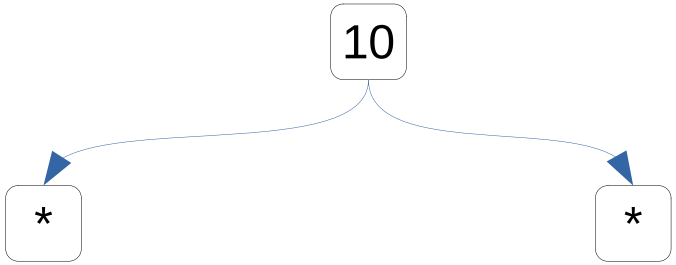
Tas

- Définition :

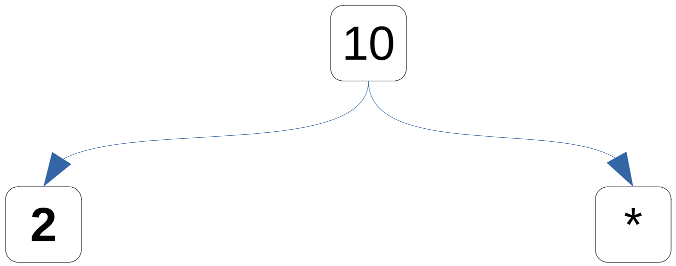
Tas

- Définition : Ensemble d'éléments auxquels sont associés des clés (entiers) structuré en arbre binaire complet tassé à gauche et tel que tout nœud possède une clé plus grande que son père

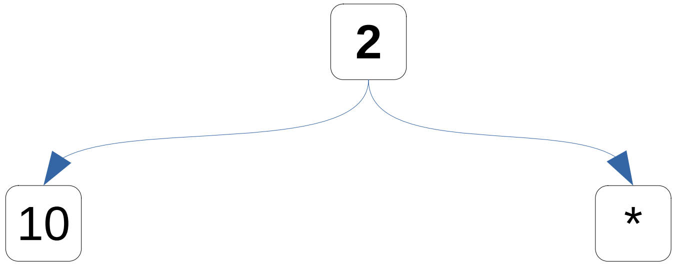
[10, 2, 5, 4, 7, 15, 1, 3]



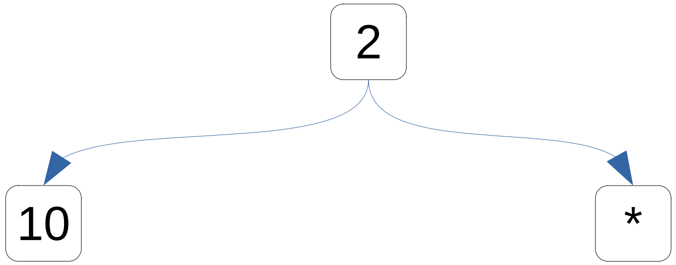
[10, 2, 5, 4, 7, 15, 1, 3]



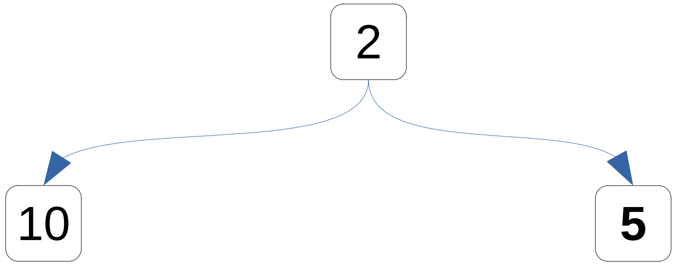
[10, 2, 5, 4, 7, 15, 1, 3]



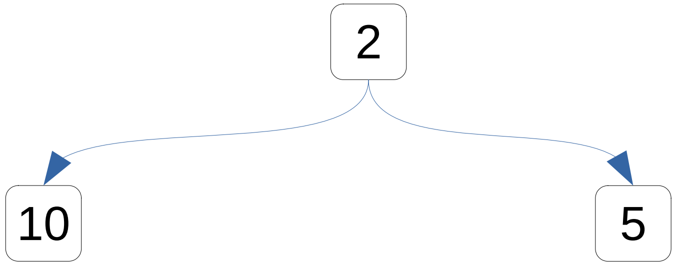
[10, 2, 5, 4, 7, 15, 1, 3]



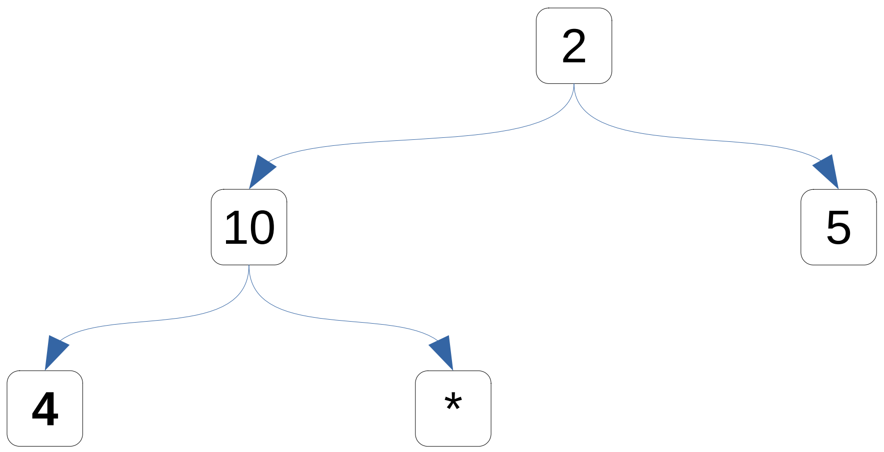
[10, 2, 5, 4, 7, 15, 1, 3]



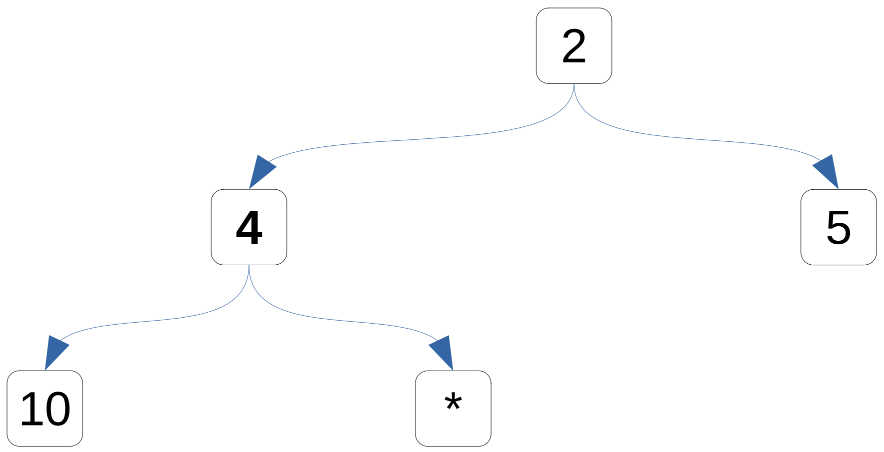
[10, 2, 5, 4, 7, 15, 1, 3]



