Design and Control Strategies for Soft Robotic Systems

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Design and Control Strategies for Soft Robotic Systems

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Brandon Jonathan Caasenbrood

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Abstract

Design and Control Strategies for Soft Robotic Systems

B.J. Caasenbrood (Brandon)   Date: March 16, 2022

In the past two decades, the field of soft robotics has kindled a major interest among many scientific disciplines. Contrary to rigid robots, soft robots explore '*soft materials*' that significantly enhance the robot's dexterity, enable a rich family of motion primitives, and enhance environmental robustness regarding contact and impact that benefactors human-robot safety. The main inspiration for soft robotic systems stems from biology with the aim to achieve similar performance and dexterity as biological creatures. Since its inception, soft robotics has exemplified its potential in diverse industrial areas such as safe robotic manipulation, adaptive grasping, aquatic and terrestrial exploration subject to environmental uncertainty, rehabilitation, and the bio-mimicry of many animals including birds, fish, elephants, octopuses, and various invertebrates. By exploring the uncharted merits of soft materials and soft actuation, soft robotics has placed the first steppingstones towards achieving biological performance in next-generation robotics.

Although some significant leaps have been made towards bridging biology and robotics, there exist major scientific challenges that hinder the advancement of the field. In particular: (I) the Design and (II) Modeling of soft robotic systems. Traditional design of rigid robotics emphasizes on maximum structural rigidity and weight minimization, as to allow for fast, repeatable motion with negligible structural flexibility. Soft robotics, on the other hand, primarily rely on minimal structural rigidity for motion – so-called '*hyper-flexibility*'. Furthermore, as soft materials undergo large nonlinear mechanical responses paired with distributed actuation, expressing the robot's workspace often leads to highly nonlinear kinematic descriptions. Using traditional engineering principles for soft materials is perhaps outdated and computer-assisted design principles for soft robotics might mandate the next steps for the field, especially with the recent advances in Additive Manufacturing (AM). As for modeling, its innate infinite-dimensionality poses fundamental problems for model-based controllers. An important question arises during the modeling of such soft robots; '*how to deal with the trade-off between accuracy and computational efficiency?*'. Besides, the presence of soft materials imbue the system with nonlinear mechanical responses that are perhaps alien to standard robot modeling. As a result, in terms of closed-loop performance, soft robots nowadays are easily outclassed by their rigid counterparts due to a lack of modeling knowledge. The diligence of achieving similar precision and speed to current state-of-the-art robots, and ultimately nature, stresses the paramount importance on design, modeling, and control tailored for soft robotics.

This thesis will address the generative design strategies for soft robots as well as model-based control strategies for a subclass – soft continuum manipulators.

In the first part of this thesis, we present a novel framework for synthesizing the design of soft robotics with various types of soft actuation, *e.g.*, tendons, hydraulics, and pneumatics. Contrary to traditional design, such as bio-mimicry, a gradient-based topology optimization is explored to find sub-optimal soft robotic morphologies that satisfy user-defined motion criteria. Two difficulties are addressed here. First, pressure-based topology optimization yields distributed adaptive loadings that changes at each optimization step; and second capturing the hyper-elastic nature of soft materials. A Finite Element Method (FEM) solver is proposed such that the physics under large nonlinear deformations of hyper-elastic materials and pneumatic actuation are accurately preserved. The optimization-driven algorithm yields generative designs for a diverse set of soft morphologies: soft rotational actuators, soft artificial muscles, and soft grippers. By assembly of smaller soft sub-components, a full soft robot can be developed and through AM of flexible materials the feasibility is validated.

The second part of the thesis will focus on the model-based control of soft continuum manipulators, where the emphasis lies on the efficiency and accuracy in low-dimensional models. The continuous dynamics of the soft robot are modeled through the differential geometry of spatial curves. Using a finite-dimensional truncation, the system can be written as a reduced port-Hamiltonian model that preserves desirable control condition, *e.g.*, passivity. However, this modeling techniques introduces gaps between the underlying material mechanics and control-structured dynamic model. Since useful information is attainable through FEM a-priori, new system identification tools are proposed that give inside into the dominant dynamic modes, the hyper-elasticity, and the reachable workspace spanned by soft materials and actuation. The approach yields accurate low-dimensional models with real-time control capabilities but also gives physical insight into optimal sensor placement applicable to proprioceptive sensing.

Following, the thesis treats the development of model-based controllers that can be employed in various control scenarios, *e.g.*, motion planning, set-point stabilization, tracking, and grasping; akin to rigid robotics. The stabilizing controller utilizes an energy-based formulation, providing robustness even when faced with material uncertainties. The controller's effectiveness is demonstrated in simulation for various soft robotic systems that share a close resemblance to biology.

Lastly, the thesis will implement the proposed computationally efficient systematic strategies for the design and control on physical soft robotic systems, including soft grippers, soft manipulators, and soft exoskeletons. As a concluding remark, this thesis contains several new techniques for design and model-based control of the increasingly fast evolving and multi-disciplinary field of soft robotics.

Keywords: Soft Robots, Flexible Robots, Design Optimization, Continuum Mechanics, Reduced-order Modeling, Model-based Control, Additive Manufacturing.

Samenvatting

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Societal summary

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Contents

Nomenclature

Vector and matrix notation

x	Scalar notation
$\boldsymbol{x}$	Vector notation
$\boldsymbol{X}$	Matrix notation
$\boldsymbol{\mathcal{X}}$	Tensor notation
$\boldsymbol{\mathcal{Q}}$	Manifold

Compact sets

$\emptyset$	Empty set
$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}_{>0}$	Strictly positive reals
$\mathbb{R}_{\geq 0}$	Positive reals
$\mathbb{N}$	Set of natural numbers
$\mathbb{T}$	Finite time horizon
$\mathbb{X}$	1-dimensional spatial set or domain (*i.e.*, line)
$\mathbb{V}$	3-dimensional spatial set or domain (*i.e.*, volume)

Groups

id	Identity
$\mathrm{SO}(n)$	Lie group of rotations on $\mathbb{R}^n$ (*i.e.*, special orthonormal matrices)
$\mathrm{SE}(n)$	Lie group of homogeneous transformations on $\mathbb{R}^n$
$\mathrm{so}(n)$	Lie algebra of $\mathrm{SO}(n)$
$\mathrm{se}(n)$	Lie algebra of $\mathrm{SE}(n)$

Vector- and matrix operations

$\dot{(\cdot)}$	First time derivative
$\ddot{(\cdot)}$	Second time derivative
$\hat{(\cdot)}, (\cdot)^\wedge$	Isomorphism from $\mathbb{R}^6 \to \mathrm{se}(3)$
$\check{(\cdot)}, (\cdot)^\vee$	Isomorphism from $\mathrm{se}(3) \to \mathbb{R}^6$
$(\cdot)_0$	Reference configuration
$(\cdot)^\top$	Transpose
$(\cdot)^{-1}$	Square matrix inverse
$(\cdot)^\dagger$	Moore-Penrose pseudo inverse
$(\cdot)^+$	Generalized matrix inverse
$(\cdot)^d$	Generalized matrix inverse

Operators and letter-like symbols

δ	Variation of a field
∂	Boundary of a set
int	Interior of a set
$\sup_t$	Supremum over continuous time t
dim	Dimension of vector
trace	Trace of matrix
$\lVert \cdot \rVert_{\mathrm{ma}}$	Mean absolute norm
$\lVert \cdot \rVert_{\mathrm{rms}}$	Root-mean-square norm

Acronyms

CoM	Center of mass
CoR	Coefficient of restitution

1
Introduction

1.1 The origin of Soft Robotics

The term '*soft robotics*' is the abbreviated form of '*soft material robotics*'. Although the words '*soft*' and '*robotics*' have a clear definitions independently, the collocation of the two has sparked vivid discussions in the robotics community for many years – even touching the territories of the philosophical. Consequently, the exponential scientific interest in soft robotics around 2011 may be seen as a historical cornerstone that has revolutionized our perspective on the branching field of robotics and rekindled its original ambition even before the term '*robot*' was introduced. Although the debate on the exact terminology is still ongoing, and perhaps may never be closed; we propose a definition for '*soft robotics*' applicable to this thesis based on an ensemble of prior literature:

> **Terminology:** **Soft robotics** *is a subclass of robotics with purposefully designed compliant elements embedded into their mechanical structure whose goal is to endow the robot with biological motion and/or compliance.*

The definition above is mostly adopted from Della Santina et al. [], yet modified to purposefully highlight the importance of soft materials to mimic biological motion – also referred to as '*bio-mimicry*'. The ambition of closely mimicking biological creatures is perhaps not often associated with the field of robotics in general, yet the inception of robotics can originally be found in bio-mimicry when regarding its rich history. We would like to implore the reader to embark with us this brief section into the history of (soft) robotics, as to show that the current trends of bio-mimicry in robotics finds roots in a period before classic robotics.

