Design, Modeling, and Control Strategies for Soft Robots

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Design, Modeling, and Control Strategies for Soft Robots

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Het onderzoek dat in dit proefschrift wordt beschreven is uitgevoerd in overeenstemming met de TU/e Gedragscode Wetenschapsbeoefening.

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Opening

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Soft Robotics – a new perspective on biomimicry

1.1 History of soft robotics: what are they?

Biological systems have long been an inspiration for roboticist to make more robust and capable machines. While holding these pages in one's hand, one might wonder *'how do we interact with so many different objects so effortlessly on a daily basis?'*. A conventional (rigid) robot, on the other hand, needs to know the exact weight, shape, and orientation of an object to interact with it safely and robustly. These philosophies adopted from nature are now presented into machines that accordingly shaped a new robotics discipline called '*soft robotics*'. Contrary to rigid robots, these machines are made from softer materials. Softness here refers to the enveloping properties of low elasticity materials. Indeed, by observing common materials in robotics and their elastic moduli (see Figure 1.1), we observe a gap between machine and nature. It is believed that the lack of material diversity in robotics might be the missing puzzle piece of achieving bio-like performance in modern machines. The idea of exploring low elasticity materials in robotics, so-called *soft material*, has sparked a new direction in robotics aimed at harmonizing robots and biology. Before continuing, we introduce the following definition:

Definition I. ***Soft materials*** *are a class of homogenous materials with a Young's modulus (i.e., the modulus of elasticity) lower than* $E \leq 10^9$ Pa. *Following, the word softness refers to the collection of mechanical properties that are often associated with these low moduli materials.*

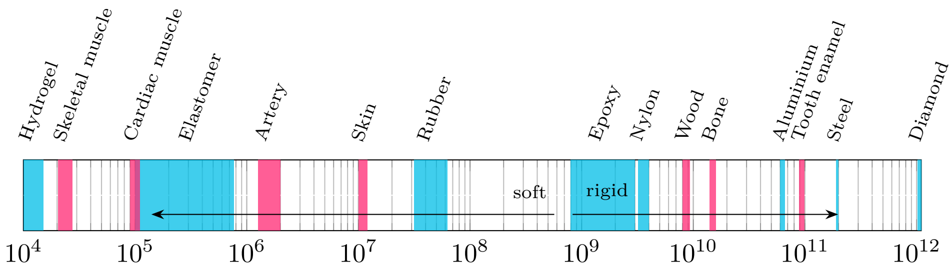


Figure 1.1. Young's modulus spectrum in (Pa) of rigid and soft materials, where ( ) are the organic (*i.e.*, biological) materials and ( ) inorganic materials.

Now, although the words '*soft*' and '*robotics*' have a clear definitions independently, the collocation of the two sparked many vivid discussions and new idealogies within the robotics community for the past two decades. Throughout its young academic life, several defintions have been coined. Early terms for soft robotics were used to indicate robots with variable joint stiffness [1], or artificial compliance through control [2]. The term was also used to underline the shift from rigid-linked robots to *bio-inspired continuum robots are inherently compliant and that exhibit large strains in normal operations* [3]. Paraphrasing the work of Robison et al. (1999, [4]): *soft robotic manipulators are continuum robots made of soft materials that undergo continuous elastic deformation and produce motion through the generation of a smooth backbone curve.* Alternatively, a broader definition was coined in a review by Kim et al. ([5], 2013) simply referring to soft-bodied robots as *an analogy to soft-bodied animals.* A concise (but generic) definition was proposed by Laschi et al. ([6], 2014), as soft robots being *any robot built by soft materials.* Rus et al. (2015, [7]) defined soft robots in terms of their structural elasticity: *Systems that are capable of autonomous behavior, and that are primarily composed of materials with moduli in the range of that of soft biological materials.*

Although the debate on the exact terminology is still ongoing, and perhaps may never be closed; we will propose a terminology applicable to this thesis. Our definition is based on the history of soft robotics and the scientific trends in modern literature related to the topics of design and control. Naturally, given its multi-disciplinary nature, the terms used in this thesis might deviate from the aforementioned literature. Nevertheless, we propose the following definition for soft robotics:

> **Definition II.** ***Soft robotics*** *is a subclass of robotics with purposefully designed compliant actuators embedded into their mechanical structure whose goal is to endow the robot with natural (or biological) motion or compliance.*

This definition is adopted from the work of Della Santina et al. (2020, [8]), but modified to highlight the importance of soft materials to mimic biological motion – also referred to as '*bio-mimicry*'. The ambition of closely mimicking biological creatures is perhaps not commonly associated with the field of robotics, given its important role in the automation industry, yet the inception of robotics can originally be found in bio-mimicry when regarding its rich history. In this section, we will present a short historical overview of soft robotics. Hereby showing that the current trends of bio-mimicry and elasticity in robotics find roots in a periods way before the soft robotic boom in the early 2010's. To guide the reader, in Figure 1.2, we provide a graphical, historic overview of soft robotic systems from 1960 to 2022. We will discuss the inception of soft actuation, early soft robotic designs, and modeling and control strategies for these continuum robotics.

Remark 1.1. *Readers familiar with the history and terminologies, like soft actuation and soft sensing, and that wish to read on modern soft robotics instead; are referred to Section 1.2 – "Challenges in modern soft robotics"*

Early soft actuation in robotics. To relate the historical progress of soft robots to rigid robots, let us begin with early rigid robots. In 1954, George Devol filed a patent describing an autonomous robotic machine that could be preprogrammed to execute step-by-step motions [9]. The machine was designed to reduce the workload on the manufacturing work floor, with a major focus on mimicking repetitive (exhausting) human labor. In 1958, those prototypes led to a robotic system under the name '*Unimate*'. An illustration of this early rigid robot is shown in Figure 1.2. The Unimate was used for manipulating metal die-casts and welding these to the main body of automobiles. In doing so revolutionizing the car industry shortly after. Much later (1969), Victor Scheinman created the Stanford Arm [10, 11], recognized as the first electronic computer-controlled robotic arm because the Unimate's instructions (*i.e.*, predefined setpoints in joint space) were prerecorded on a magnetic drum. He later developed the well-known PUMA robot in 1972 (video available at [12]) – the successor to the Unimate. Keep Scheinman in mind, as he ultimately ties to early soft robots.

Nearly four years after the Unimate was developed, Joseph L. McKibben developed a pneumatic muscle-inspired actuator capable of linear contraction – called the McKibben actuator. The McKibben muscle is a type of Pneumatic Artificial Muscle (PAM) which is to this date the most frequently used and published artificial muscle in literature. According to [13], he developed the McKibben actuators to bring motion to his little daughter's polio-paralyzed hand. His aim was that eventually such pneumatic actuators may help patients with paralyzed fingers to move, grasp, and even write. Inspired by the human muscle, the McKibben actu-

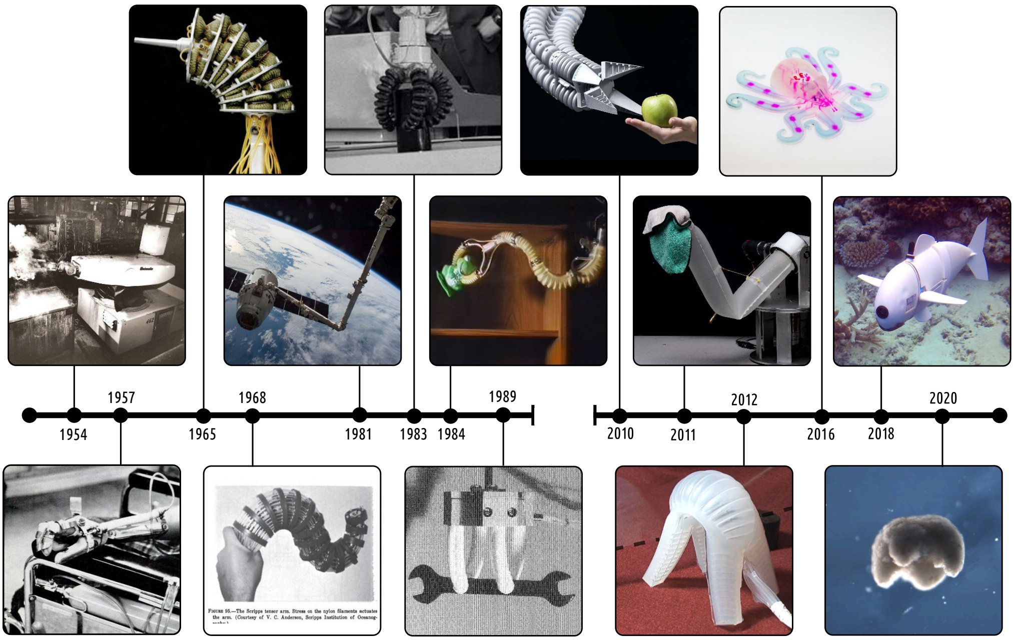


Figure 1.2. A brief timeline of the state-of-the-art of bio-inspired robotics throughout human history. (1954): Unimate, the first industrial robot . (1957): McKibben actuator, an early soft actuator inspired by the human muscle used for rehabilitation purposes [13]. (1965): The Orm, believed to be the first soft robotic system designed by Scheinman and Leifer [11]. (1968): Tensor Scripps arm developed by Anderson [22]. (1981): Canadarm-1, early flexible robotics employed on the International Space Station. (1983): Robot Arm with Pneumatic Gripper by Teleshev [23]. (1984): Bellows robotic arm by Wilson et al. [24]. (1989): The soft robotic gripper developed by Suzumori et al. [25, 26], seen as one of the earliest *academic* soft robot, developed before the word '*soft robot*' existed. (2010): Festo's Bionic arm inspired by the elephant's trunk [27]. (2011): Soft inflatable robot arm by Sanan and Atkeson [28, 29]. (2012) Multi-gait soft robot capable of terrestial locomotion [30]. (2016): Octobot, the first autonomous 3D-printed soft robot that explores a stabilizing oscillator chemical network that produces preprogrammed repetitive motion [31]. (2018): Autonomous robotic fish made by Katzschmann [32]. (2020): Xenobot, an organic soft robot composed of skin and muscle cells made by Blackiston and Kriegman [33].