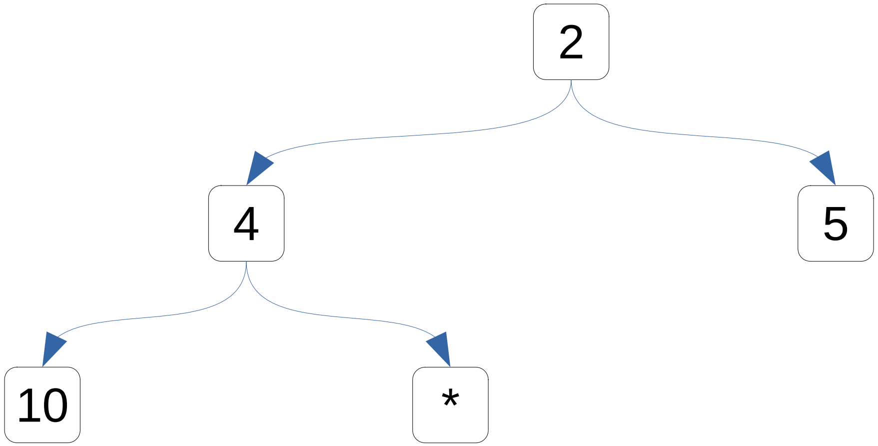
[10, 2, 5, 4, 7, 15, 1, 3]



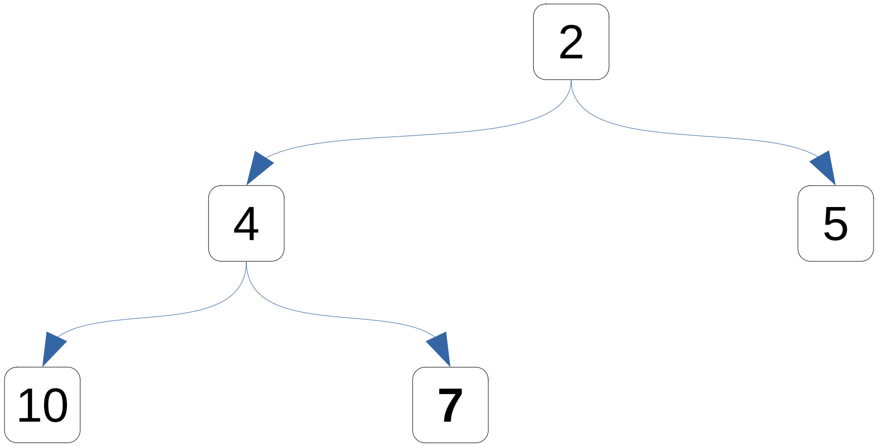
[10, 2, 5, 4, 7, 15, 1, 3]



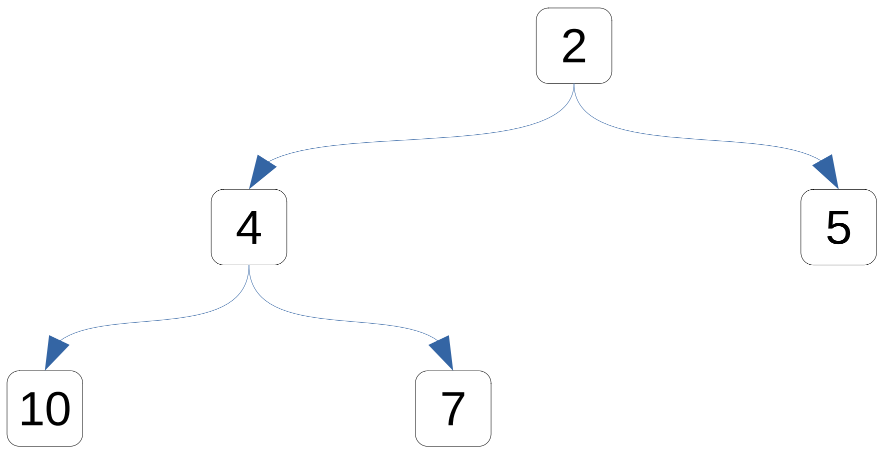
[10, 2, 5, 4, 7, 15, 1, 3]



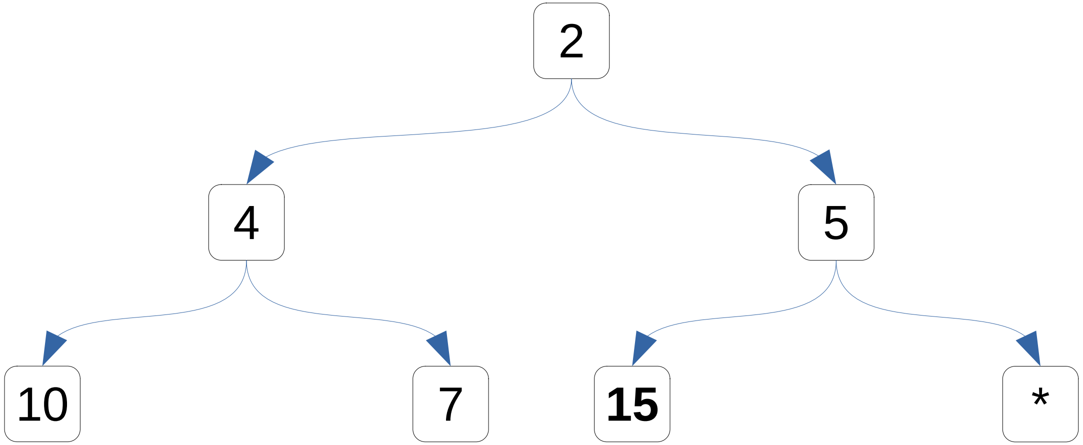
[10, 2, 5, 4, 7, 15, 1, 3]



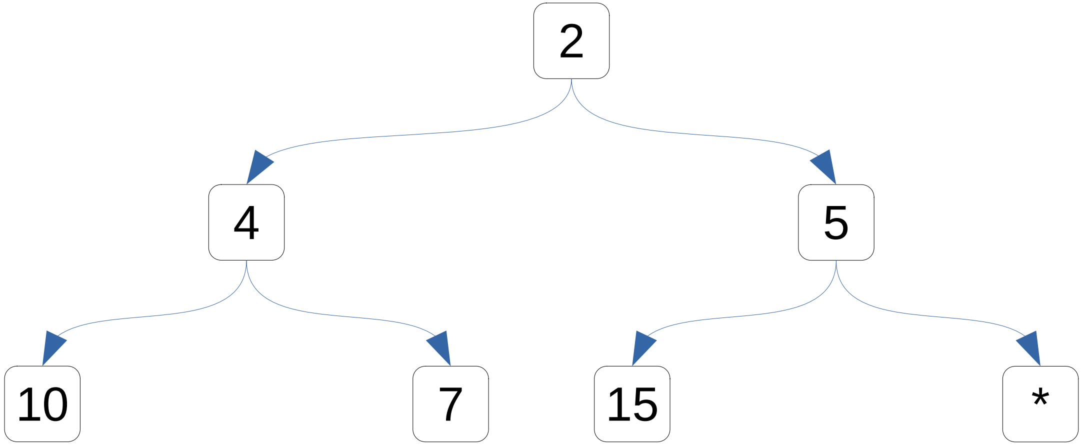
[10, 2, 5, 4, 7, 15, 1, 3]