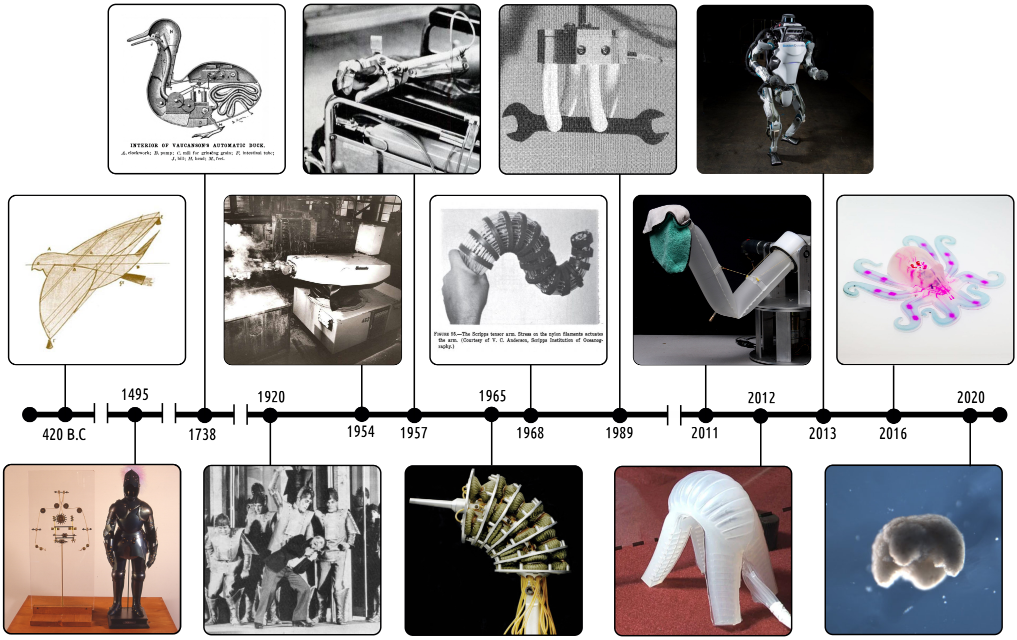

Figure 1.1. A brief timeline of the state-of-the-art of robotics throughout human history. *a)* One of the earliest examples of bio-mimicry – the flying mechanical bird using steam-powered propulsion. *b)* The Mechanical Knight by Leonardo Da Vinci. *c)* A mechanical monk adopted by the designs of Da Vinci. *d)* Digesting duck patent of Jacques de Vaucanson. *e)* The science-fiction play by Karel Čapek on robots, who introduced the word '*robot*' into the Oxford English Dictionary originally from his brother Josef Čapek.

One of the earliest examples of bio-mimicry is a mechanical wooden dove developed by mathematician Archytas of Tarentum in 350 BC. According to historians, the system was driven by compressed air or an internal steam-driven engine to achieve forward propulsion, capable of traveling distances of ~200 m (see note[1]). Archytas's invention could be considered as one of the earliest robotic systems – a machine or device that operates automatically or by remote control – whose main principles are somewhat analogous to nowadays '*drone*' technology. A millennium later, in the period of the High Renaissance, Leonardo da Vinci designed and constructed a mechanical knight around the 1490's. Such mechanical constructions were perhaps closer to conventional rigid robotics given our current robotics perspective. It is well-known that his work was built upon extensive anatomical research, which may have facilitated a deep understanding of the human body into the mechanical knight's robotic design.

In the 1920's, shortly after the second industrial revolution (1870 - 1914) and the first world war (1914), the first usage of the work '*robot*' appeared – originally meaning 'forced labor by serfs' (*i.e.*, peasants) derived from the Czech word '*robota*'. An common misconception is that robot implies slave, nonetheless, its origin is somewhat related. The word was popularized by Karel Čapek in his play R.U.R. (Rossum's Universal Robots) that involves an inventor named Rossum who discovers the secret of creating human-like machines. In his play, Rossom's robots assisted or fully alleviated mankind from any labor. Through human's ambition to assimilate man and machine, the robots ultimately gained the capacity for emotions. Shortly after, the robots, who were created to serve humans, have come to dominate mankind completely. The word '*robotics*' was later solidified by Isaac Asimov, adapting the term from Čapek. These works of science fiction are perhaps the fundamental groundwork of modern robotics which have led to the base practices of robotics and its corresponding academic field.

Only three decades later, in the 1950's, the first robotic arm called the 'Unimate' was employed in industry. The robot was used for manipulating metal die-casts and welding these to welding these to the main body of automobiles. Interestingly, the robot explored both electric as hydraulic-mechanical actuation, similar to nowadays popular Atlas robot (2013) by the company Boston Dynamics. Note that these robots were still controlled remotely, and rudimentary levels of closed-loop control were applied then. The 1950's also brought forth the McKibben actuators developed by Joseph Laws McKibben – a well-known work in the field of soft robotics. These McKibben actuator consists of an inflatable inner bladder enveloped with a double-helical weave. When pressurized, the fluidic actuator converted radial expansion into uni-axial contraction since weave inhibited extensive '*ballooning*' – a term for undesired radial expansion. The McKibben actuators are perhaps seen as one of the first fundamental technologies that enabled soft robotics and to this day it remains a framework for many soft artificial muscle. Nevertheless, besides fluidics, there exist many other technologies employed

[1]It was unclear if the devices was attached to a rope, or autonomous flight was achieved.

in soft robotic motion that predate the invention of the McKibben actuator: such as thermal or chemical expansion/contraction, re-alignment of crystals, di-electric elastomers, magnetism, and naturally electro-mechanical actuation. For instance, the earliest Dielectric Elastomer Actuators (DEA) were developed W. C. Röntgen in 1880. Although these mechanisms do not fall under the class soft robot, they are; however, categorized as soft actuators. We like to emphasize here the difference between soft actuators and soft robots in view of a terminology corresponding with the thesis:

> **Terminology: Soft actuators** *are controllable flexible components of the constitutive soft robotic system that through external stimuli allow for motion or adaptability of compliance and/or texture.*

This terminology attempts to address a common ambiguity in the field of soft robotics, that being interchangeably usage the term soft actuator and soft robot. In the mid-1950's, the work of ??? designed one of the soft robotics systems – even before rigid hyper-redundant robotics. The system consisted of three pneumatic soft actuators aligned in parallel to the backbone of the robot.

1.2 State-of-the-art in Soft Robotics

1.3 Problem statement

1.4 Research challenges and contribution

Design (Optimization).

> **Contribution I.** *blaa*

Hyper-redundant Modeling.

> **Contribution II.** *blaa*

Control and state estimation.

> **Contribution III.** *blaa*

Experimental applicability.

