

Team notebook

University of Science, VNU-HCM

October 21, 2023

Contents

1 Algorithms	1
1.1 Mo's algorithm on trees	1
1.2 Mo's algorithm	1
1.3 sliding window	2
2 DP Optimizations	2
2.1 convex hull trick	2
2.2 divide and conquer	2
2.3 dp on trees	3
3 Data structures	3
3.1 heavy light decomposition	3
3.2 ladder segment	4
3.3 lichao	4
3.4 persistent array	4
3.5 persistent seg tree	5
3.6 persistent trie	5
3.7 sparse table	6
3.8 splay tree	6
3.9 trie	7
3.10 wavelet tree	7
4 Geometry	7
4.1 center 2 points + rarious	7
4.2 closest pair problem	8
4.3 convex hull	8
4.4 squares	9
4.5 sweep line	9
4.6 triangles	10
5 Graphs	10
5.1 bridges	10
5.2 centroid	10
5.3 dijkstra	11
5.4 dinitz	11
5.5 directed mst	12
5.6 eulerian path	12
5.7 karp min mean cycle	13
5.8 konig's theorem	13
5.9 min cost max flow	13

5.10 minimum path cover in DAG	14
5.11 planar graph (euler)	14
5.12 query with lca	14
5.13 tarjan scc	15
5.14 two sat (with kosaraju)	15
6 Math	16
6.1 Lucas theorem	16
6.2 counting	16
6.3 cumulative sum of divisors	16
6.4 fft	16
6.5 fibonacci properties	17
6.6 polynomials	17
6.7 sigma function	17
6.8 special sequences	18
7 Matrix	18
7.1 matrix	18
8 Misc	18
8.1 dates	18
8.2 fraction	18
8.3 io	18
9 Number theory	19
9.1 convolution	19
9.2 crt	19
9.3 diophantine equations	19
9.4 discrete logarithm	20
9.5 ext euclidean	20
9.6 highest exponent factorial	20
9.7 miller rabin	20
9.8 mod integer	20
9.9 mod inv	20
9.10 number theoretic transform	20
9.11 pollard rho factorize	21
9.12 primes	21
9.13 totient sieve	21
9.14 totient	21

10 Strings	22
10.1 Incremental Aho Corasick	22
10.2 kmp	22
10.3 minimal string rotation	22
10.4 suffix array	23
10.5 suffix automaton	23
10.6 z algorithm	23

1 Algorithms

1.1 Mo's algorithm on trees

```
/**
problems:
- https://codeforces.com/gym/101161 problem E
*/
void flat(vector<vector<edge>>& g, vector<int>& a,
vector<int>& le, vector<int>& ri,
vector<int>& cost, int node, int pi, int& ts, int
w) {

    cost[node] = w;
    le[node] = ts;
    a[ts] = node;
    ts++;
    for (auto e : g[node]) {
        if (e.to == pi)
            continue;
        flat(g, a, le, ri, cost, e.to, node, ts,
            e.w);
    }
    ri[node] = ts;
    a[ts] = node;
    ts++;
}

/**
 * Case when the cost is in the edges.
 */
void compute_queries(vector<vector<edge>>& g) {
    // g is undirected
    int n = g.size();

    lca_tree.init(g, 0);
```

```

vector<int> a(2 * n), le(n), ri(n), cost(n);
// a: nodes in the flatten array
// le: left id of the given node
// ri: right id of the given node
// cost: cost of the edge from the node to the
//       parent

int ts = 0; // timestamp
flat(g, a, le, ri, cost, 0, -1, ts, 0);

int q;
cin >> q;
vector<query> queries(q);
for (int i = 0; i < q; i++) {
    int u, v;
    cin >> u >> v;
    u--;
    v--;
    int lca = lca_tree.query(u, v);
    if (le[u] > le[v])
        swap(u, v);
    queries[i].id = i;
    queries[i].lca = lca;
    queries[i].u = u;
    queries[i].v = v;
    if (lca == u) {
        queries[i].a = le[u] + 1;
        queries[i].b = le[v];
    } else {
        queries[i].a = ri[u];
        queries[i].b = le[v];
    }
}
solve_mo(queries, a, le, cost); // this is the usual
// algorithm
}

```

1.2 Mo's algorithm

```

const int MN = 5 * 100000 + 1;
const int SN = 708;

struct Query {
    int a, b, id;
    Query() {}
    Query(int x, int y, int i) : a(x), b(y), id(i) {}

    bool operator<(const Query& o) const {
        if (a / SN != o.a / SN)
            return a < o.a;
        return a / SN & 1 ? b < o.b : b > o.b;
    }
};

struct DS {
    DS() : {}

    void Insert(int x) {}

```

```

    void Erase(int x) {}

    long long Query() {}
};

Query s[MN];
int ans[MN];
DS active;

int main() {
    int n;
    cin >> n;
    vector<int> a(n);
    for (auto& i : a)
        cin >> i;

    int q;
    cin >> q;
    for (int i = 0; i < q; ++i) {
        int b, e;
        cin >> b >> e;
        b--;
        e--;
        s[i] = Query(b, e, i);
    }

    sort(s, s + q);

    int i = 0;
    int j = -1;
    for (int k = 0; k < (int)q; ++k) {
        int L = s[k].a;
        int R = s[k].b;

        while (j < R)
            active.Insert(a[++j]);
        while (j > R)
            active.Erase(a[j--]);
        while (i < L)
            active.Erase(a[i++]);
        while (i > L)
            active.Insert(a[--i]);

        ans[s[k].id] = active.Query();
    }

    for (int i = 0; i < q; ++i) {
        cout << ans[i] << endl;
    }

    return 0;
}

```

1.3 sliding window

```

/**
 * Given an array ARR and an integer K, the problem boils
 * down to computing for each index i: min(ARR[i],

```

```

    ARR[i-1], ..., ARR[i-K+1]).
 * if mx == true, returns the maximum.
 *
 * http://people.cs.uct.ac.za/~ksmith/articles/sliding\_window\_mi
 * */

vector<int> sliding_window_minmax(vector<int>& ARR, int K,
    bool mx) {
    deque<pair<int, int>> window;
    vector<int> ans;
    for (int i = 0; i < ARR.size(); i++) {
        if (mx) {
            while (!window.empty() &&
                window.back().first <= ARR[i])
                window.pop_back();
        } else {
            while (!window.empty() &&
                window.back().first >= ARR[i])
                window.pop_back();
        }
        window.push_back(make_pair(ARR[i], i));

        while (window.front().second <= i - K)
            window.pop_front();

        ans.push_back(window.front().first);
    }
    return ans;
}

```

2 DP Optimizations

2.1 convex hull trick

```

/**
 * Problems:
 * http://codeforces.com/problemset/problem/319/C
 * http://codeforces.com/contest/311/problem/B
 *
 * https://csacademy.com/contest/archive/task/squared-ends
 * http://codeforces.com/contest/932/problem/F
 * */

struct line {
    long long m, b;
    line(long long a, long long c) : m(a), b(c) {}
    long long eval(long long x) { return m * x + b; }
};

long double inter(line a, line b) {
    long double den = a.m - b.m;
    long double num = b.b - a.b;
    return num / den;
}

/**
 * min m_i * x_j + b_i, for all i.
 * x_j <= x_{j+1}

```

```

*   m_i >= m_{j + 1}
* */
struct ordered_chn {
    vector<line> ch;
    int idx; // id of last "best" in query
    ordered_chn() { idx = 0; }

    void insert_line(long long m, long long b) {
        line cur(m, b);
        // new line's slope is less than all the
        // previous
        while (ch.size() > 1 && (inter(cur,
            ch[ch.size() - 2]) >= inter(cur,
            ch[ch.size() - 1]))) {
            // f(x) is better in interval
            [inter(ch.back(), cur), inf)
            ch.pop_back();
        }

        ch.push_back(cur);

        long long eval(long long x) { // minimum
            // current x is greater than all the
            // previous x,
            // if that is not the case we can make
            // binary search.
            idx = min<int>(idx, ch.size() - 1);
            while (idx + 1 < (int)ch.size() && ch[idx +
                1].eval(x) <= ch[idx].eval(x))
                idx++;

            return ch[idx].eval(x);
        }
    };

    /**
     * Dynamic convex hull trick
     * */

    typedef long long int64;
    typedef long double float128;

    const int64 is_query = -(1LL << 62), inf = 1e18;

    struct Line {
        int64 m, b;
        mutable function<const Line*> succ;
        bool operator<(const Line& rhs) const {
            if (rhs.b != is_query)
                return m < rhs.m;
            const Line* s = succ();
            if (!s)
                return 0;
            int64 x = rhs.m;
            return b - s->b < (s->m - m) * x;
        }
    };

    struct HullDynamic : public multiset<Line> { // will
        maintain upper hull for maximum
        bool bad(iterator y) {

```

```

        auto z = next(y);
        if (y == begin()) {
            if (z == end())
                return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if (z == end())
            return y->m == x->m && y->b <= x->b;
        return (float128)(x->b - y->b) * (z->m -
            y->m) >= (float128)(y->b - z->b) *
            (y->m - x->m);
    }

    void insert_line(int64 m, int64 b) {
        auto y = insert({m, b});
        y->succ = [=] {
            return next(y) == end() ? 0 :
                &*next(y);
        };
        if (bad(y)) {
            erase(y);
            return;
        }
        while (next(y) != end() && bad(next(y)))
            erase(next(y));
        while (y != begin() && bad(prev(y)))
            erase(prev(y));
    }

    int64 eval(int64 x) {
        auto l = *lower_bound((Line){x, is_query});
        return l.m * x + l.b;
    }
};

```

2.2 divide and conquer

```

/**
 * recurrence:
 *   dp[k][i] = min dp[k-1][j] + c[i][j - 1], for all j >
 *   i;
 *
 * "comp" computes dp[k][i] for all i in O(n log n) (k is
 *   fixed)
 *
 * Problems:
 *   https://icpc.kattis.com/problems/branch
 *   http://codeforces.com/contest/321/problem/E
 * */

void comp(int l, int r, int le, int re) {
    if (l > r)
        return;

    int mid = (l + r) >> 1;

    int best = max(mid + 1, le);
    dp[cur][mid] = dp[cur - 1][best] + cost(mid, best -
        1);
}

```

```

for (int i = best; i <= re; i++) {
    if (dp[cur][mid] > dp[cur - 1][i] +
        cost(mid, i - 1)) {
        best = i;
        dp[cur][mid] = dp[cur - 1][i] +
            cost(mid, i - 1);
    }

    comp(l, mid - 1, le, best);
    comp(mid + 1, r, best, re);
}

```

2.3 dp on trees

```

/**
 * This trick is very useful when doing DP on trees,
 * basically, you can save
 * the answer for each node as if it was the root of the
 * tree. Partial results
 * are also stored in order to query subtrees (taking the
 * root and exclude some
 * child).
 *
 * problems:
 * - http://codeforces.com/gym/101161, problem I : Sky tax
 * - http://codeforces.com/contest/791/problem/D
 * */

struct edge {
    int to, p_id;
    edge(int a, int b) : to(a), p_id(b) {}
};

struct state {
    bool seen;
    long long missing;
    long long total;
    vector<long long> partial;

    state() { clear(); }

    void clear() {
        seen = false;
        missing = 0;
        total = 0;
        partial.clear();
    }
};

void add_edge(int u, int v) {
    int id_u_v = g[u].size();
    int id_v_u = g[v].size();
    g[u].emplace_back(v, id_v_u); // id of the parent
    // in the child's list (g[v][id] -> u)
    g[v].emplace_back(u, id_u_v); // id of the parent
    // in the child's list (g[u][id] -> v)
}

```

```

int go(int node, int id_parent) {
    state& s = dp[node];

    if (!s.seen) {
        int ans = 1;
        s.partial.assign(g[node].size(), 0); //
        // create the list of partial results.
        for (int i = 0; i < int(g[node].size());
            i++) {
            int to = g[node][i].to;
            int pid = g[node][i].p_id;
            if (i != id_parent) {
                int tmp = go(to, pid);
                ans += tmp;
                s.partial[i] = tmp;
            }
        }

        s.missing = id_parent;
        s.total = ans;
        s.seen = true;

        return ans;
    } else {

        if (s.missing == id_parent) {
            // the same id_parent than before, so
            // we can not complete the results
            // yet
            return s.total;
        }

        if (s.missing != -1) { // only one missing
            // and is different of 'id_parent'
            int tmp = go(g[node][s.missing].to,
                g[node][s.missing].p_id);
            s.partial[s.missing] = tmp;
            s.total += tmp;
            s.missing = -1;
        }

        int extra = (id_parent == -1) ? 0 :
            s.partial[id_parent];
        return s.total - extra;
    }
}

```

3 Data structures

3.1 heavy light decomposition

```

// Heavy-Light Decomposition
struct TreeDecomposition {
    vector<int> g[MAXN], c[MAXN];
    int s[MAXN]; // subtree size
    int p[MAXN]; // parent id
    int r[MAXN]; // chain root id
}

