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19CSA23

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PQT - Assignment - 2

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II - CSE - A
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UNIT-II - Two Dimensional Random Variables:

① If x and y are random variables having the joint density function $f(x,y) = \frac{1}{8}(6-x-y)$, $0 < x < 2$, $2 < y < 4$, find $P[X+Y<3]$

Soln: Given $f(x,y) = \frac{1}{8}(6-x-y)$, $0<x<2$, $2<y<4$

$P[x+y<3] = \int_2^3 \int_0^{3-y} \frac{1}{8}(6-x-y)\,dx\,dy = \frac{1}{8}\left[18y - \frac{6y^2}{2} - \frac{9}{2}y + \frac{6y^2}{4} - \frac{y^3}{6} - \frac{3y^2}{6} + \frac{y^3}{3}\right]$

y various from 2 to 3 $= 5/24$

② $E(xy) = E(x)E(y)$ if x and y are ___ variable

Soln: independent

③ If x and y are independent, then their covariance is

Soln: zero

④ If x and y are uncorrelated, then they are not necessarily statistically independent Soln: True.

⑤ The correlation between variables gives the relationship between them

Soln: True

⑥ Regression between variables gives the relationship between them

Soln: True

⑦ Regression between x and y is the same as between x and y, soln: False

⑧ Correlation between x and y is the can be infinity Soln: false

⑨ Find acute angle between two lines of regression

⑩ State central limit theorem $\tan\theta = \frac{1-r^2}{r}\left(\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma^2}\right)$

(11) The regression lines or x on y and y on x are $5x-y=22$, $6x+y=45$ find the mean. Soln: 6, 8

(12) x and y are independent random variables with variance 2 and 3. Find the variance of $3x+4y$

Soln: $var(x)=2$ $var(y)=3$. $var[3x+4y]=9var(x)+16var(y)=9\times2+16\times$

(13) The minimum and maximum values the correlation coefficient is ___ and ___ Soln: $-1, 1$

(14) Two random variables are said to be orthogonal if ___

Soln: Their correlation is zero

(15) The following table given the joint probability distribution of x and find the (a) marginal density function of x (b) marginal density of y

X/Y	1	2	3
1	0.1	0.1	0.2
2	0.2	0.3	0.1

Solution:

$P(y=1)=0.4$, $P(y=2)=0.6$, $P(x=1)=0.3$, $P(x=2)=$

$P(x=3)=0.3$.

(16) If x and y be integer valued random variables with $P[x=m, y=n]=q^2p^{m+n-2}$, $n,m=1,2,3,\ldots$ and $p+q=1$ are x and y independent

Soln: yes.

UNIT - III - Classification of Random Process

1. What are all the four types of Stochastic Process

Soln: (a) continous random process
(b) Discreate random Process
(c) Continuous random sequence
(d) Discreate Random Sequence

2. State any two perperties of Poisson Process

Soln: a). The poisson Process is not a marcor Process
b) The Sum of two Independant Poisson Process is not a Poisson Process

3. Prove That the diffrence of two Independant poisson Process is not a process

Soln: $X(t) = X_1(t) - X_2(t)$, $E(X(t)) = (\lambda_1 - \lambda_2)t$, $E\{X^2(t)\} \neq (\lambda_1 - \lambda_2)(t) - (\lambda_1 - \lambda_2)^2 t^2$

4. Define Wss : Soln: A random Process $\{X(t)\}$ is called wss. if mean is a constant and The auto correlation depands only on The time difference

5. Define SSS : Soln: A random Process is called SSS if all its finite dimensional distributions are invariant under translation of time Parameter

6. example of marcov Process : Soln: let $\{X(t)\}$ = no. of births up to time 't' so That The saquence $\{X(t), t \in [0, \infty]\}$ form a Pure birth process,

7. Define marcov chain : Soln: if $X(t)$ is a marcov process which posses marcov peroperty which takes only discreate values whether its continous or discreate called marcov chain

(8). State. The postualates of poisson Process

Soln: (i) $P[X(t) = 1$ for t in $(x, x+h)] = \lambda h + o(h)$

(ii) $P[X(t) = 0$ for t in $(x, x+h)] = 1 - \lambda h + o(h)$

(iii) $P[X(t) = 2$ for t in $(x, x+h)] = o(h)$

(9) What are the Continuous random sequence said to be regular?