Contribution IV. *blaa*

1.5 Outline of the thesis

The first robotic manipulator arm used in the orbital environment was the Space Shuttle remote manipulator system. It was successfully demonstrated in the STS-2 mission in 1981 and is still operational today.

First soft robot: Victor Scheinman and Larry Leifer developed an air-powered robot arm called Orm, which is the Norwegian word for snake.

- . F. Shulte, "The Characteristics of the Mckibben Artificial Muscle", The Application of External Power in Prosthetics and Orthetics, pp. 94-115, 1960.

- A. Chen, R. Yin, L. Cao, C. Yuan, H. K. Ding and W. J. Zhang, "Soft robotics: Definition and research issues," 2017 24th International Conference on Mechatronics and Machine Vision in Practice (M2VIP), 2017, pp. 366-370, doi: 10.1109/M2VIP.2017.8267170.

- W. C. Röntgen, "Ueber die durch Electricität bewirkten Form—und Volumenänderungen von dielectrischen Körpern," Ann Phys Chem, no. 11, pp. 771-786, 1880.

I

Modeling of Soft Robots

2

Dynamic modeling – The Piece-wise Constant Approach

Abstract - *The motion complexity and use of exotic materials in soft robotics call for accurate and computationally efficient models intended for control. To reduce the gap between material and control-oriented research, we build upon the existing Piecewise-Constant Curvature framework by incorporating hyper-elastic and visco-elastic material behavior. In this work, the continuum dynamics of the soft robot are derived through the differential geometry of spatial curves, which are then related to Finite-Element data to capture the intrinsic geometric and material nonlinearities. To enable fast simulations, a reduced-order integration scheme is introduced to compute the dynamic Lagrangian matrices efficiently, which in turn allows for real-time (multi-link) models with sufficient numerical precision. By exploring the passivity and using the parametrization of the hyper-elastic model, we propose a passivity-based adaptive controller that enhances robustness towards material uncertainty and unmodeled dynamics – slowly improving their estimates online. As a study case, a fully 3D-printed soft robot manipulator is developed, which shows good correspondence with the dynamic model under various conditions, e.g., natural oscillations, forced inputs, and under tip-loads. The solidity of the approach is demonstrated through extensive simulations, numerical benchmarks, and experimental validations.*

This chapter is based on:

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A detailed list of the differences between this chapter and the article on which it is based is provided in the '*Modifications*' chapter of this thesis.

2.1 Introduction

Traditional robots are made from rigid and dense materials that ensure accurate and repeatable motions. While rigid robotics excel at fast and precise motion, their structural rigidity lacks the compliance and mechanical robustness needed for safe and passive interaction in an unknown environment. Soft robotics is a field of robotics that aims to improve the motion complexity and environmental robustness that is generally lacking its rigid counterpart. To further promote these topics in robotics, researchers aim to mimic living creatures by developing bio-inspired robots with similar morphologies and mechanical properties [? ? ? ? ? ?]. In soft robotics, the hyper-flexible and continuum-bodied structure provide them with a rich family of motion primitives. Besides bio-mimicry, soft robotics has proven to be a prominent alternative for rigid robotics with a variety of applications, e.g., manipulation and adaptive grasping [?], untethered locomotion and exploration through uncertain environments [? ? ?], rehabilitation [?], and even minimal-invasive surgery [? ?]. Although popularity of the field has increased exponentially in recent years, one of the first soft robots date back already to the early 1990's, e.g., the work of Suzumori et al. [?]. Yet, despite years of soft robotics research, their intrinsic hyper-flexible nature still possesses numerous challenges on modeling and control.

One major challenge in modeling is that the soft robot's elastic body undergoes large, continuous deformation. Since its inception, numerous works have addressed the kinematics for soft continuum robots[? ?]; yet, its original framework stems from to hyper-redundant robotics nearly a decade earlier[? ?]. Similar to soft robots, hyper-redundant robots exploit their high joint redundancy to achieve a wider ranges of tasks (e.g., shape control and collision avoidance) besides end-effector manipulation. To some extent, soft robots can be seen as the successor to hyper-redundant robots in which rigid mechanical joints or links are substituted with hyper-flexible soft elements. As a result, their dynamics involve a continuously deformable inertial body rather than the classical notion of rigid bodies. As such, conventional modeling approaches cannot be applied directly to these continuously deformable robots, stressing the importance of novel modeling strategies. In this respect, the dynamics of a continuously deformable soft robot are in theory of an infinite-dimensional nature. This paradigm shift has further emphasized the challenges in control-oriented modeling of soft robots; as their physical description are often more suited for a Partial Differential Equations (PDEs) rather than Ordinary Differential Equations (ODEs).

Recently, some significant steps have been made towards formulating reduced-order ODE models for elastic continuum soft robots, paving a path towards model-based controllers. Perhaps one of the most popular techniques of spatial reduction is the so-called Piece-wise Constant Curvature (PCC) model. The PCC model assumes that the continuum shape can be described using a number of spatially-constant curves which are parametrized using a set of generalized coordinates.

Although PCC models can be seen as a significant oversimplification of true continuum mechanics at hand, these models have proven to be remarkably viable for various control applications. Besides its use in inverse kinematic control [? ? ?], PCC models have also shown to be suitable for feedforward controllers as demonstrated by Falkenhahn et al. [?]; and more recently, closed-loop feedback controllers by Della Santina et al. [? ?]. Although the aforementioned works utilize the lumped-mass description, others have employed PCC models with uniform mass distribution [? ? ? ?] and current models even extend beyond the constant curvature [? ? ?]. However, in the face of significant external loading or (distributed) contact with the environment, the PCC assumption is relatively conservative and leads to undesired kinematic constraints on the continuum deformation. Besides, these models often need additional identification to model the compliance as they do not originate from continuum mechanical framework.

On the other hand, Finite-Element Method (FEM) models do originate from continuum mechanics and due to their PDE description provide a more accurate representation of deformations; and are particularly suited to deal with geometric and material nonlinearities. Duriez et al.[?] and related works[? ?] showed that reduced-order FEM models can play an important role in closed-loop control – allowing accurate volumetric deformation and hyper-elastic behavior. Although such real-time simulations for FEM-based models are possible, a significant state-reduction is required to ensure sufficient computational speed. In the process, FEM-based models often loose desirable control properties, e.g., passivity preservation, which might play an important role in control. An alternative modeling strategy is the recently emerging geometrically-exact Cosserat-beam model. Similar to the PCC models, the Cosserat models have the merit benefit that they can be structured into a standard Lagrangian form – the basis for robotics control theory. Rooted in a geometric method for describing the continuum mechanics using Lie theory[?], Boyer et al. [? ?] proposed a geometrically-exact modeling framework for Cosserat beams using nonlinear parametrization of the strain field. Other examples include the work of Renda et al.[? ?] providing various options for Piecewise-Constant Strain (PCS) and Variable Strain modeling approaches. Although recent variants of the Cosserat models offer good computational performance [? ?], its use in model-based control is slowly upcoming.

In this respect, the topic of reduced-order modeling of soft robots is an active area of research. Yet, a challenge that is frequently overlooked in control-orientated research is the anisotropic material behavior, mechanical saturation, and more importantly, the nonlinear and possibly time-varying nature of the highly hyper-elastic soft materials [? ? ? ?]. This is further amplified by the fact that soft robots are known for their diversity in elastic materials and corresponding morphologies. Mustaza et al.[?] proposed modified nonlinear Kelvin–Voigt material model to embody the complex material behavior of silicone-composite manipulators (so-called STIFF-FLOP actuators). A similar silicone composite actuator was experimentally validated by Sadati et. al [?] who proposed a novel modeling approach with an appendage-dependent Hookean model and viscous power-law to describe nonlinear and time-dependent material effects, respectively. Both nonlinear material models show good correspondence with physical soft robots under various dynamic conditions, yet they lack general transferability to the soft robots with different geometries – intrinsically captured by FEM-driven models. As of today, there are little control-oriented models that both offers geometry and material versatilely similar to FEM-models and the control convenience similar to spatial curve models.

