic model, we propose a passivity-based adaptive controller that enhances robustness towards material uncertainty and unmodeled dynamics – slowly improving their estimates online. As a study case, a fully 3D-printed soft robot manipulator is developed, which shows good correspondence with the dynamic model under various conditions, e.g., natural oscillations, forced inputs, and under tip-loads. The solidity of the approach is demonstrated through extensive simulations, numerical benchmarks,

This chapter is based on:

1. Caasenbrood, B. J., Pogromsky, A. Y., and Nijmeijer, H. (2020). **Dynamic modeling of hyper-elastic soft robots using spatial curves.** IFAC-PapersOnLine, 53, 92389243.

2. Caasenbrood, B. J., Pogromsky, A. Y., and Nijmeijer, H. (2021). **Dynamic Modeling of Hyper-elastic Soft Robots through Differential Geometry of Curves** Soft Robotics, 2021. (under review).

and experimental validations.

1.1 Continuum dynamic model

As mentioned previously, soft robots are composed of soft bodies that may be regarded as a continuum body with (theoretically) infinitely many degrees-of-freedom (DOF). In this section, we aim to derive a compact and computationally efficient model that envelops the continuous dynamics of a soft robot through a small set of generalized coordinates $\boldsymbol{q} \in \mathcal{Q}$ and their respective generalized velocities $\dot{\boldsymbol{q}}(t) \in \mathbb{R}^n$ with n the number of active joint variables. We base the modeling framework on the work of Mochiyama et al. [Mochiyama and Suzuki, 2003] who outlined a theoretical foundation for continuum manipulators. Their work is extended upon by including extensibility, serial-chaining of multiple soft-links, pneumatic actuation, and the introduction of nonlinear and time-dependent material behavior. Earlier modeling strategies addressing similar issues can be found in from Godage et al. [Godage and Walker, 2015, **?**], Della Santina et al. [Santina and Rus, 2020, **?**, **?**], Renda et al. [Renda et al., 2018], and Boyer et al. [Boyer et al., 2021]. Leveraging from the aforementioned works, the continuous dynamics of a soft robot manipulator can be written in the familiar Lagrangian form:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{Q}^{\mathrm{nc}}, \tag{1.1}$$

where $\boldsymbol{M}(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ denotes the generalized inertia matrix, $\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \mathbb{R}^n$ a vector of nonlinear state-dependent force contributions. In this work, a similar modeling framework is adopted; however, we propose an extension to incorporate FEM-driven data to more accurately reflect the underlying continuum mechanics – in particular hyper-elasticity; and we propose a numerical scheme that allows for fast computation of the continuous dynamics. For completeness, we will recapitulate on the modeling approach here.

Kinematics of elastic continuum bodies

To represent the hyper-flexible configuration of the soft robot, let us consider a smooth spatial curve that passes through the geometric center of the continuously deformable body, as shown in Figure **??**. In literature, this curve is called the '*backbone curve*' as it simplifies the three-dimensional deformation imposed by distributed forces acting on the elastic body. The arc-length of the backbone corresponds to the extensible length of the soft robot denoted by the variable $l(t) \in [l_-, l_+]$ which we assume bounded $l_+ \geq l \geq l_-$, and let L be a constant denoting the total unstressed length of the soft robot. Next, let us introduce a spatial variable $\sigma \in \mathbb{X}$ that belongs to the one-dimensional material domain of the backbone curve, i.e., $\mathbb{X} = [0, L]$. Let it be clear that the spatial variable σ represents the arc-length of a material coordinate along the undeformed material domain of the soft robot manipulator.