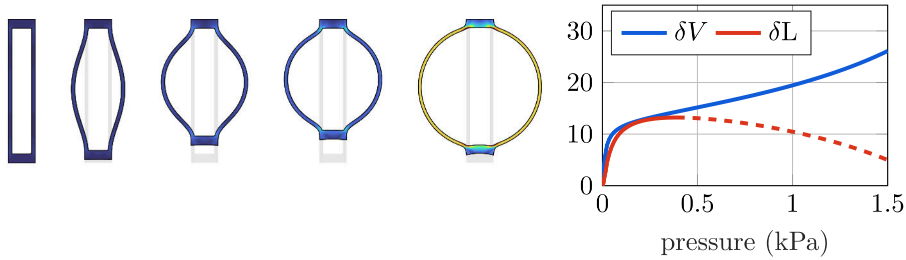
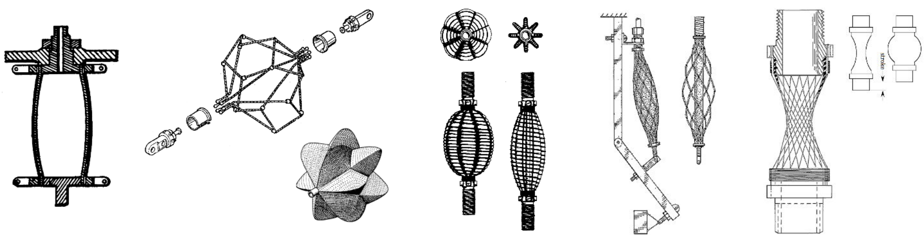


Figure 1.3. Working principle of a basic pneumatic artificial muscle (*i.e.*, Morin muscle [20]) with the internal volume (—) in mL, and the end-effector displacement (—) in mm and (- -) is the point at which the undesirable ballooning occurs.



(a) (b) (c) (d) (e)

Figure 1.4. Patent diagrams of pneumatic artificial muscle from 1953 till 1988. (a) Morin Muscle (1953, [20]); (b) ROMAN muscle (1986, [18]); (c) Yarlott muscle (1972, [17]); (d) Kukolj muscle (1988, [19]); (e) Paynter Hyperboloid (1974, [34]).

ator consists of an inflatable inner bladder enveloped with a double-helical weave. When pressurized, the fluidic actuator converts radial expansion into uni-axial contraction [14–16] since weave inhibits extensive '*ballooning*' – a term for undesired rapidly-accelerated volumetric expansion. Its material composition is often silicone rubber with a nylon-fiber exterior. A schematic representation of a general pneumatic muscle and the effect of ballooning are shown in Figure 1.3. Ballooning is an (often undesired) nonlinear effect, where the hyper-elastic pressure vessel exhibits strain-softening after a critical point is reached. As a result, further increase of the pressure leads to an exponential growth in volume, which ultimately leads to actuator tearing. At stages of ballooning, mechanical performance significantly drops and even produces adverse effects, like actuation reversal. McKibben solved this problem through a combination of soft and inextensible fiber weaves. These inextensible were placed at the exterior wall of the soft muscle, thereby limiting the radial expansion before ballooning could occur. According to Daerden (1999, [14]), there exist many variations of pneumatic muscle besides braided muscles, such as the '*netted muscles*' (e.g, Yarlott [17], ROMAC [18], and Kukolj [19]) and '*embedded muscles*' (*e.g.*, Morin [20], Paynter Hyperboloid [21]). Illustration of their patent schematics are shown in Figure 1.4.

Pneumatic muscles are perhaps one of the first fundamental technologies that enabled soft robotics, and to this day, it remains a framework for many soft robotic systems. Nevertheless, besides the many examples of fluidics [25, 32, 35–37], there exist many other technologies employed in soft robotic motion: such as thermal [38] or chemical expansion/contraction [31, 39, 40], crystal re-alignment [41–44], di-electric elastomers [45], magnetism [46–49], and naturally the use of tendons paired with electro-mechanical actuation [50–54]. Some predate the invention of the McKibben actuator. For example, a popular soft actuation principle still applied in soft robotics today are Dielectric Elastomer Actuators (DEA) developed by Röntgen in 1880 [55]. Therefore, given the abundance of soft robotic actuation, it is difficult to pinpoint the exact date of origin of soft actuation technology. Note, however, that these systems are not categorized as soft robots, they are categorized as soft actuators. Here, we emphasize the difference between soft actuators and soft robots in view of the modeling and control terminology relevant to the thesis:

Definition III. ***Soft actuators*** *are controllable flexible actuation units of the constitute soft robot that through external stimuli are responsible for natural motion and/or change in adaptive compliance.*

Remark 1.2. *The terminology above attempts to address an ambiguity common to soft robotics, namely the interchangeable use of soft actuator and soft robot. The thesis invokes that soft robots must be comprised of multiple soft actuators that connect to a passive deformable body. Here, the soft body functions as a mechanical conduit between actuators, sensors, and the environment.*

Origin of soft robotics. Returning to 1965, nearly a decade after the invention of the McKibben actuator and the Unimate robot, Scheinman and Leifer proposed a novel pneumatic robotic arm named the '*Orm*' – Norwegian for snake (recall that he also developed the popular PUMA robot [12]). The name was also an abbreviation for Object-Relational Mapping tool [56]. To the author's knowledge, this is believed to be the first instance of a soft robotic system. Surprisingly the system predates any rigid snake-like robot, like the Scripps tensor arm by Anderson (1968, [22]). Inspired by the anatomy of snakes, the system featured 28 rubber pneumatic artificial muscle (*i.e.*, bellows) distributed along a flexible backbone (*i.e.*, skeletal support). The network of artificial muscles were sandwiched between steel plates to prevent misalignment. It is worth mentioning that the technology is analogous to the pneumatic McKibben muscle, where

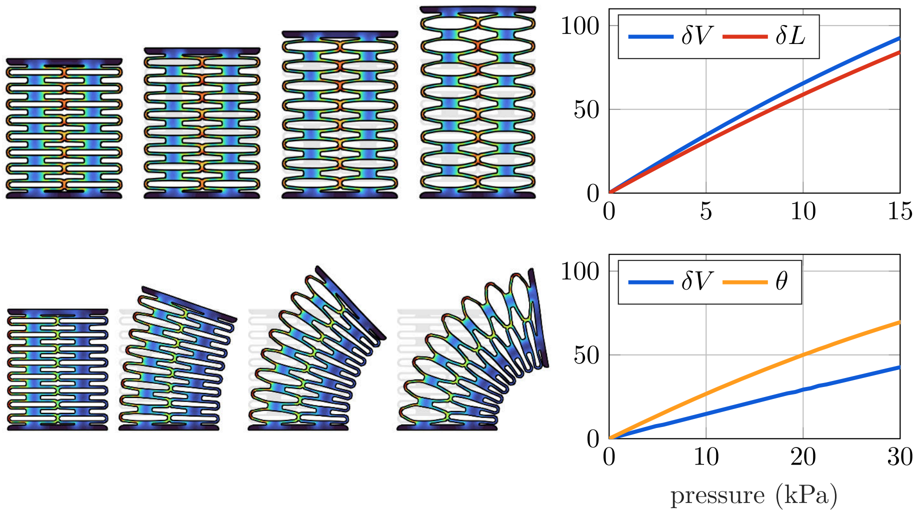


Figure 1.5. Working principle of the Orm robotic manipulator [11] with the internal volume (—) in mL, and the end-effector displacement (—) in mm and bending-angle (—) in deg. By actuation of the pneumatic network, both elongation and bending can be achieved. Observe that the response is significantly more linear than McKibben actuators in Fig. 1.3, emphasizing the importance of geometry.