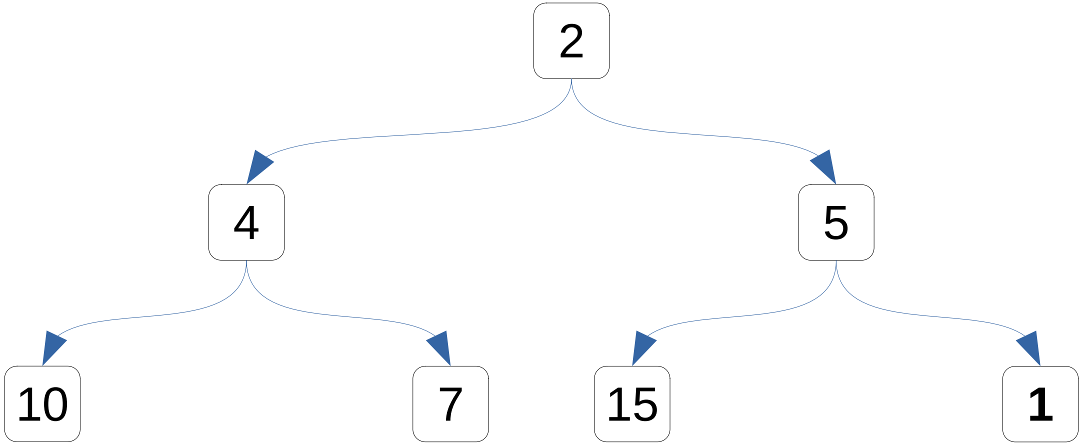
[10, 2, 5, 4, 7, 15, 1, 3]



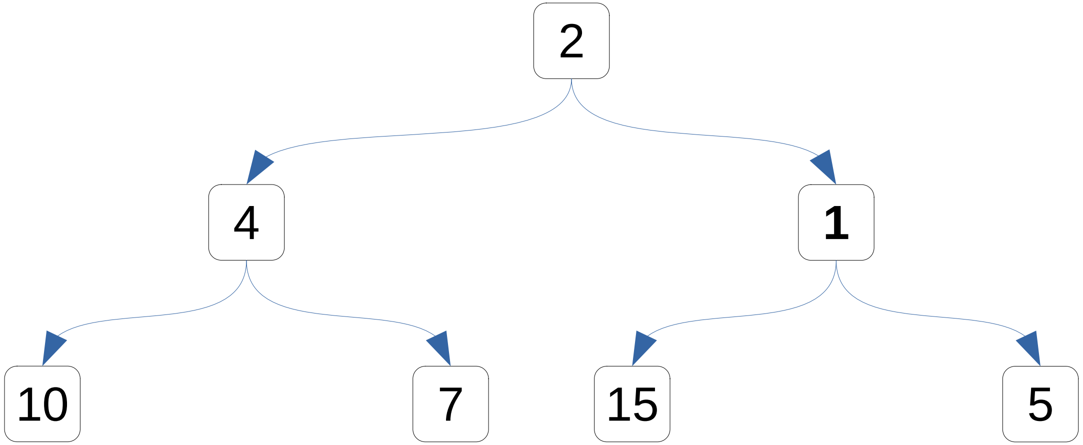
[10, 2, 5, 4, 7, 15, 1, 3]



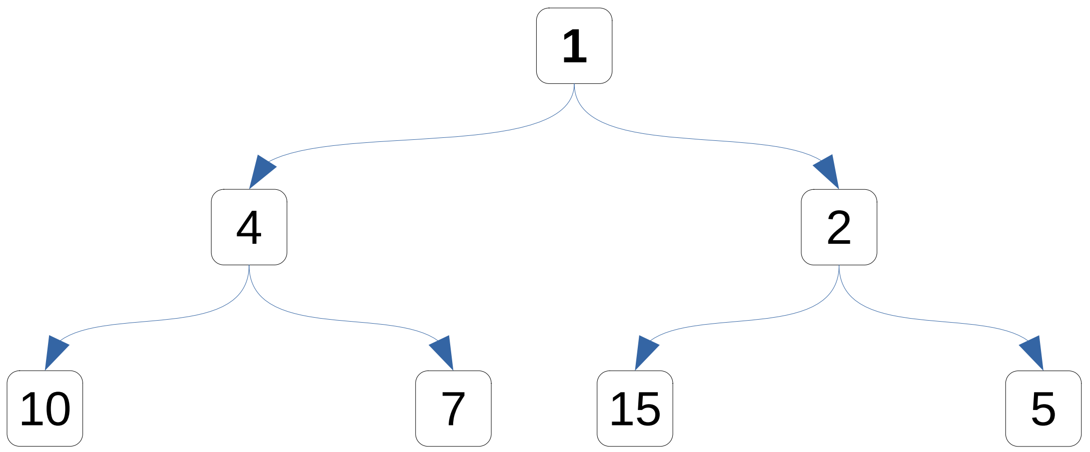
[10, 2, 5, 4, 7, 15, 1, 3]



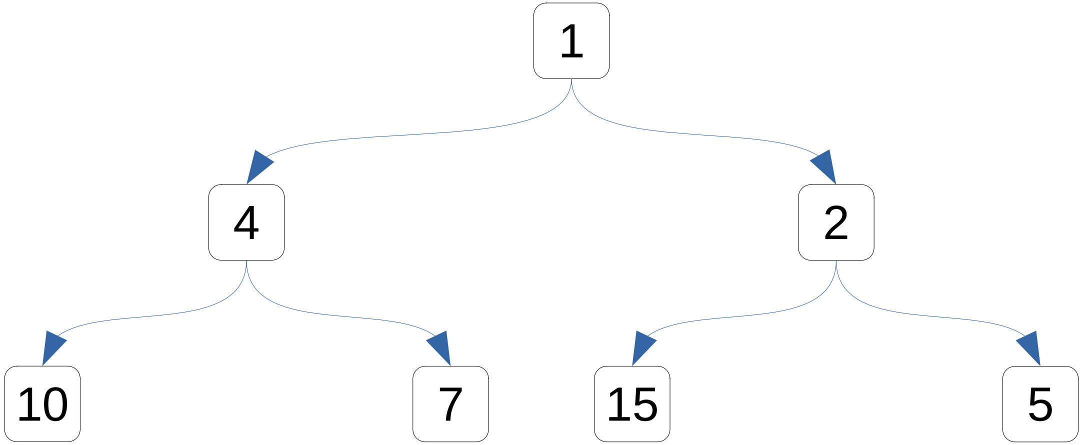
[10, 2, 5, 4, 7, 15, 1, 3]