```

```

int t[MAXN]; // index used in segtree/bit/...
int d[MAXN]; // depth
int ts;

void dfs(int v, int f) {
    p[v] = f;
    s[v] = 1;
    if (f != -1)
        d[v] = d[f] + 1;
    else
        d[v] = 0;

    for (int i = 0; i < g[v].size(); ++i) {
        int w = g[v][i];
        if (w != f) {
            dfs(w, v);
            s[v] += s[w];
        }
    }

    void hld(int v, int f, int k) {
        t[v] = ts++;
        c[k].push_back(v);
        r[v] = k;

        int x = 0, y = -1;
        for (int i = 0; i < g[v].size(); ++i) {
            int w = g[v][i];
            if (w != f) {
                if (s[w] > x) {
                    x = s[w];
                    y = w;
                }
            }
        }
        if (y != -1) {
            hld(y, v, k);
        }

        for (int i = 0; i < g[v].size(); ++i) {
            int w = g[v][i];
            if (w != f && w != y) {
                hld(w, v, w);
            }
        }
    }

    void init(int n) {
        for (int i = 0; i < n; ++i) {
            g[i].clear();
        }
    }

    void add(int a, int b) {
        g[a].push_back(b);
        g[b].push_back(a);
    }

    void build() {
        ts = 0;
        dfs(0, -1);
    }
}

```

```

        hld(0, 0, 0);
    }
};

```

3.2 ladder segment

```

#include <bits/stdc++.h>
using namespace std;

template <typename... T>
#define error(args...) \
    { \
        string _s = #args; \
        replace(_s.begin(), _s.end(), ',', ' '); \
        stringstream _ss(_s); \
        istream_iterator<string> _it(_ss); \
        err(_it, args); \
    }

void err(istream_iterator<string> it) {
}

template <typename T, typename... Args>
void err(istream_iterator<string> it, T a, Args... args) {
    cerr << *it << " = " << a << ", ";
    err(++it, args...);
}

#define int long long
#define pb push_back
#define F first
#define S second

const int inf = 1LL << 62;
const int md = 1000000007;

struct node {
    int s = 0, z0 = 0, z1 = 0;
};

struct node seg[1000005];
void build(int p, int v, int k, int x, int y) {
    if (x == y) {
        if (x == p)
            seg[k].s = v;
        return;
    }
    if (x <= p && y >= p) {
        int d = (x + y) / 2;
        build(p, v, 2 * k, x, d);
        build(p, v, 2 * k + 1, d + 1, y);
        seg[k].s = seg[2 * k].s + seg[2 * k + 1].s;
    }
}

void update(int a, int b, int k, int x, int y) {
    if (a > y || b < x)
        return;
    if (a <= x && b >= y) {
        seg[k].z0 += (1 + x - a);
        seg[k].z1++;
        // error(seg[k].z0, seg[k].z1,k);cerr<<endl;
        return;
    }
}

```

```

    }
    int xx = max(a, x), yy = min(b, y);
    seg[k].s += (yy - xx + 1) * (1 + x - min(x, a)) +
        (yy - xx) * (yy - xx + 1) / 2;
    // error(seg[k].s,k);cerr<<endl;
    int d = (x + y) / 2;
    update(a, b, 2 * k, x, d);
    update(a, b, 2 * k + 1, d + 1, y);
}

int sum(int a, int b, int k, int x, int y) {
    if (a > y || b < x)
        return 0;
    if (a <= x && b >= y) {
        //
        error("ss",k,seg[k].s,seg[k].z0,seg[k].z1*(y-x)*(y-x+1)/2);cerr<<endl;
        return seg[k].s + seg[k].z0 * (y - x + 1) +
            seg[k].z1 * (y - x) * (y - x + 1) / 2;
    }
    seg[k].s += seg[k].z0 * (y - x + 1) + seg[k].z1 *
        (y - x) * (y - x + 1) / 2;
    // error(k,seg[k].z0, seg[k].z1);
    seg[2 * k].z1 += seg[k].z1, seg[2 * k + 1].z1 +=
        seg[k].z1;
    seg[2 * k].z0 += seg[k].z0;
    seg[2 * k + 1].z0 += (y - x + 1) / 2 * seg[k].z1 +
        seg[k].z0;
    seg[k].z0 = 0, seg[k].z1 = 0;
    // error(seg[k].s,k);cerr<<endl;
    int d = (x + y) / 2;
    return sum(a, b, 2 * k, x, d) + sum(a, b, 2 * k +
        1, d + 1, y);
}

void solve() {
    int n, nn, q;
    cin >> n >> q;
    nn = n;
    n = 1 << (int)ceil(log2(n));
    for (int i = 0; i < nn; i++) {
        int x;
        cin >> x;
        build(i, x, 1, 0, n - 1);
    }
    while (q--) {
        int z;
        cin >> z;
        int x, y;
        cin >> x >> y;
        x--, y--;
        if (z == 1)
            update(x, y, 1, 0, n - 1);
        else
            cout << sum(x, y, 1, 0, n - 1) <<
                '\n';
    }
}

```

3.3 lichao

```

struct Line {
    ld m, b;
    ld operator()(ld x) { return m * x + b; }
} a[C * 4];

void insert(int l, int r, Line seg, int o = 0) {
    if (l + 1 == r) {
        if (seg(l) > a[o](l))
            a[o] = seg;
        return;
    }
    int mid = (l + r) >> 1, lson = o * 2 + 1, rson = o
        * 2 + 2;
    if (a[o].m > seg.m)
        swap(a[o], seg);
    if (a[o](mid) < seg(mid)) {
        swap(a[o], seg);
        insert(l, mid, seg, lson);
    } else
        insert(mid, r, seg, rson);
}

ld query(int l, int r, int x, int o = 0) {
    if (l + 1 == r)
        return a[o](x);
    int mid = (l + r) >> 1, lson = o * 2 + 1, rson = o
        * 2 + 2;
    if (x < mid)
        return max(a[o](x), query(l, mid, x, lson));
    else
        return max(a[o](x), query(mid, r, x, rson));
}

```

3.4 persistent array

```

struct node {
    node *l, *r;
    int val;

    node(int x) : l(NULL), r(NULL), val(x) {}
    node() : l(NULL), r(NULL), val(-1) {}
};

typedef node* pnode;

pnode update(pnode cur, int l, int r, int at, int what) {
    pnode ans = new node();

    if (cur != NULL) {
        *ans = *cur;
    }
    if (l == r) {
        ans->val = what;
        return ans;
    }
    int m = (l + r) >> 1;
    if (at <= m)
        ans->l = update(ans->l, l, m, at, what);

```

```

    else
        ans->r = update(ans->r, m + 1, r, at, what);
    return ans;
}

int get(pnode cur, int l, int r, int at) {
    if (cur == NULL)
        return 0;
    if (l == r)
        return cur->val;
    int m = (l + r) >> 1;
    if (at <= m)
        return get(cur->l, l, m, at);
    else
        return get(cur->r, m + 1, r, at);
}

```

3.5 persistent seg tree

```

/**
 * Problems:
 * http://codeforces.com/contest/813/problem/E
 *
 * Important:
 * When using lazy propagation remembert to create new
 * versions for each push_down operation!!!
 */

struct node {
    node *l, *r;
    long long acc;
    int flip;

    node(int x) : l(NULL), r(NULL), acc(x), flip(0) {}
    node() : l(NULL), r(NULL), acc(0), flip(0) {}
};

typedef node* pnode;

pnode create(int l, int r) {
    if (l == r)
        return new node();
    pnode cur = new node();
    int m = (l + r) >> 1;
    cur->l = create(l, m);
    cur->r = create(m + 1, r);
    return cur;
}

pnode copy_node(pnode cur) {
    pnode ans = new node();
    *ans = *cur;
    return ans;
}

void push_down(pnode cur, int l, int r) {
    assert(cur);
    if (cur->flip) {
        int len = r - l + 1;

```

```

    cur->acc = len - cur->acc;
    if (cur->l) {
        cur->l = copy_node(cur->l);
        cur->l->flip ^= 1;
    }
    if (cur->r) {
        cur->r = copy_node(cur->r);
        cur->r->flip ^= 1;
    }
    cur->flip = 0;
}

int get_val(pnode cur) {
    assert(cur);
    assert((cur->flip) == 0);
    if (cur)
        return cur->acc;
    return 0;
}

pnode update(pnode cur, int l, int r, int at, int what) {
    pnode ans = copy_node(cur);
    if (l == r) {
        assert(l == at);
        ans->acc = what;
        ans->flip = 0;
        return ans;
    }
    int m = (l + r) >> 1;
    push_down(ans, l, r);
    if (at <= m)
        ans->l = update(ans->l, l, m, at, what);
    else
        ans->r = update(ans->r, m + 1, r, at, what);

    push_down(ans->l, l, m);
    push_down(ans->r, m + 1, r);
    ans->acc = get_val(ans->l) + get_val(ans->r);
    return ans;
}

pnode flip(pnode cur, int l, int r, int a, int b) {
    pnode ans = new node();

    if (cur != NULL) {
        *ans = *cur;
    }
    if (l > b || r < a)
        return ans;

    if (l >= a && r <= b) {
        ans->flip ^= 1;
        push_down(ans, l, r);
        return ans;
    }

    int m = (l + r) >> 1;
    ans->l = flip(ans->l, l, m, a, b);
    ans->r = flip(ans->r, m + 1, r, a, b);
    push_down(ans->l, l, m);
    push_down(ans->r, m + 1, r);
}

```

```

    ans->acc = get_val(ans->l) + get_val(ans->r);
    return ans;
}

long long get_all(pnode cur, int l, int r) {
    assert(cur);
    push_down(cur, l, r);
    return cur->acc;
}

void traverse(pnode cur, int l, int r) {
    if (!cur)
        return;
    cout << l << " - " << r << " : " << (cur->acc) << "
        " << (cur->flip) << endl;
    traverse(cur->l, l, (l + r) >> 1);
    traverse(cur->r, 1 + ((l + r) >> 1), r);
}

```

3.6 persistent trie

```

// both tries can be tested with the problem:
// http://codeforces.com/problemset/problem/916/D

// Persistent binary trie (BST for integers)
const int MD = 31;

struct node_bin {
    node_bin* child[2];
    int val;

    node_bin() : val(0) { child[0] = child[1] = NULL; }
};

typedef node_bin* pnode_bin;

pnode_bin copy_node(pnode_bin cur) {
    pnode_bin ans = new node_bin();
    if (cur)
        *ans = *cur;
    return ans;
}

pnode_bin modify(pnode_bin cur, int key, int inc, int id = MD) {
    pnode_bin ans = copy_node(cur);
    ans->val += inc;
    if (id >= 0) {
        int to = (key >> id) & 1;
        ans->child[to] = modify(ans->child[to], key,
            inc, id - 1);
    }
    return ans;
}

int sum_smaller(pnode_bin cur, int key, int id = MD) {
    if (cur == NULL)
        return 0;
    if (id < 0)

```

```

        return 0; // strictly smaller
        // if (id == - 1) return cur->val; // smaller or
        // equal

    int ans = 0;
    int to = (key >> id) & 1;
    if (to) {
        if (cur->child[0])
            ans += cur->child[0]->val;
        ans += sum_smaller(cur->child[1], key, id - 1);
    } else {
        ans = sum_smaller(cur->child[0], key, id - 1);
    }
    return ans;
}

// Persistent trie for strings.
const int MAX_CHILD = 26;
struct node {
    node* child[MAX_CHILD];
    int val;
    node() : val(-1) {
        for (int i = 0; i < MAX_CHILD; i++) {
            child[i] = NULL;
        }
    }
};

typedef node* pnode;

pnode copy_node(pnode cur) {
    pnode ans = new node();
    if (cur)
        *ans = *cur;
    return ans;
}

pnode set_val(pnode cur, string& key, int val, int id = 0) {
    {
        pnode ans = copy_node(cur);
        if (id >= int(key.size())) {
            ans->val = val;
        } else {
            int t = key[id] - 'a';
            ans->child[t] = set_val(ans->child[t], key,
                val, id + 1);
        }
        return ans;
    }
}

pnode get(pnode cur, string& key, int id = 0) {
    if (id >= int(key.size()) || !cur)
        return cur;
    int t = key[id] - 'a';
    return get(cur->child[t], key, id + 1);
}

```

3.7 sparse table

```
// RMQ.
const int MN = 100000 + 10; // Max number of elements
const int ML = 18;          // ceil(log2(MN));

struct st {
    int data[MN];
    int M[MN][ML];
    int n;

    void init(const vector<int>& d) {
        n = d.size();
        for (int i = 0; i < n; ++i)
            data[i] = d[i];

        build();
    }

    void build() {
        for (int i = 0; i < n; ++i)
            M[i][0] = data[i];
        for (int j = 1, p = 2, q = 1; p <= n; ++j, p
            <= 1, q <= 1)
            for (int i = 0; i + p - 1 < n; ++i)
                M[i][j] = max(M[i][j - 1],
                    M[i + q][j - 1]);
    }

    int query(int b, int e) {
        int k = log2(e - b + 1);
        return max(M[b][k], M[e + 1 - (1 << k)][k]);
    }
};
```