Soln: if T, The Parameter set is discreate and S, of The state. Space is continous, The random Process. is called a continous. random sequence.

(10) what is stochastic matrix? when is. said to be regular?

Soln: $P_{ij} \geq 0$ $\sum_{ij} P_{ij} = 1$. for all i Then The tPm of a markov chain is a stochastic matrix

(11) Define irraducible markov chain?

Soln: $P_{ij}(n) \geq 0$ for some n and for all i & j Then every. State can be reached from other state. This condition is stanisfied.

(12) Chapman - kolomorgav Therom:

Soln: If 'p' is the tpm of a homo geneous markov chain, Then the n-step. tpm $P(n)$ is equal to P^n, $[P_{ij}^{(n)}] \{P_{ij}\}^n$

(13) random Process is a random variable is called Soln: all The future values can be predected from The past observations

(14) random process is a particular. time instant is a soln: dependent

(15) The random Process at a variable which is soln: random variable.

(16) A random Process with time average equal to ensemble averages. is called as soln: Ergodic Process

(17) Practically, no process is Soln: SSS

(18) A true SSS Process ranges from $-\infty$ to ∞

(19) Every ergodic Process is Soln: Stationary Process

(20) Say true or false : A stationary Process not neccessarily an ergodic Process Soln: True

(21) Every SSS is a WSS Soln: True

(22) $X(t_i, s_j)$ is a real number Soln: True

(23) $X(t_i, S)$ is a random variable Soln: True

(24) $X(t, s_j)$ is a sample function soln: True

(25) A WSS is not a second order stationary Process soln: False

(26) A WSS should be first order stationary soln: False

(27) All stationary process is ergodic soln: False

(28) All ergodic Process is stationary Soln: ~~false~~ True

(29) A markov process a unlimited historical depandancy soln: False

(30) The TPM of a finite state markov Process is not a square matrix: Soln: F

(31) The TPM of a finite states markov chain takes only ____
Soln: non-negative values

(32) All regular Markov Chain are ____ soln: ergodic

(33) The sum of all the elements in any row in the TPM of a finite state markov chain is Soln: one

(34) is it a vaild $\begin{pmatrix} 0.2 & 0.8 \\ 0.1 & 0.5 \end{pmatrix}$ TPM? Soln: No

(35) Poisson Process is a soln: Discreate random Process

(36) Say True or false : The inter arrival time of a Poisson Process is also poisson soln: false.

(37) say True or false : Poisson Process is neither stationary nor markov soln: false

(38) Say true or false : Poisson Process has a mean 6 and SD 4
Soln: false

UNIT - 2 - Two Dimensional Random Variables

① Define Two. dimensional Random variables

Ans) Let S be the sample space associated with a random experiment E. Let $X = X(S)$ and $Y = Y(S)$ be two functions each assigning a real number to each $s \in S$ then (x, y) is called ~~the~~ two dimensional Random variable.

⑤ find The marginal density functions of X and Y

Y/X	1	2	3
1	0.1	0.1	0.2
2	0.2	0.3	0.1

Soln:

The marginal density of X

$P(X = x_i)\ P_{i*} = \sum_j P_{ij}$

X	1	2	3
P(x)	0.3	0.4	0.3

The marginal density of Y

$P(Y = y_j) = P_{*j} = \sum_i P_{ij}$

Y	1	2
P(Y)	0.4	0.6

③ if $f(x, y) = Kxye^{-(x^2+y^2)}$, $x \geq 0, y \geq 0$ is The joint pdf, find K

Soln:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx = 1 \Rightarrow \int_0^{\infty}\int_0^{\infty} Kxye^{-(x^2+y^2)}\,dy\,dx = 1$$

$$K\int_0^{\infty} xe^{-x^2}\,dx \int_0^{\infty} ye^{-y^2}\,dx = 1 \Rightarrow \frac{K}{4} = 1$$

$$\therefore K = 4$$

(4) ~~Let~~ Let The joint pdf of x and y is given by $f(x,y) = \begin{cases} cx(1-x), & 0 \le x \le y \le \\ 0 & \text{otherwise} \end{cases}$

find value of c

<u>Soln:</u>

$$\int_0^1 \int_0^y cx(1-x)\,dx\,dy = 1 \Rightarrow \frac{c}{6}\int_0^1 (3y^2 - 2y^3)\,dy = 1 \Rightarrow \frac{c}{6}\left[1-\frac{1}{2}\right] = 1$$ //