Ultimately, the strong nonlinearities paired with its continuous nature encourage the use of model-based controllers. Nevertheless, regarding the aforementioned model-based control approaches [? ? ?], the stability and performance of the closed-loop system could be undermined by uncertainties in physical parameters or unmodelled dynamics. Particularly for state-feedback linearization (e.g., inverse dynamic), as the inversion of inaccurately estimated systems could lead to poor performance and even instability. Adaptive control [? ?] or energy-based controllers [?] might offer the needed robustness towards material uncertainties and unmodelled dynamics. Unfortunately, up till now, the applicability of adaptive and energy-based control techniques on soft robotics is scarcely explored. Franco et al. [?] used an adaptive energy-based controller that compensate for external disturbances on the end-effector, yet these controller can be extended to include various slowly-varying material uncertainties, e.g., hyper-elasticity and viscosity.

The contributions of the work are two-fold. First, to derive a finite-dimensional dynamic model of a continuum soft robot, where we briefly recapitulate on existing modeling technique for soft robot manipulators. To address the issue of infinite-dimensionality, we explore the PCC condition that allows for a low-dimensional description of the continuum dynamics. Although such modeling approaches have been thoroughly developed, we will address two issues that will aid the development of model-based controllers. We aim to bridge the gap between the PCC model and the underlying continuum mechanics by matching the quasi-static behavior to a Finite-Element-driven model (FEM); and we propose a reduced-order integration scheme using Matrix-Differential Equations (MDEs) to compute the spatio-temporal dynamics in real-time. Preliminary results of this work were shown in Caasenbrood et al.[?].

Second, in regards to the FEM-based hyper-elastic modeling and the possible presence of unmodelled dynamics (e.g., material uncertainties or external loads on the end-effector), a passivity-based adaptive controller is proposed that enhances robustness towards material uncertainties and unmodelled dynamics in closed-loop, slowly improving their estimates online. All source code is made publicly available at Caasenbrood et al.[?] (see the open software repository)

2.2 Design and fabrication

By using additive manufacturing, we developed a soft and flexible robot manipulator that is suitable for pick-and-place application. The 3-DOF soft robot can be seen in Figure **??**. The soft robot manipulator in this work is loosely inspired by the elephant whose trunk-appendage consist mainly of parallel muscles without skeletal support. The anatomy of elephant's trunk provides an excellent study case, as they naturally exhibit continuum-body bending and moderate elongation[**? ? ?**]. Similar to the earlier soft robotic designs [**? ?**], the developed soft robot can undergo three-dimensional movement by inflation or deflation of embedded pneumatic bellow network. The soft robot can achieve bending in any preferred direction by differential pressurization of each channel (<0.1 MPa). Whereas, simultaneous pressurization accomplishes moderate elongation.

The soft robot is exclusively composed of a printable, flexible thermoplastic elastomer (Young's modulus $\leq$ 80 MPa), which intrinsically promotes softness and dexterity. The elastomer material is developed explicitly for Selective Laser Sintering (SLS), a 3-Dimensional (3D) printing method that uses a laser to solidify powdered material. The main advantage of SLS printing over other techniques is that the printed parts are fully self-supported, which allows for complex and highly detailed structures. It should be mentioned that the layer-by-layer material deposition will introduce undesired anisotropic mechanical effects. To mitigate anisotropy, the bellows are printed orthogonal to the printing plane, thereby ensuring mechanical symmetry. For the majority of this work, the 3D-printed soft robot in Figure **??** will form the basis of the dynamical model. The 3D-model is made available at the open repository [**?**].

2.3 Continuum dynamic model

As mentioned previously, soft robots are composed of soft bodies that may be regarded as a continuum body with (theoretically) infinitely many degrees-of-freedom (DOF). In this section, we aim to derive a compact and computationally efficient model that envelops the continuous dynamics of a soft robot through a small set of generalized coordinates $\boldsymbol{q} \in \mathcal{Q}$ and their respective generalized velocities $\dot{\boldsymbol{q}}(t) \in T_{\boldsymbol{q}}\mathcal{Q}$ with n the number of active joint variables. We base the modeling framework on the work of Mochiyama et al.[**?**] who outlined a theoretical foundation for continuum manipulators. Their work is extended upon by including extensibility, serial-chaining of multiple soft-links, pneumatic actuation, and the introduction of nonlinear and time-dependent material behavior. Earlier modeling strategies addressing similar issues can be found in from Godage et al. [**?** **?**], Della Santina et al. [**?** **?** **?**], Renda et al. [**?**], and Boyer et al. [**?**]. Leveraging from the aforementioned works, the continuous dynamics of a soft robot manipulator can be written in the familiar Lagrangian form:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{q} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{Q}^{\text{nc}}, \tag{2.1}$$

where $\boldsymbol{M}(\boldsymbol{q}) \in \mathbb{R}^{n \times n}$ denotes the generalized inertia matrix, $\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \mathbb{R}^n$ a vector of nonlinear state-dependent force contributions. In this work, a similar modeling framework is adopted; however, we propose an extension to incorporate FEM-driven data to more accurately reflect the underlying continuum mechanics – in particular hyper-elasticity; and we propose a numerical scheme that allows for fast computation of the continuous dynamics. For completeness, we will recapitulate on the modeling approach here.

2.3.1 Kinematics of elastic continuum bodies

To represent the hyper-flexible configuration of the soft robot, let us consider a smooth spatial curve that passes through the geometric center of the continuously deformable body, as shown in Figure **??**. In literature, this curve is called the '*backbone curve*' as it simplifies the three-dimensional deformation imposed by distributed forces acting on the elastic body. The arc-length of the backbone corresponds to the extensible length of the soft robot denoted by the variable $l(t) \in [l_-, l_+]$ which we assume bounded $l_+ \geq l \geq l_-$, and let L be a constant denoting the total unstressed length of the soft robot. Next, let us introduce a spatial variable $\sigma \in \mathbb{X}$ that belongs to the one-dimensional material domain of the backbone curve, i.e., $\mathbb{X} = [0, L]$. Let it be clear that the spatial variable σ represents the arc-length of a material coordinate along the undeformed material domain of the soft robot manipulator.

Figure here of smooth curve for p and Phi

Given each material coordinate, we wish to find a suitable low-dimensional joint representation $q(t)$ such that the position vector ${}^0p$ anywhere on the continuous backbone can be written as a mapping from generalized coordinates and space into $\mathbb{R}^3$:

$$ {}^0\boldsymbol{\gamma} : \mathbb{X} \times \mathcal{Q}(t) \to \mathbb{R}^3; \tag{2.2} $$

and similarly the rotation matrix ${}^0\boldsymbol{\Phi}(\sigma, \boldsymbol{q})$ by a mapping from the generalized coordinates and space into SO(3):

$$ {}^0\boldsymbol{\Phi} : \mathbb{X} \times \mathcal{Q}(t) \to \mathrm{SO}(3), \tag{2.3} $$

where SO(3) denotes the special orthogonal group for rotations about the origin of $\mathbb{R}^3$, and $n = \dim(\boldsymbol{q})$ the state dimension. Under this notion, the position vectors ${}^0p(q, 0)$ and ${}^0\gamma(L, \boldsymbol{q})$ relate to the base and the end-effector of the soft robot, respectively. Please note that left-sided superscript are used to indicate the frame of reference. The set of all points on the backbone $\mathcal{P} = \left\{ {}^0\gamma \in \mathbb{R}^3 \,|\, \sigma \in \mathbb{X} \right\}$ draws a possible spatial configuration of the soft robot given a time instance $t \in \mathbb{T}$ on a finite horizon $\mathbb{T} = [0, T]$.