3.8 splay tree

```
using namespace std;
#include <bits/stdc++.h>
#define D(x) cout << x << endl;

typedef int T;

struct node {
    node *left, *right, *parent;
    T key;
    node(T k) : key(k), left(0), right(0), parent(0) {}
};

struct splay_tree {
    node* root;

    void right_rot(node* x) {
        node* p = x->parent;
        if (x->parent == p->parent) {
            if (x->parent->left == p)
                x->parent->left = x;
            if (x->parent->right == p)
```

```
                x->parent->right = x;
        }
        if (p->left == x->right)
            p->left->parent = p;
        x->right = p;
        p->parent = x;
    }

    void left_rot(node* x) {
        node* p = x->parent;
        if (x->parent == p->parent) {
            if (x->parent->left == p)
                x->parent->left = x;
            if (x->parent->right == p)
                x->parent->right = x;
        }
        if (p->right == x->left)
            p->right->parent = p;
        x->left = p;
        p->parent = x;
    }

    void splay(node* x, node* fa = 0) {
        while (x->parent != fa and x->parent != 0) {
            node* p = x->parent;
            if (p->parent == fa)
                if (p->right == x)
                    left_rot(x);
                else
                    right_rot(x);
            else {
                node* gp = p->parent; //grand
                parent
                if (gp->left == p)
                    if (p->left == x)
                        right_rot(x),
                        right_rot(x);
                    else
                        left_rot(x),
                        right_rot(x);
                else if (p->right == x)
                    right_rot(x),
                    left_rot(x);
                else
                    left_rot(x),
                    left_rot(x);
            }
        }
        if (fa == 0)
            root = x;
    }

    void insert(T key) {
        node* cur = root;
        node* pcur = 0;
        while (cur) {
            pcur = cur;
            if (key > cur->key)
                cur = cur->right;
            else
                cur = cur->left;
```

```
        }
        cur = new node(key);
        cur->parent = pcur;
        if (!pcur)
            root = cur;
        else if (key > pcur->key)
            pcur->right = cur;
        else
            pcur->left = cur;
        splay(cur);
    }

    node* find(T key) {
        node* cur = root;
        while (cur) {
            if (key > cur->key)
                cur = cur->right;
            else if (key < cur->key)
                cur = cur->left;
            else
                return cur;
        }
        return 0;
    }

    splay_tree() { root = 0; };
};
```

3.9 trie

```
const int MN = 26; // size of alphabet
const int MS = 100010; // Number of states.

struct trie {
    struct node {
        int c;
        int a[MN];
    };

    node tree[MS];
    int nodes;

    void clear() {
        tree[nodes].c = 0;
        memset(tree[nodes].a, -1, sizeof
            tree[nodes].a);
        nodes++;
    }

    void init() {
        nodes = 0;
        clear();
    }

    int add(const string& s, bool query = 0) {
        int cur_node = 0;
        for (int i = 0; i < s.size(); ++i) {
            int id = gid(s[i]);
            if (tree[cur_node].a[id] == -1) {
```

```

        if (query)
            return 0;
        tree[cur_node].a[id] = nodes;
        clear();
    }
    cur_node = tree[cur_node].a[id];
}
if (!query)
    tree[cur_node].c++;
return tree[cur_node].c;
}
};

```

3.10 wavelet tree

```

// this can be tested in the problem:
// http://www.spoj.com/problems/ILKQUERY/

struct wavelet {
    vector<int> values, ori;
    vector<int> map_left, map_right;
    int l, r, m;
    wavelet *left, *right;
    wavelet() : left(NULL), right(NULL) {}
    wavelet(int a, int b, int c) : l(a), r(b), m(c),
        left(NULL), right(NULL) {}
};

wavelet* init(vector<int>& data, vector<int>& ind, int lo,
    int hi) {
    if (lo > hi || (data.size() == 0))
        return NULL;
    int mid = ((long long)(lo) + hi) / 2;
    if (lo + 1 == hi)
        mid = lo; // handle negative values

    wavelet* node = new wavelet(lo, hi, mid);

    vector<int> data_l, data_r, ind_l, ind_r;
    int ls = 0, rs = 0;
    for (int i = 0; i < int(data.size()); i++) {
        int value = data[i];
        if (value <= mid) {
            data_l.emplace_back(value);
            ind_l.emplace_back(ind[i]);
            ls++;
        } else {
            data_r.emplace_back(value);
            ind_r.emplace_back(ind[i]);
            rs++;
        }
        node->map_left.emplace_back(ls);
        node->map_right.emplace_back(rs);
        node->values.emplace_back(value);
        node->ori.emplace_back(ind[i]);
    }

    if (lo < hi) {
        node->left = init(data_l, ind_l, lo, mid);

```

```

        node->right = init(data_r, ind_r, mid + 1,
            hi);
    }
    return node;
}

int kth(wavelet* node, int to, int k) {
    // returns the kth element in the sorted version of
    // (a[0], ..., a[to])
    if (node->l == node->r)
        return node->m;
    int c = node->map_left[to];
    if (k < c)
        return kth(node->left, c - 1, k);
    return kth(node->right, node->map_right[to] - 1, k
        - c);
}

int pos_kth_occurrence(wavelet* node, int val, int k) {
    // returns the position on the original array of
    // the kth occurrence of the value "val"
    if (!node)
        return -1;

    if (node->l == node->r) {
        if (int(node->ori.size()) <= k)
            return -1;
        return node->ori[k];
    }

    if (val <= node->m)
        return pos_kth_occurrence(node->left, val, k);
    return pos_kth_occurrence(node->right, val, k);
}

```

4 Geometry

4.1 center 2 points + radius

```

vector<point> find_center(point a, point b, long double r)
{
    point d = (a - b) * 0.5;
    if (d.dot(d) > r * r) {
        return vector<point>();
    }
    point e = b + d;
    long double fac = sqrt(r * r - d.dot(d));
    vector<point> ans;
    point x = point(-d.y, d.x);
    long double l = sqrt(x.dot(x));
    x = x * (fac / l);
    ans.push_back(e + x);
    x = point(d.y, -d.x);
    x = x * (fac / l);
    ans.push_back(e + x);
    return ans;
}

```

4.2 closest pair problem

```

struct point {
    double x, y;
    int id;
    point() {}
    point(double a, double b) : x(a), y(b) {}
};

double dist(const point& o, const point& p) {
    double a = p.x - o.x, b = p.y - o.y;
    return sqrt(a * a + b * b);
}

double cp(vector<point>& p, vector<point>& x,
    vector<point>& y) {
    if (p.size() < 4) {
        double best = 1e100;
        for (int i = 0; i < p.size(); ++i)
            for (int j = i + 1; j < p.size(); ++j)
                best = min(best, dist(p[i],
                    p[j]));
        return best;
    }

    int ls = (p.size() + 1) >> 1;
    double l = (p[ls - 1].x + p[ls].x) * 0.5;
    vector<point> xl(ls), xr(p.size() - ls);
    unordered_set<int> left;
    for (int i = 0; i < ls; ++i) {
        xl[i] = x[i];
        left.insert(x[i].id);
    }
    for (int i = ls; i < p.size(); ++i) {
        xr[i - ls] = x[i];
    }

    vector<point> yl, yr;
    vector<point> pl, pr;
    yl.reserve(ls);
    yr.reserve(p.size() - ls);
    pl.reserve(ls);
    pr.reserve(p.size() - ls);
    for (int i = 0; i < p.size(); ++i) {
        if (left.count(y[i].id))
            yl.push_back(y[i]);
        else
            yr.push_back(y[i]);

        if (left.count(p[i].id))
            pl.push_back(p[i]);
        else
            pr.push_back(p[i]);
    }

    double dl = cp(pl, xl, yl);
    double dr = cp(pr, xr, yr);
    double d = min(dl, dr);
    vector<point> yp;
    yp.reserve(p.size());
    for (int i = 0; i < p.size(); ++i) {

```



```

        if (fabs(y[i].x - l) < d)
            yp.push_back(y[i]);
    }
    for (int i = 0; i < yp.size(); ++i) {
        for (int j = i + 1; j < yp.size() && j < i + 7; ++j) {
            d = min(d, dist(yp[i], yp[j]));
        }
    }
    return d;
}

double closest_pair(vector<point>& p) {
    vector<point> x(p.begin(), p.end());
    sort(x.begin(), x.end(), [](const point& a, const point& b) { return a.x < b.x; });
    vector<point> y(p.begin(), p.end());
    sort(y.begin(), y.end(), [](const point& a, const point& b) { return a.y < b.y; });
    return cp(p, x, y);
}

```

4.3 convex hull

```

const int N = 5e4 + 1;
int n;
int k = 0;

struct Points {
    long double x, y;
    Points(long double _x, long double _y) {
        x = _x;
        y = _y;
    }

    Points() {}
} points[N], poly[N];

bool ccw(Points a, Points b, Points c) {
    return a.x * (b.y - c.y) + b.x * (c.y - a.y) + c.x * (a.y - b.y) > 0;
}

bool cw(Points a, Points b, Points c) {
    return a.x * (b.y - c.y) + b.x * (c.y - a.y) + c.x * (a.y - b.y) < 0;
}

long double Get_Area(Points a, Points b, Points c) {
    return (a.x * b.y) - (b.x * a.y) + (b.x * c.y) - (c.x * b.y) + (c.x * a.y) - (c.y * a.x);
}

void convex_hull() {
    k = 0;
    sort(points + 1, points + 1 + n, [](Points& x, Points& y) {
        if (x.x == y.x) {
            return x.y < y.y;

```

```

        }
        return x.x < y.x;
    });
    for (int i = 1; i <= n; i++) {
        while (k >= 2 && cw(poly[k - 1], poly[k], points[i])) {
            k--;
        }
        poly[++k] = points[i];
    }

    for (int i = n - 1, t = k; i >= 1; i--) {
        while (k - t >= 1 && cw(poly[k - 1], poly[k], points[i])) {
            k--;
        }
        poly[++k] = points[i];
    }

    long double res = 0;
    for (int i = 1; i <= k; i++) {
        for (int j = i + 1; j <= k; j++) {
            int l = j + 1;
            int r = k - 1;
            while (l <= r) {
                int mid = (l + r) >> 1;
                if (Get_Area(poly[i], poly[j], poly[mid]) > Get_Area(poly[i], poly[j], poly[mid + 1])) {
                    Maximize(res, Get_Area(poly[i], poly[j], poly[mid]));
                    r = mid - 1;
                    continue;
                }
                l = mid + 1;
            }
        }
    }

    cout << setprecision(1) << fixed << res / 2.0 << '\n';
}

```

4.4 squares

```

typedef long double ld;

const ld eps = 1e-12;
int cmp(ld x, ld y = 0, ld tol = eps) {
    return (x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1;
}

struct point {
    ld x, y;
    point(ld a, ld b) : x(a), y(b) {}
    point() {}

```

```

};

struct square {
    ld x1, x2, y1, y2, a, b, c;
    point edges[4];
    square(ld _a, ld _b, ld _c) {
        a = _a, b = _b, c = _c;
        x1 = a - c * 0.5;
        x2 = a + c * 0.5;
        y1 = b - c * 0.5;
        y2 = b + c * 0.5;
        edges[0] = point(x1, y1);
        edges[1] = point(x2, y1);
        edges[2] = point(x2, y2);
        edges[3] = point(x1, y2);
    }
};

ld min_dist(point& a, point& b) {
    ld x = a.x - b.x, y = a.y - b.y;
    return sqrt(x * x + y * y);
}

bool point_in_box(square s1, point p) {
    if (cmp(s1.x1, p.x) != 1 && cmp(s1.x2, p.x) != -1 && cmp(s1.y1, p.y) != 1 && cmp(s1.y2, p.y) != -1)
        return true;
    return false;
}

bool inside(square& s1, square& s2) {
    for (int i = 0; i < 4; ++i)
        if (point_in_box(s2, s1.edges[i]))
            return true;
    return false;
}

bool inside_vert(square& s1, square& s2) {
    if ((cmp(s1.y1, s2.y1) != -1 && cmp(s1.y1, s2.y2) != 1) || (cmp(s1.y2, s2.y1) != -1 && cmp(s1.y2, s2.y2) != 1))
        return true;
    return false;
}

bool inside_hori(square& s1, square& s2) {
    if ((cmp(s1.x1, s2.x1) != -1 && cmp(s1.x1, s2.x2) != 1) || (cmp(s1.x2, s2.x1) != -1 && cmp(s1.x2, s2.x2) != 1))
        return true;
    return false;
}

ld min_dist(square& s1, square& s2) {
    if (inside(s1, s2) || inside(s2, s1))
        return 0;

    ld ans = 1e100;
    for (int i = 0; i < 4; ++i)
        for (int j = 0; j < 4; ++j)