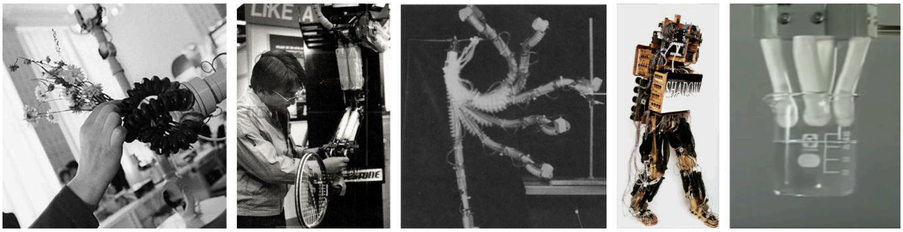
fiber weaves are used to prevent ballooning. Yet, contrary to a single McKibben actuator, the soft robotic system could undergo three-dimensional movement by inflation or deflation of an embedded pneumatic network. This led to a rich set of movements previously unseen in rigid robotics. As an illustrative example, we provided the mechanics of the Orm soft robot in Figure 1.5. The soft robot could achieve bending in any preferred direction by differential pressurization of each channel, and elongation through synchronized actuation. Most notably, comparing the volume-strain response of the Orm with respect to the McKibben actuator, *i.e.*, comparing Figure 1.3 against 1.5, it is noticeably more linear in nature. Although not documented at the time, the comparison highlights the importance of structural geometry in pneumatic muscle networks.

According to an interview with Scheinman led by Asaro et al. [57] in 2010, the positional accuracy of the system was poor, yet the concepts of pneumatically-driven soft arms continued for many years. The positional inaccuracy of pneumatic soft actuators at the time may have caused its lost of academic interest in the 60's. Three years later, in 1968, an improved hyper-redundant robot manipulator was proposed and patented by Anderson and Horn [22] (see Figure 1.2). Improving upon the Orm, which was deemed slow and had limited positional accuracy, Anderson proposed an array of nylon tendons that were connected to rigid discs distributed along the redundant backbone of the robot. The configurable backbone was comprised of universal spherical joints that allow for pivoting motion with respect to other discs – totalling 16 Degrees-of-Freedom (DOFs). The entire arm was actuated hydraulically, yet the (soft) actuators were placed outside the robot's body rather than placed at each joint, like the Orm. To improve positional accuracy further, Anderson placed sensor tendons parallel the actuator tendons which allowed for operator-based positional feedback. Although Anderson's robot does not categorize as a soft robot since it relies mostly on rigid materials, its flexibility arose from thin nylon tendons that were used for both actuation and sensing. Anderson showed that a network of distributed sensors are necessary to control the complex morphological shapes in hyper-redundant robotic systems, while also mitigating the sensor's effect on mobility. Within this context, let us define soft sensors:

> **Definition IV.** ***(Proprioceptive) soft sensors*** *are flexible measurements units embedded into the soft robotic body that through external stimuli measure the (local) changes of the system. Softness here implies that the sensor minimally alters the global mechanical behavior of the robot.*

Remark 1.3. *As the emphasize lies on "minimally alters the global mechanical behavior", soft sensors may be composed of stiff (perhaps even rigid) components. Our definition infers that these sensors must be placed into or onto the soft body, minimally affecting the operational workspace of the soft actuator network in static or dynamic condition.*

Soft robotics in the 80's. Following the fundamental works of McKibben, Scheinman and Anderson, the field of soft Pneumatic Artificial Muscles (PAMs) in robotics evolved rapidly in the early 80's. A few soft robotic systems are shown in Figure 1.6. Teleshev (1981, [23]) developed a soft gripper reminiscent of modern PneuNet actuators [30, 37, 58] – a rectangular bellow-shaped soft actuator. Unlike uniaxial PAMs, which are radially symmetric, these soft grippers explored an asymmetrical design of bellows. The geometry led to a stiffness differential around the circumference, resulting in their icon bending motion. Still popular today, these pneumatic bending actuators find their origin back in early 1974, see Andorf et al. (1974, [59]). A decade later, Takagi et al. ([60], 1983) developed a soft multi-joint robot manipulator that resembles the human arm with its movements and antagonistic muscle pairs. Although, their PAMs – called *Rubbertuator* – had a function and design identical to McKibben's PAMs, their system showed the merits of combining soft and rigid. They observed not only a high-degree of positional control of the robot arm, force control was easily regulated by the pressure control.



(a) (b) (c) (d) (e)

Figure 1.6. Early robotic systems that explored soft PAMs for various tasks. (a) Soft robotic grippers by Teleshev (1981, [23]). (b) The soft arm using *Rubbertuator* actuators by Takagi and Sakaguchi (1983, [60]). (c) Three-link soft robotic manipulator with gripper reminiscent of the elephant's trunk, developed by Wilson at Duke University (1984, [24, 66]). (d) Shadow bipedal walker by Buckley et al. (1988, [67]) using McKibben muscle in antagonistic pairs to produce locomotion.

This naturally had safety benefits. The soft robot arm could perform delicate low-force tasks while simultaneously blocking motion when encountering a human. These (soft) properties were lacking in rigid robotic manipulators at the time but reminiscent in its biological counterpart – the human arm. Note that, at that time, force and impedance control for rigid robotics had been topics of academic research for years [61–64], dating back to the early 1970's. (*e.g.*, see also [65]). Yet achieving similar properties without control were rarely explored at the time.

Shortly after, Wilson (1984, [24]) developed a soft robot manipulator based on the elephant's trunk at Duke University, Durham. His design effectively combined the works of Teleshev [23] and Takagi et al. [60] into a robot with similar dexterity but minimal use of rigid components. According to [66], his idea stemmed from the work of Kier and Smith (1985, [68]) who studied the biomechanics of muscular-hydrostats in animals, like cephalopods (*e.g.*, squids). The work of Kier et al. [68] studied how complex motions are produces in muscular organs, like elongation, shortening, bending and torsion. Inspired by the muscular hydrostat in the elephant's trunk, Wilson developed a soft arm composed of polyurethane tubes that work as half-bellows, which enabled expansion and bending under positive pressurization [66]. To accommodate for three-dimensional movement, each soft pneumatic link was placed at a $\phi = \frac{\pi}{2}$ twist offset w.r.t. to the previous link. To illustrate the motion of the soft arm, a few snapshots are provided in Figure 1.7. Wilson hypothesized that these highly-complaint robots will be more mechanically robust and be sufficiently dexterous for tight workspaces, contrary to its rigid counterparts. Although the dexterity was novel, the positional accuracy was poor. The main problem stemmed from the soft arm being controlled

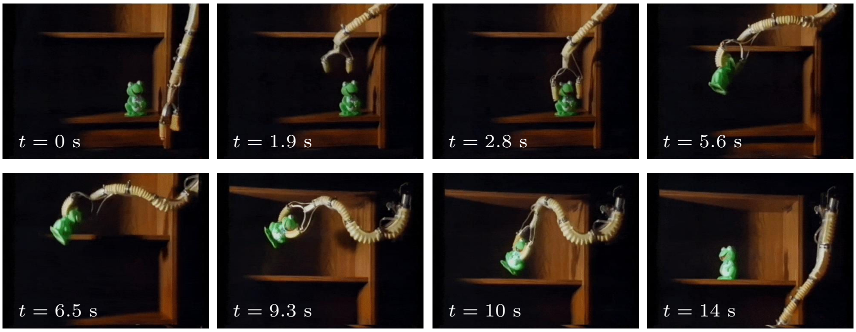


Figure 1.7. Three-link soft robotic manipulator with two-fingered soft gripper by James Wilson from Stanford university (1984, [24]). Unlike classic manipulators, where links and joints are separated, Wilson's robot consisted of three pneumatic bending actuators – being link and joint simultaneously.

in open-loop (*i.e.*, remote tele-operation) without proprioceptive sensing nor any positional feedback control. An issue akin to the Orm (1965).

A few years later, Buckley et al. (1988, [67]) developed the Shadow walker – a bipedal rigid robot comprised of antagonistic McKibben muscle pairs. Although not fully soft, their system did explore proprioceptive sensing. The hip, knee, and ankle joints were equipped with resistance-variable potentiometers for position feedback, whereas all the muscles had tension sensors for force feedback. Later on, these resistive sensors were replaced by analog optical sensors to improve robustness [67]. Although the system was top-heavy, due to the pneumatic control hardware (*e.g.*, valves and piping), rudimentary locomotion was possible. Interestingly, a similar artificial muscle system is still explored in nowadays humanoid robotics, like the Atlas from Boston Dynamics. The success of pairing soft muscles with proprioceptive sensing eventually led to the development of the McKibben Shadow hand [67, 69], comprised of 40 uniquely addressable soft muscles.

