Design, Modeling, and Control Strategies for Soft Robots

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Abstract

Design, Modeling, and Control Strategies for Soft Robots

In the past two decades, the field of soft robotics has sparked significant interest among many scientific disciplines. Contrary to rigid robots, soft robots explore soft materials that significantly enhance the robot's dexterity, enable a rich family of motion primitives, and enhance environmental robustness regarding contact and impact. Since its inception, soft robotics has exemplified its potential in diverse areas such as safe manipulation, adaptive grasping, exploration under environmental uncertainty, rehabilitation, and the bio-mimicry of many animals. By exploring the uncharted versatile nature of soft materials, soft robotics places the first steppingstones towards achieving biological performance in modern-day's robotics. This thesis aims to further the advances in soft robotics by addressing some of the open multi-disciplinary challenges within this young field of research.

Although soft materials harbor many advantages akin to biology, which are difficult to achieve for rigid robotics, it also roots many fundamental problems. First is the issue of soft robotic design. Traditional robotic design emphasizes high structural rigidity and weight minimization – a well-established practice in engineering. On the other hand, soft robotic design relishes minimal structural rigidity for motion, leading to complex, highly nonlinear relations between the input and output. Besides, distributed soft actuation, imparted by gravitational and inertial forces acting on the continuum elastic body, introduce joint mobilities that are in many cases uncontrollable nor aligned with the control objective, *e.g.*, precise grasping and manipulation. Since describing the underlying continuum mechanics and applying such mathematical theory to systematic design is challenging, a large number of soft robotic systems are still developed *ad hoc*.

Second, a direct duality of the previous challenge is dealing with the innate infinite-dimensionality from a control perspective – particularly with model-based feedback in mind. The transition from rigid to flexible has introduced a new control paradigm: the trade-off between precision and speed in a numerical setting. Not only is control theory for soft robotics in stages of inception, but deriving accurate and numerically efficient model-based controllers is challenging due to large nonlinear deformations of the soft robotic continuum.

In light of these challenges, this thesis proposes a set of systematic tools with theoretical and experimental applications for (*i*) the structural design and fabrication of continuum-deformable soft actuators optimized for user-defined joint motion, (*ii*) the development of efficient dynamic models of soft continuum manipulators, and (*iii*) applying such mathematical models to model-based controllers for a subclass of (pneumatic) soft continuum manipulators and soft grippers.

The first part of the thesis addresses the design problem by proposing novel computer-automated design algorithms for developing efficient soft actuators. These algorithms account for the underlying continuum mechanics described by a set of partial differential equations, which respect the aforementioned nonlinearities between the input and output motion. Tailoring a user-defined objective to a desired motion and control reachability, an implicit representation of the optimal soft material distribution can be found within a fixed design space. Several generative designs for a diverse subset of soft actuation morphologies are produced including, but not limited to, soft rotational actuators, soft artificial muscles, and soft grippers. In what follows, an optimal design for a soft robotic manipulator with an adaptive gripper is synthesized; and through Additive Manufacturing (AM) of printable flexible material, the sim-to-real boundary is passed. The proposed approach does not only accelerate design convergence but also builds upon the vast library of soft robot morphologies currently unexplored in literature.

The second part of the thesis addresses the question of modeling for control applicable to a class of soft robotic systems – most notably soft continuum manipulators. The thesis proposes a reduced-order modeling strategy for soft robotics, whose dynamics are derived through the differential geometric theory on spatial beams. Besides discussing earlier modeling strategies, the thesis also proposes a new strain-based parametrization approach that ensures the structural information and the underlying continuum mechanics are preserved when synthesizing the reduced-order beam models – a possible solution to the aforementioned control paradigm of precision vs. speed. To enhance numerical performance further, spatio-temporal integration schemes are also proposed that exploit the geometric structure of such soft beam models, resulting in real-time simulation with sufficient numerical precision purposefully tailored for control.

The third part of the thesis treats the development of model-based controllers that can be employed in various control scenarios akin to control for traditional rigid robotics, *e.g.*, inverse kinematics and motion planning, set-point stabilization, trajectory tracking, and multi-point grasping of objects. The stabilizing controller is rooted in an energy-based formulism, providing robustness even when faced with material uncertainties. The controller's effectiveness is demonstrated both in simulation and experiments for various soft robotic systems that share a resemblance to biology, *e.g.*, the elephant's trunk or the tentacle of an octopus.

The main contribution of the thesis is a collection of multi-disciplinary tools compressed into one general framework for the design, modeling, and control of a class of soft robots, ranging from the theoretical to the experimental domain.

Keywords: Soft Robots, Hyper-redundant Robots, Design Optimization, Continuum Mechanics, Reduced-order Modeling, Model-based Control, 3D Printing.

Samenvatting

In de afgelopen twee decennia heeft het veld van de zachte robotica veel interesse opgewekt binnen breed spectrum van wetenschappelijke disciplines. In tegenstelling tot rigide robots verkennen zachte robots zachte materialen die de behendigheid van de robot aanzienlijk verbeteren, een rijke collectie van bewegingsprimitieven mogelijk maken en de omgevingsbestendigheid ten aanzien van contact en impact vergroten. Sinds de oorspong heeft de zachte robotica heeft het haar potentieel aangetoond in diverse gebieden zoals veilige manipulatie, adaptief grijpen, verkenning onder omgevingsonzekerheid, revalidatie en de biomimetica van vele dieren. Door de veelzijdige aard van zachte materialen te verkennen, legt de zachte robotica de eerste stappen naar het bereiken van biologische prestaties in de moderne robotica. Deze scriptie heeft als doel de vooruitgang in de zachte robotica verder te bevorderen door enkele van de open multidisciplinaire uitdagingen binnen dit jonge onderzoeksgebied aan te pakken.

Hoewel zachte materialen, zoals systemen in de biologie, veel voordelen hebben, die soms moeilijk te bereiken zijn voor rigide robotica, brengt het ook veel fundamentele problemen met zich mee. Het eerste probleem is het ontwerp van zachte robots. Traditioneel robotica-ontwerp legt de nadruk op hoge structurele stijfheid en gewichtsminimalisatie - een goed doordachte discipline in de engineering. Aan de andere kant houdt het ontwerp van zachte robots van minimale structurele stijfheid voor beweging, wat leidt tot complexe, zeer niet-lineaire relaties tussen input en output. Bovendien leiden gedistribueerde zachte activering, toegepast door zwaartekracht- en traagheidskrachten die op het continu elastische lichaam werken, tot gewrichtsmobiliteiten die in veel gevallen niet te controleren zijn of niet zijn afgestemd op de controle-doelstelling, zoals nauwkeurig grijpen en manipulatie. Omdat het beschrijven van de onderliggende continue mechanica en het toepassen van dergelijke wiskundige theorie op systematisch ontwerp uitdagend is, worden nog steeds een groot aantal zachte robotsystemen *ad hoc* ontwikkeld.

Ten tweede vormt de directe dualiteit van de vorige uitdaging het omgaan met de intrinsieke oneindige-dimensionaliteit vanuit een controleperspectief - met name met modelgebaseerde feedback in gedachten. De overgang van rigide naar flexibel heeft een nieuw controleparadigma geïntroduceerd: de afweging tussen precisie en snelheid in een numerieke omgeving. Niet alleen bevindt de controletheorie voor zachte robotica zich in de beginfase, maar het afleiden van nauwkeurige en numeriek efficiënte modelgebaseerde controllers is uitdagend vanwege de grote niet-lineaire vervormingen van de zachte roboticacontinuüm.

Gezien deze uitdagingen stelt deze scriptie een reeks systematische tools voor

met theoretische en experimentele toepassingen voor (*i*) het structurele ontwerp en de fabricage van continuüm-deformeerbare zachte actuators geoptimaliseerd voor door de gebruiker gedefinieerde gezamenlijke beweging, (*ii*) de ontwikkeling van efficiënte dynamische modellen voor zachte continuüm manipulatoren, en (*iii*) het toepassen van wiskundige modellen op modelgebaseerde controllers voor een subklasse van (pneumatische) zachte continuüm manipulatoren en zachte grijpers.