(5) if x and y are independant Rvs with variances 8 and 5. fin the variance of $3x+4y$

<u>Soln:</u>

Given $var(x) = 8$ and $var(y) = 5$

<u>To find:</u> $var(3x-4y)$

We know That $var(x \quad . by) = a\,var(x) + b\,var(y)$

$var(3x-4y) = 3\,var(x) + 4\,var(y) = (9)(8) + (16)(5) = 152$ //

(6) if x and y radom variables having The joint P.d.f.

$f(x) = \frac{1}{8}(6-x-y),\ 0<x<2,\ 2<y<4$ find $P(x<1, y<3)$

<u>Soln:</u>

$$P(x<1, y<3) = \frac{1}{8}\int_0^1 \int_2^3 (6-x-y)\,dy\,dx$$

$$= \frac{1}{8}\int_0^1 \left(\frac{7}{2} - x\right)dx = 3/8$$ //

(7) if The joint P.d.f of (x,y) is $f(x,y) = 1/4,\ 0 \le x, y \le 2$, find $P[x+y \le 1]$

<u>Soln:</u>

$$P[x+y \le 1] = \int_0^1 \int_0^{1-y} \frac{1}{4}\,dx\,dy$$

$$= \frac{1}{4}\int_0^1 (1-y)\,dy = 1/8$$ //

(8) The conditional p.d.f. of X and $Y=y$ given by $f\left(\frac{x}{y}\right) = \frac{x+y}{1+y} e^{-x}$, $0 < x < \infty$, $0 < y < \infty$, find $P[X<1/Y=2]$.

soln: when $y=2$ $f(x|y=2) = \frac{x+2}{3} e^{-x}$

$\therefore P[X<1/y=2] = \int_0^1 \frac{x+2}{3} e^{-x}\,dx = \frac{1}{3}\int_0^1 x e^{-x}\,dx + \frac{2}{3}\int_0^1 e^{-x}\,dx.$

$= 1 - \frac{4}{3} e^{-1}$ //

(9) The joint p.d.f of (X,Y) is given by $f(x,y) = 6e^{-(x+y)}$, $0 \le x$, $y \le \infty$ are X and Y independent?

soln: marginal density of x:

$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^{\infty} 6e^{-(x+y)}\,dy = e^{-x}, 0 \le x$

marginal density of y:

$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^{\infty} 6e^{-(x+y)}\,dx = e^{-y}, y \le \infty$

$\Rightarrow f(x)\, f(y) = f(x,y)$

(10) define co-variables:

soln: if x and y are two two r.v.s The co-variable between them is defined as $\text{cov}(x,y) = E\{X - E(X)\}\{Y - E(Y)\}$

(ie) $\text{Cov}(x,y) = E(XY) - E(X)\,E(Y)$

(11) State the properties of Co-variance;

Soln:

i) If x and y are two independent variables, the $cov(x,y) = 0$. But the converse need not be true

ii) $Cov(aX, bY) = ab\, Cov(X,Y)$

iii) $cov(X+a, Y+b) = cov(X,Y)$

iv) $Cov\left(\frac{X-\bar{X}}{\sigma_x}, \frac{Y-\bar{Y}}{\sigma_y}\right) = \frac{1}{\sigma^x \sigma^y} Cov(X,Y)$

v) $cov(aX+b, cY+d) = ac\, Cov(X,Y)$

vi) $Cov(X+Y, Z) = Cov(X,Z) + Cov(Y,Z)$

vii) $Cov(aX+bY, cX, dY) = ac\sigma_x^2 + bd\sigma_y^2 + (ad+bc)\, Cov(x,y)$

where $\sigma_x^2 = Cov(X,X) = Var(X)$ and $\sigma_x^2 = Cov(Y,Y) = var(y)$

(12) If X & y are independent R.v's, what are the value of $Var(X_1+X_2)$ and $Var(X_1-X_2)$

Soln:

$Var(X_1 \pm X_2) = Var(X_1) \pm var(X_2)$ (since X and y are independent Rv than $Var(aX \pm bX) = aVar(X) \pm bVar(X)$)