Intermezzo 2.1. *Despite the inherent flexibility in soft robotics, it is sometimes sufficient to express the kinematics according to the Piecewise Constant Curvature (PCC) condition. Mathematically, it implies that the curvature of the continuous body satisfies $\kappa(q, \sigma_1) = \kappa(q, \sigma_2)$ for a neighboring region of points $\sigma_1, \sigma_2 \subseteq \mathbb{X}$. As a result, this condition allows us to describe the full forward kinematics with a significantly reduced set of generalized coordinates, mitigating kinematic complexity in the model. Numerous works employ PCC models [**? ? ? ? ? ?**], and depending on the degrees of elasticity, the PCC condition has been proven to be consistent for various soft robotic systems.* ▲

Following this Piecewise Constant Curvature (PCC) description, let us assign a coordinate frame that twists minimally along the backbone – a Bishop frame [**?**]– parametrized by the following generalized coordinate vector:

$$ \boldsymbol{q} = \begin{pmatrix} \varepsilon & \kappa_x & \kappa_y \end{pmatrix}^\top \in \mathcal{Q}, \tag{2.4} $$

where $\varepsilon \in \mathbb{R}$ is the elongation strain, and κ_x, $\kappa_y \in \mathbb{R}$ are the curvatures or angular strains in x-z and y-z plane, respectively; and $\mathcal{Q} \subset \mathbb{R}^3$ is an admissible space on which q evolves.It is worth mentioning that the joint description above is somewhat related to Renda. et al. [**?**] who proposed a Piece-wise Constant Strain (PCS) parametrization with the exception of including the twist along the tangent.

By exploring the differential geometry of the smooth backbone curve similar to Mochiyama et al.[**?**], we can express the spatial change of the position vector ${}^0\boldsymbol{p}(0, \boldsymbol{q})$ and the orientation matrix ${}^0\boldsymbol{\Phi}(q, \sigma)$ for each material point σ along the smooth backbone by

$$\frac{\partial\, {}^0\boldsymbol{\Phi}}{\partial \sigma}(\sigma, \boldsymbol{q}) = {}^0\boldsymbol{\Phi}(\sigma, \boldsymbol{q})\ [\boldsymbol{\Gamma}(\sigma, \boldsymbol{q})]_\times\,, \tag{2.5}$$

$$\frac{\partial\, {}^0\boldsymbol{p}}{\partial \sigma}(\sigma, \boldsymbol{q}) = {}^0\boldsymbol{\Phi}(\boldsymbol{q}, \sigma)\, \boldsymbol{U}(\sigma, \boldsymbol{q}), \tag{2.6}$$

where $[\boldsymbol{\Gamma}]_\times \in \mathrm{so}(3)$ is a skew-symmetric matrix composed of the entries of the vector $\boldsymbol{\Gamma} \in \mathbb{R}^3$, and $\boldsymbol{U} \in \mathbb{R}^3$ a vector representing the tangent along the extensible backbone. The vectors $\boldsymbol{\Gamma}$ and $\boldsymbol{U}$ are vectors that define the differential geometry of the backbone, which are unique entries that lives in the tangent space of the rigid-body transformation group SE(3). Given the Bishop parametrization as described by (2.4), these geometric entities yield

$$\boldsymbol{\Gamma} = \begin{pmatrix} -\kappa_y \\ \kappa_x \\ 0 \end{pmatrix}; \qquad \boldsymbol{U} = \begin{pmatrix} 0 \\ 0 \\ \varepsilon \end{pmatrix} + \boldsymbol{U}_0, \tag{2.7}$$

with $\boldsymbol{U}_0 = (0, 0, 1)^\top$ the unit-tangent. Now, given an initial configuration of backbone's base, i.e., ${}^0\boldsymbol{\Phi}(0, \boldsymbol{q}) = \boldsymbol{\Phi}_0$ and ${}^0\boldsymbol{p}(0, \boldsymbol{q}) = 0_3$, we can now solve for the position and orientation for each material coordinate σ along the backbone:

$${}^0\boldsymbol{\Phi}(\sigma, \boldsymbol{q}) = \boldsymbol{\Phi}_0 \exp(\sigma[\boldsymbol{\Gamma}(\boldsymbol{q})]_\times), \tag{2.8}$$

$${}^0\boldsymbol{p}(\sigma, \boldsymbol{q}) = \int_0^\sigma {}^0\boldsymbol{\Phi}(\eta, \boldsymbol{q})\, \boldsymbol{U}(\boldsymbol{q})\ d\eta, \tag{2.9}$$

where $\exp : \mathrm{so}(3) \to \mathrm{SO}(3)$ is the exponential map. Let it be clear that the closed-form solutions (2.8) and (2.9) form the forward configuration kinematics of the backbone curve. To express the forward velocity kinematic, let $\boldsymbol{V}(\sigma, \boldsymbol{q}, \dot{\boldsymbol{q}}) = \left({}^\sigma\boldsymbol{\omega}^\top, {}^\sigma\boldsymbol{v}^\top\right)^\top \in \mathbb{R}^6 \cong \mathrm{se}(3)$ be the aggregate of the angular velocity and linear velocity components relative to an inertial frame at σ (the frame of reference is denoted by a left superscript), where the space se(3) denotes the Lie algebra of SE(3). The velocity twist is computed by the following integration procedure:

$$\boldsymbol{V}(\sigma, \boldsymbol{q}, \dot{\boldsymbol{q}}) = \mathrm{Ad}^{-1}_{\boldsymbol{g}(\sigma,\cdot)} \int_0^\sigma \mathrm{Ad}_{\boldsymbol{g}(\eta,\cdot)}\, J^* \dot{q}\ d\eta \ =:\ J(q, \sigma)\dot{q}, \tag{2.10}$$

where $\mathrm{Ad}_g : \mathrm{SE}(3) \to \mathbb{R}^{6\times 6}$ denotes the adjoint transformation matrix regarding the rigid body transformation $g \in \mathrm{SE}(3)$ that maps local velocities (i.e., twist) to a frame located at σ, and J^* a constant joint-axis matrix. The joint-axis matrix for an extensible and bendable PCC segment parametrized by the Bishop parameters is given by

$$J^* := \begin{pmatrix} \dfrac{\partial \Gamma}{\partial q}^\top & \dfrac{\partial U}{\partial q}^\top \end{pmatrix}^\top = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^\top. \tag{2.11}$$

Although we based the forward kinematics on the work of Mochiyama et al.[?], the derived expression for the velocity twist in (2.10) is analogous to the work of Renda et al.[? ?], and Boyer et al. [? ?]. Please also note that (2.10) gives rise to the geometric manipulator Jacobian $J(q,\sigma)$ that defines the mapping from joint velocities to the velocity twist for a particular material point σ on the continuous body. In continuation, let us also introduce the acceleration twist[? ? ?] – obtained through time differentiation of (2.10):

$$\begin{aligned}\dot{V}(q,\dot{q},\ddot{q},\sigma) &= J\ddot{q} + \mathrm{Ad}^{-1}_{g(\cdot,\sigma)} \int_0^{\sigma} \mathrm{Ad}_{g(\cdot,\eta)} \mathrm{ad}_{V(\cdot,\eta)}\, J^{*}\dot{q}\; d\eta \\ &:= J(q,\sigma)\ddot{q} + \dot{J}(q,\dot{q},\sigma)\dot{q},\end{aligned} \tag{2.12}$$

where $\mathrm{ad}_V \in \mathbb{R}^{6\times 6}$ denotes the adjoint transformation regarding the velocity twist $V \in \mathrm{se}(3)$. The reader is referred to Appendix A for more detailed expressions on the adjoint transformations.