Following, Suzumori and Saiko (1989, [25, 26]) developed a micro flexible soft actuator driven by an electro-hydraulic system (intrinsic length $L \approx 12$ mm). Each end-effector enables three DOFs including pitch, yaw, and stretch – making it ideal for fingers, arms and legs. Figure 1.8 shows the level of dexterity in their system. By placing the four PAMs parallel on a gripper mount, and assigning a pre-defined trajectory, they showed their soft robotic system has sufficient dexterity to mount an hex-bolt at an incredible speed and precision. To achieve such dexterity and precision, Suzumori et al. [25] employed various modeling and control strategies to account for the dynamical characteristics under high-frequency, fluid compressibility, and the closing mechanics of pressure valves. Futhermore, the kinematics of each finger was derived using generalized homogenous transformation, not unlike traditional robotics. Knowing both the compliance characteristics (pressure-

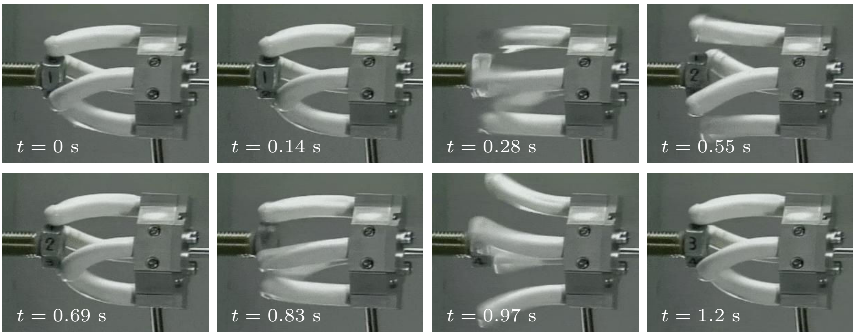


Figure 1.8. Four-fingered soft robotic gripper by Suzumori and Saiko (1989, [25, 26]). Each finger possess three pneumatic chambers that allow for directional bending – analogous the Orm. Through proper coordination of the set of soft fingers various gripping complexity can be achieved, such as the clock-wise turning a mechanical hex bolt (as shown above). Suzumori et al. showed these intricate finger motions can be easily achieved by careful modeling of the fingertip dynamics, and exploring the adaptability of soft materials.

deformation relations) and the forward kinematics, a Jacobian-based positional controller was employed to regulate the Cartesian coordinates of the finger-tips (successfully one might add).

Early controllers for hyper-redundant (soft) robots. Following the increasing interest in highly-flexible robots around the late 80's, academic research into controlling these *hyper-redundant* robots boomed shortly after. At the time, the term hyper-redundancy – being an extension to redundancy in robotics [70] – was defined as the relative degree of kinematic and/or actuator redundancy which is large or even infinite [71, 72]. The term was first introduced by Chirikjian and Burdick (1989, [73]). Others referred to these robots as *highly redundant* [74, 75] or *High Degree-of-Freedom* (HDOF) manipulator [76, 77]. Around that time, Chirikjian and Burdick provided a plenary of mathematical foundation [71, 72, 78–80] focussed on the kinematics and motion planning of hyper-redundant manipulators. Their work presented a modal discretization approach to describe the shape of the deformable backbone [78], and from this geometric approaches were introduced to solve obstacle avoiding trajectories using generalized *follow-the-leader* strategies [80]. Especially the latter showed the limitation in rigid redundant manipulators. Although Chirikjian laid the foundation for the control of hyper-redundant robot, basic principles of motion planning in pneumatic hyper-redundant robots were already presented by Wilson et al. (1988, [74, 81]) – yet were not called hyper-redundant robots nor soft robots yet. Recall also that the work of Wilson et al. has been sown earlier in Figure 1.2 and Figure 1.7. In Brock et al. (1991, [82]), a similar analysis was used for optimal shape design of thin elastic rods to realize

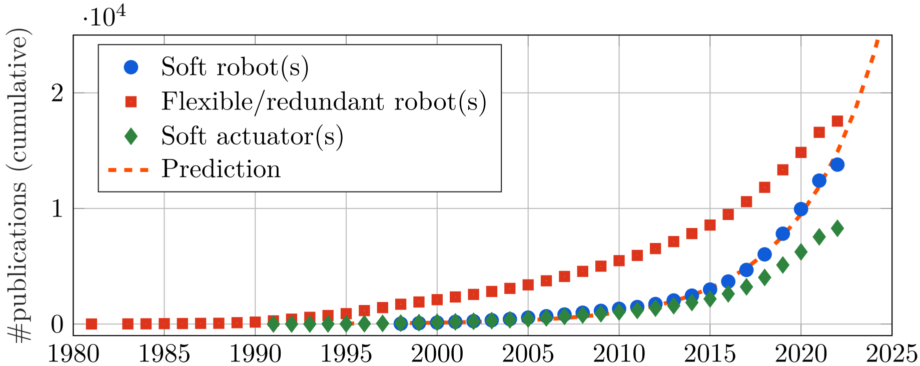


Figure 1.9. Cumulative number of (scientific) publications on the topic of '*soft robots*' or related topics. Data acquired from the Web of Sciences data repository.

desired robotic compliance. Besides, there exist an abundance of literature prior to [71] on Variable Geometry Truss Manipulators (VGTMs) – a variant of hyper-redundant tensegrity robotics – that dealt with motion planning for such systems [75, 76, 83]. Later, Mochiyama et al. [77, 84], built upon Chirikjian's work by extending it to a dynamic formulation for elastic rods such that classic controller design is possible. They proposed shape-regulation controllers for HDOF manipulator by projecting them onto time-invariant curves, thereby showing that the estimation the desired curve parameters is the crucial key to solving the problem by Lyapunov design [84]. Although there existed a variety of modeling and control strategies, computational power in relation to modeling complexity was the limiting factor for the simulation-to-reality transfer at the time.

1.2 Challenges in modern soft robotics

The exact date of the academic boom of soft robotics is unclear, but it is believed to be the mid 2000's. To motivate the argument, we have provided a graph of scientific publications on the topic *soft robot(s)* from 1980 till 2022 (see Figure 1.9). It shows the cumulative number of publications related to the term soft robot in the titles, keywords, and abstracts. The publication data from Figure 1.9 is obtained using the Web of Sciences data repository. Let it be clear that the term *soft robot* was introduced decades after hyper-redundant robots, thus some older academic work might be lost simply due to different terminology. Hence, we also provided related topics such as '*flexible*' and '*redundant robot(s)*'. In the following section, we will discuss modern trends and challenges in the field of soft robotics, that continue the research paths of highly-flexible robots in early 90's.

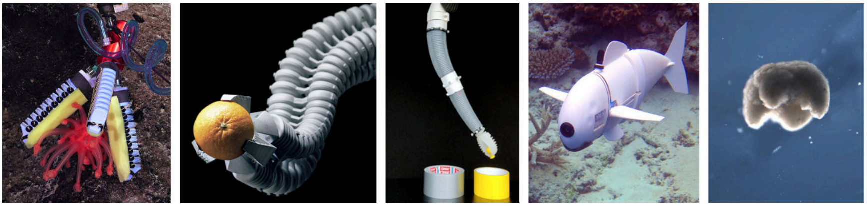
Remark 1.4. *In retrospect to the previous section, soft robotics is an extensive and diverse academic field since early 90's and it has been growing ever since. Given its* *vastness, we will limit our scope here. Within the upcoming introductory material, we will primarily focus on topics related to: (i) categories in soft robotics, e.g., soft grippers and manipulators, (ii) design and fabrication of fluidic soft actuators, and (iii) modelling and control for the soft manipulator subclass.*

1.2.1 Labelling of soft robotic systems

In the past two decades, soft robotics has undergone an evolutionary-like transformation alike biological systems. In this section, we humbly attempt to categorize some soft robotic systems.

Soft grippers. Although the origin of soft robots might stem from manipulators, modern soft robotics is perhaps most known for its diversity in soft grippers. Many systems explore redundancy and adaptability in grasping – and its subsequent manipulation of objects of various sizes, densities, compliance, and fragility. This subfield is a direct response to the hurdles in rigid robotics, where sensory force feedback and strict control laws are necessary for simultaneous safe and firm grasping. Soft grippers, on the other hand, have the natural advantage of adapting to shape and compliance of objects without the need for advanced controllers. Recalling the work of Suzumori et al. [25, 26] and earlier the work of Teleshev [23], many soft grippers explore pneumatics or hydraulics for joint motion. These inherently under-actuated soft grippers, often composed of silicone with embedded pneumatic, have shown impressive adaptability and dexterity in recent years [85, 86]. Galloway et al. (2016, [58]) developed an underwater soft gripper to delicately manipulate and sample fragile species on the deep reef (see Figure 1.10a). Their deep-reef grippers are inspired by the PneuNet actuators from Mosadegh et al. (2014, [37]). Li et al. (2017, [87]) developed enveloping soft grippers by combining collapsible origami structures enveloped in flexible skin. Hong et al. (2022, [88]) explored kirigami soft grippers, delicate enough to handle the yolk of an egg. Others explore electro-adhesive soft grippers, like Shintake et al. (2016, [89]) and Dadkhah et al. (2016, [90]), that can manipulate an unprecedented range of object types. By far, these systems have matured the most, even employing the technology in many industrial and commercial applications [91, 92].