(13) X and Y are independent random variables with variances 2 and 3 find the variance $3X+4Y$

Soln:

Given $Var(X) = 2$, $Var(y) = 3$

we know that $Var(aX+y) = aVar(X) + Var(Y)$

And $var(aX + bY) = aVar(X) + bVar(Y)$

$Var(3X+4Y) = 3Var(X) + 4Var(Y) = 9(2) + 16(3) = 66$ //

(14) Define Correlation:

Soln:

The correlation between two Rvs X and y is defined as $E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f(xy)\, dx\, dy$

(15) Define un correlated

Soln:

Two Rvs are un correlated with each other, if the correlation between x and y is equal to the product of their means i.e, $E[XY] = E[X] \cdot E[Y]$

(16) If the joint pdf of (X, Y) is given by $f(x,y) = e^{-(x+y)}$, $x \geq 0$, $y \geq 0$. find $E(XY)$.

Soln:

$$E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f(x,y)\, dy\, dx$$

$$= \int_{0}^{\infty}\int_{0}^{\infty} xy e^{-(x+y)}\, dx\, dy$$

$$= \int_{0}^{\infty} x e^{-x}\, dx \int_{0}^{\infty} y e^{-y}\, dy = 1/1$$

(17) A R.V X is uniformly distributed over $(-1, 1)$ and $Y = x^2$. Check if X and Y are correlated?

Soln:

Given X is uniformly distributed in $(-1, 1)$, pdf of X is

$$f(x) = \frac{1}{b-a} = \frac{1}{2}, \quad -1 \leq x \leq 1$$

$$E(X) = \frac{1}{2}\int_{-1}^{1} x\, dx = 0 \text{ and } E(XY) = E(X^3) = 0$$

$\therefore Cov(x,y) = E(xy) - E(x)E(y) = 0 \Rightarrow r(x,y) = 0$

Hence X and Y are un correlated.

(18) X and Y are discreate random variable if $var(x) = Var(y) = \sigma^2$ $Cov(X,Y) = \frac{\sigma^2}{2}$. find $Var(2X-3Y)$

Soln:

$$Var(2X-3Y) = 4\,Var(X) + 9\,Var(Y) - 12\,Cov(x,y)$$

$$= 13\sigma^2 - 12\frac{\sigma^2}{2} = 7\sigma^2$$

(19) Two independant random variables. X and Y have 36 and 16. find The Correlation co-efficient between X+Y and X-Y.

Soln:

$$\therefore r(X+Y, X-Y) = \frac{\sigma_x^2 - \sigma_Y^2}{\sigma_x^2 + \sigma_Y^2} = \frac{36-16}{36+16} = \frac{20}{52} = \frac{4}{13}$$

(20) Define. Statistical Properties

Soln:

Two jointly distributed Rvs X and Y are statistical independent of each other if and only if the joint probability density function equals the product of the two marginal probability density function

ie., $f(x,y) = f(x).f(y)$

UNIT - 3

Markov Processes and markov chains

① Define Random processes, give an example.

Soln: A Random Process is a collection of R.V $\{x(s,t)\}$ That are functions of a real variable namely time t where $s \in S$ and $t \in T$

ex: $X(t) = A\cos(\omega t + \theta)$ wher θ is uniformly distributed in where A and ω are constants

② Define Stationary random Processes

Soln: If certain Probability distributions or averages donot depend on t, then The random Process $\{X(t)\}$ is called Stationary

③ Define first order Stationary Random Process

Soln: A random Processes $\{X(t)\}$ is said to be a first order SSS Process if $f(x_1, t_1 + \delta) = f(x_1, t_1)$ The first order density of a Stationary Process $\{X(t)\}$ is independent of time t.