2.3.2 Euler-Lagrange equations

Given the forward kinematics in (2.8), (2.9), (2.10) and (2.12), we can shift our attention to formulating the finite-dimensional dynamics of the soft robot. Our goal here is to write the spatio-temporal dynamics of the hyper-elastic soft robot as a second-order ODE into the Lagrangian form:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = Q^{\mathrm{nc}}, \tag{2.13}$$

where $\mathcal{L}(q,\dot{q}) :=^{\top} (q,\dot{q}) - \mathcal{U}(q)$ is the Lagrangian function, $^{\top} \in \mathbb{R}_{\geq 0}$ and $\mathcal{U} \in \mathbb{R}$ the kinetic and potential energy, respectively; and $Q^{\mathrm{nc}} \in \mathbb{R}^n$ a vector of generalized non-conservative forces. To apply the Lagrangian formalism to a continuum dynamical system, regard an infinitesimal slice of the continuum body for each material coordinate σ along the backbone curve. Given this notion, we embody this infinitesimal slice with an inertia tensor $\mathcal{M} = \mathrm{blkdiag}(\rho I_3, \mathcal{J}_\sigma)$ with $\rho = m/L$ the line-density and J_σ a tensor for the second moment of inertia. The kinetic energy can be obtained through spatial integration of its respective kinetic energy densities[? ? ?], i.e., $\mathfrak{T} = \frac{1}{2}V^{\top}\mathcal{M}V$:

$$\begin{aligned}\mathcal{T}(q,\dot{q}) &= \frac{1}{2}\int_{\mathbb{X}} V(q,\dot{q},\sigma)^{\top}\,\mathcal{M}\,V(q,\dot{q},\sigma)\; d\sigma, \\ &= \frac{1}{2}\dot{q}^{\top}\int_{\mathbb{X}} J(q,\sigma)^{\top}\,\mathcal{M}\,J(q,\sigma)\; d\sigma\,\dot{q}, \\ &= \frac{1}{2}\dot{q}^{\top}M(q)\dot{q}.\end{aligned} \tag{2.14}$$

Note that expression for the kinetic energy naturally gives rise to the generalized inertia matrix $M(q)$ of the Lagrangian model. By substitution of the kinetic energy into the Euler-Lagrange equation (2.13), we find $M(q)\ddot{q} + C(q,\dot{q})q$ where $C(q,\dot{q})$ denotes the Coriolis matrix. Instead of computing the Coriolis matrix through the conventional Christoffel symbols[**?**], we adopt a computational scheme by Garofalo et al. [**?**] used for serial-chain rigid manipulators, in which we replaced the finite summation of N rigid-bodies by a spatial integration over the continuum domain $\mathbb{X}$:

$$C(q,\dot{q}) = \int_{\mathbb{X}} J(q,\sigma)^\top \mathcal{C}_{V(q,\dot{q},\sigma)} J(q,\sigma) \; + J(q,\sigma)^\top \mathcal{M} \dot{J}(q,\dot{q},\sigma)\, d\sigma, \tag{2.15}$$

where $\mathcal{C}_V = -\mathcal{C}_V^\top := \mathcal{M}\mathrm{ad}_V - \mathrm{ad}_V^\top \mathcal{M}$ is a skew-symmetric matrix. The computation above is slight different from existing literature[**? ?**] to ensure that the matrix $\dot{M} - 2C$ is skew-symmetric; the so-called the passivity condition[**?**] for Euler-Lagrange systems (see Appendix B for proof). The importance of this property will become apparent later in the energy-based controller design. Lastly, the potential energy is given by sum of gravitational potential energy and internal elastic potential, i.e., $\mathcal{U}(q) = \mathcal{U}_g(q) + \mathcal{U}_e(q)$. Since gravitational potential energy density is given by $\mathfrak{U}_g = -\rho\, {}^0p(q,\sigma)\gamma_g$ with $\gamma_g \in \mathbb{R}^3$ is a vector of body accelerations, the potential energy related to gravity is obtained by spatial integration of their respective energy densities:

$$\mathcal{U}_g(q) = -\rho \int_{\mathbb{X}} {}^0p(q,\sigma)^\top \gamma_g \, d\sigma. \tag{2.16}$$

To model the hyper-elastic nature, lets introduce two nonlinear stiffness functions for both stretching and bending, denoted by $k_e : \mathbb{R} \mapsto \mathbb{R}_{>0}$ and $k_b : \mathbb{R} \mapsto \mathbb{R}_{>0}$, respectively. These functions allow us to describe a collective elastic behavior imposed by the hyper-elastic materials and the continuum-bodied deformation. It shall be clear that these entities are unique to the soft robot's geometry and soft material choice, and thus finding a suitable candidate model requires further analysis. Later, we will sculpt these nonlinear stiffness functions through Finite Element Methods (FEM). For now, we assume that these analytical nonlinear stiffness functions are known, and thus the (hyper)-elastic potential energy takes the form

$$\mathcal{U}_e(q) = \int_0^{\varepsilon} k_e(\eta)\,\eta\, d\eta + \int_0^{\beta(q)} k_b(\eta)\,\eta\, d\eta, \tag{2.17}$$

where ε is the elongation strain, and $\beta(q) = \kappa L(\varepsilon + 1)$ is the bending angle with the total curvature of the soft segment $\kappa = \sqrt{{\kappa_x}^2 + {\kappa_y}^2}$ (see Figure **??**).

Overall dynamics

Finally, by combining (2.13), (2.14), (2.15), (2.16), and (2.17), the continuum dynamics of the soft robot can be casted into the familiar closed form [**? ? ? ?**] similar to aforementioned model (1):

$$M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + P(q,\dot{q}) + G(q) = \tau(u,\delta), \tag{2.18}$$

where $P = d\mathcal{U}_e/dq + R\dot{q}$ is a vector of generalized forces imposed by the deformation of the soft materials with $R \in \mathbb{R}^{n\times n}$ the Rayleigh damping matrix, $G = \partial\mathcal{U}_g/\partial q$ a vector of generalized gravitational forces, and $u \in \mathbb{R}^m$ the control input with the index m the number of pressure inputs. The generalized input vector is chosen of the form: $\tau(u,\delta) = Hu + \delta$ with $H : \mathbb{R}^m \mapsto \mathbb{R}^n$ a mapping from the input space to the joint actuation space, and $\delta(t)$ an external disturbance (e.g., unmodelled material uncertainties).

Remark 2.1. *Given the context of manipulators, a possible disturbance $\delta(t)$ could be an external mass applied to the tip of the soft robot. Given the kinematic relations in* (2.10) *and* (2.12)*, one can describe the disturbance (modeled here as a point-mass located at L) by a state-dependent vector:*

$$\delta_m = m_\delta \lfloor J(\cdot, L)\rfloor_3^\top \left(\mathrm{Ad}^{-1}_{g(\cdot,L)}\gamma_g + \lfloor \dot{V}(\cdot, L)\rfloor_3\right), \tag{2.19}$$

where $\lfloor\cdot\rfloor_3$ extracts the last three rows of a matrix or vector, and $m_\delta > 0$ the applied mass to the end-effector. It is worth recalling that the acceleration twist can be computed through the geometric Jacobian and its time derivative, i.e., $\dot{V} = J\ddot{q} + \dot{J}\dot{q}$. Indeed, the PCC condition for a soft body can only accurately describe the true dynamics if external forces produced by mass m_δ do not excessively exceed the intrinsic elastic balancing forces $P(q)$. Alternatively, a soft body can be modeled using multiple PCC curves of smaller size, similar to standard Finite Element discretization. △

The actuation mapping H depends on the geometry, placement, and orientation of the (pneumatic) soft actuators. Since the pneumatic chambers are aligned parallel to the backbone curve and are equally spaced along the circumference, we propose the following ansatz:

$$H := \begin{pmatrix} \alpha_\varepsilon & \dots & \alpha_\varepsilon \\ -\alpha_\kappa \cos(\phi_1) & \dots & -\alpha_\kappa \cos(\phi_m) \\ \alpha_\kappa \sin(\phi_1) & \dots & \alpha_\kappa \sin(\phi_m) \end{pmatrix}, \tag{2.20}$$

where $\alpha_\varepsilon, \alpha_\kappa > 0$ are system parameters representing the effective transferal of differential pressure to joint forces, and $\phi_i = (i-1)\cdot\frac{2\pi}{m}$ the angular inter-distance between the m-number of pneumatic bellows. Please note that the parameters α_ε and α_κ are dependent on the bellow area and radius from the bellow to the backbone curve.