Given each material coordinate, we wish to find a suitable low-dimensional joint representation $q(t)$ such that the position vector ${}^0p$ anywhere on the continuous backbone can be written as a mapping from generalized coordinates and space into $\mathbb{R}^3$:

$$ {}^0\boldsymbol{p} : \mathbb{X} \times \mathcal{Q}(t) \mapsto \mathbb{R}^3; \tag{1.2} $$

and similarly the rotation matrix ${}^0\boldsymbol{\Phi}(\sigma, \boldsymbol{q})$ by a mapping from the generalized coordinates and space into $\mathbb{SO}(3)$:

$$ {}^0\boldsymbol{\Phi} : \mathbb{X} \times \mathcal{Q}(t) \mapsto \mathbb{SO}(3), \tag{1.3} $$

where $\mathbb{SO}(3)$ denotes the special orthogonal group for rotations about the origin of $\mathbb{R}^3$, and $n = \dim(\boldsymbol{q})$ the state dimension. Under this notion, the position vectors ${}^0p(q, 0)$ and ${}^0p(q, L)$ relate to the base and the end-effector of the soft robot, respectively. Please note that left-sided superscript are used to indicate the frame of reference. The set of all points on the backbone $\mathcal{P} = \left\{ {}^0p \in \mathbb{R}^3 \,|\, \sigma \in \mathbb{X} \right\}$ draws a possible spatial configuration of the soft robot given a time instance $t \in \mathbb{T}$ on a finite horizon $\mathbb{T} = [0, T]$.

Intermezzo 1. *Despite the inherent flexibility in soft robotics, it is sometimes sufficient to express the kinematics according to the Piecewise Constant Curvature (PCC) condition. Mathematically, it implies that the curvature of the continuous body satisfies $\kappa(q, \sigma_1) = \kappa(q, \sigma_2)$ for a neighboring region of points $\sigma_1, \sigma_2 \subseteq \mathbb{X}$. As a result, this condition allows us to describe the full forward kinematics with a significantly reduced set of generalized coordinates, mitigating kinematic complexity in the model. Numerous works employ PCC models [Falkenhahn et al., 2015, Katzschmann et al., 2019, Tatlicioglu et al., 2007, Marchese and Rus, 2016, ?, ?], and depending on the degrees of elasticity, the PCC condition has been proven to be consistent for various soft robotic systems.*

Following this Piecewise Constant Curvature (PCC) description, let us assign a coordinate frame that twists minimally along the backbone – a Bishop frame [**?**]– parametrized by the following generalized coordinate vector:

$$ q = \begin{pmatrix} \varepsilon & \kappa_x & \kappa_y \end{pmatrix}^\top \in \mathcal{Q}, \tag{1.4} $$

where $\varepsilon \in \mathbb{R}$ is the elongation strain, and κ_x, $\kappa_y \in \mathbb{R}$ are the curvatures or angular strains in x-z and y-z plane, respectively; and $\mathcal{Q} \subset \mathbb{R}^3$ is an admissible space on which q evolves.It is worth mentioning that the joint description above is somewhat related to Renda. et al. [Renda et al., 2018] who proposed a Piecewise Constant Strain (PCS) parametrization with the exception of including the twist along the tangent.

By exploring the differential geometry of the smooth backbone curve similar to Mochiyama et al. [Mochiyama and Suzuki, 2003], we can express the spatial change of the position vector ${}^0p(q, \sigma)$ and the orientation matrix ${}^0\Phi(q, \sigma)$ for each material point σ along the smooth backbone by

Chapter 2

Reduced-order modeling of Soft Robots using Shape Identification

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This chapter is based on B. Caasenbrood, A. Pogromsky, and H. Nijmeijer. **Reduced-order Modeling of Soft Robotic Systems use Shape Identification** Mechatronics, 2021. (under review).

2.1 Brief introduction into Finite Element Method

In this section, we briefly touch upon the method of nonlinear finite-element analysis. Although the section might be straight-forward for some readers, our aim is to bridge the field of continuum mechanics and control theory. First, we introduce the notion of continuum deformation, and explore Galerkin-reduction to solve a reduced-form (or discretized weak form) of the continuum mechanical problem subject to hyper-elastic soft robotics.