Soft continuum manipulators. Following soft grippers, soft continuum manipulators are the soft-equivalent of rigid continuum manipulators. Building upon their predecessors [96–99], soft continuum manipulators are a class of hyper-redundant manipulators composed of mostly soft materials with virtually infinite DOFs. Let it be clear that not all continuum manipulator are necessarily soft. It has been a study prior to soft robotics [4, 71, 72]. For example, Cieslak and Morecki (1999, [100]) developed an elastic elephant trunk manipulator composed of coil-shaped elastic elements as a deformable backbone. Rucker et al. (2011, [97]) developed a semi-rigid continuum robot actuated by tendons. On the other hand,



(a) (b) (c) (d) (e)

Figure 1.10. Various categories in soft robotic systems. (a) Soft grippers by Galloway et al. (2016, [58]). (b) Commercial soft continuum manipulator by Festo Inc. [93] (c) Soft continuum Proprioceptive Arm (SoPrA) by Toshimitsu et al. (2021, [94, 95]). (d) Soft robotic fish by Katzschmann et al. (2018, [32]). Organic robot composed of skin and muscle cells by Kriegman et al. (2019, [33])

examples of truly soft manipulators included the work of Pritts and Rahn (2004, [101]), Marchese and Rus (2015, [102]) and their follow-up work (2015, [36]). Note that the aforementioned soft manipulators had limited embedded sensing, and many systems preferred fluidic or pneumatic actuation over tendon systems – a modern take on PAM robotics in the early 70's-80's. In response, Katzschmann et al. (2019, [103]) proposed a pneumatic soft manipulator with optical markers for model-based feedback control. Following their work, Toshimitsu et al. (2021, [94]) developed a soft robotic arm with proprioceptive sensors which explored capacitive flex sensor – called *SoPrA* (see Figure 1.10c). Paired with robust analytical model, their system allowed for the measurement of external forces acting on the manipulator. Mitchell et al. (2019, [104]) proposed a soft manipulator build from hydraulically amplified self-healing electrostatic (HASEL) actuators with muscle-like performance. Besides their incredible high-actuation speeds (>100 Hz) – that are difficult to achieve for any fluidic or tendon alternative – HASEL are intrinsically self-sensing. Coevoet et al. [53] developed a soft continuum manipulator actuated by tendons. As for sensing, the tendon forces are fed into an inverse Finite Element Model (FEM) to compute the deformation accordingly. Given the rich extent of rigid manipulators in classical control; its soft counterpart has become a well-known topic within recent control-oriented research [105].

Soft mobile robots. Due to the robustness of soft materials, the technology offers several advantages in terrestrial and aquatic locomotion. A popular example of terrestrial locomotion is the soft quadruped crawler by Shepherd et al. (2011, [106]). An illustation of their system can be seen in Figure 1.2. Their system proposed a configuration of four PneuNets (as legs) connected to a central soft PneuNet body. Through a periodic sequence of pneumatic activation, soft-bodied locomotion is achieved with relative ease. A similar yet improved approach is followed by Tolley et al. (2014, [107]) which presented a fully untethered 0.65

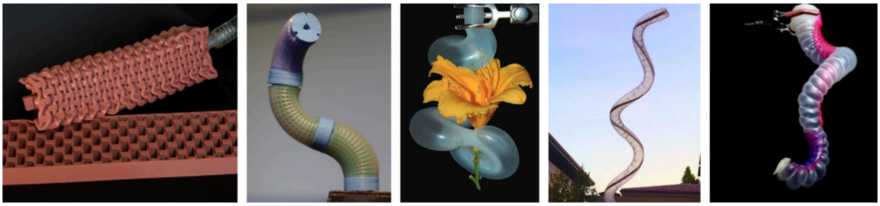
meter long mobile soft robot. Dortman et al. (2017, [108]) 3D-printed a soft robot with bellowed compliant legs that added more mobility by enabling yaw-pitch rotation. Katzschmann et al. (2018, [32]) developed a fully-autonomous soft robotic fish suited for aquatic exploration.

Soft cyborgs. The term *soft cyborgs* is a recent development in the field of soft robotics that relates to a subclass fully composed of biological material [7]. An example is the popular work by Kriegman et al. (2019, [33]) which produced a *living organic* robot made from frog skin and muscle cells. They named these robots *Xenobots* after the origin of the biological tissue – the *Xenopus laevis* frog. Not only are these system capable of autonomous locomotion, recent studies show that these systems are fully capable of artificial (self)-reproduction [109]. Other examples of bio-engineered soft robots are [110, 111]. To this day, these systems stand the furthest apart from any classic robotics, and maybe seen as a successful attempt at filling the fringes between machine and nature.

1.2.2 Tailoring design of fluidic soft actuation

As the name soft robots arises from its use of soft materials, it follows that design and fabrication using soft materials play a huge role in their technological development. Contrary to rigid robots, many soft robots explore whole body movement rather than localized regions undergoing motion – called *joints*. In classic robotics, robots are composed of a countable number of rigid links and joints [112–114], either arranged in series or parallel. Together they span a workable range of motion called the *workspace* [112]. Focussing on rigid manipulators, whose base is often structurally fixated, their workspace can be obtained through a system of kinematics, often derived through a set of geometrical equalities. Rigid manipulators often have a bounded workspace (assuming actuation limits). In robotic locomotion, similar kinematic descriptions can be obtained for the legs and feet, with the exception of an additional free-floating base. In these cases, however, the workspace is of less interest, rather the different possibility of *gait cycles* that arise from the link-joint configuration and actuator dynamics determine system's success for locomotion.

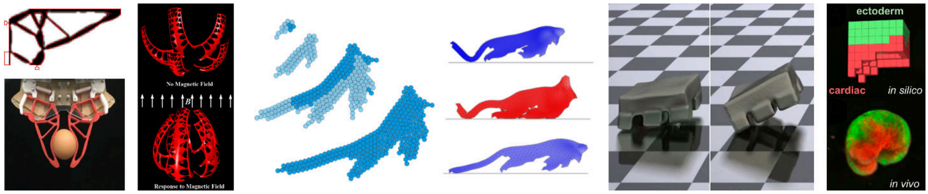
Returning to soft robots, the definitions such as joints, workspace and gait cycles also apply here. Yet, the high flexibility allows for many non-restricted joint displacements which make deriving closed-form mathematical descriptions challenging. The shape of workspace and locomotion patterns are majorly influenced by geometry of the soft actuator, its flexibility modes, and how forces are transferred with the continuum soft body. Controlling the motion within soft actuation – so to speak reducing parasitic mobility and tailoring motion based on structural geometry – is an active topic in soft robotics research for decades.



(a) (b) (c) (d) (e)

Figure 1.11. Various examples of continuum-bodied joint motions in modern soft actuation. (a) Soft actuator undergoing contraction by Yang et al. (2016, [115]). (b) Set of serial-chain of bending soft actuator by Cianchetti et al. (2013, [116, 117]) (c) Soft tentacle composed of twisting soft actuators. (d) Vine-inspired soft actuators capable of growth by Hawkes et al. (2017, [118]). (e) Soft manipulator composed of hybrid bending and twisting soft actuators through laminate materials by Kim et al. (2019, [119]).

Engineering principles in fluidic soft actuators. In the past decade, researchers have developed various techniques of exploiting the high-elasticity of soft materials for *controllable* actuation. One key development, similar working principles to the pneumatic muscle groups (see [13, 20] or Figure 1.3), are Soft Pneumatic Actuators (SPAs). A few examples are shown in Figure 1.11. SPAs undergo similar mechanics akin to McKibben actuators [13] or Morin actuators [20], yet they envelop a diverse collection of motion besides uniaxial. Examples include: contraction and elongation [115], axial growth [118], bending [36, 37, 58], helical and twisting, and a hybridization of all the aforementioned motions. An example of soft actuators capable of contraction is the Vacuum-Actuated Muscle-inspired Pneumatic (VAMP) structures by Yang et al. (2016, [115]). Their work proposes a tailored geometrical structure embedded into a soft elastomer medium that is highly sensitive towards buckling. When subjected to a sufficiently large negative differential pressure, the internal structure undergoes a (reversible) mechanically instable leading to uniaxial contraction, we see in Figure 1.11. Their work is inspired by a similar buckling behavior of patterned elastomer [120–122] subjected to axial loads. These muscle-inspired vacuum soft actuators are fast, produce a stable, repeatable motion; and more importantly, explore structural geometry to reduce parasitic motion. An example of soft bending actuators is the FLIP-FLOP system [116]. Akin the ORM system [11], it has three pressure chambers embedded into a soft cylindrical-shaped elastomer. To prevent ballooning, inextensible rings are placed orthogonal to the deformable backbone. It design is also reminiscent of [25, 26]. Hawkes et al. (2017, [118]) developed a soft manipulator inspired by the growing behavior of vines. Kim et al. (2019, [119]) used laminates that adhere to the volumetrically expanding soft body as to govern the