2.4 Extension to multi-link dynamics

We previously expressed the position and velocity kinematics as explicit functions of the generalized coordinates (i.e., Bishop parameters) and their time-derivatives. This explicit dependency stems from the PCC conditions inferring the curvature is non-varying along the spatial domain $\mathbb{X}$, i.e., $\kappa(q,\sigma) = \kappa(q)$. Although sufficient for some cases, the condition is generally restrictive, and to some extent inconvenient, since the inclusion of multiple links demands piece-wise integration of the kinematics (2.9), (2.8), (2.10), and (2.12). Rather than separation of integration, we can extend this PCC description by using piece-wise continuous spatial function to distinguishes multiple soft-bodied links along the continuous body of the soft robot. The idea of parametrization through shapes functions has been explored earlier by Chirikjian et al.[**?** **?**], and later by Boyer et al. [**?**], Della Santina et al. [**?**]. A similar discontinuous shape function series was used by Berthet-Rayne et al. [**?**] to pursue multi-body dynamics for growing continuum robots; and proposed by Chirikjian [**?**] for hyper-redundant robots earlier.

Following the aforementioned works, let us parameterize the the geometric vectors Γ and U for a N-link soft robot through the product of a basis of orthonormal functions $\{s_i\}_{i\in\mathbb{N}}$ and the Bishop parametrization as follows

$$\Gamma(q,\sigma) = \sum_{i=1}^{N} s_i(\sigma)\lceil J^* \rceil_3 \, \tilde{q}_i, \tag{2.21}$$

$$U(q,\sigma) = \sum_{i=1}^{N} s_i(\sigma)\lfloor J^* \rfloor_3 \, \tilde{q}_i + U_0, \tag{2.22}$$

where J^* is the joint-axis matrix as in (**??**), the mathematical operators $\lceil\cdot\rceil_3$ and $\lfloor\cdot\rfloor_3$ extract the first or last three rows of a matrix, respectively; $\tilde{q}_i$ the joint variables of the i-th link, and $s_i : \mathbb{X} \mapsto \{0,1\}$ is a piece-wise continuous shape function, whose purpose is to be non-zero for a given interval on $\mathbb{X}$. The new generalized coordinate vector becomes the aggregate of all joint variables of the multi-body soft robotic system $q = \left(\tilde{q}_1^\top, \tilde{q}_2^\top, ..., \tilde{q}_N^\top\right)^\top$ with the vector $\tilde{q}_i = (\varepsilon_i,\, \kappa_{x,i},\, \kappa_{y,i})^\top$ relating to the Bishop parametrization of the ith-link. Given (2.21) and (2.22), we may now rewrite the velocity-twist as

$$V(q,\dot{q},\sigma) = \mathrm{Ad}_g^{-1} \int_0^\sigma \mathrm{Ad}_g J^* S(\sigma)\, d\sigma \dot{q} := J(q,\sigma)\dot{q} \tag{2.23}$$

where $S = (s_1,\, s_2,\, ..., s_N) \otimes I_n$ is an unitary selection matrix derived from the basis of piece-wise continuous shape functions $\{s_i\}_{i=1}^N$. To be less ambiguous about this selection matrix S, lets consider a spatial coordinate $\sigma_2 \in [L_1, L_1 + L_2]$ that lies on the spatial interval of the second link. Consequently, the operation $S(\sigma_2)q = \tilde{q}_2$ returns the corresponding joint variable of the second link. This selection of generalized coordinates follows analogously for other links along the serial-chain of the soft manipulator. We provided a small library of piece-wise continuous shape functions upto $1 \leq N \leq 8$ links under `./src/pwf` on the open repository[?]. Now, substitution of the discontinuous variation of the geometric Jacobian in (2.23) into (2.14) leads to the dynamic model of a N-link soft robot manipulator in the Lagrangian form similar to (2.18).

2.5 Efficient solver of the soft robotic dynamics through Matrix-Differential Equations

Due to the partial differential nature of soft robots, obtaining a closed-form expression for the projected Lagrangian model in (2.18) can become notoriously long and complex (especially for multi-link systems). The origin of this problem stems from the integrands of inertia matrix $M(q)$ in (2.14) and Coriolis forces $C(q, \dot{q})$ in (2.15); which become highly nonlinear and therefore difficult to calculate a-priori. As a result, solving the forward dynamics using traditional solvers often deteriorates the real-time performance, and in turn its usability for closed-loop control. Inspired by Boyer et al. [?] and Godage et al [?], instead of finding an exact solution to the dynamic entries $M(q)$, $C(q, \dot{q})$ and $G(q)$, let us introduce a similar reduced-order integration scheme that produces an approximate of the dynamic model (2.18). Yet, instead of using an inverse Newton-Euler algorithm (i.e., Featherstone or Hollerbach scheme) in which the Lagrangian entries are built column-wise, we propose an explicit integration scheme that efficiently computes all Lagrangian entities in parallel through a so-called Matrix-Differential Equation (MDE).

The idea here is to replace all necessary spatial integrations for the computation of the Lagrangian entities by an equivalent Matrix-Differential Equation of the form:

$$\frac{\partial Z}{\partial \sigma} = F(Z, \sigma), \tag{2.24}$$

where $Z(\cdot, \sigma)$ is a matrix-valued function composed of the necessary elements for the forward kinematics and forward dynamics, and $F(Z, \sigma)$ a matrix-valued flow function that describes the spatial evolution of Z. Then, by choosing the appropriate initial condition for $Z(\cdot, 0) = Z_0$ and numerically solving (2.24) over a finite horizon $\mathbb{X}$, we can retrieve an approximate of the Lagrangian model in (2.18) by extracting the necessary elements from the solution $Z(\cdot, L)$.

Before describing the MDE, let us first introduce two intermediate matrices related to the computation of the manipulator Jacobian and its time-derivative, namely:

$$\frac{\partial B_1}{\partial \sigma} = \mathrm{Ad}_{g(\cdot,\sigma)}\, J^* S(\sigma), \tag{2.25}$$

$$\frac{\partial B_2}{\partial \sigma} = \mathrm{Ad}_{g(\cdot,\sigma)} \mathrm{ad}_{V(\cdot,\sigma)}\, J^* S(\sigma) \tag{2.26}$$

such that they satisfy $J\dot{q} = \mathrm{Ad}_g^{-1} B_1 \dot{q}$ and $\dot{J}\dot{q} = \mathrm{Ad}_g^{-1} B_2 \dot{q}$. Given the expressions above, we can now include a partial computation Jacobians into the MDE. By collecting all the differential relation for the forward kinematics (5), (6) and forward dynamics (14), (15), and (16), we can assign a flow function $F := \mathrm{blkdiag}\,(F_1, F_2)$ composed of two matrices:

$$F_1 = \left(\begin{matrix} {}^0\Phi[\Gamma]_\times & {}^0\Phi U \\ 0_{3\times 3} & 0_3 \end{matrix}\middle|\ \mathrm{Ad}_g J^* S \ \middle|\ \mathrm{Ad}_g \mathrm{ad}_V J^* S\right), \tag{2.27}$$

$$F_2 = \begin{pmatrix} \frac{\partial M}{\partial \sigma} & \frac{\partial C}{\partial \sigma} & \frac{\partial G}{\partial \sigma} \end{pmatrix}, \tag{2.28}$$

in which the differential form of the dynamic entities $M(q)$, $C(q, \dot{q})$, and $G(q)$ of the Lagrangian model are given by

$$\frac{\partial M}{\partial \sigma} = (\mathrm{Ad}_g^{-1} B_1)^\top \mathcal{M} (\mathrm{Ad}_g^{-1} B_1), \tag{2.29}$$

$$\frac{\partial C}{\partial \sigma} = (\mathrm{Ad}_g^{-1} B_1)^\top \left[\mathcal{C}_V (\mathrm{Ad}_g^{-1} B_1) + \mathcal{M} (\mathrm{Ad}_g^{-1} B_2)\right], \tag{2.30}$$

$$\tfrac{\partial G}{\partial \sigma} = (\lfloor B_1 \rfloor_3)^\top \rho \gamma_g, \tag{2.31}$$