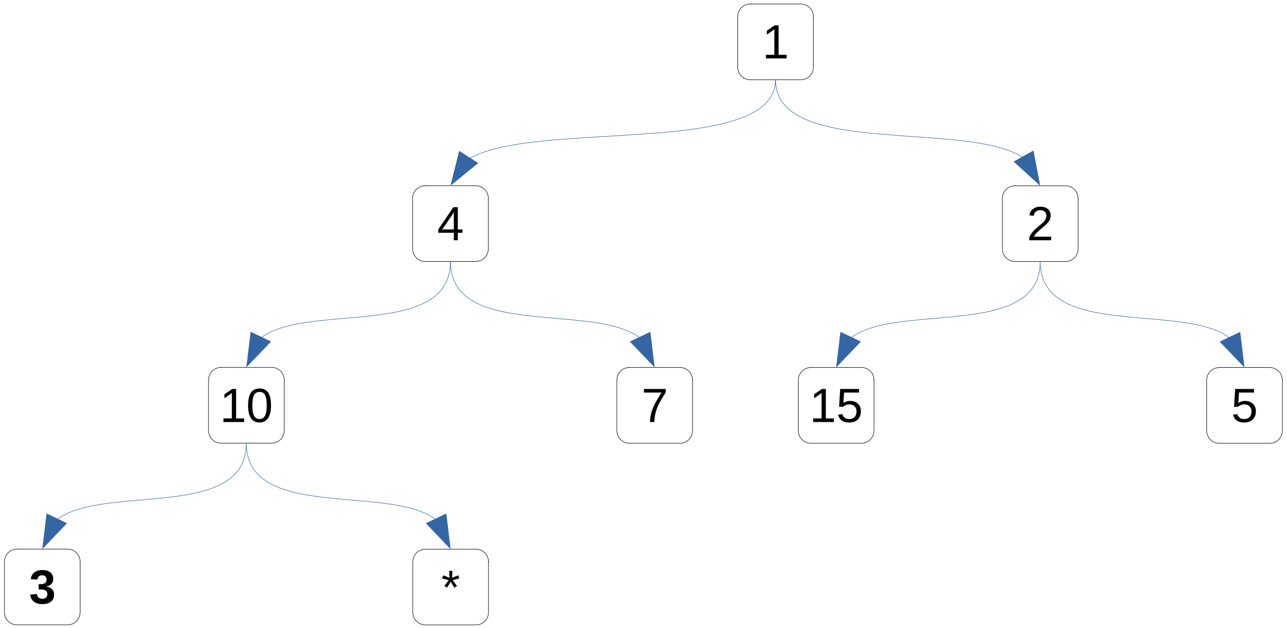
[10, 2, 5, 4, 7, 15, 1, 3]



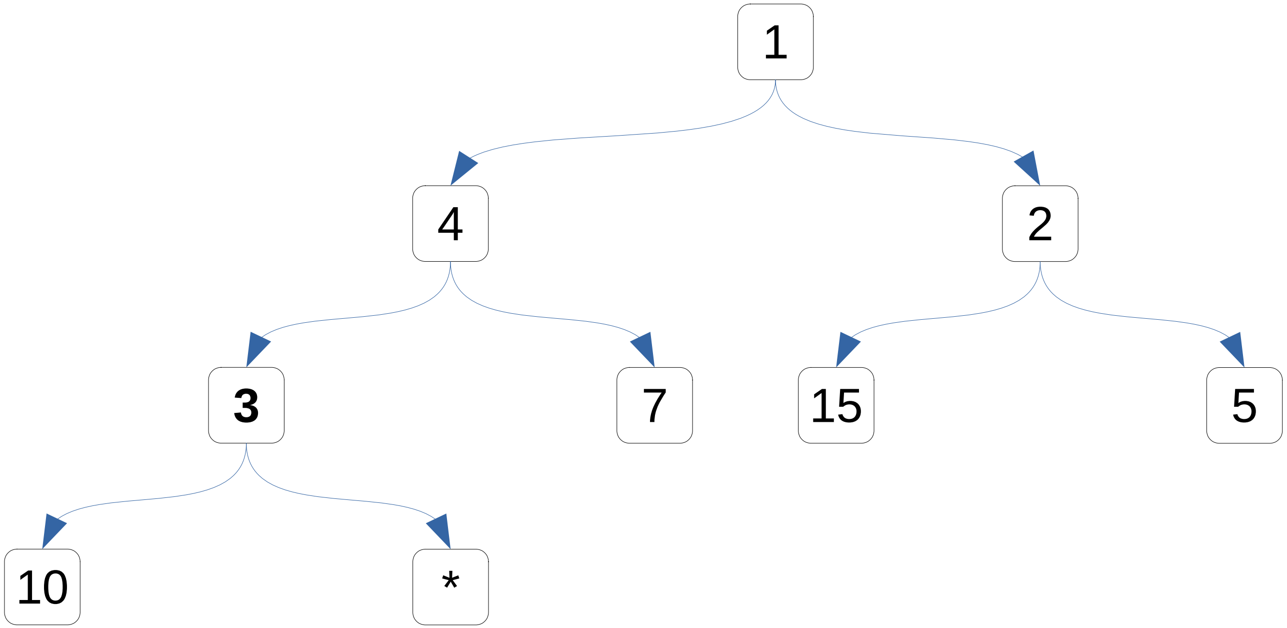
[10, 2, 5, 4, 7, 15, 1, 3]



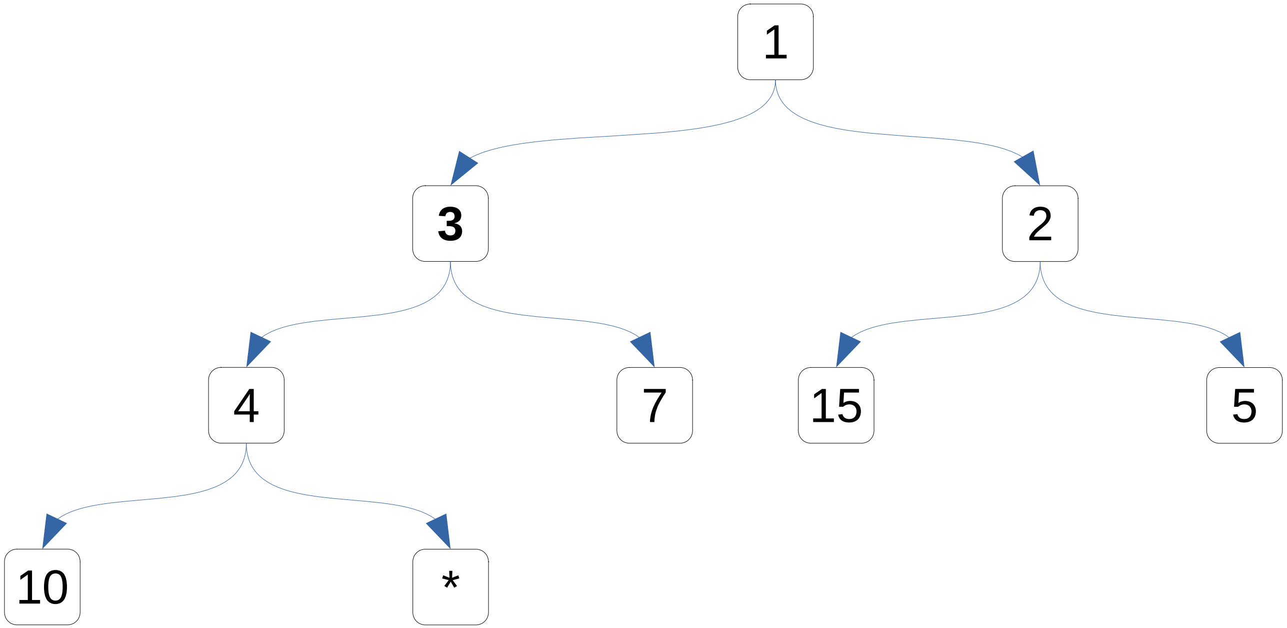
[10, 2, 5, 4, 7, 15, 1, 3]