(a) (b) (c) (d) (e)

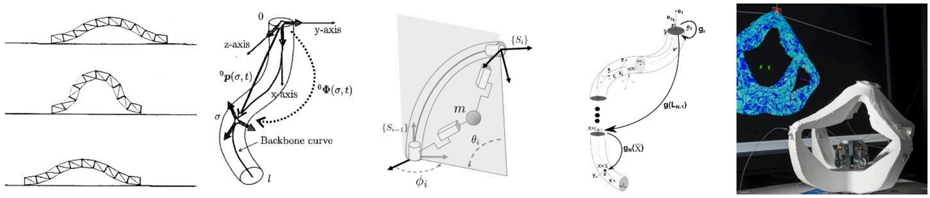
Figure 1.12. Optimization for design and motion of soft robots. (a) Soft gripper by Wang et al. (2020, [123]). (b) Topology optimization for ferromagnetic grippers by Tian et al. (2020, [124]). (c) Evolutionary algorithms for multicellular soft-bodied robots by Joachimczak et al. (2015, [125, 126]). (d) `DiffTachi` result for soft crawler by Hu et al. (2019, [127]). Voxel-based optimization for xenobots [33].

motion trajectory through bending and twisting.

Exploring optimization and evolutionary algorithms. Besides design through engineering principles, optimization in soft robotics is slowly gaining momentum recent years. Wang et al. (2020, [123]) used a topology optimization to find the optimal design for a cable-driven soft gripper (Figure 1.12a). Similarly, Tian et al. (2020, [124]) explored topology optimization for ferro-magnetic soft grippers. Besides soft grippers, evolutionary design algorithms are also employed for soft mobile like crawler and swimmers. Joachimczak et al. (2015, [125, 126]) explored evolutionary search algorithm with the purpose of automatically designing complex morphologies and controllers of multi-cellular, soft-bodied robots (Figure 1.12c). Hu et al. (2019, [127]) used a differential physics simulator called `DiffTachi` that efficiently computes gradient information for each simulation timestep. The gradient information can then be fed into neural network controllers to solve, for instance, the appropriate gait-cycles required in the locomotion of soft-bodied crawlers (Figure 1.12d).

1.2.3 Gaining performance through modelling and control

As the inherent properties in soft materials bring forth many benefits, *e.g.*, adaptability, hyper-redundancy, and passivity w.r.t. the environment; it too hinders the progress in model-based controllers. Earlier, we have touched upon these subject with the rise of kinematic and dynamic models for hyper-redundant robotics in the late 80's – early 90's. Chirikjian et al. (1992, [71]) provided a kinematic framework for hyper-redundant manipulators with application to motion planning (see Figure 1.13a). Here the elastic backbone is approximated using a modal formulation. Such modeling framework are one-to-one transferable to soft continuum manipulators. Mochiyama et al. (1998, [84, 128]) extended this work to a dynamics formulation – even providing Lyapunov-based control strategies for shape regula-



(a) (b) (c) (d) (e)

Figure 1.13. Popular modeling strategies for soft robotics. (a) Hyper-redundant modeling description through tensegrity by Chirikjian (1992, [71, 72]). (b) Analytical continuum beam description by Mochiyama (1999, [77, 128]). (c) Augmented rigid-body model subjected to PCC kinematics by Katzschmann et al. (2019, [103]). (d) Geometric Cosserat model subjected to PCS kinematics by Renda et al. (2018, [50]). (e) Diamond-shaped soft robot manipulator controlled using the FEM-based `SOFA` software by Duriez et al. (2013, [129]) and related [130, 131].

tions (Figure 1.13b). However, both modeling frameworks were computationally inefficient – lacking transferability to real-time control. The root problem stems from the fact that soft continuum robots, belonging in their exact formulation to the field of continuum mechanics, lead to infinite-dimensional models often expressed as Partial Differential Equations (PDEs). Rigid multi-body systems, like robot manipulators or mobile robots on the other hand, have convenient Ordinary Differential Equation (ODEs) structures as they are rooted in Lagrangian or Newtonian mechanical principles. Rigid-body models were (and still are) fast computationally and their literature on controller design is vast and well-established [112–114]. The computational issues in early continuum robots may be reflected by its literature gap between the 1990's to 2010's.

Modern control-oriented models for soft robots. In the past decade, significant steps have been made to address the issues of infinite dimensionality [105]. The key is to formulate a finite-dimensional approximation of the soft robot's dynamic such that they can be written as standard ODEs. Reduced-Order Models – ROM for short – have paved the path of model-based controller for soft robots whose reduced formulations are both tractable and precise. In the years shortly after its academic boom, many different assumptions and model approximations have come forth to address the issue of control.

Remark 1.5. *A majority of the reduced-order formulations for soft robotics are only applicable to slender robots. Consequently, soft robot manipulators have become the main subject for many control-oriented studies. This scope reduction is well-motivated however as many soft robots have one physical dimension that is more dominant than the other two [105]. As such, the thesis focusses majorly on modeling and control of soft manipulators rather than a broader scope.*

Focussing on soft robot manipulators first, a popular choice of finite-dimensional reduction is the so-called *Piecewise Constant Curvature* (PCC) soft beam model. The PCC modeling approach is by far the most adopted in the soft robotics community [96]. As the name implies, the soft robot is modelled as an elastically deformable beam with all strains but curvature neglected. Examples of this approximation include [32, 36, 96, 103, 132–135]. Highlighting a few, Katzschmann et al. (2018, [32]) proposed to connect the PCC formulation to an augmented rigid robot dynamical model with parallel elastic actuation (see Figure 1.13c). A similar approach was proposed earlier by Falkenhanh et al. (2015, [132]) and applied to Festo's bionic arm [27]. Although such lumped models may seem like a major over-simplification, the proposed model allows sufficient speed and accuracy such that model-based feedback is applicable. This formulation was also employed later in an adaptive sliding mode control scheme [136] for the SoPra soft arm [94]. Following, Renda et al. (2018, [50]) extended the PCC model to *Piecewise Constant Strain* (PCS) formulation. This formulation allowed for all strain if and only if considered spatially piece-wise constant (Figure 1.13d). Rooted in SE(3) geometry of the Cosserat approach [137], it provided a closer relation with the rigid body geometry of the traditional robotics. The formulation also extends to soft manipulators with fluidic actuation [138, 139] The PCS model was later employed to feedforward controllers by Thuruthel et al. (2018, [140]) using model-based policy learning algorithms. To improve efficiency, a recurrent neural network was trained using an offline PCS model. Grazioso et al. (2019, [141]) explored a similar path of geometric Cosserat beams using helical strain functions. Nevertheless, Constant Strain (CS) models have severe limitations. They often do not originate from continuum mechanics and thus are only applicable in restrictive settings. Although computationally performance might surpass continuous models, due to intrinsic kinematic restrictions, they are unable to capture important continuum phenomena, like buckling, environmental interaction, or wave propagation.

In response to its limitation, many researchers continued their search for efficient and more generalizable alternative. Della Santina et al. (2020, [142]) proposed a polynomial description to described the continuum dynamics, a description analogous to [71]. In their work, they expressed the curvature function of the soft robot in terms of a standard polynomial bases. Not only can an exact infinite dimensional formulation of the problem be obtained (in theory), truncation at any level is changed easily. The technique is also widely used for flexible-link robot manipulators to capture small vibrations, see DeLuca et al. (2016, [143]). Della Santina et al. also showed that PCC-rooted assumptions as control output produce a minimum phase system [142] – a fundamental stepping-stone for nonlinear control. Following, Boyer et al. (2021, [144]) extended upon their prior Cosserat models [50, 145], and presented a tractable and generalizable beam model for slender soft manipulators. A similar approach to [142] was followed but all strains are discretized using a finite set of strain basis functions. Renda et al. (2020, [145]) improved computation by introducing a two-stage Gauss quadrature [146] to derive the Magnus expansion [147]. Other examples of the Cosserat beam descriptions, but more focussed on the continuum mechanics rather than control, is the work Gazzola et al. (2018, [148]). Their work allowed for efficient Cosserat beam models suitable for self-collision, thus providing various simulation for twirling and coiling of beams under increasing torsional loads.

Another popular alternative, better suited for general soft robotic systems like soft mobile robots, are reduced-order finite element models [53, 129–131, 149–154] or neural-network trained using offline FEM simulations [155]. Starting from high-order FEM data (*e.g.*, state dimensions of the order 10k) that capture the whole workspace spanned by the network of soft actuators, Proper Orthogonal Decomposition (POD) techniques are employed to drastically to reduce the state dimension of the soft robot model. These techniques can even retain external loads (*e.g.*, contact and friction) with precision as long as they are included in the offline data set [131]. By far, this FEM-driven method has shown the most success in the experimental control regime.