We wish to stress that F_1 collects all elements related to the forward kinematics, whereas F_2 contains the dynamic entities related to the Lagrangian model. Following the spatial Matrix-Differential equation in (2.24) above, its solution will be a matrix $Z := \mathrm{blkdiag}\,(Z_1, Z_2)$ composed of two smaller state matrices Z_1 and Z_2:

$$Z_1 := \left(\begin{matrix} {}^0\Phi & {}^0p \\ 0_{3\times 3} & 0_3 \end{matrix}\middle|\ B_1 \ \middle|\ B_2 \right), \tag{2.32}$$

$$Z_2 := \begin{pmatrix} M & C & G \end{pmatrix}, \tag{2.33}$$

Such a Matrix-Differential equation as in (2.24) are not supported natively by standard ODE solvers. Therefore, an explicit second-order Runge-Kutta solver for MDEs is developed such that efficiently computes the evolution of the state matrix Z along $\mathbb{X}$. The solver is written in `MATLAB` and can be found under `./src/Model.m` at Caasenbrood [?].

As for state trajectories along the temporal regime $\mathbb{T} = [0, T]$, an implicit trapezoidal integration scheme is proposed to solve the approximated continuum dynamics, which are generally less conservative on discretization to preserve numerical stability. Here implicit schemes are favored over explicit scheme, since a coarser time integration can significantly increase real-time performance. In addition, to further boost performance of the temporal integration, a cost-effective approximation of the Hessian is introduced. For more detail, see Appendix C for more detail.

Preliminary titles, committee members, and list of publications

Preliminary titles

1. A Control-oriented Perspective on Design and Modeling of Soft Robotic Systems;
2. Towards a Unified Framework for Design and Model-based Control of Soft Robots;
3. Addressing the Open Challenges in Soft Robotics: from Design to Model-based Control
4. Design and Control Strategies for Soft Robotic Systems;
5. Design, Modeling, Simulation and Control of Soft Robots.
6. Design, Modeling and Control Strategies for Soft Robotics Systems;
7. (Soft Manipulators/Soft Robotic Manipulators?)

Preliminary committee members

- prof. J. den Toonder (TU/e, Microsystems, ICMS)
- dr. R. Luttge (TU/e, Microsystems) - Backup for Jaap

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- prof. G. Krijnen (Twente University) Technologies)
- prof. C.C.L. Wang (Delft University) [1], [2]
- prof R. Carloni (Rijksuniversiteit Groningen) - backup for Gijs

....

- dr. E. Franco (Enrico, Imperial College London) [1], [2]
- prof. H. Mochiyama (University of Tsukuba) [1], [2]
- dr. C. Duriez (INRIA Lille) [1],[2]
- prof. R. Katzschmann (ETH Zurich) [1],[2]
- dr. S. Grazioso (University of Naples) [1]
- dr. M. Bächer (ETH Zurich) [1],[2]
- Antonio Bicchi - Italian
- Annibal Olero - Spain
- Bas Overvelde??

Peer-reviewed journal articles

- B. Caasenbrood, A. Pogromsky and H. Nijmeijer, "*Reduced-order Cosserat Models for Soft Robotic Systems using FEM-driven Shape Reconstruction*", Robotics and Automation Letters. (*in preparation for journal submission*);

- B. Caasenbrood, A. Amoozandeh Nobaveh, M. Janssen, A. Pogromsky, J. Herder, and H. Nijmeijer "*An Energy-efficient Gravity-balancing Wrist Exoskeleton by exploring Compliant Beams and Soft Robotic Actuation*," Wearable Technologies. (*in preparation for journal submission*);

- A. Amiri, B. Caasenbrood, D. Liu, N. van de Wouw, and I. Lopez Arteaga, "*An Electric Circuit Model for the Nonlinear Dynamics of Electro-active Liquid Crystal Coatings*", Applied Physics Letters, 2022. (*under review*);

- B. Caasenbrood, A. Pogromsky and H. Nijmeijer, "*Energy-shaping Controllers for Soft Robot Manipulators through Port-Hamiltonian Cosserat Models*", SN Computer Science Springer, 2022. (*under review*);

- B. Caasenbrood, A. Pogromsky and H. Nijmeijer, "*Control-oriented Models for Hyper-elastic Soft Robots through Differential Geometry of Curves*", Soft Robotics, 2022.

Peer-reviewed articles in conference proceedings

- B. Caasenbrood, F.E. van Beek, H. Khanh Chu, and I.A. Kuling, "*A Desktop-sized Platform for Real-time Control Applications of Pneumatic Soft Robots*," IEEE International Conference on Soft Robotics, RoboSoft 2022. (accepted)

- A. Amoozandeh Nobaveh, and B. Caasenbrood, "*Design Feasibility of an Energy-efficient Wrist Exoskeleton using Compliant Beams and Soft Actuators*", Proceedings of the 18th International Consortium for Rehabilitation Robotics, 2022 (accepted).

- B. Caasenbrood, A. Pogromsky and H. Nijmeijer, "*Energy-based control for Soft Robots using Cosserat-beam models*", Proceedings of the 18th International Conference on Informatics in Control, Automation and Robotics, 2021, pp. 311–319.

- B. Caasenbrood, A. Pogromsky and H. Nijmeijer, "*A Computational Design Framework for Pressure-driven Soft Robots through Nonlinear Topology Optimization*," 2020 3rd IEEE International Conference on Soft Robotics, 2020, pp. 633-638.

- B. Caasenbrood, A. Pogromsky and H. Nijmeijer, "*Dynamic modeling of hyper-elastic soft robots using spatial curves*," IFAC World Congress, IFAC-PapersOnLine, 2020, pp. 9238-9243.

Invited Talks and Non Peer-reviewed Abstracts

- B. Caasenbrood, "`SOROTOKI`*: an Open-source Toolkit for Soft Robotics written in MATLAB*," IEEE International Conference on Soft Robotics, RoboSoft 2022 (abstract).
- B. Caasenbrood, C. Della Santina, and A. Pogromsky, "*Workshop on Model-based Control of Soft Robots*," European Control Conference (ECC), 2021. (main organizer).
- B. Caasenbrood, talk on "*Towards Desing and Control of Soft Robotics*," 4TU Symposium on Soft Robotics, 2020. (invited speaker).
- B. Caasenbrood, talk on "*3D-printed Soft Robotics*," Symposium on Robotic Technologies, 2019. (invited speaker).
- B. Caasenbrood, A. Pogromsky and H. Nijmeijer, talk on "*Forward Dynamics of Hyper-elastic Soft Robotics*," 39th Benelux Meeting on Systems and Control, 2019. (abstract).
- B. Caasenbrood, A. Pogromsky and H. Nijmeijer, talk on "*Dynamical modeling and control of continuum soft robots*," 37th Benelux Meeting on Systems and Control, 2018. (abstract).