Het eerste deel van deze scriptie richt zich op het ontwerpprobleem door het voorstellen van nieuwe computer-geautomatiseerde ontwerpalgoritmen voor de ontwikkeling van efficiënte zachte actuators. Deze algoritmen houden rekening met de onderliggende continue mechanica die wordt beschreven door een set partiële differentiaalvergelijkingen, die de eerder genoemde niet-lineariteiten tussen de input en output beweging respecteren. Door een door de gebruiker gedefinieerd doel aan te passen aan een gewenste beweging en controlebereik, kan een impliciete representatie van de optimale zachte materiaalverdeling worden gevonden binnen een vast ontwerpruimte. Verschillende generatieve ontwerpen voor een diverse subset van zachte actuator-morfologieën worden geproduceerd, waaronder, maar niet beperkt tot, zachte rotatie-actuators, zachte kunstmatige spieren en zachte grijpers. Vervolgens wordt een optimaal ontwerp voor een zachte robotmanipulator met een adaptieve grijper gesynthetiseerd. Door middel van Additive Manufacturing (AM) van printbaar flexibel materiaal wordt de grens tussen simulatie en realiteit overschreden. De voorgestelde aanpak versnelt niet alleen het convergeren van het ontwerp, maar bouwt ook voort op de enorme bibliotheek van zachte robot-morfologieën die momenteel onontdekt zijn in de literatuur.

Het tweede deel van de scriptie richt zich op de modellering voor controle die van toepassing is op een klasse van zachte robotica-systemen - met name zachte continuüm manipulatoren. De scriptie stelt een modelleerstrategie voor met gereduceerde orde voor zachte robotica, waarvan de dynamica worden afgeleid door middel van differentiële geometrische theorie op ruimtelijke balken. Naast het bespreken van eerdere modelleerstrategieën stelt de scriptie ook een nieuwe spanning-gebaseerde parameterisatiebenadering voor die ervoor zorgt dat de structurele informatie en de onderliggende continue mechanica behouden blijven bij het synthetiseren van de gereduceerde balkmodellen - een mogelijke oplossing voor het eerder genoemde controleparadigma van precisie versus snelheid. Om de numerieke prestaties verder te verbeteren, worden ook spatio-temporele integratieschema's voorgesteld die de geometrische structuur van dergelijke zachte balkmodellen benutten, wat resulteert in real-time simulatie met voldoende numerieke precisie die specifiek is afgestemd op controle.

Het derde deel van de scriptie behandelt de ontwikkeling van op modellen gebaseerde controllers die kunnen worden gebruikt in verschillende controle scenario's vergelijkbaar met de controle voor traditionele rigide robotica, bijvoorbeeld inverse kinematica en bewegingsplanning, set-point stabilisatie, traject volgen en multi-point grijpen van objecten. De stabiliserende controller is geworteld in een op energie gebaseerde formulering, die robuustheid biedt, zelfs wanneer er sprake is van materiaalonzekerheden. De effectiviteit van de controller wordt zowel in simulatie als in experimenten aangetoond voor verschillende zachte robotica-systemen die een gelijkenis vertonen met de biologie, bijvoorbeeld de slurf van een olifant of de tentakel van een octopus.

De belangrijkste bijdrage van de scriptie is een verzameling multidisciplinaire tools gecomprimeerd in één algemeen framework voor het ontwerp, de modellering en de controle van een klasse van zachte robots, variërend van het theoretische tot het experimentele domein.

Trefwoorden: Trefwoord 1, Trefwoord 2, Trefwoord 3, ...

Societal summary

Soft robotics is an emerging field that focuses on the design, development, and implementation of robots made from soft, flexible materials. Unlike traditional rigid robots, soft robots are able to mimic the movements and behaviors of living organisms, making them well-suited for a wide range of applications in fields such as healthcare, manufacturing, and exploration.

One of the main benefits of soft robotics is their ability to interact with humans and delicate objects in a safe and gentle way. Soft robots can be designed to be compliant and adaptable, allowing them to work in close proximity to people without causing harm. They can also be used in healthcare applications, such as wearable devices and prosthetics, where their flexibility and ability to conform to the human body can provide greater comfort and functionality than traditional rigid devices. Another advantage of soft robotics is their ability to operate in unstructured and unpredictable environments, such as disaster zones or outer space. Soft robots can change shape and adapt to their surroundings, making them more versatile and capable of performing a wide range of tasks. For example, soft robots can be designed to crawl through small spaces, squeeze through tight openings, and manipulate objects with greater precision than rigid robots.

However, there are also challenges associated with the development and implementation of soft robotics. One major challenge is the complexity of designing and controlling soft robots, which often require sophisticated algorithms and advanced materials. Additionally, there is a need for more research into the long-term durability and reliability of soft robots, particularly in harsh or extreme environments.

Despite these challenges, the potential benefits of soft robotics are significant, and the field continues to grow and evolve. As more researchers and engineers develop new materials, technologies, and applications for soft robots, we can expect to see these versatile and adaptable machines play an increasingly important role in our lives and society.

Nomenclature

Vector and matrix notations

x	Scalar notation
$\boldsymbol{x}$	Vector notation
$\boldsymbol{X}$	Matrix notation
$\boldsymbol{\mathcal{X}}$	Tensor notation
$\mathcal{Q}$	Manifold
$T_{\mathcal{Q}}$	Tangent space of $\mathcal{Q}$

Set notations

$\emptyset$	Empty set
$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}_{>0}$	Strictly positive reals
$\mathbb{R}_{\geq 0}$	Positive reals
$\mathbb{N}$	Set of natural numbers
$\mathbb{T}$	Compact time domain
$\mathbb{X}$	Compact set in $\mathbb{R}$ (*i.e.*, line segment)
$\mathbb{V}$	Compact set in $\mathbb{R}^2$ or $\mathbb{R}^3$ (*i.e.*, volume)

Groups

id	Identity group
SO(3)	Lie group of rotations on $\mathbb{R}^3$ (*i.e.*, special orthonormal matrices)
SE(3)	Lie group of homogeneous transformations on $\mathbb{R}^3$
se(3)	Lie algebra of SO(3)
so(3)	Lie algebra of SE(3)

Vector- and matrix operations

$(\cdot)^\top$	Transpose
$(\dot{\cdot})$	First time derivative
$(\ddot{\cdot})$	Second time derivative
$(\hat{\cdot}), (\cdot)^\wedge$	Isomorphism from $\mathbb{R}^6 \to \mathrm{se}(3)$
$(\check{\cdot}), (\cdot)^\vee$	Isomorphism from $\mathrm{se}(3) \to \mathbb{R}^6$
$(\cdot)^\circ$	Reference or rest configuration
$(\cdot)^\star$	Optimal solution
$(\cdot)^{-1}$	Square matrix inverse
$(\cdot)^\dagger$	Moore-Penrose pseudo inverse
$(\cdot)^+$	Generalized matrix inverse
$(\cdot)^\perp$	Annihilator

Operators and letter-like symbols

δ	Variation of a field
∂	Boundary of a set
int	Interior of a set
$\sup_t$	Supremum over continuous time t
dim	Dimension of vector
trace	Trace of matrix
diag	Diagonal of matrix
blkdiag	Block diagonal of matrices
$\mathrm{Ad}_{(\cdot)}$	Adjoint action on Lie group
$\mathrm{ad}_{(\cdot)}$	Adjoint action on Lie algebra
$\lVert \cdot \rVert$	Euclidean norm
$\lVert \cdot \rVert_X$	Matrix norm
$\lVert \cdot \rVert_{\mathrm{rms}}$	Root-mean-square norm
$\lfloor \cdot \rfloor_n$	Floor operator
$\lceil \cdot \rceil_n$	Ceiling operator

Acronyms

CoM	Center of mass
DoF	Degree of freedom
FEM	Finite element method (or model)
MDE	Matrix differential equation
ODE	Ordinary differential equation
PDE	Partial differential equation
PMDE	Partial matrix differential equation
PneuNet	Pneumatic network
SRM	Soft robotic manipulator
TopoOpt	Topology Optimization

Contents

1

Soft Robotics – a new perspective on biomimicry

1.1 Soft robots: what are they?

Biological systems have long been a source of inspiration for roboticists seeking to create machines that are more resilient and capable. While humans can effortlessly interact with a wide variety of objects, conventional rigid robots require precise knowledge of an object's weight, shape, and orientation to safely and reliably interact with it. The field of soft robotics has emerged as a new discipline that incorporates methodologies commonly found in nature. Soft robots are constructed from highly compliant materials, and the term "*soft*" is used in contrast to rigid robots, but also refers to the flexible, compressible, and environmentally robust properties associated with soft materials. To achieve bio-like performance in modern machines, researchers believe that the general lack of material diversity in robotics may be the missing piece of the puzzle.