Soft robot simulation and programming environments. The rapid development of soft robotic models in recent years have also increased the demand for (open-access) software packages. Especially since many of the aforementioned ROM models require an advanced level of mathematical understanding. In an attempt to help the soft robotics community, many researchers have provided open-source, documented simulators interwoven in their soft robotics research. A popular FEM-based software on soft robotic modeling and control is `SOFA` by Duriez et al. (2013, [129, 130]). `DiffTachi` by Hu et al. [127, 156] explores differential simulations to produce soft machine capable of locomotion. Bern et al. (2022, [51, 157]) developed `SoftIK`, a software for soft deformable plushy robots. Among beam or rod-based models there exist many options. Examples included `Elastica` (or `pyElastica` [158]) by Gazzola et al. [148, 159], `TMTDyn` by Sadati et al. [160], `SimSOFT` by Grazioso et al. [141], and `SoRoSim` by Mathew et al. [161] based on the work of Renda et al. [145] and Boyer et al. [144]. The `Sorotoki` toolkit by Caasenbrood et al. [162] – open-source software package presented as a part of this thesis – explores a combination of FEM models and soft beam models. The toolkit written in Matlab aims to bridge the gaps between design, modeling, and control of (hyper-elastic) soft robots.

Closing the loop in soft robotics. Following the many developments in computational efficiency of reduced-order models (and accordingly the advances in soft sensing), academic research in model-based or model-free control for soft robots is significantly growing since early 2019. Note that feedforward controllers for soft manipulators have been proposed years prior, *e.g.*, [132, 163–165].

By far, experimental validation of PCC model has been used more intensively than other models. In Della Santina et al. (2019. [135], the augmented rigid-body PCC model is used to design a closed-loop controller for a continuous soft manipulator, presenting two architectures designed for dynamic trajectory tracking and surface following. Prior work is provided here [103]. A similar approach was followed by Milana et al. (2021, [166]) and applied to an artificial soft cilia. They showed that soft bending actuators could mimic the asymmetric motion of the cilia through model-based control. Cao et al. (2021, [167]) explored a reduced analytical model [168], somewhat equivalent to a linear pendulum model (apart from quadratic terms in the potential force); to develop robust tracking controllers without velocity observers. Their controller was tested experimentally on a soft PneuNet actuator. Wang et al. (2022, [169]) developed a computed torque controller (see [112]) using the augmented rigid-body PCC model and applied it to a soft Honeycomb Pneumatic Network Arm [170]. Franco et al. (2020, [134, 171]) used a port-Hamiltonian modeling framework akin to rigid-body PCC model (*i.e.*, three-link pendulum) and applied such principles to energy-shaping controllers. The performance of their controller was assessed via simulations and via experiments on two soft continuum prototypes. On a side note, Franco et al. (2022, [172]) also developed an energy-shaping control law together with nonlinear observers for the control of soft growing robots [118] with pneumatic actuation subject to the (ideal) gas laws.

As mentioned previously, the PCC model has significant limitation that impose questions on the useability, dexterity, and robustness of their control derivation. Although majorly focussed on simulation, higher-order dynamics have been used for the development of feedback controller in soft manipulators. Della Santina et al. ([105]) developed swing-up controllers for a soft pendulum modelled by the affine curvature models (*i.e.*, a polynomial curvature model [142] of order $k = 2$). Their approach mirrors the path of classic control problem of inverted pendulums in the 90's and early 00's [173–176]. Later, Weerakoon et al. (2021, [177]) extended upon their work by introducing a revolute base. A common control problem here is under-actuation [112, 113, 178] – implying that not all control actions can be realized to steer the configuration space to a desired position. In the work of Borja et al. (2022, [179]) developed a general control framework that can stabilize soft manipulators based on potential energy shaping based on the affine curvature model. Their work showed that some linear matrix inequalities can be derived based on the gradient of the potential energy related to the passive and active states, such that local stability can be proven. In laymen terms, elasticity must dominate the forces resulting from the gravity in the underactuated states for (potential) energy-shaping controllers (and mostly-like others) to work.

1.3 Research objectives and contributions

In this section, the research objectives and contributions are specified. The research is divided into three unique branches each related to a specific subproblem within the field of soft robotics: (I) design synthesis of soft actuators, (II) modeling for control of soft robot manipulators, and (III) the development of software aimed to support the soft robotics community. For each research objective, a number of contributions of the thesis are presented.

I: Design synthesis of soft actuators. As presented by abundance of literature on soft robotic design, either rooted in engineering principles or through optimization, achieving an optimal structural geometry that fully accounts for hyper-redundancies in soft materials is no easy feat. Unlike their rigid counterparts and many biological systems for that matter, the kinematics are inherently encoded in the topological structure of the soft materials and where actuation is presented in the system. This implies the workspace cannot be characterized analytically in closed from through joint motions stemming from one point (unlike joints in rigid robotics), see Figure 1.14a. Futhermore, any external inputs will result in parasitic motion, which are mainly induced by distributed continuum deformations that are antagonistic to the input. These parasitic motions – or better phrased *passive joint displacements* – arise from the redundant flexibility of the system that are often unaccounted for during design. As such, they should be kept at as minimum as they lead to imprecisions in the mechanical operation and lost of mechanical efficiency (*e.g.*, the balloon effect [14]). Futhermore, there exist many elasticity moduli for various options of soft material, ranging from soft gels to hard rubber [7]. It is therefore of paramount importance that the nonlinear behaviors of soft materials, both in hyper-elasticity and nonlinear geometrical deformation, is understood and accounted for during the design process. This deepens the research problem by imposing questions on optimality regarding material choice.

(Optimality in soft material design). Our first research objective therefore focusses on the design. The main design principles for any compliant mechanical device can be described rigorously through continuum mechanics. In the thesis we focus on harmonizing the underlying continuum theory with the automated design of efficient soft actuators with user-defined objectives in mind (Figure 1.14b). Rooted in continuum mechanics [180, 181] (and its subsequent discretization through finite elements), optimization algorithms are employed that aim to seek an optimal layout of soft materials such that user-specified objective functions are minimized (Figure 1.14c). Such problems are often referred to an *inverse design problem* – solving shape by knowing deformation, rather than solving for deformation based on the shape. The study of combining continuum mechanics and free-form optimization in compliant structures is well-established field called

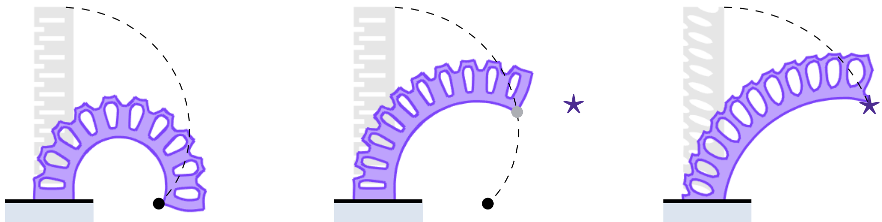


Figure 1.14. Schematic illustration of the inverse design problem in soft robotics. (a) Bending behavior of PneuNet actuator under linearly increasing pressure. (b) Desired end-effector position ($\star$) outside the robot's workspace, changing the input is not sufficient will not improve objective – the workspace is *a-priori* encoded into the soft topology. (c) Solution to inverse design problem by finding a soft topology that contains ($\star$) in its workspace.

topology optimization [182]. Yet, such practices are not easily transferred to soft actuation, since soft materials introduce hyper-elasticity and nonlinear geometric deformations. Besides, fluidic or pneumatic actuation complicates the optimization, as these loads become both design and state-dependent. This brings us to the first contribution:

> **Contribution I.** *Development of efficient algorithms, applicable to the general design of fluidic soft actuators, solving the inverse design problem: Given a desired motion and input, what is accordingly the optimal soft material distribution within a design domain to realize such joint motion?*

(Fabrication through Additive Manufacturing). Following these automated algorithms, many variations of soft actuation system with various joint mobility can be developed with relative ease. Yet, being of simulation origin, this raises the questions on the transferability of the numerical studies to practice. Hence, our second research objective therefore focusses on the transferring simulation design to feasible, functional soft actuators. Through the recent advances in Additive Manufacturing, many complex three-dimensional geometries can be fabricated with relative ease and effort.