```

```

    ans = min(ans, min_dist(s1.edges[i],
        s2.edges[j]));

    if (inside_hori(s1, s2) || inside_hori(s2, s1)) {
        if (cmp(s1.y1, s2.y2) != -1)
            ans = min(ans, s1.y1 - s2.y2);
        else if (cmp(s2.y1, s1.y2) != -1)
            ans = min(ans, s2.y1 - s1.y2);
    }

    if (inside_vert(s1, s2) || inside_vert(s2, s1)) {
        if (cmp(s1.x1, s2.x2) != -1)
            ans = min(ans, s1.x1 - s2.x2);
        else if (cmp(s2.x1, s1.x2) != -1)
            ans = min(ans, s2.x1 - s1.x2);
    }

    return ans;
}

```

4.5 sweep line

```

/*
Solution for : CSES - Area of Rectangles
*/

#include <bits/stdc++.h>
using namespace std;
#define reu(i, a, b) for (int i = (a); i <= (b); ++i)
typedef long long ll;
typedef vector<int> vi;

/* Main solution */
int n;
struct Event {
    int x, y1, y2, val;

    bool operator<(const Event& other) const { return x <
        other.x; }
};
vector<Event> events;
const int MAX = 1e6;

class SegTree {
    int n;
    vi sum;
    vi cover;

    void update(int L, int R, int val, int p, int b, int e) {
        if (e < L || R < b || L > R)
            return;
        if (L <= b && e <= R) {
            cover[p] += val;
            if (cover[p] > 0)
                sum[p] = e - b + 1;
            else if (b < e)
                sum[p] = sum[p << 1] + sum[(p << 1) | 1];
            else
                sum[p] = 0;
        }
    }
}

```

```

    } else {
        int mid = (b + e) >> 1;
        update(L, R, val, p << 1, b, mid);
        update(L, R, val, (p << 1) | 1, mid + 1, e);

        if (cover[p] > 0)
            sum[p] = e - b + 1;
        else
            sum[p] = sum[p << 1] + sum[(p << 1) | 1];
    }
}

public:
    SegTree(int n) : n(n), sum(4 * n + 4, 0), cover(4 * n +
        4, 0) {}

    void update(int L, int R, int val) { update(L, R, val,
        1, 0, n - 1); }

    int get() { return sum[1]; }
};

void Input() {
    cin >> n;
    for (int i = 0; i < n; ++i) {
        int x1, y1, x2, y2;
        cin >> x1 >> y1 >> x2 >> y2;
        events.push_back({x1, y1 + MAX, y2 + MAX, 1});
        events.push_back({x2, y1 + MAX, y2 + MAX, -1});
    }
}

void Solve() {
    sort(events.begin(), events.end());
    SegTree ST(2 * MAX + 1);
    ll ans = 0;
    for (int i = 0; i < (int)events.size() - 1; ++i) {
        ST.update(events[i].y1, events[i].y2 - 1,
            events[i].val);
        ans += 1ll * (events[i + 1].x - events[i].x) *
            ST.get();
    }
    cout << ans;
}

int main() {
    ios::sync_with_stdio(0);
    cin.tie(0);
    Input(), Solve();
    return 0;
}

```

4.6 triangles

Let a, b, c be length of the three sides of a triangle.

$$p = (a + b + c) * 0.5$$

The inradius is defined by:

$$iR = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

The radius of its circumcircle is given by the formula:

$$cR = \frac{abc}{\sqrt{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}}$$

5 Graphs

5.1 bridges

```

struct Graph {
    vector<vector<Edge>> g;
    vector<int> vi, low, d, pi, is_b;
    int bridges_computed;

    int ticks, edges;

    Graph(int n, int m) {
        g.assign(n, vector<Edge>());
        is_b.assign(m, 0);
        vi.resize(n);
        low.resize(n);
        d.resize(n);
        pi.resize(n);
        edges = 0;
        bridges_computed = 0;
    }

    void AddEdge(int u, int v) {
        g[u].push_back(Edge(v, edges));
        g[v].push_back(Edge(u, edges));
        edges++;
    }

    void Dfs(int u) {
        vi[u] = true;
        d[u] = low[u] = ticks++;
        for (int i = 0; i < (int)g[u].size(); ++i) {
            int v = g[u][i].to;
            if (v == pi[u])
                continue;
            if (!vi[v]) {
                pi[v] = u;
                Dfs(v);
                if (d[u] < low[v])
                    is_b[g[u][i].id] = true;

                low[u] = min(low[u], low[v]);
            } else {
                low[u] = min(low[u], d[v]);
            }
        }
    }

    // Multiple edges from a to b are not allowed.
}

```

```

// (they could be detected as a bridge).
// If you need to handle this, just count
// how many edges there are from a to b.
void CompBridges() {
    fill(pi.begin(), pi.end(), -1);
    fill(vi.begin(), vi.end(), 0);
    fill(low.begin(), low.end(), 0);
    fill(d.begin(), d.end(), 0);
    ticks = 0;
    for (int i = 0; i < (int)g.size(); ++i)
        if (!vi[i])
            Dfs(i);
    bridges_computed = true;
}

map<int, vector<Edge>> BridgesTree() {
    if (!bridges_computed)
        CompBridges();
    int n = g.size();
    Dsu dsu(g.size());
    for (int i = 0; i < n; i++)
        for (auto e : g[i])
            if (!is_b[e.id])
                dsu.Join(i, e.to);

    map<int, vector<Edge>> tree;
    for (int i = 0; i < n; i++)
        for (auto e : g[i])
            if (is_b[e.id])
                tree[dsu.Find(i)].emplace_back(dsu.Find(e.to),
                e.id);

    return tree;
}
};

```

5.2 centroid

```

const int N = 2e5 + 1;
vector<pair<int, int>> vertices[N];
int sub_size[N];
int k;
bool dead[N];
int res = -1;

void AddEdge(int u, int v, int w) {
    vertices[u].push_back(make_pair(v, w));
    vertices[v].push_back(make_pair(u, w));
}

int DFS(int u, int daddy) {
    sub_size[u] = 1;
    foreach (pair<int, int> adj in vertices[u]) {
        int v = adj.first;
        if (v == daddy || dead[v]) {
            continue;
        }
        sub_size[u] += DFS(v, u);
    }
}

```

```

        return sub_size[u];
    }

    int centroid(int u, int daddy, int lim) {
        foreach (pair<int, int> adj in vertices[u]) {
            int v = adj.first;
            if (v == daddy || dead[v]) {
                continue;
            }
            if (sub_size[v] > lim) {
                return centroid(v, u, lim);
            }
        }
        return u;
    }

    void dfs2(int u, int daddy, int distance, map<int, int>&
    cur, int s) {
        if (cur.count(s)) {
            Minimize(cur[s], distance);
        } else {
            //cout << distance << '\n';
            cur[s] = distance;
        }
        foreach (pair<int, int> adj in vertices[u]) {
            int v = adj.first;
            if (v == daddy || dead[v]) {
                continue;
            }
            if (s + adj.second > k) {
                continue;
            }
            dfs2(v, u, distance + 1, cur, s +
            adj.second);
        }
    }

    void Build(int u, int daddy) {
        int lim = DFS(u, daddy);
        int _c = centroid(u, daddy, lim >> 1);

        dead[_c] = true;

        map<int, int> dict;
        foreach (pair<int, int> adj in vertices[_c]) {

            int v = adj.first;
            if (dead[v]) {
                continue;
            }

            map<int, int> cur;
            dfs2(v, _c, 1, cur, adj.second);
            foreach (auto val in cur) {
                if (dict.count(k - val.first)) {
                    Minimize(res, dict[k -
                    val.first] + val.second);
                }
            }
        }
    }
}

```

```

        foreach (auto val in cur) {
            if (dict.count(val.first)) {
                Minimize(dict[val.first],
                val.second);
            } else {
                dict[val.first] = val.second;
            }
        }
    }
    if (dict.count(k)) {
        foreach (auto val in dict) {
            if (val.first == k)
                Minimize(res, val.second);
        }
    }
    dict.clear();

    foreach (pair<int, int> adj in vertices[_c]) {
        int v = adj.first;
        if (dead[v]) {
            continue;
        }
        Build(v, _c);
    }
}

int best_path(int n, int m, int H[N][2], int L[N]) {
    k = m;
    for (int i = 0; i < n - 1; i++) {
        AddEdge(H[i][0] + 1, H[i][1] + 1, L[i]);
    }
    Build(1, -1);
    return res;
}

```

5.3 dijkstra

```

struct edge {
    int to;
    long long w;
    edge() {}
    edge(int a, long long b) : to(a), w(b) {}
    bool operator<(const edge& o) const { return w > o.w; }
};

typedef vector<vector<edge>> graph;

const long long inf = 1000000LL * 10000000LL;

pair<vector<int>, vector<long long>> dijkstra(graph& g,
    int start) {
    int n = g.size();
    vector<long long> d(n, inf);
    vector<int> p(n, -1);
    d[start] = 0;
    priority_queue<edge> q;
    q.push(edge(start, 0));

    while (!q.empty()) {

```

```

int node = q.top().to;
long long dist = q.top().w;
q.pop();

if (dist > d[node])
    continue;

for (int i = 0; i < (int)g[node].size(); i++) {
    int to = g[node][i].to;
    long long w_extra = g[node][i].w;

    if (dist + w_extra < d[to]) {
        p[to] = node;
        d[to] = dist + w_extra;
        q.push(edge(to, d[to]));
    }
}

return {p, d};
}

```

5.4 dinitz

```

const int N = 2207;
int subtask, n, m;
int s, t;
struct Edges {
    int u, v;
    long long capa, flow = 0;
    Edges(int _u, int _v, long long _capa) {
        u = _u;
        v = _v;
        capa = _capa;
    }

    long long residual() { return capa - flow; }
    Edges() {}
};
vector<Edges> edge;
int dist[N * 2 + 100], cnt[N * 2 + 100];
vector<int> vertices[N * 2 + 100];

void Input() {
    cin >> subtask;
    cin >> n >> m;
    s = n + m + 1;
    t = n + m + 2;
    for (int i = 1; i <= n; i++) {
        int x;
        cin >> x;
        edge.push_back(Edges(s, i, x));
        vertices[s].push_back((int)edge.size() - 1);
        edge.push_back(Edges(i, s, 0));
        vertices[i].push_back((int)edge.size() - 1);
    }
    for (int i = 1; i <= m; i++) {
        int x;
        cin >> x;

```

```

        edge.push_back(Edges(n + i, t, x));
        vertices[n + i].push_back((int)edge.size() - 1);
        vertices[t].push_back((int)edge.size());
        edge.push_back(Edges(t, n + i, 0));
    }

    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= m; j++) {
            char c;
            cin >> c;
            if (c == '1') {
                edge.push_back(Edges(i, j + n, INF));
                vertices[i].push_back((int)edge.size() - 1);
                edge.push_back(Edges(j + n, i, 0));
                vertices[j + n].push_back((int)edge.size() - 1);
            }
        }
    }

    bool BFS() {
        for (int i = 1; i <= t; i++) {
            dist[i] = -1;
            cnt[i] = 0;
        }
        dist[s] = 0;
        queue<int> q;
        q.push(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            foreach (int id in vertices[u]) {
                if (edge[id].residual() > 0) {
                    int v = edge[id].v;
                    if (dist[v] < 0) {
                        dist[v] = dist[u] + 1;
                        q.push(v);
                    }
                }
            }
        }
        return dist[t] >= 0;
    }

    long long DFS(int u, long long flow) {
        if (flow == 0) {
            return 0;
        }
        if (u == t) {
            return flow;
        }
        for (; cnt[u] <= (int)vertices[u].size() - 1; cnt[u]++) {
            int id = vertices[u][cnt[u]];
            int v = edge[id].v;
            if (dist[v] != dist[u] + 1 || edge[id].residual()
                <= 0) {
                continue;
            }
            long long new_flow = DFS(v, min(flow,
                edge[id].residual()));
            if (new_flow == 0) {

```

```

                continue;
            }
            edge[id].flow += new_flow;
            edge[id ^ 1].flow -= new_flow;
            return new_flow;
        }

        return 0;
    }

    long long Max_Flow() {
        long long tot = 0;
        while (BFS()) {
            while (true) {
                long long new_flow = DFS(s, INF);
                tot += new_flow;
                if (new_flow == 0) {
                    break;
                }
            }
            return tot;
        }
    }

    void Track() {
        set<int> List_1;
        set<int> List_2;
        for (int i = 1; i <= t; i++) {
            if (dist[i] < 0) {
                continue;
            }
            foreach (int id in vertices[i]) {
                int v = edge[id].v;
                if (dist[v] < 0) {
                    if (i == s) {
                        List_1.insert(v);
                    }
                    if (v == t) {
                        List_2.insert(i - n);
                    }
                }
            }
        }
        cout << (int)List_1.size() << ' ';
        foreach (int value in List_1) {
            cout << value << ' ';
        }
        cout << '\n';
        cout << (int)List_2.size() << ' ';
        foreach (int value in List_2) {
            cout << value << ' ';
        }
    }

    void Process() {
        cout << Max_Flow() << '\n';
        Track();
    }
}