In engineering, a common way to convey a measure of elasticity is the Young's modulus. While limited to homogeneous materials subject to small deformations, it can still be applied to classify rigidity in robotics. To assist the reader, we have included a spectrum of different materials in Figure 1.1. A careful observation of this spectrum reveals that biological organisms are primarily composed of low-modulus materials in the range of 10^4 to 10^7 Pa, such as muscle and skin tissue, with rigid materials (such as bone) being much less prevalent. In contrast, conventional robotics predominantly rely on hard materials such as metals and hard plastics. Furthermore, it is worth noting that materials which undergo repeated deformation during motion possess correspondingly low elastic moduli, as opposed to the use of rigid materials in classic robotics. The concept of exploring

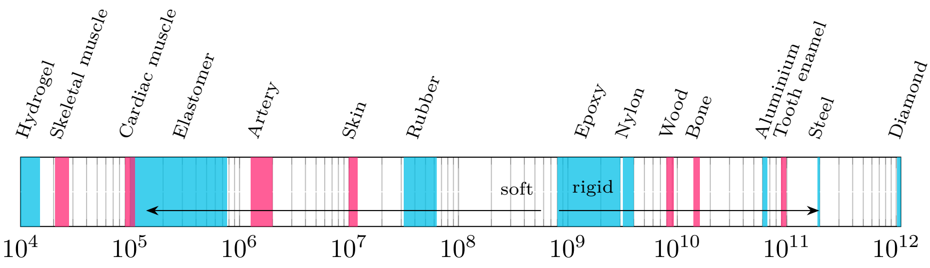


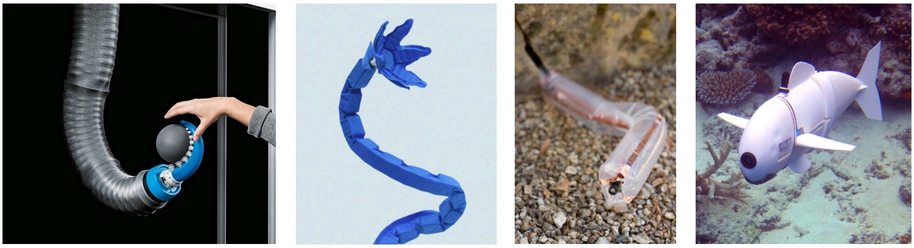
Figure 1.1. Young's modulus spectrum in (Pa) of rigid and soft materials, where (**—**) are the organic (*i.e.*, biological) materials and (**—**) inorganic materials.

low-elasticity materials, collectively referred to as *"soft materials"*, has fostered a new philosophy in robotics aimed at unifying robots and biology. Although there exist many definitions of soft materials, we propose the following description:

> ***Soft materials*** *are a class of homogenous materials with a Young's modulus (i.e., the modulus of elasticity) typically lower than* $E \leq 10^9$ Pa*. Following, the word 'soft' or 'softness' refers to the collection of mechanical properties that are often associated with these low moduli materials.*

Now, despite the fact that the words *"soft"* and *"robotics"* have clear definitions independently, the collocation of the two has sparked many new ideas and perspectives within the robotics community for the past two decades. Figure 1.2 highlights a few of these soft robots that are inspired by various biological creatures. Throughout its young academic life, several definition have been coined. Initially, soft robotics referred to robots with variable joint stiffness [2] or artificial compliance achieved through control [1]. The term was also used to underline the shift from rigid-linked robots to *"bio-inspired continuum robots are inherently compliant and that exhibit large strains in normal operations"* [123]. Paraphrasing the work of Robison et al. [104]: *"soft robotic manipulators are continuum robots made of soft materials that undergo continuous elastic deformation and produce motion through the generation of a smooth backbone curve"*. Alternatively, a broader definition was coined in a review by Kim et al. [69] simply referring to soft-bodied robots as *"an analogy to soft-bodied animals"*. A concise definition was proposed by Laschi et al. [76], as soft robots being *"any robot built by soft materials"*. Rus et al. [105] defined soft robots in terms of their structural elasticity: *"Systems that are capable of autonomous behavior, and that are primarily composed of materials with moduli in the range of that of soft biological materials"*.

The ongoing debate regarding the precise terminology for soft robotics may never reach a definitive conclusion. However, it is crucial to establish consistent terminology not only for the purpose of this thesis but also for proper communication within the broader scientific community. Previous definitions coined by the scientific community have placed great emphasis on the natural motion that arises



(a) (b) (c) (d)

Figure 1.2. Examples of soft robotic systems that draw inspiration from nature. (a) Festo's Bionic arm inspired by the elephant's trunk but with octopus-inspired gripper [51]. (b) Vacuum-driven origami-inspired artificial muscles by Li et al. [80]. Vine-inspired inspection soft robot by Hawkes et al. [52]. Soft robotic fish by Katzschmann et al. [60] composed of fluid-driven soft actuators [85].

from soft materials with high similarities to nature. In this thesis, we propose our definition of *"soft robots"* based on the historical development of soft robotics and current scientific trends in literature (discussed in Chapter 2) with particular focus on design and control. Given the interdisciplinary nature of the field, the terms used in this thesis may differ from those used in existing literature. Throughout the thesis, we will refer to the following definition when discussing "*soft robotics*":

> ***Soft robotics*** *is a subclass of robotics with purposefully designed compliant actuators embedded into their soft body, that enable control over the soft robot's natural ability to perform bio-inspired morphological behavior.*

The formulation above, modified from an early definition proposed in [29], emphasizes the significance of soft materials in replicating biological motion, also known as "*biomimicry*" or "*biomimetics*". Soft robotics represents the first stepping stone in the pursuit of harmonizing robotics and biological principles. This newly adopted field of soft robotics delves into the use of soft materials not only from a design perspective but also with regard to their implications for control, with the aim of recovering biological morphologies.

1.2 Soft robotics from a biometic perspective

The octopus is a fascinating subject of study in the field of soft robotics [15, 70, 76, 89, 102, 105, 129]. Unlike animals with rigid skeletons, the octopus has compact arrays of muscle tissue that stiffen and soften when they move. Its eight soft appendages have virtually infinite degrees of freedom (DOF) [63, 65, 114]. The exceptional dexterity of the octopus arms results from their behavior as muscular hydrostats. These arms are composed of densely packed muscle fibers whose ori-

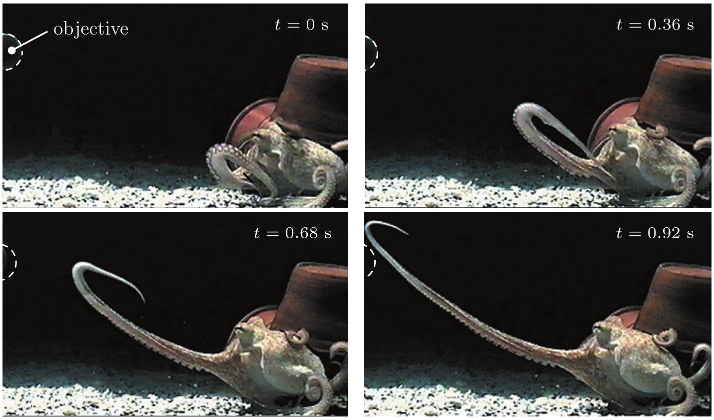


Figure 1.3. Recording of an octopus extending its tentacle towards an object of interest using coordinated activation of a tightly packed network of muscle fibers. The highly flexible appendage allows for traveling bending wave propagation, while the octopus orients its suckers towards the object to ensure secure grip. Source: Sumbre et al. [114].