> **Contribution II.** *Fabrication of an array of computer-optimized pneumatic soft actuators through Additive Manufacturing methods, whose collective assembly can be explored for soft robotic manipulation.*

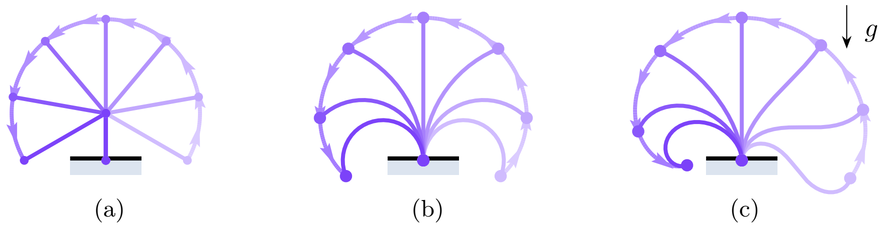


Figure 1.15. Stages of kinematic complexity in (soft) manipulators. (a) Standard 1-DOF rigid manipulator with analytic workspace. (b) Ideal soft manipulator deformed under uniform curvature (*i.e.*, PCC model). Workspace can be analytically expressed for non-complexity cases, *e.g.*, stiffness dominates gravity or homogenous deformation. (c) Truly under-actuated soft manipulator starting from initial (gravity-balanced) condition. Workspace can often not be derived analytically and depends highly on the initial conditions and actuation constraints.

II: Modeling for control of soft manipulators. Besides design, modeling for control is another important topic in soft robotics. In particular with the aim of achieving biological performance in modern soft robotic systems, models must be both accurate and fast. However, the infinite-dimensionality that are inherent to these soft continuum robots present some challenges. Augmented rigid-body models, like [103, 134, 166], offer exceptional computation speeds but do not respect the fundamental continuum mechanics, therefore invoking strict operation constraints on any soft system, *i.e.*, small deformation as to limit hyper-elastic nonlinearities; or slow actuation to prevent inertial mismatching.

(Accurate control-oriented models through PCC condition). Our second research objective therefore focusses on the reduced-order modeling, aiming to balance precision and speed for control. By building upon the original works of Chirikjian et al. [71] and Mochiyama et al. [183] in the 90's, the thesis presents a dynamic modeling formulation for soft manipulators that respects its continuum nature. To address the issue of infinite-dimensionality, a reduced-order modeling strategy for soft robot manipulators is proposed, whose mathematical framework is based on the differential geometric theory of spatial curves. Such framework allows for easy transferability to classical ROM models in soft robotics, like the PCC strain [103, 132, 142] (see Figure 1.15b). However, the thesis proposes two improvements that are essential development of model-based controllers. Inspired by the success of FEM-based models in soft robotics [129, 131], we bridge the gap between the PCC model and the underlying continuum mechanics by matching the quasi-static behavior to a Finite Element Model (FEM). The numerical FEM approaches used are analogous to Contribution I. Second, to enhance computational efficiency, we propose a reduced-order integration scheme using Matrix Differential Equations (MDEs) to compute the spatio-temporal dynamics in real-time. The

proposed reduced-order soft beam model is tested rigorously in simulation; however, for practical control tests demand experimental rigorousness. As such, a 3-DOF soft robot manipulator is developed *ad-hoc* through Additive Manufacturing. The system is loosely inspired by the *Orm* soft manipulator from the early 60's [11]. The proposed dynamic model is tested under various experimental conditions. For example, natural oscillations, forced inputs, and under tip-disturbances of various inertial mass. Various performance measures are introduced to compare the proposed model objectively with respect to linear elastic alternatives. This brings us to our third contribution:

> **Contribution III.** *Development of fast, efficient, and accurate dynamic models for soft manipulators composed of hyper-elastic soft materials that are directly applicable to classical control theory akin to rigid robotics.*

(Models beyond the PCC). As mentioned in Section 1.2.3, the piecewise continuity inhibits many kinematic redundancies – and therefore the hyper-redundant nature of these soft robotic systems cannot be explored to its full potential. For example, any large nonlinear deflection due to gravity (see Figure 1.15c) cannot be captured using these constant descriptions. Also, the study of (true) underactuation through such kinematically lumped models is (nearly) impossible as the modeling framework infers homogenous strain and actuation. Herein lies the third research objective that focusses majorly on relaxing the PCC condition, to closely pursue its true infinite-dimensionality. Adopting the prior modeling strategies of differential curves, spatially-varying strain fields are considered and approximated (closely) through sets of orthogonal shape functions rather than prior piecewise representations. Building upon prior models presented in the thesis, the underactuated and hyper-redundant soft system can be casted into a port-Hamiltonian framework [184, 185]. This allows for classic controller design through energy-shaping by modifying the closed-loop potential energy of the system – a well-known practice in classic robotics [175, 184, 185]. Exploring the hyper-redundancy in soft robotics, more advanced control objectives can be realized such as shape regulation and full-body grasping of rigid objects. However, spatial discretization in these model-based controller play a crucial part in their dexterity to achieve various control tasks. Within this context, the fourth contribution reads:

> **Contribution IV.** *Investigation of spatial discretization in low-order energy-shaping controllers applied to high-order soft robotic models with a focus on closed-loop performance in shape and full-body grasping control.*

(Exploiting geometry for reduction). Contribution IV provides a stable modeling platform for a variety of possible shape functions tailored to unique joint mobilities in soft robotic systems. Yet, many works in modeling literature choose such functions in *ad-hoc* fashion, *e.g.*, polynomial bases [79, 142, 144]. Within this context, the thesis explores a (geometric) modal decomposition approach. Similar to the eigenmode analysis in continuum mechanical systems, geometric strain modes are extracted from higher-order (volumetric) FEM models and used to construct optimal soft beam models. The approach leads to fast, accurate, and generic low-dimensional models that encode the geometric features and elasticity of the true soft body into a new strain parameterization - we call a Geometry-Informed Variable Strain (GIVS) basis. A merit benefit of the approach can be naturally expanded to identify the hyper-elastic material parameters and the actuation map of the reduced beam model. Robustness of the technique is investigated experimentally for several soft robotic systems, including PneuNets, soft grippers, and soft manipulators. In context of robustness, the nonlinearities with increasing actuation frequency, under environmental contact modeled by signed distance functions, and multi-input pneumatic actuation are also considered. The thesis continues with a qualitative comparison between existing strategies, demonstrating that our approach can improve accuracy and speed compared traditional techniques. Our fifth contribution therefore reads:

> **Contribution V.** *A novel method for finite-dimensional model reduction in soft manipulators that explores the mechanical interconnection between structural geometry and flexibility modes of the soft manipulator body.*

III: Software development. Finally, the thesis collects all theoretical material into one unifying soft robotics software package called `Sorotoki` – short for SOft RObotics TOolKIt. The software aims to bridge the gaps between different disciplines of soft robotics, applicable to design, modeling and control. Using a minimal programming framework, complex problems can be coded using minimal lines of code. The toolkit is heavily interwoven with Contributions I to V of the thesis, and is publicly available at https://github.com/BJCaasenbrood/SorotokiCode [162]. The last contribution therefore reads:

> **Contribution VI.** *Developement of a versatile, user-friendly, open-source software called* `Sorotoki` *that envelops the presented theory on design, modeling and control of the thesis into one coherent Matlab toolkit.*

1.4 Outline of the thesis

This thesis discusses the design, modeling and control of soft robotic systems. Including this introductory materials, the thesis consists of seven chapters.

Chapter 2 presents the design algorithm for soft actuators that aim to solve the inverse design problem. The chapter start with a brief introduction into continuum mechanics applied to three-dimensional deformation of hyper-elastic materials, followed by its numerical implementation using finite elements. From here, numerical optimization procedures are introduced that solve the inverse design within the context of (fluidic) soft actuation. Chapter 3 follows with the second objective of the thesis, namely modeling for control. Instead of volumetric soft robotic models, lower-dimensional soft beam models are introduced that are tailored for fast and accurate model-based controllers. The chapter focusses primarily on PCC soft beam models. Chapter 4 address the limitation of the PCC model, and instead extends upon it. The chapter formulates a finite-dimensional port-Hamiltonian modeling approach for soft beams, where spatial shape functions are used to discretize the modal flexibilities of the soft robot. From here, energy-shaping controller are introduced that allow for shape control and grasping of objects. A major emphasize on the chapter is the effects of discretization in soft robotics and choice of control gain, and their subsequent effects on the closed-loop solutions. Chapter 5 focusses on the Geometry-informed Variable Strain description that ties together Chapter 2 and Chapter 4 into an (optimal) reduced-order modeling and control framework. Chapter 6 presents the culmination of all theoretical material presented in the thesis into a concise, user-friendly, toolkit called `Sorotoki`. The chapter presents an overview of the included programming tools for the design, modeling and control of soft robots. Finally, Chapter 7 closes the main body of the thesis by summarizing the research deliverable of prior chapters, and provides a list of recommendations that could sculp future work.

Note for the reader. Chapters 2-6 are all based on published or submitted researcher articles and can therefore be read independently. A reference to the corresponding research paper is provided at every beginning of these chapters. This thesis, however, provide some minor modifications to these works, either in the context of mathematical or material improvement, or connection between other chapters. An overview of these modifications can be found as a supplementary chapter at the end of the thesis in the chapter named *Modifications*.

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