2.1.1 Deformation gradient

To describe the continuum-bodied deformation of a soft robot, consider a generalized three-dimensional (3D) solid as shown in Figure 2.1. Let $\mathbb{V}_0 \subset \mathbb{R}^3$ be a open subset of Euclidean space that envelops the undeformed continuum body $V_0 \subseteq \mathbb{V}_0$. Throughout this work, we will use subscript $(\cdot)_0$ to denote the '*reference*' configuration. Now suppose there are forces acting on the boundary of the undeformed material domain. As these external forces induce motion in the solid, the reference solid V_0 will transition from an initial configuration to a new configuration – denoted by $V(t) \subseteq \mathbb{V} \subset \mathbb{R}^3$ at time t.

Let us focus an infinitesimal material point located at $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)^\top$ that belongs to the reference configuration $\mathbb{V}_0$ (see Fig. 2.1). Now assume that the motion of the particle $\boldsymbol{\sigma}$ can be described by a smooth mapping $\boldsymbol{\phi}(\cdot, t)$: $V_0 \to V(t)$ such that it relates to a corresponding material point at time t. As such, considering the collection of all material points that make up the solid V_0,

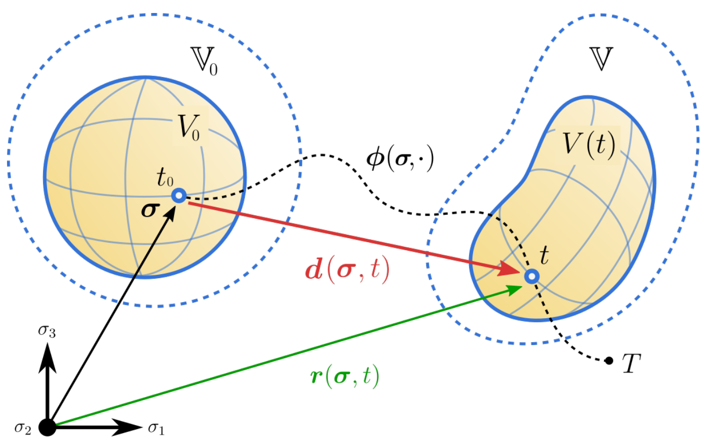

Figure 2.1: The continuum solid occupies a volume V_0 in the reference configuration, and a volume $\mathbb{V}$ post-deformation. The material points in a volume $V_0 \subseteq \mathbb{V}_0$ occupy a volume $V(t) \subseteq \mathbb{V}(t)$ at time $t \in [0, T]$

we can define that space that the continuum body occupies during motion by $V(t) := \{\boldsymbol{r} \in \mathbb{R}^3 \ : \ \exists \boldsymbol{\sigma} \in V_0 \text{ s.t. } \boldsymbol{r} = \boldsymbol{\phi}(\boldsymbol{\sigma}, t)\}$.

Assumption 1. *The motion of the particle described by the mapping $\boldsymbol{\phi}(\boldsymbol{\sigma}, t)$ is sufficiently smooth, orientation preserving, and invertible for all space $\boldsymbol{\sigma} \in V_\circ$ and time $t \in [0, T]$, even when the system is subjected to external force contributions that are spatio-temporally non-smooth in nature.*

Given the motion of the continuum body, we can expression the displacement of the continuum solid relative to its reference configuration as:

$$\boldsymbol{d}(\boldsymbol{\sigma}, t) := \boldsymbol{\phi}(\boldsymbol{\sigma}, t) - \boldsymbol{\sigma}. \tag{2.1}$$

Please note that the expression defines local displacement of a material point located at $\boldsymbol{\sigma}$ at time instance t. Now, given the expression for the local displacement in (2.1), we can introduce the notation of continuum deformation. We introduce a first-order tensor called the *deformation gradient*:

$$\boldsymbol{D}(\boldsymbol{\sigma}, t) := \frac{\partial \boldsymbol{d}}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}, t), \tag{2.2}$$

$$\boldsymbol{F}(\boldsymbol{\sigma}, t) := \frac{\partial \boldsymbol{\phi}}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}, t) = \boldsymbol{I}_3 + \boldsymbol{D}(\boldsymbol{\sigma}, t). \tag{2.3}$$

The deformation gradient in (2.3) holds useful information about the deformation at a local level, i.e., it describes the deformation of an infinitesimal subvolume of the continuum. Since a volume in the reference configuration cannot shrink to a point, i.e., zero volume, its holds that $\det(\boldsymbol{F}) := J > 0$. This is an important property that can be explored as $\boldsymbol{\phi}(\boldsymbol{\sigma}, t)$ undergoes large deformations [Kim, 2018].