entation can be grouped into transverse, longitudinal, and oblique axes [66]. Each arm can control itself semi-independently from the central brain and is controlled by a decentralized peripheral nervous system that runs axially along each tentacle. Motor control in the octopus arises from approximately 500 million neurons in its body, two-thirds of which are distributed among its limbs, enabling semi-independent control from the central brain. When a tentacle receives a command from the central brain, such as "find food," it gathers its own sensory and position data, processes it, and issues its own motor coordination on how to start motion by stiffening or relaxing its muscular network [65, 114]. This remarkable ability to coordinate movement and control each arm independently has inspired soft robotics researchers to develop new approaches to robotic design and control. An example of the amazing behavioral motor abilities of the octopus is shown in Figure 1.3 (obtained from [114]). The figure presents a visual of an octopus attempting to grasp a food particle, which denote a possible inspiration for many soft robotic systems. Interestingly, the octopus exhibits a remarkable behavior of simultaneous bending and twisting propagation motion to orient the bottom side of its tentacles towards the target.

This exemplifies the remarkable ability of animals to accomplish a hierarchy of tasks, attributed to the hyper-redundancies in their morphological structure. Hyper-redundancy [17, 19, 105] implies that the system possesses many additional DOFs than are strictly necessary to complete a given task. Consequently, free joints can be assigned to sub-tasks that run in parallel. This ability allows many organisms to passively adapt to their environment without affecting the primary motion. These attributes are highly sought after in modern robotic systems [27, 93, 112] as they enable robustness in unstructured environments and environmental durability, especially regarding impact.

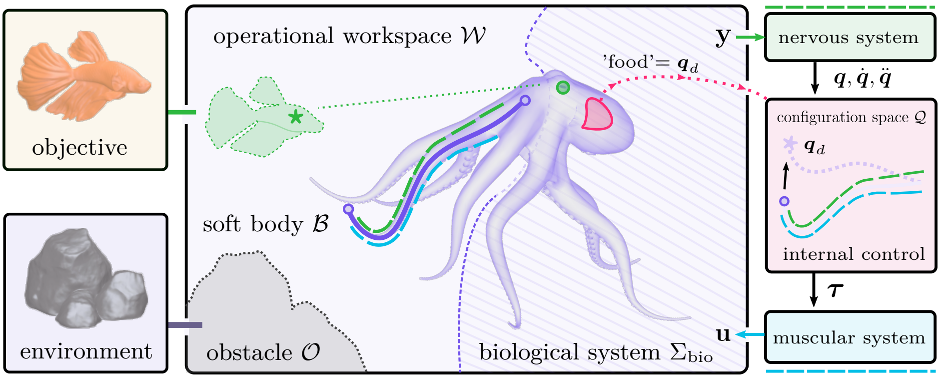


Figure 1.4. A schematic representation of the control architecture of a octopus-inspired soft robots with embodied intelligence. The architecture illustrates the flow of information among significant biological components, including the body (*e.g.*, soft, deformable arm), actuators (*e.g.*, a network of muscle fibers), sensors and decentralized controller (*i.e.*, peripheral nervous system), and the brain that coordinates these elements.

As shown in Figure 1.3, the success of biological systems cannot be attributed to their morphological design alone; rather it is the interplay between physical structure and coordinated motor control that enables their functionality. To effectively implement embodied intelligence in soft robotics, it is crucial to consider the entire embodiment of the bio-inspired system, which encompasses both design and control [52, 105]. Given the multidisciplinary nature of soft robotics, it is important to clarify the distinction between these two aspects.

> ***Design*** *is the process of developing the structure of the soft robot that enable it to perform specific tasks within a predefined workspace.* ***Control****, on the other hand, refers to the process of finding control laws that steer the robotic system towards a desired static or dynamic behavior.*

Recognizing the fact that body and brain are equal partners in supporting intelligent behavior, we hypothesize a deconstruction of the morphological behavior that underlies the grasping example in Figure 1.3. This is illustrated in Figure 1.4, which presents a schematic representation of a biological system Σ_{bio}. The control objective is to reduce the distance between the prey and one tentacle, denoted by continuum body $\mathcal{B} \subset \mathbb{R}^3$. Such soft appendage consists of discrete bundles of muscle fibers for motion and nerve fibers for sensing, which are denoted by the inputs $\boldsymbol{u}$ (*e.g.*, muscle activation) and outputs $\boldsymbol{y}$ (*e.g.*, nerve potentials) as shown in Figure 1.4, respectively. Due to physical design limitations, there is only a finite number of actuators and sensors that can be composed within the body. This yields a compact region called the *"operational workspace"* $\mathcal{W}$ [93, 94, 112] in which the system can operate. Even though the body has virtually infinite DOF, it can only be controlled via a finite number of inputs $\boldsymbol{u}$, and thus soft robots are inherently underactuated [113, 118]. This is further emphasized by possible input saturations, such as the inability for uniaxial extension of the muscle fiber network, which often resolved using antagonistic design. Part of the soft robot design problem is therefore finding an appropriate composition of the inputs $\boldsymbol{u}$ such that the system's reachability space $\mathcal{W}$ coincides with the desired task with sufficient kinematic redundancies. A variation where the structural deformations of $\mathcal{B}$ is tuned is called *"optimal shape design"* [8]. This involves altering the shape of $\mathcal{B}$ to achieve the desired deformation but keeping $\boldsymbol{u}$ is fixed *a-priori* in space and time. Given the continuum nature of soft robots, as well as their distributed actuation, sensing, and mechanical saturations, such design problems are not solved trivially [9, 22, 111, 121, 132].

On the other hand, control involves determining a control law for the network of actuators $\mathbf{u}$ that steers the octopus' arm towards its desired goal. Regarding Figure 1.4, two mechanisms of closed-loop control can be observed. The first mechanism employs proprioceptive sensors, which are strain-sensing neurons responsible for controlling muscle contractions and measuring joint position. The second mechanism utilizes visual feedback to inform the soft appendage about the desired setpoint relative to the task. To coordinate motion, the octopus must possess a fundamental understanding of the relationship between the inputs and outputs of its soft arm. From an engineering perspective, the dynamics associated with the motion of the continuum body $\mathcal{B}$ often require Partial Differential Equations (PDEs) that characterize the deformation in both space and time [6, 30, 105]. This leads to the well-known issue of infinite-dimensionality [31, 54, 88], which infer that such models often lack closed-form solutions. However, this raises a fundamental question: *How do octopi and other invertebrate animals accurately predict their continuous motion without any apparent difficulty?*

A solution might be found in dimensional reduction. Assuming that the motor neurons controlling the movement have limited memory and cognitive capabilities, the octopus likely perceives its soft arm as having only a finite number of DOFs to enable online motion prediction. This may suggests that the continuum joints of the arms, processed by the decentralized controllers, are possibly represented by a reduced state $\boldsymbol{q}$ belonging to a finite-dimensional configuration space $\mathcal{Q}$. As such, biological systems may be able to identify internal dynamics models based on a reduced representation alone, e.g., $\ddot{\boldsymbol{q}} = \boldsymbol{f}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{u})$, that predict how their continuum appendages will evolve over time given the inputs $\boldsymbol{u}$ and the initial conditions $\boldsymbol{q}_0$ and $\dot{\boldsymbol{q}}_0$. Notably, these dynamic models can serve as a control framework for introducing stabilizing feedback terms to the input $\boldsymbol{u}$ that ensures convergences towards a desired setpoint. This, however, highlights the main challenge in modeling soft robots; namely (*i*) identifying an appropriate dimensional reduction that preserves both accuracy and computational tractability [30], and (*ii*) finding control structures applicable to such reduced-order models.