④ Define Second order Stationary Random Process:

Soln: A RP $\{X(t)\}$ is said to be Second order SSS if $f(x_1, x_2, t_1, t_2) = f(x_1, x_2, t_1 + h, t_2 + h)$ where $f(x_1, x_2, t_1, t_2)$ is the Joint PDF of $\{X(t_1), X(t_2)\}$

(5) Define wide Sense Stationary Random Processes

Soln: A RP $\{x(t)\}$ is called WSS if $E\{x(t)\}$ is constant and $E[X(t)X(t+\tau)] = R(\tau)$ (i.e) ACF is a function of τ only

(6) Define evolutionary Random Process and give an example

Soln:
A Random Processes that is not stationary in any sense is called an Evolutionary Process.
(ex) Poisson process

(7) Define markov Process

Soln: If for $t_1 < t_2 < t_3 < t_4 \cdots\cdots < t_n < t$ then
$P(X(t) \le x / X(t_1) = x_1, X(t_2) = x_2 \cdots\cdots X(t_n) = x_n)$
$= P(X(t) \le x / X(t_n) = x_n)$ The n Process $\{X(t)\}$ is called a markov Process

(8) Define markov Chain:

Soln:
A Discreate Parameter markov Process is Called markov chain

(9) Define one step transition Probability

Soln: The one step probability $P[X_n = a_j / X_{n-1} = a_i]$ is called the one step probability from the state a_i to a_j at the n^{th} step and is denoted by $P_{ij}(n-1, n)$

(10) The one step tpm of a markov chain with states 0 and 1 is given as $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Draw the transition diagram. Is it irreducible. Markov chain?

Soln: yes it is irreducible since each state can be reached from any other state

(11) Prove that the matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is the tpm of an irreducible Markov chain

Soln: $P^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$ $P^3 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$

here $P_{11}^{(3)} > 0$, $P_{13}^{(2)} > 0$, $P_{21}^{(2)} > 0$, $P_{22}^{(2)} > 0$, $P_{33}^{(2)} > 0$ and for all other $P_{ij}^{(1)} > 0$ Therefore the chain is irreducible

(12) State any two properties of Poisson Process:

Soln: i) The poisson process is a Markov Process

ii) Sum of two independant poisson process is a Poisson Processes.

(13) Consider a markov chain with two states and transition probability matrix $P = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$. find stationary probabilities of chain

Soln: $(\pi_1, \pi_2) \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix} = (\pi_1, \pi_2)$ $\pi_1 + \pi_2 = 1$

$\frac{3}{4}\pi_1 + \frac{\pi_2}{4} = \pi_1 \Rightarrow \frac{\pi_1}{4} - \frac{\pi_2}{2} = 0$ $\therefore \pi_1 = 2\pi_2$

$\therefore \pi_1 = 2/3, \pi_2 = 1/3$

(14) Customers arrive a large store randomly at an average rate of 240 per hour. What is the probability of that during a two minute interval no one will arrive

Soln: $P(X(t) = n) = \frac{e^{-4t}(4t)^n}{n!}, n = 0, 1, 2, \ldots$

since $\lambda = \frac{240}{60} = 4$

$\therefore P(X(2) = 0) = e^{-8} = 0.0003$

(15) Define the Auto covariance of Poisson Process

Soln: $C(t_1, t_2) = R(t_1, t_2) - E[X(t_1)]\,E[X(t_2)]$

$= \lambda^2 t_1 t_2 + \lambda t_1 - \lambda^2 t_1 t_2 = \lambda t_1$ if $t_2 \geq t_1$

$\therefore C(t_1, t_2) = \lambda \min\{t_1, t_2\}$

(16) Define Ergodic State of a markov Chain

Soln: A non null presistent and aperiodic state is called an ergodic state.

(17) Define Ergodic Random Process

Soln: A random process $\{X(t)\}$ is said to be Ergodic Random Process if its ensembled averages to equal to appropriate time averages

(18) Define Absorbing State of a markov chain

Soln: A state i is called an absorbing state if and only if $P_{ii} = 1$ and $P_{ij} = 0$ for $i \neq j$

(19) Define Bernoulli Process:

Soln: The Bernoulli random variables is defined as $(X(t))$ which takes two values 0 and 1 with the time index t such that $\{X(t,s) : t = \ldots -1, 0, 1, \ldots ; s = 1, 0\}$

(20) Properties of Bernoulli Process:

Soln: (i) it is a Discreate process
(ii) it is a SSS Process
(iii) $E(X) = P$, $E(X_i^2) = P$ and $Var(X_i) = P(1-P)$

(21) Define Sine Wave Process:

Soln: A sine process is represented as $X(t) = A\sin(\omega t + \theta)$ where the amplitude A, or frequency ω or Phase θ or any combination of these three may be random

it is also represented as $X(t) = A\cos(\omega t + \theta)$