2.1.2 Green-Lagrange strain and its energy conjugate

Following a Lagrangian description, in which we describe the deformation relative to the reference configuration, let us introduce the second-order Green-Lagrange strain tensor given as follows:

$$\boldsymbol{\mathcal{E}} := \frac{1}{2}\left(\boldsymbol{F}^\top \boldsymbol{F} - \boldsymbol{I}\right) = \frac{1}{2}\left[\boldsymbol{D} + \boldsymbol{D}^\top + \boldsymbol{D}^\top \boldsymbol{D}\right], \tag{2.4}$$

From (2.4) we can clearly see that $\boldsymbol{\mathcal{E}}$ is a symmetric second-order tensor. Due to this symmetry, we can exploit its isomorphism $(\cdot)^\vee : \mathbb{R}^3 \otimes \mathbb{R}^3 \mapsto \mathbb{R}^6$ to rewrite the Lagrangian strain in column vector representation. Following the standard Kelvin-Voight notation, we write

$$\boldsymbol{\mathcal{E}} := \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \cdot & \varepsilon_{22} & \varepsilon_{23} \\ \cdot & \cdot & \varepsilon_{33} \end{pmatrix} \quad \Rightarrow \quad \boldsymbol{\mathcal{E}}^\vee := \left(\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 2\varepsilon_{12}, 2\varepsilon_{23}, 2\varepsilon_{13}\right)^\top \tag{2.5}$$

which can be compactly written as a matrix operation:

$$\boldsymbol{\mathcal{E}}^{\vee} = \left[\boldsymbol{B}_0 + \frac{1}{2}\boldsymbol{B}_{\text{nl}}(\boldsymbol{d})\right]\boldsymbol{d}, \tag{2.6}$$

where $\boldsymbol{B}_0$ is a the linear part of the displacement-strain mapping, and $\boldsymbol{B}_{\text{nl}}$ the nonlinear part of the displacement-strain mapping. Please note that for linear FEM analysis, only the $\boldsymbol{B}_0$ part is considered for the strain computation, i.e., $\delta\boldsymbol{\mathcal{E}}^{\vee} \cong \boldsymbol{B}_0\,\delta\boldsymbol{d}$. Let it be clear that this approximation only holds for small deformations. In the context of soft robotics, the nonlinear part is most definitely of importance, and plays an integral part in the system's dynamics.

$$\delta\boldsymbol{\mathcal{E}}^{\vee} := \left.\frac{d\boldsymbol{\mathcal{E}}^{\vee}}{d\epsilon}(\boldsymbol{d} + \epsilon\delta\boldsymbol{d})\right|_{\epsilon=0} = \left[\boldsymbol{B}_0 + \boldsymbol{B}_{\text{nl}}(\boldsymbol{d})\right]\delta\boldsymbol{d} \tag{2.7}$$