```

5.5 directed mst

```
const int inf = 1000000 + 10;

struct edge {
    int u, v, w;
    edge() {}
    edge(int a, int b, int c) : u(a), v(b), w(c) {}
};

/**
 * Computes the minimum spanning tree for a directed graph
 * - edges : Graph description in the form of list of
 *           edges.
 * - each edge is: From node u to node v with cost w
 * - root : Id of the node to start the DMST.
 * - n : Number of nodes in the graph.
 */

int dmst(vector<edge>& edges, int root, int n) {
    int ans = 0;
    int cur_nodes = n;
    while (true) {
        vector<int> lo(cur_nodes, inf),
            pi(cur_nodes, inf);
        for (int i = 0; i < edges.size(); ++i) {
            int u = edges[i].u, v = edges[i].v, w
                = edges[i].w;
            if (w < lo[v] and u != v) {
                lo[v] = w;
                pi[v] = u;
            }
        }

        lo[root] = 0;
        for (int i = 0; i < lo.size(); ++i) {
            if (i == root)
                continue;
            if (lo[i] == inf)
                return -1;
        }

        int cur_id = 0;
        vector<int> id(cur_nodes, -1),
            mark(cur_nodes, -1);
        for (int i = 0; i < cur_nodes; ++i) {
            ans += lo[i];
            int u = i;
            while (u != root and id[u] < 0 and
                mark[u] != i) {
                mark[u] = i;
                u = pi[u];
            }
            if (u != root and id[u] < 0) { //
                Cycle
                for (int v = pi[u]; v != u; v
                    = pi[v])
                    id[v] = cur_id;
                id[u] = cur_id++;
            }
        }
    }
}
```

```
if (cur_id == 0)
    break;

for (int i = 0; i < cur_nodes; ++i)
    if (id[i] < 0)
        id[i] = cur_id++;

for (int i = 0; i < edges.size(); ++i) {
    int u = edges[i].u, v = edges[i].v, w
        = edges[i].w;
    edges[i].u = id[u];
    edges[i].v = id[v];
    if (id[u] != id[v])
        edges[i].w -= lo[v];
}

cur_nodes = cur_id;
root = id[root];

return ans;
}
```

5.6 eulerian path

```
// Taken from
// https://github.com/lbv/pc-code/blob/master/code/graph.cpp
// Eulerian Trail

struct Euler {
    ELV adj;
    IV t;
    Euler(ELV Adj) : adj(Adj) {}
    void build(int u) {
        while (!adj[u].empty()) {
            int v = adj[u].front().v;
            adj[u].erase(adj[u].begin());
            build(v);
        }
        t.push_back(u);
    }
};

bool eulerian_trail(IV& trail) {
    Euler e(adj);
    int odd = 0, s = 0;
    /*
    for (int v = 0; v < n; v++) {
        int diff = abs(in[v] - out[v]);
        if (diff > 1) return false;
        if (diff == 1) {
            if (++odd > 2) return false;
            if (out[v] > in[v]) start = v;
        }
    }
    */
    e.build(s);
    reverse(e.t.begin(), e.t.end());
    trail = e.t;
    return true;
}
```

5.7 karp min mean cycle

```
/**
 * Finds the min mean cycle, if you need the max mean cycle
 * just add all the edges with negative cost and print
 * ans * -1
 *
 * test: uva, 11090 - Going in Cycle!!
 */

const int MN = 1000;
struct edge {
    int v;
    long long w;
    edge() {}
    edge(int v, int w) : v(v), w(w) {}
};

long long d[MN][MN];
// This is a copy of g because increments the size
// pass as reference if this does not matter.
int karp(vector<vector<edge>> g) {
    int n = g.size();

    g.resize(n + 1); // this is important

    for (int i = 0; i < n; ++i)
        if (!g[i].empty())
            g[n].push_back(edge(i, 0));
    ++n;

    for (int i = 0; i < n; ++i)
        fill(d[i], d[i] + (n + 1), INT_MAX);

    d[n - 1][0] = 0;

    for (int k = 1; k <= n; ++k)
        for (int u = 0; u < n; ++u) {
            if (d[u][k - 1] == INT_MAX)
                continue;
            for (int i = g[u].size() - 1; i >= 0; --i)
                d[g[u][i].v][k] =
                    min(d[g[u][i].v][k],
                        d[u][k - 1] + g[u][i].w);
        }

    bool flag = true;

    for (int i = 0; i < n && flag; ++i)
        if (d[i][n] != INT_MAX)
            flag = false;

    if (flag) {
        return true; // return true if there is no a
            cycle.
    }

    double ans = 1e15;

    for (int u = 0; u + 1 < n; ++u) {
```

```

    if (d[u][n] == INT_MAX)
        continue;
    double W = -1e15;

    for (int k = 0; k < n; ++k)
        if (d[u][k] != INT_MAX)
            W = max(W, (double)(d[u][n] -
                                d[u][k]) / (n - k));

    ans = min(ans, W);
}

// printf("%.2lf\n", ans);
cout << fixed << setprecision(2) << ans << endl;

return false;
}

```

5.8 konig's theorem

In any bipartite graph, the number of edges in a maximum matching equals the number of vertices in a minimum vertex cover

5.9 min cost max flow

```

const int N = 1e2 + 2;
int n, m;
int s, t;
int dist[N];
int path[N];
vector<int> List;

struct Edges {
    int u, v, w, capa, flow = 0;
    bool is_used = false;
    Edges(int _u, int _v, int _w, int _capa) {
        u = _u;
        v = _v;
        w = _w;
        capa = _capa;
    }

    Edges() {}

    int residual() { return capa - flow; }
};

vector<int> vertices[N];
vector<Edges> edge;
void AddEdge(int u, int v, int w, int capa) {
    edge.push_back(Edges(u, v, w, capa));
    edge.push_back(Edges(v, u, -w, 0));
    vertices[u].push_back((int)edge.size() - 2);
    vertices[v].push_back((int)edge.size() - 1);
}

bool Find_Path() {

```

```

    queue<int> q;
    vector<bool> InQueue(n + 1, false);
    for (int i = 1; i <= n; i++) {
        dist[i] = 1e9 + 7;
    }
    dist[s] = 0;
    InQueue[s] = true;
    q.push(s);
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        InQueue[u] = false;
        for (int id : vertices[u]) {
            if (edge[id].residual() > 0) {
                int v = edge[id].v;
                if (Minimize(dist[v], dist[u]
                            + edge[id].w)) {
                    path[v] = id;
                    if (InQueue[v]) {
                        continue;
                    }
                    q.push(v);
                    InQueue[v] = true;
                }
            }
        }
        return dist[t] < 1e9 + 7;
    }
    int tot_cost = 0;
    int maxFlow() {
        int tot = 0;

        foreach (Edges& e in edge) {
            e.flow = 0;
        }
        while (Find_Path()) {
            int delta = 1e9 + 7;
            for (int u = t; u != s; u = edge[path[u]].u)
                Minimize(delta,
                        edge[path[u]].residual());
            tot += delta;
            for (int u = t; u != s; u = edge[path[u]].u)
                edge[path[u]].flow += delta;
                edge[path[u] ^ 1].flow -= delta;
            tot_cost += delta * dist[t];
        }
        return tot;
    }

    void Prepare() {}

    void Input() {
        cin >> n >> m;
        cin >> s >> t;
        for (int i = 1; i <= m; i++) {
            int u, v, w;
            cin >> u >> v >> w;

```

```

            AddEdge(u, v, w, 1);
            AddEdge(v, u, w, 1);
        }
        AddEdge(n + 1, s, 0, 2);
        s = n + 1;
    }

    void Process() {
        int res = maxFlow();
        if (res < 2) {
            cout << -1;
            return;
        }
        cout << tot_cost << '\n';
        s = edge[vertices[s][0]].v;
        for (int i = 2; i >= 1; i--) {

            List.clear();
            int u = s;
            while (u != t) {
                List.push_back(u);
                for (int id : vertices[u]) {
                    if (edge[id].is_used ||
                        edge[id].v > n ||
                        edge[id].u > n) {
                        continue;
                    }
                    int v = edge[id].v;
                    if (edge[id].flow > 0) {
                        u = v;
                        edge[id].is_used =
                            true;
                        break;
                    }
                }
            }
            cout << (int)List.size() + 1 << ' ';
            for (int v : List) {
                cout << v << ' ';
            }
            cout << t;
            cout << '\n';
        }
    }
}

```

5.10 minimum path cover in DAG

Given a directed acyclic graph $G = (V, E)$, we are to find the minimum number of vertex-disjoint paths to cover each vertex in V .

We can construct a bipartite graph $G' = (V_{out} \cup V_{in}, E')$ from G , where :

$$V_{out} = \{v \in V : v \text{ has positive out-degree}\}$$

$$V_{in} = \{v \in V : v \text{ has positive in-degree}\}$$

$$E' = \{(u, v) \in V_{out} \times V_{in} : (u, v) \in E\}$$

Then it can be shown, via König's theorem, that G' has a matching of size m if and only if there exists $n - m$ vertex-disjoint paths that cover each vertex in G , where n is the number of vertices in G and m is the maximum cardinality bipartite matching in G' .

Therefore, the problem can be solved by finding the maximum cardinality matching in G' instead.

NOTE: If the paths are not necessarily disjoint, find the transitive closure and solve the problem for disjoint paths.

5.11 planar graph (euler)

Euler's formula states that if a finite, connected, planar graph is drawn in the plane without any edge intersections, and v is the number of vertices, e is the number of edges and f is the number of faces (regions bounded by edges, including the outer, infinitely large region), then:

$$f + v = e + 2$$

It can be extended to non connected planar graphs with c connected components:

$$f + v = e + c + 1$$

5.12 query with lca

```

struct lowest_ca {
    int T[MN], L[MN], W[MN];
    int P[MN][ML], MI[MN][ML], MA[MN][ML];

    void dfs(vector<vector<edge>>& g, int root, int pi
        = -1) {
        if (pi == -1) {
            L[root] = W[root] = 0;
            T[root] = -1;
        }
        for (int i = 0; i < (int)g[root].size(); ++i) {
            int to = g[root][i].v;
            if (to != pi) {
                T[to] = root;
                W[to] = g[root][i].w;
                L[to] = L[root] + 1;
                dfs(g, to, root);
            }
        }
    }

    void init(vector<vector<edge>>& g, int root) {
        // g is undirected
        dfs(g, root);
        int N = g.size(), i, j;

```

```

        for (i = 0; i < N; i++) {
            for (j = 0; j < N; j++) {
                P[i][j] = -1;
                MI[i][j] = inf;
            }
        }

        for (i = 0; i < N; i++) {
            P[i][0] = T[i];
            MI[i][0] = W[i];
        }

        for (j = 1; j < N; j++)
            for (i = 0; i < N; i++)
                if (P[i][j - 1] != -1) {
                    P[i][j] = P[P[i][j - 1]][j - 1];
                    MI[i][j] = min(MI[i][j - 1], MI[P[i][j - 1]][j - 1]);
                }
        }

    int query(int p, int q) {
        int tmp, log, i;

        int mmin = inf;
        if (L[p] < L[q])
            tmp = p, p = q, q = tmp;

        for (log = 1; 1 <= log <= L[p]; log++)
            ;
        log--;

        for (i = log; i >= 0; i--)
            if (L[p] - (1 <= i) >= L[q]) {
                mmin = min(mmin, MI[p][i]);
                p = P[p][i];
            }

        if (p == q)
            // return p;
            return mmin;

        for (i = log; i >= 0; i--)
            if (P[p][i] != -1 && P[q][i] != -1) {
                mmin = min(mmin, min(MI[p][i], MI[q][i]));
                p = P[p][i], q = P[q][i];
            }

        // return T[p];
        return min(mmin, min(MI[p][0], MI[q][0]));
    }

    int get_child(int p, int q) { // p is ancestor of q
        if (p == q)
            return -1;

        int i, log;
        for (log = 1; 1 <= log <= L[q]; log++) {}

```

```

        log--;

        for (i = log; i >= 0; i--)
            if (L[q] - (1 <= i) > L[p]) {
                q = P[q][i];
            }

        assert(P[q][0] == p);
        return q;
    }

    int is_ancestor(int p, int q) {
        if (L[p] >= L[q])
            return false;

        int dist = L[q] - L[p];

        int cur = q;
        int step = 0;
        while (dist) {
            if (dist & 1)
                cur = P[cur][step];
            step++;
            dist >>= 1;
        }

        return cur == p;
    }
};

```

5.13 tarjan scc

```

const int MN = 20002;

struct tarjan_scc {
    int scc[MN], low[MN], d[MN], stacked[MN];
    int ticks, current_scc;
    deque<int> s; // used as stack.

    tarjan_scc() {}

    void init() {
        memset(scc, -1, sizeof scc);
        memset(d, -1, sizeof d);
        memset(stacked, 0, sizeof stacked);
        s.clear();
        ticks = current_scc = 0;
    }

    void compute(vector<vector<int>>& g, int u) {
        d[u] = low[u] = ticks++;
        s.push_back(u);
        stacked[u] = true;
        for (int i = 0; i < g[u].size(); ++i) {
            int v = g[u][i];
            if (d[v] == -1)
                compute(g, v);
            if (stacked[v]) {
                low[u] = min(low[u], low[v]);
            }
        }
    }
};