In the field of soft robotics, the aforementioned paradigms have attracted a large audience from the robotics control community. Researchers explore various controllers, including model-based approaches [6, 30, 42, 87], data-driven methods [3, 13], and machine learning techniques [67, 107, 119, 120]. The use of dynamical models, whether data-driven or physics-based, in open-loop or closed-loop systems; is essential in enabling effective control strategies that strive for superior performance and robustness. The absence of such models would pose significant challenges in designing efficient control strategies.

1.3 State-of-the-art solutions in soft robotics

Driven by the aspiration for bio-like performance in robotics, there has been significant advances in the development of continuum soft robots since the early 2000s. Besides octopi [102], various researchers draw inspiration from a plethora of biological organisms such as fish [60, 84], elephants' trunks [48, 58, 129], snakes [46, 84, 100], birds [46, 137], and even the human hand [43, 124]. Researchers have discovered several ways to mimic biology by harmonizing soft materials and robotics. This section offers a brief overview of the state-of-the-art solutions concerning design and control.

Remark 1.1 *Considering the extensive scope of soft robotics, an additional chapter (Chapter 2) has been included to provide a more comprehensive overview of its origins dating back to the early 1980s, as well as the various research aspects currently associated with this field. Thus, the following section serves as a preliminary introduction, ensuring brevity in the following section while setting the stage for the research problems relevant to the scope of the thesis.*

1.3.1 Soft fluidic actuators inspired by muscular hydrostats

Conventional soft robots are characterized by their continuum-bodied motion that arises from a network of compliant actuators embedded throughout its soft body. There exist many options for such embodied actuation, including tendons [58, 102], chemical reactions [7, 55], light-driven liquid crystals [25, 26, 125], and ferromagnetic materials [71, 126], and hydrogels [57, 77]. However, the most common approach is fluidic actuation [60, 84, 96, 124] that mimics muscular hydrostats found in animals. The latter are commonly referred to as *"Soft Fluidic Actuators"* (SFAs), and the majority are designed via human-driven techniques. In this section, we aim to provide a concise explanation of the SFAs technology and explore the potential benefits of incorporating structural optimization into the design process alongside the common human-driven design techniques.

Soft Fluidic Actuators (SFAs) are inflatable fluidic channels that are embedded into an elastic soft body. When pressurized fluid is applied, the elastic pressure vessel uniformly distributes the internal stresses along the interior, resulting in motion due to strain relaxation of the surrounding continuum body. Since elasticity is key, silicone rubbers are frequently employed for their notable material properties. Specifically, silicone rubbers exhibit a low Young's modulus, only display material fatigue at high strains exceeding 100%, and a commercially available. By

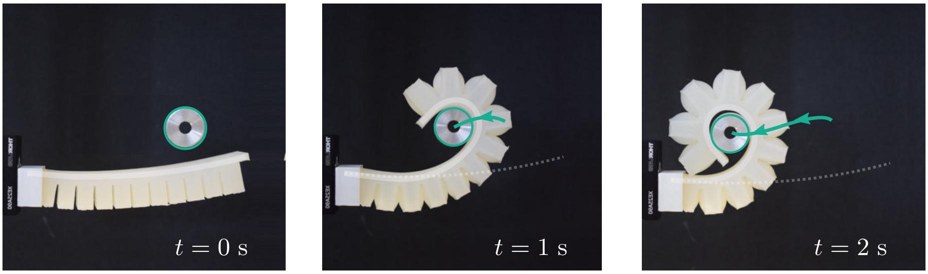


Figure 1.5. An illustrative example of a fluidic PneuNet actuator reaching and grasping a 20 mm aluminum cylinder, which is inspired by the morphological grasping motion of the octopus' soft arm as seen in Figure 1.3. The main body is composed of Dragonskin™ 10A silicone, and 30A for the bottom layer. It is worth noting that despite using open-loop motion control (*i.e.*, linear pressure ramp of 40 kPa), the enveloping properties of soft materials can lead to the emergence of complex "intelligent" behavior.

exploring purposefully designed asymmetrical geometries, often created by hand, predictable motion can be guided [56, 105, 132]. The aforementioned principles are analogous to those of (semi-rigid) compliant mechanisms, which flexible structures that facilitate motion or force transmission through elastic deformation of their components, as opposed to conventional rigid-body joints. Exploring structural geometries or purposefully introducing stiffer material, a wide family of motion primitives, including bending, twisting, and elongation behavior are possible.

1.3.2 Systematic design of soft fluidic actuation: human-driven versus design optimization

A popular example of a soft fluidic actuator (SFA) is the PneuNet actuator, which was proposed by Mosadegh et al. [92]. This actuator has a consistent linear input-output bending behavior and its design is simple, easy to fabricate, and repeatable, making it a standard in the academic field. To illustrate the capabilities, in Figure 1.5, we present an example of it grasping an aluminum cylinder, inspired by the octopus example mentioned earlier [114]. Despite using rudimentary open-loop control, the system can emulate complex behaviors such as reaching, grasping, and pulling objects closer through manipulation, similar to those exhibited by the octopus. As shown in Figure 1.5, an important property is their inherent mechanical impedance allow for safe and stable interactions with objects of varying rigidity, even without environmental perception. This contrasts rigid robots interacting with unknown environments, where impedance control appear to be a necessary requirement [27, 93].

Given its success in open-loop, common applications for SFA technology are therefore soft grippers [4, 45, 53, 56] that are useful in handling delicate objects. Pushing the technology further, system dexterity has been extended by incorporated many different SFAs into one single body, similar to muscle groups in animals. Consequently, researchers have explored their potential for enabling highly dexterous robots capable of in-hand manipulation [50, 115], as well as autonomous terrestrial and aquatic locomotion [20, 32, 60, 116]. Although SFAs provide a wide range of motion, they require significant amounts of supplied volume inflow, which adversely affects their speed, compactness, and efficiency [96, 105, 132]. A common nonlinear phenomenon called "*ballooning*" occurs when the elastic membrane of the changes deforms significantly, further increasing the channels volumetric capacitance. This behavior is particularly evident in soft materials that exhibit strain softening nonlinearities, which intensifies this inefficient expansion. In order to address the aforementioned limitations, various researchers have proposed two potential solutions. The first involves the use of hand-designed geometrical features, such as ribs, which increase structural compliance. The second solution involves the utilization of composites made up of different materials. One example of this approach is demonstrated in the work of Polygerinos et al. [98, 99], as well as other similar studies [21, 21, 43, 115], which employ fiber-reinforced SFAs that incorporate fiber weaves to create new soft material composites. The introduction of such weaves results in anisotropic mechanical properties due to the fibers have a low bending-twist modulus but high elongation modulus. Additionally, various weave patterns can be used to steer the deformation characteristics towards a desired kinematic profile [24, 70], or as proprioceptive sensors using magnetic inductance [38, 39] or strain sensors from EGaIn (Eutectic Gallium Indium) [97]. Recently, also multi-material 3D-printing approach are employed combining actuation with integrated sensing [130, 136]. It is noteworthy that the advent of Additive Manufacturing (AM) technology has significantly contributed to the advancement of soft robotic technology, enabling flexibility in design and materials, surpassing what was previously achievable with subtractive techniques.

Like nature, there are often no unique design solutions when finding the optimal geometrical shape, material composites, and actuation type, or a combination of the aforementioned. While such design freedom has its benefits, it also poses challenges in establishing effective hand-driven design solutions that are tailored towards functionality. Furthermore, predicting structural deformation *a-priori* is challenging and often requires time-consuming numerical simulations. In response, optimization-based solutions are being slowly considered. Cheney et al. [16] employed compositional pattern-producing network (CPPN) to determine the most efficient arrangement of soft material voxels and activation patterns, thereby facilitating the synthesis of soft robots capable of terrestrial and aquatic locomotion in simulation. Following, in Kriegman et al. [73, 74], auto-generative processes are explored for a variety of candidate creatures in silico, with the aim of achieving specific motor functions like locomotion, and object manipulation and transport.