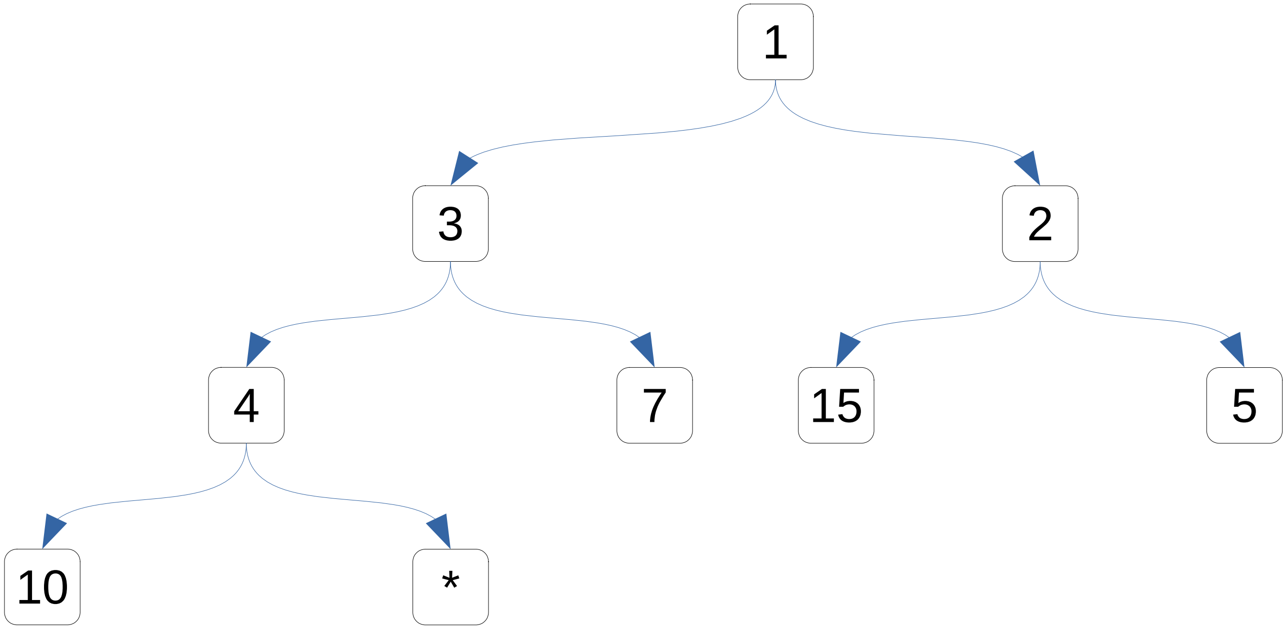
[10, 2, 5, 4, 7, 15, 1, 3]



[10, 2, 5, 4, 7, 15, 1, 3]

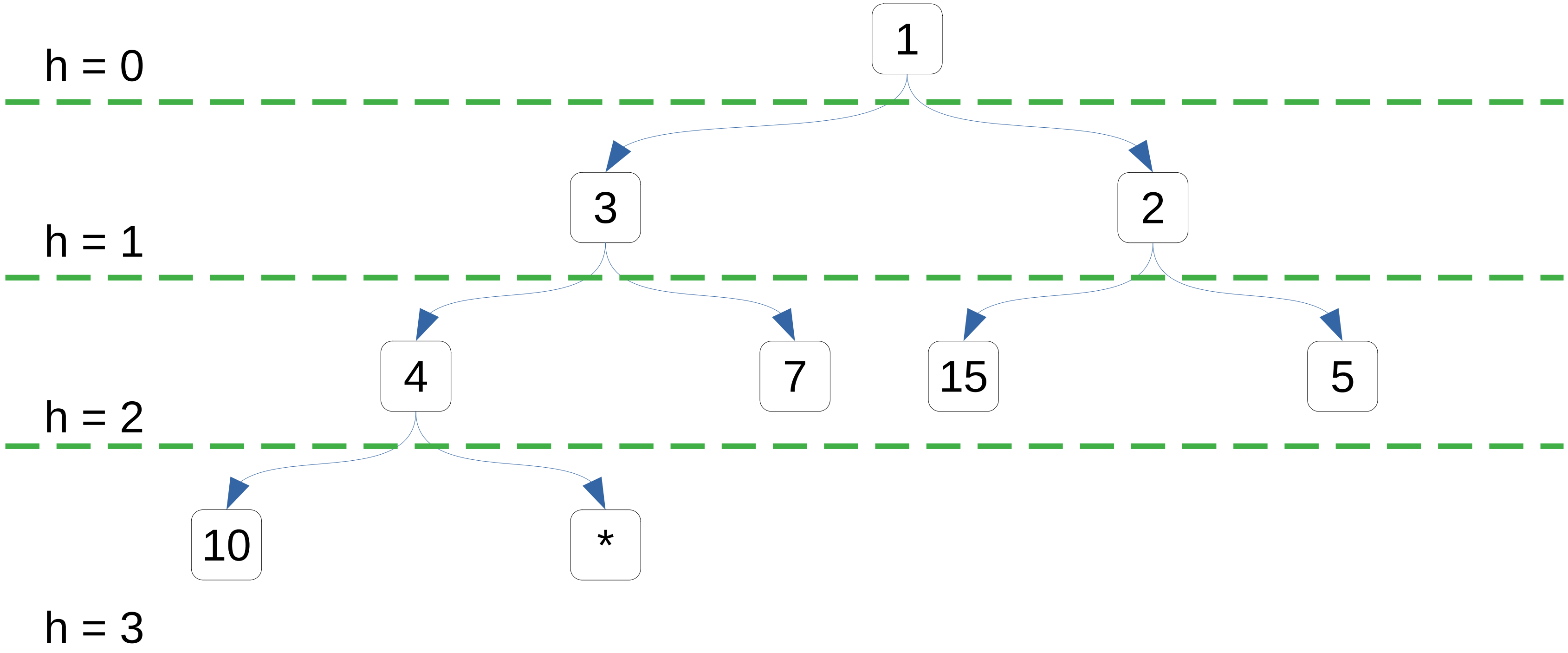


[10, 2, 5, 4, 7, 15, 1, 3]