```

```

    }
}

if (d[u] == low[u]) { // root
    int v;
    do {
        v = s.back();
        s.pop_back();
        stacked[v] = false;
        scc[v] = current_scc;
    } while (u != v);
    current_scc++;
}
}
};

```

5.14 two sat (with kosaraju)

```

/**
 * Given a set of clauses (a1 v a2)^(a2 v a3)....
 * this algorithm find a solution to it set of clauses.
 * test:
 * http://lightoj.com/volume_showproblem.php?problem=1251
 */

#include <bits/stdc++.h>
using namespace std;
#define MAX 100000
#define endl '\n'

vector<int> G[MAX];
vector<int> GT[MAX];
vector<int> Ftime;
vector<vector<int>> SCC;
bool visited[MAX];
int n;

void dfs1(int n) {
    visited[n] = 1;

    for (int i = 0; i < G[n].size(); ++i) {
        int curr = G[n][i];
        if (visited[curr])
            continue;
        dfs1(curr);
    }

    Ftime.push_back(n);
}

void dfs2(int n, vector<int>& scc) {
    visited[n] = 1;
    scc.push_back(n);

    for (int i = 0; i < GT[n].size(); ++i) {
        int curr = GT[n][i];
        if (visited[curr])
            continue;
        dfs2(curr, scc);
    }
}

```

```

}

void kosaraju() {
    memset(visited, 0, sizeof visited);

    for (int i = 0; i < 2 * n; ++i) {
        if (!visited[i])
            dfs1(i);
    }

    memset(visited, 0, sizeof visited);
    for (int i = Ftime.size() - 1; i >= 0; i--) {
        if (visited[Ftime[i]])
            continue;
        vector<int> _scc;
        dfs2(Ftime[i], _scc);
        SCC.push_back(_scc);
    }
}

/**
 * After having the SCC, we must traverse each scc, if in
 * one SCC are -b y b, there is not a solution.
 * Otherwise we build a solution, making the first "node"
 * that we find truth and its complement false.
 */

bool two_sat(vector<int>& val) {
    kosaraju();
    for (int i = 0; i < SCC.size(); ++i) {
        vector<bool> tmpvisited(2 * n, false);
        for (int j = 0; j < SCC[i].size(); ++j) {
            if (tmpvisited[SCC[i][j] ^ 1])
                return 0;
            if (val[SCC[i][j]] != -1)
                continue;
            else {
                val[SCC[i][j]] = 0;
                val[SCC[i][j] ^ 1] = 1;
            }
            tmpvisited[SCC[i][j]] = 1;
        }
    }
    return 1;
}

// Example of use

int main() {
    int m, u, v, nc = 0, t;
    cin >> t;
    // n = "nodes" number, m = clauses number

    while (t--) {
        cin >> m >> n;
        Ftime.clear();
        SCC.clear();
        for (int i = 0; i < 2 * n; ++i) {
            G[i].clear();
            GT[i].clear();
        }
    }
}

```

```

}

// (a1 v a2) = (a1 -> a2) = (a2 -> a1)
for (int i = 0; i < m; ++i) {
    cin >> u >> v;
    int t1 = abs(u) - 1;
    int t2 = abs(v) - 1;
    int p = t1 * 2 + ((u < 0) ? 1 : 0);
    int q = t2 * 2 + ((v < 0) ? 1 : 0);
    G[p ^ 1].push_back(q);
    G[q ^ 1].push_back(p);
    GT[p].push_back(q ^ 1);
    GT[q].push_back(p ^ 1);
}

vector<int> val(2 * n, -1);
cout << "Case " << ++nc << ": ";
if (two_sat(val)) {
    cout << "Yes" << endl;
    vector<int> sol;
    for (int i = 0; i < 2 * n; ++i)
        if (i % 2 == 0 and val[i] == 1)
            sol.push_back(i / 2 + 1);
    cout << sol.size();

    for (int i = 0; i < sol.size(); ++i) {
        cout << " " << sol[i];
    }
} else {
    cout << "No" << endl;
}

return 0;
}

```

6 Math

6.1 Lucas theorem

For non-negative integers m and n and a prime p , the following congruence relation holds :

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p},$$

where :

$$m = m_k p^k + m_{k-1} p^{k-1} + \dots + m_1 p + m_0,$$

and :

$$n = n_k p^k + n_{k-1} p^{k-1} + \dots + n_1 p + n_0$$

are the base p expansions of m and n respectively. This uses the convention that $\binom{m}{n} = 0$ if $m \leq n$.

6.2 counting

```
const int MN = 1e5 + 100;
long long fact[MN];

void fill_fact() {
    fact[0] = 1;
    for (int i = 1; i < MN; i++) {
        fact[i] = mult(fact[i - 1], i);
    }
}

long long perm_rep(vector<int>& frec) {
    int total = 0;
    long long den = 1;
    for (int i = 0; i < (int)frec.size(); i++) {
        den = mult(den, mod_inv(fact[frec[i]]));
        total += frec[i];
    }
    return mult(fact[total], den);
}
```

6.3 cumulative sum of divisors

```
/**
The function SOD(n) (sum of divisors) is defined
as the summation of all the actual divisors of
an integer number n. For example,

    SOD(24) = 2+3+4+6+8+12 = 35.

The function CSOD(n) (cumulative SOD) of an integer n, is
defined as below:

    csod(n) = \sum_{i=1}^n sod(i)

It can be computed in O(sqrt(n)):
*/

long long csod(long long n) {
    long long ans = 0;
    for (long long i = 2; i * i <= n; ++i) {
        long long j = n / i;
        ans += (i + j) * (j - i + 1) / 2;
        ans += i * (j - i);
    }
    return ans;
}
```

6.4 fft

```
/**
* Fast Fourier Transform.
* Useful to compute convolutions.
* computes:
```

```
* C(f star g)[n] = sum_m(f[m] * g[n - m])
* for all n.
* test: icpc live archive, 6886 - Golf Bot
*/

using namespace std;
#include <bits/stdc++.h>
#define D(x) cout << #x " = " << (x) << endl
#define endl '\n'

const int MN = 262144 << 1;
int d[MN + 10], d2[MN + 10];

const double PI = acos(-1.0);

struct cpx {
    double real, image;
    cpx(double _real, double _image) {
        real = _real;
        image = _image;
    }
    cpx() {}
};

cpx operator+(const cpx& c1, const cpx& c2) {
    return cpx(c1.real + c2.real, c1.image + c2.image);
}

cpx operator-(const cpx& c1, const cpx& c2) {
    return cpx(c1.real - c2.real, c1.image - c2.image);
}

cpx operator*(const cpx& c1, const cpx& c2) {
    return cpx(c1.real * c2.real - c1.image * c2.image,
        c1.real * c2.image + c1.image * c2.real);
}

int rev(int id, int len) {
    int ret = 0;
    for (int i = 0; (1 << i) < len; i++) {
        ret <<= 1;
        if (id & (1 << i))
            ret |= 1;
    }
    return ret;
}

cpx A[1 << 20];

void FFT(cpx* a, int len, int DFT) {
    for (int i = 0; i < len; i++)
        A[rev(i, len)] = a[i];
    for (int s = 1; (1 << s) <= len; s++) {
        int m = (1 << s);
        cpx wm = cpx(cos(DFT * 2 * PI / m), sin(DFT * 2 *
            PI / m));
        for (int k = 0; k < len; k += m) {
            cpx w = cpx(1, 0);
            for (int j = 0; j < (m >> 1); j++) {
                cpx t = w * A[k + j + (m >> 1)];
                cpx u = A[k + j];
                A[k + j] = u + t;
```

```
                A[k + j + (m >> 1)] = u - t;
                w = w * wm;
            }
        }
    }
    if (DFT == -1)
        for (int i = 0; i < len; i++)
            A[i].real /= len, A[i].image /= len;
    for (int i = 0; i < len; i++)
        a[i] = A[i];
    return;
}

cpx in[1 << 20];

void solve(int n) {
    memset(d, 0, sizeof d);
    int t;
    for (int i = 0; i < n; ++i) {
        cin >> t;
        d[t] = true;
    }
    int m;
    cin >> m;
    vector<int> q(m);
    for (int i = 0; i < m; ++i)
        cin >> q[i];

    for (int i = 0; i < MN; ++i) {
        if (d[i])
            in[i] = cpx(1, 0);
        else
            in[i] = cpx(0, 0);
    }

    FFT(in, MN, 1);
    for (int i = 0; i < MN; ++i) {
        in[i] = in[i] * in[i];
    }
    FFT(in, MN, -1);

    int ans = 0;
    for (int i = 0; i < q.size(); ++i) {
        if (in[q[i]].real > 0.5 || d[q[i]]) {
            ans++;
        }
    }
    cout << ans << endl;
}

int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    int n;
    while (cin >> n)
        solve(n);
    return 0;
}
```

6.5 fibonacci properties

Let A, B and n be integer numbers.

$$k = A - B \quad (1)$$

$$F_A F_B = F_{k+1} F_A^2 + F_k F_A F_{A-1} \quad (2)$$

$$\sum_{i=0}^n F_i^2 = F_{n+1} F_n \quad (3)$$

$ev(n)$ = returns 1 if n is even.

$$\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - ev(n) \quad (4)$$

$$\sum_{i=0}^n F_i F_{i-1} = \sum_{i=0}^{n-1} F_i F_{i+1} \quad (5)$$

6.6 polynomials

```
const double pi = acos(-1);
struct poly {
    deque<double> coef;
    double x_lo, x_hi;

    double evaluate(double x) {
        double ans = 0;
        for (auto it : coef)
            ans = (ans * x + it);
        return ans;
    }

    double volume(double x, double dx = 1e-6) {
        dx = (x_hi - x_lo) / 1000000.0;
        double ans = 0;
        for (double ix = x_lo; ix <= x; ix += dx) {
            double rad = evaluate(ix);
            ans += pi * rad * rad * dx;
        }
        return ans;
    }
};
```

6.7 sigma function

the sigma function is defined as:

$$\sigma_x(n) = \sum_{d|n} d^x$$

when $x = 0$ is called the divisor function, that counts the number of positive divisors of n .

Now, we are interested in find

$$\sum_{d|n} \sigma_0(d)$$

if n is written as prime factorization:

$$n = \prod_{i=1}^k P_i^{e_k}$$

we can demonstrate that:

$$\sum_{d|n} \sigma_0(d) = \prod_{i=1}^k g(e_k + 1)$$

where $g(x)$ is the sum of the first x positive numbers:

$$g(x) = (x * (x + 1)) / 2$$

6.8 special sequences

Name	Elements	Description
Euler's totient function $\phi(n)$	1, 1, 2, 2, 4, 2, 6, 4, 6, 4	$\phi(n)$ is the number of the positive integers not greater than n that are coprime to n
Lucas number	2, 1, 3, 4, 7, 11, 18, 29, 47, 76	$L(n) = L(n-1) + L(n-2)$
Prime number	2, 3, 5, 7, 11, 13, 17, 19, 23, 29	The prime numbers
Sylvester's sequence	2, 3, 7, 43, 1807, 3263443, 10650056950807, 113423713055421844361000443	$a(n+1) = a(n)^2 - a(n) + 1$, with $a(0) = 2$
Tribonacci number	0, 1, 1, 2, 4, 7, 13, 24, 44, 81	$T(n) = T(n-1) + T(n-2) + T(n-3)$ with $T(0) = 0, T(1) = T(2) = 1$
Catalan number	1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862	$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!} = \prod_{k=2}^n \frac{n+k}{k}$ for $n \geq 0$
Jacobsthal number	0, 1, 1, 3, 5, 11, 21, 43, 85, 171, 341	$a(n) = a(n-1) + 2a(n-2)$, with $a(0) = 0, a(1) = 1$
Padovan sequence	1, 1, 1, 2, 2, 3, 4, 5, 7, 9	$P(0) = P(1) = P(2) = 1, P(n) = P(n-2) + P(n-3)$

7 Matrix

7.1 matrix

```
const int MN = 111;
const int mod = 10000;

struct matrix {
    int r, c;
    int m[MN][MN];

    matrix(int _r, int _c) : r(_r), c(_c) {
        memset(m, 0, sizeof m);
    }

    void print() {
        for (int i = 0; i < r; ++i) {
            for (int j = 0; j < c; ++j)
                cout << m[i][j] << " ";
            cout << endl;
        }
    }

    int x[MN][MN];
    matrix & operator *=(const matrix &o) {
        memset(x, 0, sizeof x);
        for (int i = 0; i < r; ++i)
            for (int k = 0; k < c; ++k)
                if (m[i][k] != 0)
                    for (int j = 0; j < c; ++j) {
                        x[i][j] = (x[i][j] + (m[i][k] * o.m[k][j]) %
                                mod) % mod;
                    }
        memcpy(m, x, sizeof(m));
        return *this;
    }
};

void matrix_pow(matrix b, long long e, matrix &res) {
    memset(res.m, 0, sizeof res.m);
    for (int i = 0; i < b.r; ++i)
        res.m[i][i] = 1;

    if (e == 0) return;
    while (true) {
        if (e & 1) res *= b;
        if ((e >= 1) == 0) break;
        b *= b;
    }
}
```