Although voxel-based [16, 74] and commonly parametric shape optimization [23, 83, 91] approaches have demonstrated success in soft robot design, they are inherently imposed by design restriction that do perhaps limit the design possibilities in soft robotics, especially given the recent advances in Additive Manufacturing. In contrast, topology optimization [8, 44, 134] (TopOpt) is a versatile technique that enables the design of structures with desired functionality without imposing significant design constraints. Furthermore, TopOpt approaches explore gradient-based optimizers derived from continuum theory that allow accurate descriptions of nonlinear deformations, which are simplified in many schemes in favor of computation speed. Wang et al. [127] and Tian et al. [121] explored TopOpt for soft grippers using tendons and ferromagnetics, respectively. Yuhn et al. [133] extended density-based optimization to include time as an additional variable, allowing for simultaneous optimization of structure and movement with gradient-based methods through 4-dimensional TopOpt. However, research on TopoOpt applied fluidic soft actuators is limited, perhaps due to the challenges associated with modeling fluid-structure interaction with adaptive structures.

1.3.3 Model-based control for soft robots

In a previous discussion, we showed that open-loop approach can accomplish various complex tasks such as locomotion [20, 60, 116], grasping [116], and manipulation [86]. While open-loop control has been successfully demonstrated, it ultimately relies on *a-priori* system knowledge that determines the control logic based on numerical surrogate models or experimental regression. This, independent of using open-loop strategies, further establishes the importance of modeling which is a crucial ingredient for modern conventional robotics [64, 93, 112]. In the early days of soft robotics, model-based control strategies were deemed unfeasible due to the infinite-dimensional nature of these systems [31]. Over the last two decades, however, a plethora of modeling solutions have surfaced, paving the way for sophisticated model-based controllers, which are expected to push the dexterity of soft robotics systems even beyond what is already achievable in open-loop.

A common modeling approach is the Finite Element Method (FEM), which involves spatially discretizing continuum solids into a group of "*finite elements*," permitting the underlying Partial Differential Equation to be rewritten as an approximate Ordinary Differential Equation (ODE) [54, 68]. These can be solved straightforwardly using standard numerical integration (*e.g.*, `ode45`, `ode23t` in MATLAB®). While primarily explored for quasi-static behavior, FEM models have demonstrated their efficacy in handling hyper-elasticity, geometric nonlinearities due to large deformation, fluid-structure interactions and other multi-physical domains [56, 82, 90, 110, 132]. Motivated by the assumption that quasi-static models provide sufficiently accuracy to describe soft robots under slow-varying dynamics, open-loop forward kinematic controllers are used that inversely search the input space for the desired deformation profile [9, 84, 86]. Alternatively, high-fidelity FEM data can be fed into neural networks approximators [37, 135] or Quadratic Programming (QP) algorithms [9] that tackle the inverse kinematic problem directly. Yet, the high-dimensionality of these models, sometimes millions of DOFs, poses a significant challenge when considering dynamic behavior [34, 49], thereby limiting their applicability for closed-loop feedback control. Undoubtedly, under dynamic conditions, is where the benefits of soft robots are indisputably evident.

A viable solution for reducing dimensionality of FEM is found in Proper Orthogonal Decomposition (POD). Here, dynamic snapshots of observable data is collected, and through singular values decomposition, principal dynamic modes are identified that combined to represent the reduced linear dynamical model. Not only does this approach improve speed, but it also preserves accurate, robust, and efficient models suited for closed-loop controller design [49]. Recent extensions [108] preserve even nonlinear deformations and self-contact. The proposed modeling approach is encapsulated in an open-source software called *SOFA* [22, 35], which facilitated a wide range of closed-loop controller designs contributed by an active community in past years [35, 49, 75, 79, 131]. Furthermore, it has been successfully implemented in physical systems, and recently Reinforcement Learning (RL) methods have been explored [107]. Alternatively, Koopman system identification tools are a data-driven approach that can be applied directly to experimental data (*i.e.*, optical markers), leading to discrete-time dynamical systems. By gathering measured data alone, accurate models of the true system can be identified [13, 72]. These models are often followed by Model-Predictive Control (MPC) approaches [12, 13].

Another common approach for modeling soft robots is based on the Elastica theory, where the volumetric soft body is represented by a smooth backbone which captures its geometric features. The method, often referred to as *"soft beam"* models, are applicable to a specific subclass of soft robots, including soft manipulators [36, 58, 86, 128], fins [61, 85], and soft legs [33, 124]. Elastica is a general mathematical framework developed by Euler in the 1800s to describe the behavior of elastic rods subjected to external forces [5, 78]. Early approaches include the seminal work of Chirikjian et al. [17–19] presenting hyper-redundant continuum robot model represented by backbone curve. Using a modal parametrization, their work presented inverse kinematics solutions, path planning, and grasping strategies [18]. Later, Mochiyama et al. [88, 89] extended their work by developing dynamic formulations that led to shape regulation controllers derived from Lyapunov theory.

A modern approach for soft beams is the "*Piecewise-Constant Curvature*" model (PCC) [128]. This generalizable modeling structure discretizes the one-dimensional backbone curve into finite segments of constant curvature, Hence, all strains except curvature are neglected. Following either Lagrangian or Newton-Euler formulations, the approach leads to soft-bodied formulations which are often synonymous those of rigid robot models. The PCC approximation exhibits modeling structures that closely resemble those of classical rigid serial-link manipulators. With slight modifications involving compliance, similar to rigid robots with joint compliance [27, 81], it enables the direct implementation of a collection of classic control theories [31, 40, 42, 47, 48, 58, 60, 62] applied to soft robots.

However, despite the dimensional advantages over FEM, the PCC approach presents some limitations. Unlike planar description, one such limitation is the introduction of kinematic singularities and discontinuities when representing 3D motion [58, 59]. These singularities and discontinuities are byproducts of the bending parametrization, which can potentially lead to destabilization in closed-loop when approaching zero-curvature. Some solutions have been proposed by lifting the state parametrization [28], or singularity avoidance [36, 117]. Second, although PCC allows for analytic closed-form dynamics, the resulting models are highly nonlinear, complex and large. A solution is found in augmented rigid body models, which explore PCC kinematics to describe a lumped-mass model of the continuum robot. These mitigate expensive spatial integration required for the inertia. Although rigid-body surrogates are seemingly an oversimplification compared to FEM, such models have proven to be rather effective [31, 41, 42, 61, 62]. For example, Kazemipour et al. [62] propose an adaptive sliding mode control scheme, which is robust against model parameter uncertainties and unknown input disturbances. The last and perhaps largest limitation of PCC is that they are only applicable in restrictive settings where constant curvature holds. They are unfit to capture important continuum phenomena, such as gravitational deflection, buckling, environmental interaction, or wave propagation.

In contrast to PCC surrogate models, Cosserat beam-models have demonstrated an ability to capture a broad range of continuum phenomena [11, 46, 102, 103, 122]. Unlike PCC models, they provide truncatable models derived from continuum mechanics. They also provide precise representation of the hyper-flexible nature of materials under large deformations, and even allow for self-collision [46]. Seminal work by Renda et al. [101, 102] and Boyer et al. [10] proposed computational models of Cosserat beams using Geometrically-Exact finite elements on the Lie group SE(3) [109]. While these models have gained popularity in the soft robotics community, literature on model-based control for Cosserat beam models is slowly emerging when compared to PCC. Especially those that consider underactuation and the hyper-redundancy that often identified with these systems.

1.4 Research objectives and contributions

In short, soft robotics is a rapidly emerging subfield within the broader domain of robotics that focuses on the use of soft materials to create systems capable of dexterous bio-inspired morphological behavior akin to animals, capable of locomotion, grasping, and manipulation. Such features are made possible by the unique mechanical properties intrinsic to soft materials and soft fluidic actuation. Although open-loop control strategies have already established bio-like features, soft robotics still lags behind in terms of precision and speed compared to rigid robotics. To address this gap, new design principles for soft fluidic actuation and accurate, fast control-oriented models are believed to be crucial missing components.

The aforementioned statement leads to a few open questions:

- How do we design fluidic-interacting mechanical structures made from soft materials that deform according to a user-defined morphological pattern?
- How do we derive dynamic models that offer a reasonable trade-off between the accuracy and its applicability for control?
- Can we adopt classical control methods from rigid robotics to soft robotics?
- Can we effectively explore the intrinsic morphological properties of the soft robots, similar to biology, that are meaningful for modeling and control?