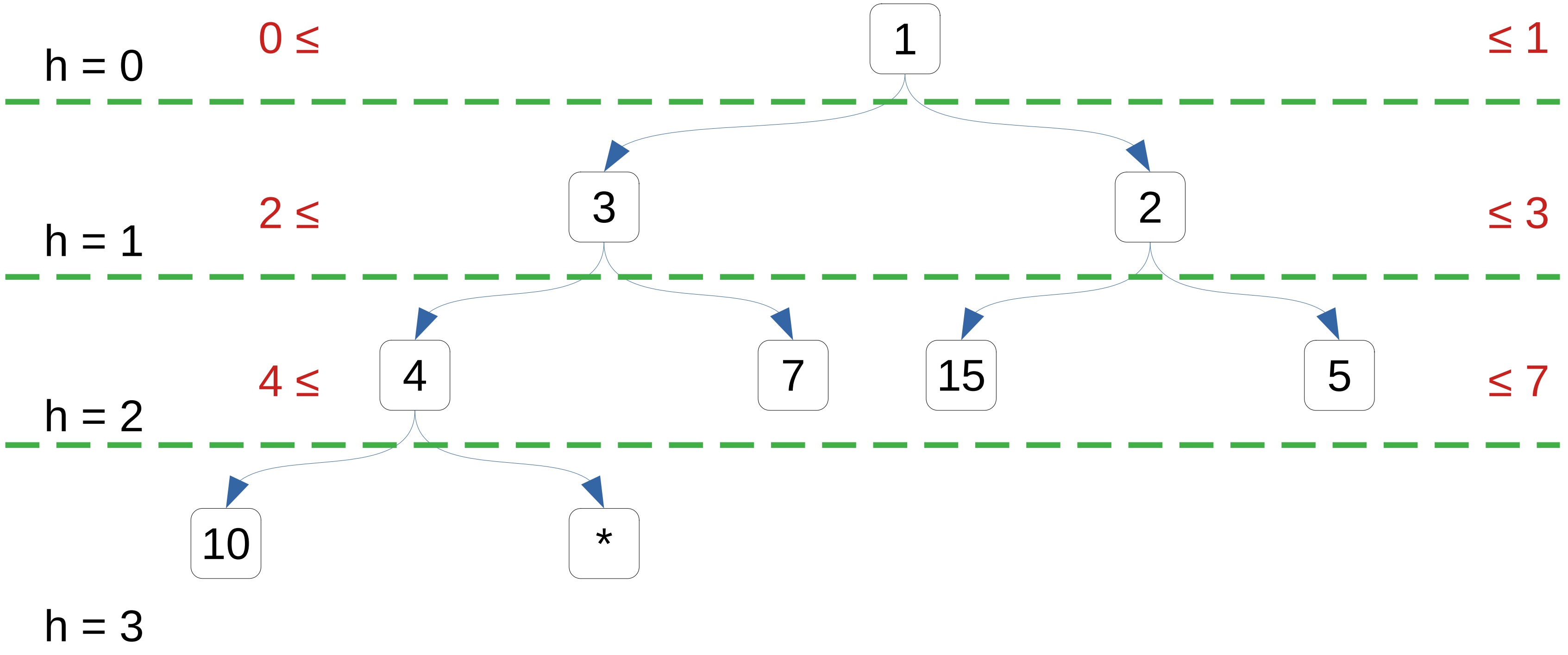


Nombre d’éléments d’un tas de hauteur h ?

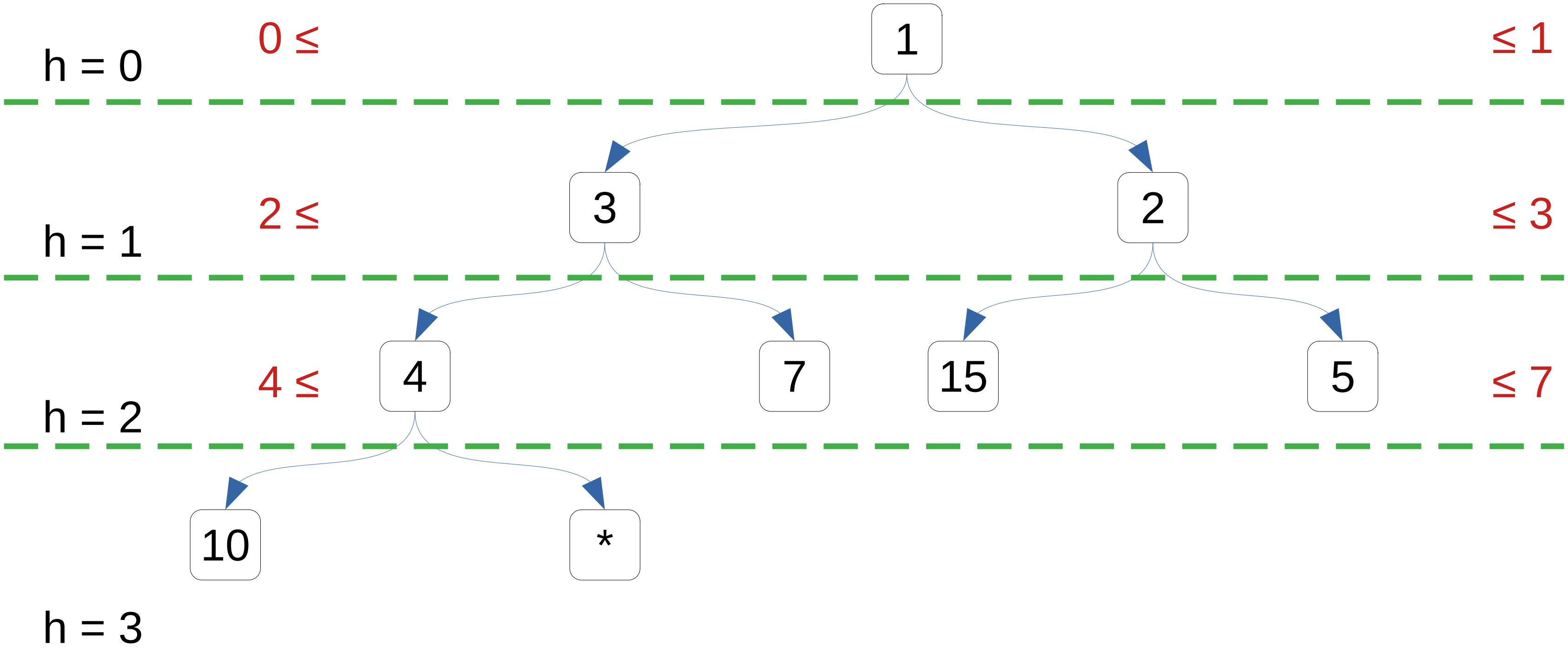
Nombre d'éléments d'un tas de hauteur h ?



Nombre d’éléments d’un tas de hauteur h ?



Nombre d’éléments d’un tas de hauteur h ?