8 Misc

8.1 dates

```
//
// Time - Leap years
//
// A[i] has the accumulated number of days from months
// previous to i
const int A[13] = { 0, 0, 31, 59, 90, 120, 151, 181, 212,
    243, 273, 304, 334 };
// same as A, but for a leap year
const int B[13] = { 0, 0, 31, 60, 91, 121, 152, 182, 213,
    244, 274, 305, 335 };
// returns number of leap years up to, and including, y
int leap_years(int y) { return y / 4 - y / 100 + y / 400; }
bool is_leap(int y) { return y % 400 == 0 || (y % 4 == 0
    && y % 100 != 0); }
// number of days in blocks of years
const int p400 = 400*365 + leap_years(400);
const int p100 = 100*365 + leap_years(100);
const int p4 = 4*365 + 1;
const int p1 = 365;
int date_to_days(int d, int m, int y)
{
    return (y - 1) * 365 + leap_years(y - 1) + (is_leap(y) ?
        B[m] : A[m]) + d;
}
void days_to_date(int days, int &d, int &m, int &y)
{
    bool top100; // are we in the top 100 years of a 400
        block?
    bool top4;   // are we in the top 4 years of a 100 block?
    bool top1;   // are we in the top year of a 4 block?

    y = 1;
    top100 = top4 = top1 = false;

    y += ((days-1) / p400) * 400;
    d = (days-1) % p400 + 1;

    if (d > p100*3) top100 = true, d -= 3*p100, y += 300;
    else y += ((d-1) / p100) * 100, d = (d-1) % p100 + 1;

    if (d > p4*24) top4 = true, d -= 24*p4, y += 24*4;
    else y += ((d-1) / p4) * 4, d = (d-1) % p4 + 1;

    if (d > p1*3) top1 = true, d -= p1*3, y += 3;
    else y += (d-1) / p1, d = (d-1) % p1 + 1;

    const int *ac = top1 && (!top4 || top100) ? B : A;
    for (m = 1; m < 12; ++m) if (d <= ac[m + 1]) break;
    d -= ac[m];
}
```

8.2 fraction

```
struct frac{
    long long x, y;
    frac(long long a, long long b) {
        long long g = __gcd(a, b);
        x = a / g;
        y = b / g;
    }
    bool operator < (const frac &o) const {
        return (x * o.y < y * o.x);
    }
};
```

8.3 io

```
// taken from :
// https://github.com/lbv/pc-code/blob/master/solved/c-e/diablo/diablo.cpp
// this is very fast as well :
// https://github.com/lbv/pc-code/blob/master/code/input.cpp

typedef unsigned int u32;
#define BUF 524288
struct Reader {
    char buf[BUF]; char b; int bi, bz;
    Reader() { bi=bz=0; read(); }
    void read() {
        if (bi==bz) { bi=0; bz = fread(buf, 1, BUF, stdin); }
        b = bz ? buf[bi++] : 0; }
    void skip() { while (b > 0 && b <= 32) read(); }
    u32 next_u32() {
        u32 v = 0; for (skip(); b > 32; read()) v = v*10 +
            b-48; return v; }
    int next_int() {
        int v = 0; bool s = false;
        skip(); if (b == '-') { s = true; read(); }
        for (; 48<=b&&b<=57; read()) v = v*10 + b-48; return s
            ? -v : v; }
    char next_char() { skip(); char c = b; read(); return c;
    }
};
```

9 Number theory

9.1 convolution

```
typedef long long int LL;
typedef pair<LL, LL> PLL;

inline bool is_pow2(LL x) {
    return (x & (x - 1)) == 0;
}

inline int ceil_log2(LL x) {
    int ans = 0;
    --x;
    while (x != 0) {
```

```
        x >= 1;
        ans++;
    }
    return ans;
}

/* Returns the convolution of the two given vectors in
time proportional to n*log(n).
* The number of roots of unity to use nroots_unity must
be set so that the product of the first
* nroots_unity primes of the vector nth_roots_unity is
greater than the maximum value of the
* convolution. Never use sizes of vectors bigger than
2^24, if you need to change the values of
* the nth roots of unity to appropriate primes for those
sizes.
*/
vector<LL> convolve(const vector<LL>& a, const vector<LL>&
    b, int nroots_unity = 2) {
    int N = 1 << ceil_log2(a.size() + b.size());
    vector<LL> ans(N, 0), fA(N), fB(N), fC(N);
    LL modulo = 1;
    for (int times = 0; times < nroots_unity; times++) {
        fill(fA.begin(), fA.end(), 0);
        fill(fB.begin(), fB.end(), 0);
        for (int i = 0; i < a.size(); i++)
            fA[i] = a[i];
        for (int i = 0; i < b.size(); i++)
            fB[i] = b[i];
        LL prime = nth_roots_unity[times].first;
        LL inv_modulo = mod_inv(modulo % prime,
            prime);
        LL normalize = mod_inv(N, prime);
        ntfft(fA, 1, nth_roots_unity[times]);
        ntfft(fB, 1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++)
            fC[i] = (fA[i] * fB[i]) % prime;
        ntfft(fC, -1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++) {
            LL curr = (fC[i] * normalize) % prime;
            LL k = (curr - (ans[i] % prime) +
                prime) % prime;
            k = (k * inv_modulo) % prime;
            ans[i] += modulo * k;
        }
        modulo *= prime;
    }
    return ans;
}
```

9.2 crt

```
/**
* Chinese remainder theorem.
* Find z such that z % x[i] = a[i] for all i.
* */
long long crt(vector<long long>& a, vector<long long>& x) {
    long long z = 0;
    long long n = 1;
```

```

for (int i = 0; i < x.size(); ++i)
    n += x[i];

for (int i = 0; i < a.size(); ++i) {
    long long tmp = (a[i] * (n / x[i])) % n;
    tmp = (tmp * mod_inv(n / x[i], x[i])) % n;
    z = (z + tmp) % n;
}

return (z + n) % n;
}

```

9.3 diophantine equations

```

long long gcd(long long a, long long b, long long& x, long long& y) {
    if (a == 0) {
        x = 0;
        y = 1;
        return b;
    }
    long long x1, y1;
    long long d = gcd(b % a, a, x1, y1);
    x = y1 - (b / a) * x1;
    y = x1;
    return d;
}

bool find_any_solution(long long a, long long b, long long c, long long& x0, long long& y0, long long& g) {
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % g) {
        return false;
    }

    x0 *= c / g;
    y0 *= c / g;
    if (a < 0)
        x0 = -x0;
    if (b < 0)
        y0 = -y0;
    return true;
}

void shift_solution(long long& x, long long& y, long long a, long long b, long long cnt) {
    x += cnt * b;
    y -= cnt * a;
}

long long find_all_solutions(long long a, long long b, long long c, long long minx, long long maxx, long long miny, long long maxy) {
    long long x, y, g;
    if (!find_any_solution(a, b, c, x, y, g))
        return 0;
    a /= g;
    b /= g;
}

```

```

long long sign_a = a > 0 ? +1 : -1;
long long sign_b = b > 0 ? +1 : -1;

shift_solution(x, y, a, b, (minx - x) / b);
if (x < minx)
    shift_solution(x, y, a, b, sign_b);
if (x > maxx)
    return 0;
long long lx1 = x;

shift_solution(x, y, a, b, (maxx - x) / b);
if (x > maxx)
    shift_solution(x, y, a, b, -sign_b);
long long rx1 = x;

shift_solution(x, y, a, b, -(miny - y) / a);
if (y < miny)
    shift_solution(x, y, a, b, -sign_a);
if (y > maxy)
    return 0;
long long lx2 = x;

shift_solution(x, y, a, b, -(maxy - y) / a);
if (y > maxy)
    shift_solution(x, y, a, b, sign_a);
long long rx2 = x;

if (lx2 > rx2)
    swap(lx2, rx2);
long long lx = max(lx1, lx2);
long long rx = min(rx1, rx2);

if (lx > rx)
    return 0;
return (rx - lx) / abs(b) + 1;
}

```

9.4 discrete logarithm

```

// Computes x which a ^ x = b mod n.

long long d_log(long long a, long long b, long long n) {
    long long m = ceil(sqrt(n));
    long long aj = 1;
    map<long long, long long> M;
    for (int i = 0; i < m; ++i) {
        if (!M.count(aj))
            M[aj] = i;
        aj = (aj * a) % n;
    }

    long long coef = mod_pow(a, n - 2, n);
    coef = mod_pow(coef, m, n);
    // coef = a ^ (-m)
    long long gamma = b;
    for (int i = 0; i < m; ++i) {
        if (M.count(gamma)) {

```

```

            return i * m + M[gamma];
        } else {
            gamma = (gamma * coef) % n;
        }
    }
    return -1;
}

```

9.5 ext euclidean

```

void ext_euclid(long long a, long long b, long long& x, long long& y, long long& g) {
    x = 0, y = 1, g = b;
    long long m, n, q, r;
    for (long long u = 1, v = 0; a != 0; g = a, a = r) {
        q = g / a, r = g % a;
        m = x - u * q, n = y - v * q;
        x = u, y = v, u = m, v = n;
    }
}

```

9.6 highest exponent factorial

```

int highest_exponent(int p, const int& n) {
    int ans = 0;
    int t = p;
    while (t <= n) {
        ans += n / t;
        t *= p;
    }
    return ans;
}

```

9.7 miller rabin

```

const int rounds = 20;

// checks whether a is a witness that n is not prime, 1 < a < n
bool witness(long long a, long long n) {
    // check as in Miller Rabin Primality Test described
    long long u = n - 1;
    int t = 0;
    while (u % 2 == 0) {
        t++;
        u >>= 1;
    }
    long long next = mod_pow(a, u, n);
    if (next == 1)
        return false;
    long long last;
    for (int i = 0; i < t; ++i) {
        last = next;

```

```

        next = mod_mul(last, last, n);
        if (next == 1) {
            return last != n - 1;
        }
    }
    return next != 1;
}

// Checks if a number is prime with prob 1 - 1 / (2 ^ it)
// D(miller_rabin(9999999999999999LL) == 1);
// D(miller_rabin(9999999999971LL) == 1);
// D(miller_rabin(7907) == 1);
bool miller_rabin(long long n, int it = rounds) {
    if (n <= 1)
        return false;
    if (n == 2)
        return true;
    if (n % 2 == 0)
        return false;
    for (int i = 0; i < it; ++i) {
        long long a = rand() % (n - 1) + 1;
        if (witness(a, n)) {
            return false;
        }
    }
    return true;
}

```

9.8 mod integer

```
template<class T, T mod>
struct mint_t {
    T val;
    mint_t() : val(0) {}
    mint_t(T v) : val(v % mod) {}

    mint_t operator + (const mint_t& o) const {
        return (val + o.val) % mod;
    }
    mint_t operator - (const mint_t& o) const {
        return (val - o.val) % mod;
    }
    mint_t operator * (const mint_t& o) const {
        return (val * o.val) % mod;
    }
};

typedef mint_t<long long, 998244353> mint;
```

9.9 mod inv

```
long long mod_inv(long long n, long long m) {
    long long x, y, gcd;
    ext_euclid(n, m, x, y, gcd);
    if (gcd != 1)
        return 0;
}
```

```
    return (x + m) % m;
}
```

9.10 number theoretic transform

```

typedef long long LL;
typedef pair<LL, LL> PLL;

/* The following vector of pairs contains pairs (prime,
generator)
* where the prime has an Nth root of unity for N being a
power of two.
* The generator is a number g s.t  $g^{p-1} \equiv 1 \pmod{p}$ 
* but is different from 1 for all smaller powers */
vector<PLL> nth_roots_unity {
    {1224736769, 330732430}, {1711276033, 927759239}, {167772161, 167489322},
    {469762049, 343261969}, {754974721, 643797295}, {1107296257, 883868065}};

PLL ext_euclid(LL a, LL b) {
    if (b == 0)
        return make_pair(1, 0);
    pair<LL, LL> rc = ext_euclid(b, a % b);
    return make_pair(rc.second, rc.first - (a / b) *
        rc.second);
}

//returns -1 if there is no unique modular inverse
LL mod_inv(LL x, LL modulo) {
    PLL p = ext_euclid(x, modulo);
    if ( (p.first * x + p.second * modulo) != 1 )
        return -1;
    return (p.first + modulo) % modulo;
}

//Number theory fft. The size of a must be a power of 2
void ntfft(vector<LL> &a, int dir, const PLL &root_unity) {
    int n = a.size();
    LL prime = root_unity.first;
    LL basew = mod_pow(root_unity.second, (prime-1) / n,
        prime);
    if (dir < 0) basew = mod_inv(basew, prime);
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        LL w = 1;
        for (int i = 0; i < mh; i++) {
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                LL x = (a[j] - a[k] + prime) % prime;
                a[j] = (a[j] + a[k]) % prime;
                a[k] = (w * x) % prime;
            }
            w = (w * basew) % prime;
        }
        basew = (basew * basew) % prime;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >= 1; k >= 1);
    }
}