- To enable better designs and controllers for soft robots, how can we bridge the interdisciplinary aspects intrinsic to this field?

In the pursuit of finding systematic answers to these questions, we have divided our research into three branches: (I) automated design synthesis of soft fluidic actuators, (II) modeling for control of soft robot manipulators, and (III) the development of software aimed to support the soft robotics community. For each research objective, a number of contributions are listed.

I: Automated design synthesis of soft actuators. As presented by abundance of literature on soft robotic design, either rooted in hand-driven principles or through optimization, achieving an optimal structural geometry that fully accounts for hyper-redundancies in soft materials is no easy feat. Unlike their rigid counterparts and many biological systems for that matter, the kinematics are inherently encoded in the topological structure of the soft materials and where actuation is presented in the system. This implies the workspace cannot be characterized analytically in closed from through joint motions stemming from one point, as commonly done for joints in rigid robotics. It is therefore of paramount importance that the nonlinear behaviors of soft materials, both in hyper-elasticity and nonlinear geometrical deformation, is understood and accounted for during the design process.

(Optimality in soft material design). Our first research objective is centered around the design aspect of soft robotics. The fundamental principles of compliant mechanical devices can be rigorously described through continuum mechanics. In this thesis, we aim to integrate the underlying theory of continuum mechanics with the automated design of efficient soft actuators that enable user-defined objectives. To achieve this, we utilize optimization algorithms that seek to minimize user-specified objective functions by optimizing the layout of soft materials. This approach is rooted in continuum mechanics [54, 68] and its subsequent discretization through FEM method. The aforementioned issue is commonly referred to as an "inverse design problem" - determining the shape from final deformation, rather than deducing the deformation from the shape. The field of combining continuum mechanics and free-form optimization in compliant structures is a well-established area known as "topology optimization" [8]. However, such techniques are not easily applicable to soft actuation due to the presence of hyper-elasticity and nonlinear geometric deformations in soft materials. Furthermore, fluidic or pneumatic actuation further complicates the optimization process, as these loads become both design and state-dependent. This leads us to our first contribution:

> **Contribution I.** *Development of efficient algorithms, applicable to the general design of fluidic soft actuators, solving the inverse design problem: Given a desired motion and input, what is accordingly the optimal soft material distribution within a design domain to realize such joint motion?*

(Fabrication through Additive Manufacturing). After applying such automated

algorithms, a plethora of soft actuation systems with varying joint mobility can be developed with ease. However, since these designs originate from simulations, concerns arise regarding their transferability to practical applications. Therefore, our second research objective is to focus on the translation of simulation-based designs into functional and feasible soft actuators. Recent advancements in Additive Manufacturing have made it possible to fabricate complex 3D geometries with minimal difficulty and effort. Our second contribution therefore reads:

> **Contribution II.** *Testing and validation of optimized soft fluidic actuators by exploring Additive Manufacturing methods of printable soft materials.*

II: Modeling for control of soft manipulators. Parallel to design, modeling for control is a crucial aspect in achieving biological performance. Accurate and fast models are required to achieve this goal. However, the infinite-dimensionality inherent in soft continuum robots poses significant challenges. While the PCC approach and its augmented rigid-body variations have been proposed as a solution, such models do not respect fundamental continuum mechanics. As a result, they impose strict operational constraints on any soft system, such as limiting hyperelastic nonlinearities or slowing down actuation to prevent dynamic mismatching.

(Accurate control-oriented models through PCC condition). Our second research objective therefore focuses on the reduced-order modeling (ROM), aiming to balance precision and speed for control. By building upon the original works of Chirikjian et al. [18] and Mochiyama et al. [88] in the 90's, the thesis presents a dynamic modeling formulation for soft manipulators that better preserves its continuum nature. To address the issue of infinite-dimensionality, a reduced-order modeling strategy for soft robot manipulators is proposed, whose mathematical framework is based on the differential geometric theory of spatial curves. Such framework allows for easy transferability to conventional ROM models in soft robotics, like the PCC strain [31, 36, 61]. However, the thesis proposes two improvements that are essential development of model-based controllers. Inspired by the success of FEM-based models in soft robotics [34, 49], we bridge the gap between the PCC model and the underlying continuum mechanics by matching the quasi-static behavior to FEM data. Second, to enhance computational efficiency, new reduced-order integration scheme are required that compute the spatio-temporal dynamics in real-time, thus enabling controller design. This brings us to our third contribution:

> **Contribution III.** *Development of fast, efficient, and accurate dynamic models for soft manipulators composed of hyper-elastic soft materials that are directly applicable to classical control theory akin to rigid robotics.*

(Models beyond the PCC). As mentioned in Section 1.3.3, the piecewise con-

tinuity in PCC approach inhibits kinematic redundancies, which restricts the exploration of the hyper-redundant nature that is a key attribute of soft robotics. For example, large nonlinear deflections due to gravity cannot be captured using constant strains alone, nor can environmental interactions be described accurately. Therefore, the third research objective focuses on relaxing the PCC condition to pursue its true infinite-dimensionality more closely. This is achieved by adopting prior modeling strategies of differential curves, where spatially-varying strain fields are considered and approximated through sets of orthogonal shape functions rather than piecewise representations. Building upon prior models presented in the thesis, the underactuated and hyper-redundant soft system is written in a port-Hamiltonian framework [95, 106], which enables energy-shaping techniques by modifying the closed-loop potential energy of the system - a well-known practice in classic robotics [94, 95, 106]. By exploring the hyper-redundancy in soft robotics, more advanced control objectives can be achieved, allowing for multimodal shape regulation and full-body grasping. However, spatial discretization in these model-based controllers plays a crucial role in their dexterity to achieve various control tasks. Within this context, the fourth contribution reads:

> **Contribution IV.** *Investigation of spatial discretization in low-order energy-shaping controllers applied to high-order soft robotic models with a focus on closed-loop performance in shape and full-body grasping control.*

(Exploiting structural geometry for reduction). Contribution IV provides a stable modeling platform for a variety of possible shape functions tailored to unique joint mobilities in soft robotic systems. Yet, many works in modeling literature choose such functions in *ad-hoc* fashion, *e.g.*, polynomial bases [10, 19, 31]. Within this context, the thesis explores a (geometric) modal decomposition approach. Similar to eigenmode analysis in structural dynamics, geometric strain modes are extracted from higher-order (volumetric) FEM models and used to construct optimal soft beam models. The approach leads to fast, accurate, and generic low-dimensional models that encode the geometric features and elasticity of the true soft body into a new strain parametrization, we call the *Data-driven Variable Strain* (DVS) basis. A merit benefit of the approach can be naturally expanded to identify the hyper-elastic material parameters and the actuation map of the reduced beam model, which is often miss in PCC models.

> **Contribution V.** *A novel method for finite-dimensional model reduction in soft manipulators that explores the mechanical interconnection between structural geometry and flexibility modes of the soft manipulator body.*

C: Software development. Finally, the thesis collects all theoretical material into one unifying soft robotics software package called `Sorotoki` – short for SOft RObotics TOolKIt. The software aims to bridge the gaps between different disciplines of soft robotics, applicable to design, modeling and control. Using a minimal programming framework, complex problems can be coded using minimal lines of code. The toolkit is heavily interwoven with Contributions I to V of the thesis, and is publicly available at `https://github.com/BJCaasenbrood/SorotokiCode` [14]. The last contribution therefore reads:

> **Contribution VI.** *Developement of a versatile, user-friendly, open-source software called* `Sorotoki` *that envelops the presented theory on design, modeling and control of the thesis into one coherent Matlab toolkit.*

1.5 Outline of the thesis

This thesis discusses the design, modeling and control of soft robotic systems. Including this introductory materials, the thesis consists of seven chapters.