2.1.3 Energy minimization on a finite-dimensional domain

The weak form of the nonlinear elastic continuum problem can be obtained through the principle of minimum potential energy, i.e., the difference between the stored elastic strain energy $\mathcal{U}^{\text{in}} : \boldsymbol{\mathcal{E}} \mapsto \mathbb{R}$ and the external work $\mathcal{U}^{\text{ext}} : \boldsymbol{\mathcal{E}} \mapsto \mathbb{R}$ done by the forces acting solid and its surface. Using the previously defined strain energy, the potential energy of the elastic continuum system can be obtained as

$$\begin{aligned}\mathcal{U}(\boldsymbol{d},t) &= \mathcal{U}^{\text{in}}(\boldsymbol{d},t) - \mathcal{U}^{\text{ext}}(\boldsymbol{d},t),\\ &= \iiint_{\mathbb{V}_0} \Psi(\boldsymbol{\mathcal{E}})\; dV - \iiint_{\mathbb{V}_0} \boldsymbol{d}^{\top}\boldsymbol{\mathcal{F}}^{\text{b}}\; dV - \iint_{\partial\mathbb{V}_0} \boldsymbol{d}^{\top}\boldsymbol{\mathcal{F}}^{\text{t}}\; dS,\end{aligned} \tag{2.8}$$

where $\boldsymbol{\mathcal{F}}^{\text{b}}$ are body forces, and $\boldsymbol{\mathcal{F}}^{\text{t}}$ are surface traction forces on the boundary $\partial\mathbb{V}_0$.

Given the definition on Lagrangian strain, we can explore its expression to tie the mathematical entity to elasticity. To be more explicit, in nonlinear continuum mechanics, each constitutive model that describes elasticity (and its corresponding change due to deformation) in terms of the Lagrangian strain tensor $\boldsymbol{\mathcal{E}}$. More conveniently, the elastic energy density is often represented in its strain invariants J_1, J_2, and J_3 – scalars that encode the magnitude of a particular deformation (e.g., isochoric deformation). Hence, let write down the energy balance for a continuum post-deformation:

$$\mathcal{H} := \mathcal{K}(t) + \mathcal{U}(t) - \mathcal{W}(t) = \iiint_{\mathbb{V}} \Psi(\cdot,t)\; dV \tag{2.9}$$

In this work, we consider a modified Yeoh material model Ψ_{YH} which depends exclusively on the first strain invariant $J_1 = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$:

$$\Psi^{\text{yh}} = \sum_{i=1}^{3} c_i\,(J_1 - 3)^i\,, \tag{2.10}$$

with $J_1 = V^{-2/3} J_1$ the first strain-invariant, and c_i and d_i the elastic and volumetric material coefficients, respectively.

Up till now, the formulations have been on the continuum domain. In practice, however, these continuum domains are discretized by so-called 'finite elements' in which we discretization the continuum domain $\mathbb{X}$ into smaller subregions $\boldsymbol{\mathcal{X}}_i \subset \mathbb{X}$. As such, we can approximate the continuum domain using discretized representation $V \cong \{V_0^{(i)}\}_{i=1}^{N_e}$ with N_e the number of finite elements in the tesselation.

Given the ith node of an arbitrarily shaped finite element, let us define $\tilde{U}_i \in \mathbb{R}^n$ be the vector of nodal displacements living in a n-dimensional space. Alternatively, we can write the displacements of the finite element using an interpolation scheme:

$$\boldsymbol{d}_i\,(\boldsymbol{\sigma}, t) \cong \sum_{j=1}^{N_p} \theta_j(\boldsymbol{\sigma}) \boldsymbol{q}_{i,j}^{\mathrm{F}}(t) := \Theta(\boldsymbol{\sigma}) \boldsymbol{q}_i^{\mathrm{F}} \qquad \forall \boldsymbol{\sigma} \in \mathcal{X}_i, \tag{2.11}$$

where $\{\theta_i\}_{i=1}^{N_p}$ is the set of shape functions with the N_p the number of points that span the finite element.

2.1.4 Dynamic Finite-Element model

$$\Sigma_F : \quad \boldsymbol{M}(\boldsymbol{q}^{\mathrm{F}})\ddot{\boldsymbol{q}}^{\mathrm{F}} + \boldsymbol{f}^{\mathrm{in}}(\boldsymbol{q}^{\mathrm{F}}) + \boldsymbol{R}\dot{\boldsymbol{q}}^{\mathrm{F}} = \boldsymbol{H}\boldsymbol{u} \tag{2.12}$$

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