```

```
    if (j < i) swap(a[i], a[j]);
  }
}
```

9.11 pollard rho factorize

```

long long pollard_rho(long long n) {
    long long x, y, i = 1, k = 2, d;
    x = y = rand() % n;
    while (1) {
        ++i;
        x = mod_mul(x, x, n);
        x += 2;
        if (x >= n) x -= n;
        if (x == y) return 1;
        d = __gcd(abs(x - y), n);
        if (d != 1) return d;
        if (i == k) {
            y = x;
            k *= 2;
        }
    }
    return 1;
}

// Returns a list with the prime divisors of n
vector<long long> factorize(long long n) {
    vector<long long> ans;
    if (n == 1)
        return ans;
    if (miller_rabin(n)) {
        ans.push_back(n);
    } else {
        long long d = 1;
        while (d == 1)
            d = pollard_rho(n);
        vector<long long> dd = factorize(d);
        ans = factorize(n / d);
        for (int i = 0; i < dd.size(); ++i)
            ans.push_back(dd[i]);
    }
    return ans;
}

```

9.12 primes

```
namespace primes {
const int MP = 100001;
bool sieve[MP];
long long primes[MP];
int num_p;
void fill_sieve() {
    num_p = 0;
    sieve[0] = sieve[1] = true;
    for (long long i = 2; i < MP; ++i) {
```

```

        if (!sieve[i]) {
            primes[num_p++] = i;
            for (long long j = i * i; j < MP; j += i)
                sieve[j] = true;
        }
    }

// Finds prime numbers between a and b, using basic primes
// up to sqrt(b)
// a must be greater than 1.
vector<long long> seg_sieve(long long a, long long b) {
    long long ant = a;
    a = max(a, 3LL);
    vector<bool> pmap(b - a + 1);
    long long sqrt_b = sqrt(b);
    for (int i = 0; i < num_p; ++i) {
        long long p = primes[i];
        if (p > sqrt_b)
            break;
        long long j = (a + p - 1) / p;
        for (long long v = (j == 1) ? p : j * p; v <=
            b; v += p) {
            pmap[v - a] = true;
        }
    }
    vector<long long> ans;
    if (ant == 2)
        ans.push_back(2);
    int start = a % 2 ? 0 : 1;
    for (int i = start, I = b - a + 1; i < I; i += 2)
        if (pmap[i] == false)
            ans.push_back(a + i);
    return ans;
}

vector<pair<int, int>> factor(int n) {
    vector<pair<int, int>> ans;
    if (n == 0)
        return ans;
    for (int i = 0; primes[i] * primes[i] <= n; ++i) {
        if ((n % primes[i]) == 0) {
            int expo = 0;
            while ((n % primes[i]) == 0) {
                expo++;
                n /= primes[i];
            }
            ans.emplace_back(primes[i], expo);
        }
    }

    if (n > 1) {
        ans.emplace_back(n, 1);
    }
    return ans;
}
} // namespace primes

```

9.13 totient sieve

```

for (int i = 1; i < MN; i++)
    phi[i] = i;

for (int i = 1; i < MN; i++)
    if (!sieve[i]) // is prime
        for (int j = i; j < MN; j += i)
            phi[j] -= phi[j] / i;

```

9.14 totient

```

long long totient(long long n) {
    if (n == 1)
        return 0;
    long long ans = n;
    for (int i = 0; primes[i] * primes[i] <= n; ++i) {
        if ((n % primes[i]) == 0) {
            while ((n % primes[i]) == 0)
                n /= primes[i];
            ans -= ans / primes[i];
        }
    }
    if (n > 1) {
        ans -= ans / n;
    }
    return ans;
}

```

10 Strings

10.1 Incremental Aho Corasick

```

class IncrementalAhoCorasick {
    static const int Alphabets = 26;
    static const int AlphabetBase = 'a';
    struct Node {
        Node* fail;
        Node* next[Alphabets];
        int sum;
        Node() : fail(NULL), next{}, sum(0) {}
    };

    struct String {
        string str;
        int sign;
    };

public:
    //totalLen = sum of (len + 1)
    void init(int totalLen) {
        nodes.resize(totalLen);
        nNodes = 0;
        strings.clear();
        roots.clear();
        sizes.clear();
        que.resize(totalLen);
    }
}

```

```

}

void insert(const string& str, int sign) {
    strings.push_back(String{str, sign});
    roots.push_back(nodes.data() + nNodes);
    sizes.push_back(1);
    nNodes += (int)str.size() + 1;
    auto check = [&]() {
        return sizes.size() > 1 && sizes.end()[-1] ==
            sizes.end()[-2];
    };
    if (!check())
        makePMA(strings.end() - 1, strings.end(),
            roots.back(), que);
    while (check()) {
        int m = sizes.back();
        roots.pop_back();
        sizes.pop_back();
        sizes.back() += m;
        if (!check())
            makePMA(strings.end() - m * 2,
                strings.end(), roots.back(), que);
    }
}

int match(const string& str) const {
    int res = 0;
    for (const Node* t : roots)
        res += matchPMA(t, str);
    return res;
}

private:
    static void makePMA(vector<String>::const_iterator
        begin, vector<String>::const_iterator end,
        Node* nodes, vector<Node*>& que) {
        int nNodes = 0;
        Node* root = new (&nodes[nNodes++]) Node();
        for (auto it = begin; it != end; ++it) {
            Node* t = root;
            for (char c : it->str) {
                Node*& n = t->next[c - AlphabetBase];
                if (n == nullptr)
                    n = new (&nodes[nNodes++]) Node();
                t = n;
            }
            t->sum += it->sign;
        }
        int qt = 0;
        for (Node*& n : root->next) {
            if (n != nullptr) {
                n->fail = root;
                que[qt++] = n;
            } else {
                n = root;
            }
        }
        for (int qh = 0; qh != qt; ++qh) {
            Node* t = que[qh];
            int a = 0;
            for (Node* n : t->next) {
                if (n != nullptr) {

```

```

        que[qt++] = n;
        Node* r = t->fail;
        while (r->next[a] == nullptr)
            r = r->fail;
        n->fail = r->next[a];
        n->sum += r->next[a]->sum;
    }
    ++a;
}
}
}

static int matchPMA(const Node* t, const string& str) {
    int res = 0;
    for (char c : str) {
        int a = c - AlphabetBase;
        while (t->next[a] == nullptr)
            t = t->fail;
        t = t->next[a];
        res += t->sum;
    }
    return res;
}

vector<Node> nodes;
int nNodes;
vector<String> strings;
vector<Node*> roots;
vector<int> sizes;
vector<Node*> que;
};

int main() {
    int m;
    while (~scanf("%d", &m)) {
        IncrementalAhoCorasic iac;
        iac.init(600000);
        rep(i, m) {
            int ty;
            char s[300001];
            scanf("%d%s", &ty, s);
            if (ty == 1) {
                iac.insert(s, +1);
            } else if (ty == 2) {
                iac.insert(s, -1);
            } else if (ty == 3) {
                int ans = iac.match(s);
                printf("%d\n", ans);
                fflush(stdout);
            } else {
                abort();
            }
        }
    }
    return 0;
}

```

10.2 kmp

```

vector<int> kmp(string& s) {
    vector<int> next;
    int j;
    j = next[1] = 0;
    for (int i = 2; s[i]; ++i) {
        while (j > 0 && s[j + 1] != s[i])
            j = next[j];
        if (s[j + 1] == s[i])
            ++j;
        next[i] = j;
    }
    return next;
}

```

10.3 minimal string rotation

```

// Lexicographically minimal string rotation
int lmsr() {
    string s;
    cin >> s;
    int n = s.size();
    s += s;
    vector<int> f(s.size(), -1);
    int k = 0;
    for (int j = 1; j < 2 * n; ++j) {
        int i = f[j - k - 1];
        while (i != -1 && s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1])
                k = j - i - 1;
            i = f[i];
        }
        if (i == -1 && s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1]) {
                k = j;
            }
            f[j - k] = -1;
        } else {
            f[j - k] = i + 1;
        }
    }
    return k;
}

```

10.4 suffix array

```

/**
 * O (n log^2 (n))
 * See
 * http://web.stanford.edu/class/cs97si/suffix-array.pdf
 * for reference
 */

struct entry {
    int a, b, p;
    entry() {}
    entry(int x, int y, int z) : a(x), b(y), p(z) {}
}

```

```

bool operator<(const entry& o) const {
    return (a == o.a) ? (b == o.b) ? (p < o.p) :
        (b < o.b) : (a < o.a);
}

};

struct SuffixArray {
    const int N;
    string s;
    vector<vector<int>> P;
    vector<entry> M;

    SuffixArray(const string& s) : N(s.length()), s(s),
        P(1, vector<int>(N, 0)), M(N) {
        for (int i = 0; i < N; ++i)
            P[0][i] = (int)s[i];

        for (int skip = 1, level = 1; skip < N; skip
            *= 2, level++) {
            P.push_back(vector<int>(N, 0));
            for (int i = 0; i < N; ++i) {
                int next = ((i + skip) < N) ?
                    P[level - 1][i + skip] :
                    -10000;
                M[i] = entry(P[level - 1][i],
                    next, i);
            }
            sort(M.begin(), M.end());
            for (int i = 0; i < N; ++i)
                P[level][M[i].p] = (i > 0 and
                    M[i].a == M[i - 1].a and
                    M[i].b == M[i - 1].b)
                    ?
                    P[level][M[i - 1].p]
                    : i;
        }
    }

    vector<int> getSuffixArray() {
        vector<int>& rank = P.back();
        vector<pair<int, int>> inv(rank.size());
        for (int i = 0; i < rank.size(); ++i)
            inv[i] = make_pair(rank[i], i);
        sort(inv.begin(), inv.end());
        vector<int> sa(rank.size());
        for (int i = 0; i < rank.size(); ++i)
            sa[i] = inv[i].second;
        return sa;
    }

    // returns the length of the longest common prefix
    // of s[i...L-1] and s[j...L-1]
    int lcp(int i, int j) {
        int len = 0;
        if (i == j)
            return N - i;
        for (int k = P.size() - 1; k >= 0 && i < N
            && j < N; --k) {
            if (P[k][i] == P[k][j]) {
                i += 1 << k;
            }
        }
    }
}

```

```

        j += 1 << k;
        len += 1 << k;
    }
    }
    return len;
}
};

```

10.5 suffix automaton

```

/*
 * Suffix automaton:
 * This implementation was extended to maintain (online)
 * the
 * number of different substrings. This is equivalent to
 * compute
 * the number of paths from the initial state to all the
 * other
 * states.
 *
 * The overall complexity is O(n)
 * can be tested here:
 * https://www.urionlinejudge.com.br/judge/en/problems/view/1530
 */

struct state {
    int len, link;
    long long num_paths;
    map<int, int> next;
};

const int MN = 200011;
state sa[MN << 1];
int sz, last;

```

```

long long tot_paths;

void sa_init() {
    sz = 1;
    last = 0;
    sa[0].len = 0;
    sa[0].link = -1;
    sa[0].next.clear();
    sa[0].num_paths = 1;
    tot_paths = 0;
}

void sa_extend(int c) {
    int cur = sz++;
    sa[cur].len = sa[last].len + 1;
    sa[cur].next.clear();
    sa[cur].num_paths = 0;
    int p;
    for (p = last; p != -1 && !sa[p].next.count(c); p =
        sa[p].link) {
        sa[p].next[c] = cur;
        sa[cur].num_paths += sa[p].num_paths;
        tot_paths += sa[p].num_paths;
    }

    if (p == -1) {
        sa[cur].link = 0;
    } else {
        int q = sa[p].next[c];
        if (sa[p].len + 1 == sa[q].len) {
            sa[cur].link = q;
        } else {
            int clone = sz++;
            sa[clone].len = sa[p].len + 1;
            sa[clone].next = sa[q].next;
            sa[clone].num_paths = 0;
            sa[clone].link = sa[q].link;

```

```

        for (; p != -1 && sa[p].next[c] == q; p =
            sa[p].link) {
            sa[p].next[c] = clone;
            sa[q].num_paths -= sa[p].num_paths;
            sa[clone].num_paths += sa[p].num_paths;
        }
        sa[q].link = sa[cur].link = clone;
    }
    last = cur;
}

```

10.6 z algorithm

```

vector<int> z_function(string s) {
    int n = s.size();
    vector<int> z(n);
    int l = 0, r = 0;
    for (int i = 1; i < n; i++) {
        if (i < r) {
            z[i] = min(r - i, z[i - l]);
        }
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            z[i]++;
        }
        if (i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
    }
    return z;
}

```