Chapter 2 presents a historical overview of the field of soft robotics, and is complementarity to introduction presented earlier. Chapter 3 presents the design algorithm for soft actuators that aim to solve the inverse design problem. The chapter start with a brief introduction into continuum mechanics applied to three-dimensional deformation of hyper-elastic materials, followed by its numerical implementation using finite elements. From here, numerical optimization procedures are introduced that solve the inverse design within the context of (fluidic) soft actuation. Chapter 4 follows with the second objective of the thesis, namely modeling for control. Instead of volumetric soft robotic models, lower-dimensional soft beam models are introduced that are tailored for fast and accurate model-based controllers. The chapter focuses primarily on PCC soft beam models. Chapter 5 address the limitation of the PCC model, and instead extends upon it. The chapter formulates a finite-dimensional port-Hamiltonian modeling approach for soft beams, where spatial shape functions are used to discretize the modal flexibilities of the soft robot. From here, energy-shaping controller are introduced that allow for shape control for underactuated soft robots. Chapter 6 presents the culmination of all theoretical material presented in the thesis into a concise, user-friendly, toolkit called `Sorotoki`. The chapter presents an overview of the included programming tools for the design, modeling and control of soft robots. It also presents the Data-driven Variable Strain approach that lead to efficient low-dimensional models. Finally, Chapter 7 closes the main body of the thesis by summarizing the research deliverable of prior chapters, and provides a list of recommendations that could sculp future work.

Note for the reader. Chapters 3-6 are all based on published or submitted researcher articles and can therefore be read independently. A reference to the corresponding research paper is provided at every beginning of these chapters.

`Sorotoki`

2

A brief historical overview on the progress of soft robotics

Abstract - This chapter presents a detailed chronology of the evolution of soft robotics, from its inception in the early 1950s to current research trends. The aim is to provide readers with an insightful introduction to the extensive field of soft robotics. The chapter commences by tracing the origins of soft robotics in pneumatic muscles, including the pioneering work of Joseph McKibben and Victor Scheinman. Subsequently, we explore the emergence of novel concepts and technologies that facilitated the development of increasingly sophisticated soft robots through the utilization of exotic material properties. Throughout the chapter, significant milestones in the development of soft robotics are highlighted, such as the creation of the first soft gripper, the integration of robotics with soft actuations, and the introduction of design and modeling principles for these systems, and early control approaches. Additionally, the chapter highlights some of the key challenges that lie ahead for the field, which serve as a basis for standardization of terminology.In short, this chapter offers an insightful and comprehensive perspective on the history of soft robotics, and will aid the reader in solify the thesis's objectives introduced in the chapter prior.

3

Optimal Design of Soft Robots – a Gradient-based Approach

Abstract - In this chapter, we present a novel framework for synthesizing the design of pressure-driven soft robots. Contrary to traditional design methods, a topology optimization scheme is employed to find the optimal soft robotic structure given user-defined requirements. To our knowledge, the combination of pressure-driven topology optimization and soft robotics is, to date of this thesis, unexplored. Two difficulties are related to this problem. First, pressure-based topology optimization is challenging since the adaptive topology changes the pneumatic load at each optimization step. To deal with this issue, we exploit the facial connectivity in mesh tesselations to efficiently simulate the physics involving pneumatic actuation in soft robotics. The second issue is describing the hyper-elastic nature of soft materials. Here, nonlinear finite element is explored such that large deformations can be described accurately. Numerical investigation shows that the framework can produce meaningful and insight-full material layouts with little to no prior knowledge of soft robotic design. This framework does not only accelerates design convergence, but it could also extend to the development of new and unexplored soft robot morphologies.

This chapter is based on: B.J. Caasenbrood, A.Y. Pogromsky, and H. Nijmeijer. *A Computational Design Framework for Pressure-drivenSoft Robots through Nonlinear Topology Optimization.* IEEE International Conference on Soft Robotics (RoboSoft), 2020. `doi: 10.1109/RoboSoft48309.2020.9116010`

4

Dynamic Modeling of soft robots – The Constant Strain Approach

Abstract - In this chapter, we derive continuum dynamic models for pneumatic soft robot manipulators through the differential geometry of spatial curves. These models are then related tothe Finite Element Method (FEM) to capture the intrinsic geometric and material nonlinearities. To accelerate numerical simulation, a reduced-order integration scheme is introduced to compute the dynamic Lagrangian matrices efficiently. This, in turn, allows for high-speed and (multi-link) dynamic models for soft manipulators with a minimal sacrifice in numerical precision. By exploring passivity ideas and a linear parametrization of hyper-elastic material coefficients, we also propose a passivity-based adaptive controller that enhances robustness towards material uncertainty and unmodelled dynamics – slowly improving their estimates online. As a study case, a fully 3D-printed soft robot manipulator is developed, which shows good correspondence with the dynamic model under various conditions, *e.g.*, natural oscillations, forced inputs, and subjected to external disturbances like tip-loads. The solidity of the approach is demonstrated through extensive simulations, numerical benchmarks, and experimental validations.

This chapter is based on: B.J. Caasenbrood, A.Y. Pogromsky, and H. Nijmeijer. *Control-oriented Models for Hyper-elastic Soft Robots through Differential Geometry of Curves.* Soft Robotics, 2022. `doi: 10.1089/soro.2021.0035`.

5

Port-Hamiltonian Cosserat models for Soft Robots – Beyond the Constant Strain Approach

Abstract - In this chapter, we address some of the limitation of the previous *Piecewise-Constant-Curvature* (PCC) model presented in Chapter 4, in particular the infinite-dimensionality of the soft robot's deformable body. The continuous dynamics of the soft robot are modeled through the differential geometry of Cosserat beams. Contrary to the PCC model, a wide variety of spatial discretization can be used that better respect the spatial continuity and continuum mechanics of the system. Using a finite-dimensional truncation, the infinite-dimensional system can be written as a reduced-order port-Hamiltonian (pH) model that preserves desirable passivity conditions. Then,a model-based controller is introduced that produces a local minimizer of closed-looppotential energy for the desired end-effector configuration. The stabilizing control utilizes an energy-based approach and exploits the passivity of soft robotic system. The effectiveness of the controller is demonstrated through extensive simulations of various soft manipulators that share a resemblance with biology.

This chapter is based on: B.J. Caasenbrood, A.Y. Pogromsky, and H. Nijmijer. *Energy-shaping Controllers for Soft Robot Manipulators through Port-Hamiltonian Cosserat Models.* SN Computer Science, Springer, 2022. `doi: 10.1089/soro.2021.0035`.

6

Sorotoki: A Matlab Toolkit for Design, Modeling, and Control of Soft Robots

Abstract - In this chapter, we present *Sorotoki*, an open-source toolkit in MATLAB that offers a comprehensive suite of tools for the design, modeling, and control of soft robots. The complexity associated with the study and development of soft robots frequently arises from the interdependence between the areas of design and control, which are rarely treated as a joint problem. To address the complex interdependencies in soft robotics, the *Sorotoki* toolkit provides a comprehensive and modular programming environment composed of seven Object-Oriented classes. These classes are designed to work together to solve a wide range of soft robotic problems, offering versatility and flexibility for its users. We provide a comprehensive overview of the *Sorotoki* software architecture to highlight its usage and capacities. The details and interconnections of each module are thoroughly described, collectively explaining how to build up complexity of modeling in soft robotics. The effectiveness of *Sorotoki* is also demonstrated through a range of case studies, including novel problem scenarios and established works widely recognized in the soft robotics community. These case studies cover a broad range of research problems in the field of soft robotics, including: inverse design of soft actuators, passive and active soft locomotion, object manipulation with soft grippers, meta-materials, model reduction, model-based control of soft robots, and shape estimation. Additionally, the toolkit provides access to four open-hardware soft robotic systems that can be fabricated using commercially available 3D printers.

This chapter is based on: B.J. Caasenbrood, A.Y. Pogromsky, and H. Nijmijer. *Energy-shaping Controllers for Soft Robot Manipulators through Port-Hamiltonian Cosserat Models.* SN Computer Science, Springer, 2022. `doi: 10.1089/soro.2021.0035`.

7
Conclusions and recommendations

7.1 Conclusions

I

Appendices

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