$$2^h \leq n \leq 2^{h+1} - 1$$

Hauteur h en fonction de n

$$2^h \leq n \leq 2^{h+1} - 1$$

$$2^h \leq n \text{ et } n + 1 \leq 2^{h+1}$$

Hauteur h en fonction de n

$$2^h \leq n \leq 2^{h+1} - 1$$

$$2^h \leq n \text{ et } n + 1 \leq 2^{h+1}$$

$$\log(2^h) \leq \log(n) \qquad \log(n+1) \leq \log(2^{h+1})$$

$$h \log(2) \leq \log(n) \qquad \log(n+1) \leq (h+1)\log(2)$$

$$h \leq \log(n) \qquad \log(n+1) \leq h + 1$$

$$\log(n+1) - 1 \leq h$$

$$\lceil \log(n+1) - 1 \rceil \leq h \leq \lfloor \log(n) \rfloor$$

Hauteur h en fonction de n

$$2^h \leq n \leq 2^{h+1} - 1$$

$$2^h \leq n \text{ et } n + 1 \leq 2^{h+1}$$

$$\log(2^h) \leq \log(n) \qquad \log(n+1) \leq \log(2^{h+1})$$

$$h \log(2) \leq \log(n) \qquad \log(n+1) \leq (h+1)\log(2)$$

$$h \leq \log(n) \qquad \log(n+1) \leq h + 1$$

$$\log(n+1) - 1 \leq h$$

$$\lceil \log(n+1) - 1 \rceil \leq h \leq \lfloor \log(n) \rfloor$$

Démontrer que $\log(n) - (\log(n+1) - 1) < 1$

Hauteur h en fonction de n

$$2^h \leq n \leq 2^{h+1} - 1$$

$$2^h \leq n \text{ et } n + 1 \leq 2^{h+1}$$

$$\log(2^h) \leq \log(n) \qquad \log(n+1) \leq \log(2^{h+1})$$

$$h \log(2) \leq \log(n) \qquad \log(n+1) \leq (h+1)\log(2)$$

$$h \leq \log(n) \qquad \log(n+1) \leq h + 1$$

$$\log(n+1) - 1 \leq h$$

$$\lceil \log(n+1) - 1 \rceil \leq h \leq \lfloor \log(n) \rfloor$$

Démontrer que $\log(n) - (\log(n+1) - 1) < 1$

$$\log(n) - \log(n+1) < 0$$

$$\log(\, n/(n+1) \,) < 0$$

$$n/(n+1) < 1$$

$$n < n + 1$$

$$n = (n+1) - 1$$

Hauteur h en fonction de n

$$2^h \leq n \leq 2^{h+1} - 1$$

$$2^h \leq n \text{ et } n + 1 \leq 2^{h+1}$$

$$\log(2^h) \leq \log(n) \qquad \log(n+1) \leq \log(2^{h+1})$$

$$h \log(2) \leq \log(n) \qquad \log(n+1) \leq (h+1)\log(2)$$

$$h \leq \log(n) \qquad \log(n+1) \leq h + 1$$

$$\log(n+1) - 1 \leq h$$

$$\lceil \log(n+1) - 1 \rceil \leq h \leq \lfloor \log(n) \rfloor$$

Démontrer que $\log(n) - (\log(n+1) - 1) < 1$

$$\log(n) - \log(n+1) < 0$$

$$\log(\, n/(n+1) \,) < 0$$

$$n/(n+1) < 1$$

$$n < n + 1$$

$$n = (n+1) - 1$$

$$\lceil \log(n+1) - 1 \rceil = \mathbf{h} = \boldsymbol{\lfloor \log(n) \rfloor}$$

Complexité insertion

Pire cas ?

Complexité insertion

Pire cas ?

Insertion d'un nouveau min

→ h swaps

Donc ?

Complexité insertion

Pire cas ?

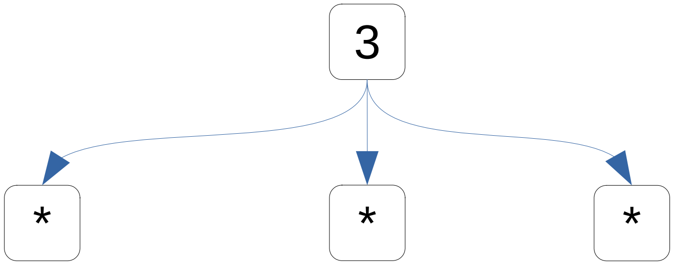
Insertion d’un nouveau min

→ h swaps

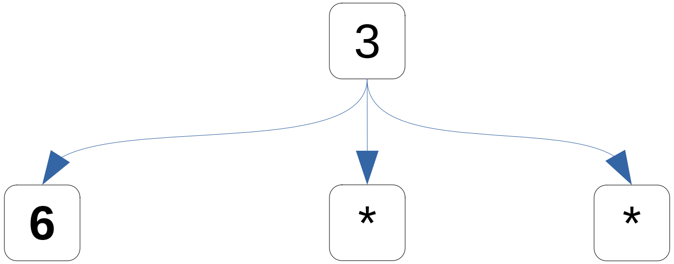
Donc ?

O(h) = O(log(n))

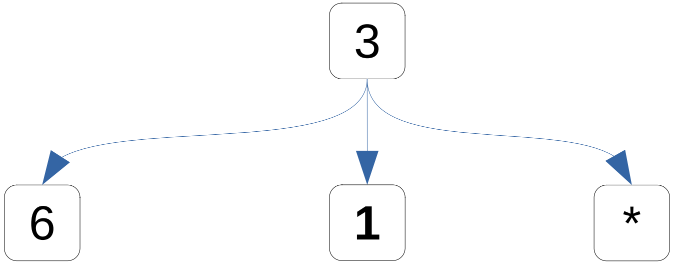
Tas ternaire
[3,6,1,13,17,18,2]



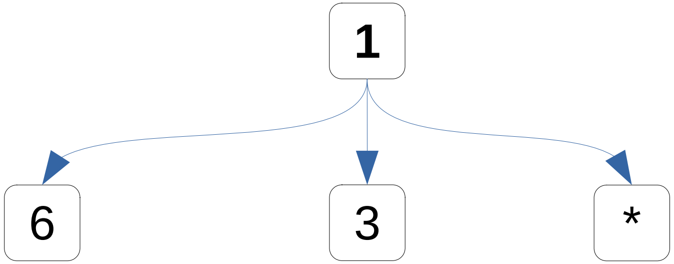
Tas ternaire [3,6,1,13,17,18,2]



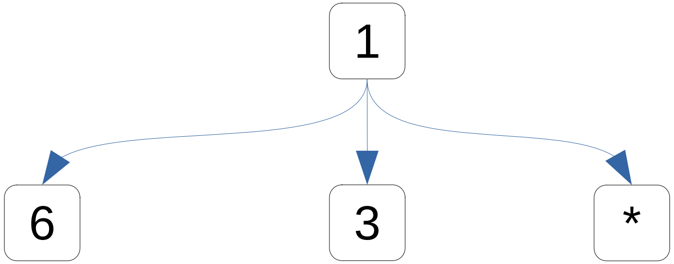
Tas ternaire
[3,6,1,13,17,18,2]



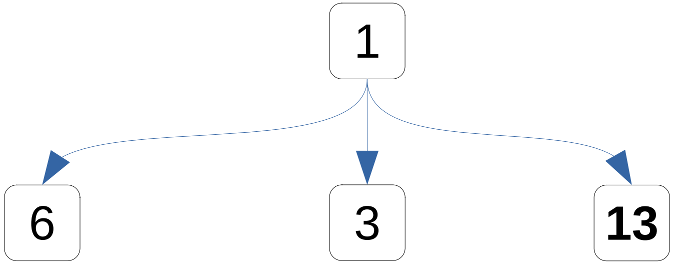
Tas ternaire [3,6,1,13,17,18,2]



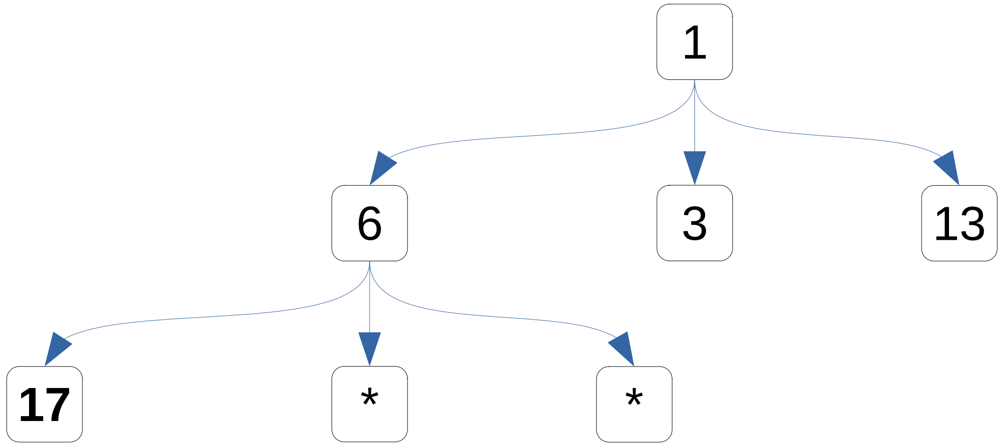
Tas ternaire [3,6,1,13,17,18,2]



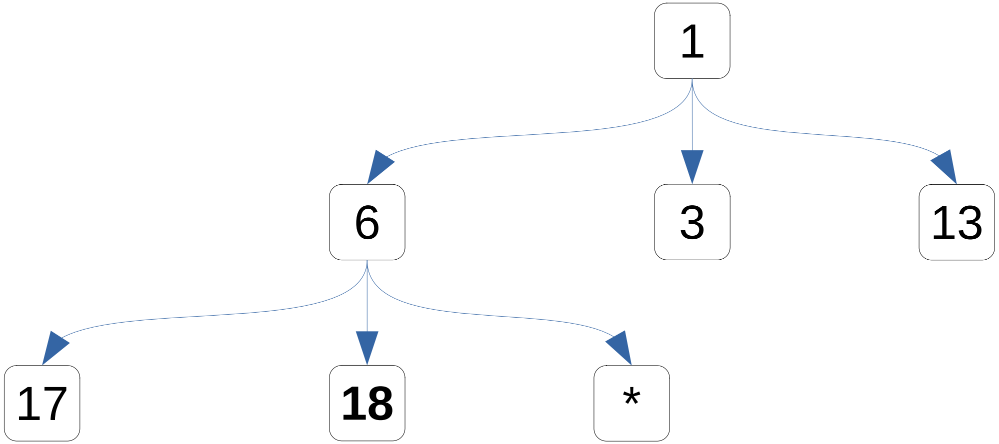
Tas ternaire [3,6,1,13,17,18,2]



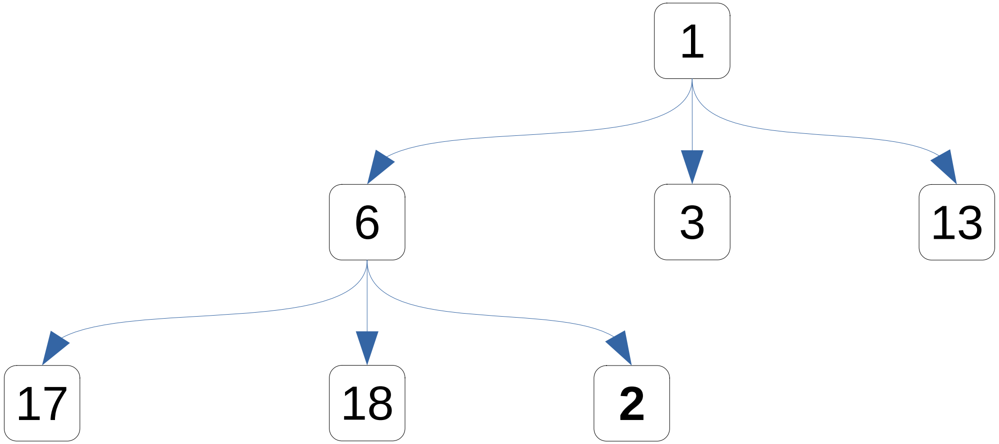
Tas ternaire
[3,6,1,13,17,18,2]



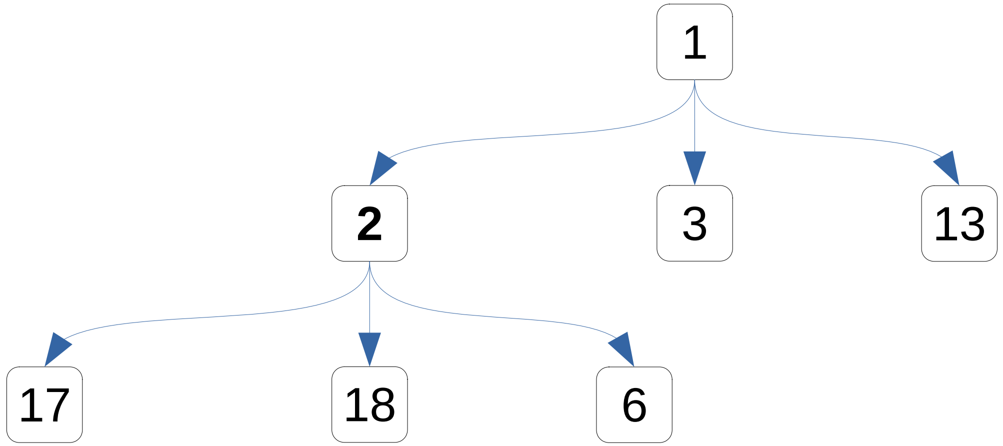
Tas ternaire
[3,6,1,13,17,18,2]



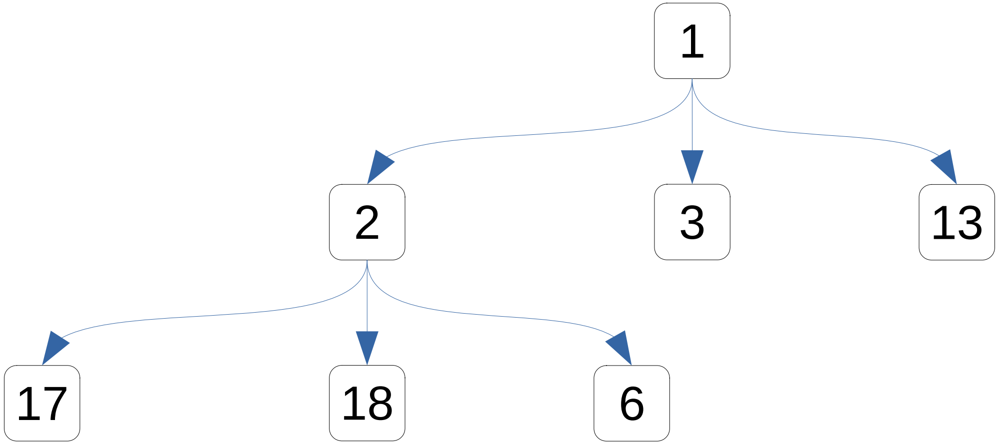
Tas ternaire
[3,6,1,13,17,18,2]



Tas ternaire
[3,6,1,13,17,18,2]

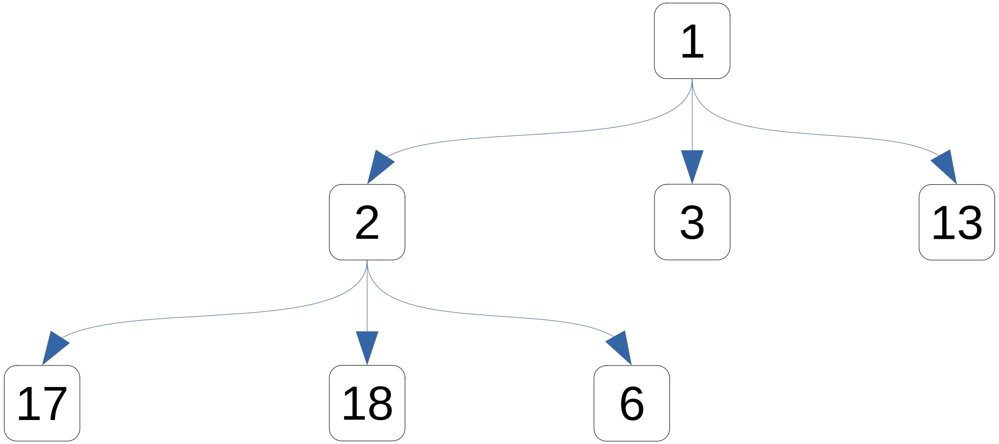


Tas ternaire
[3,6,1,13,17,18,2]



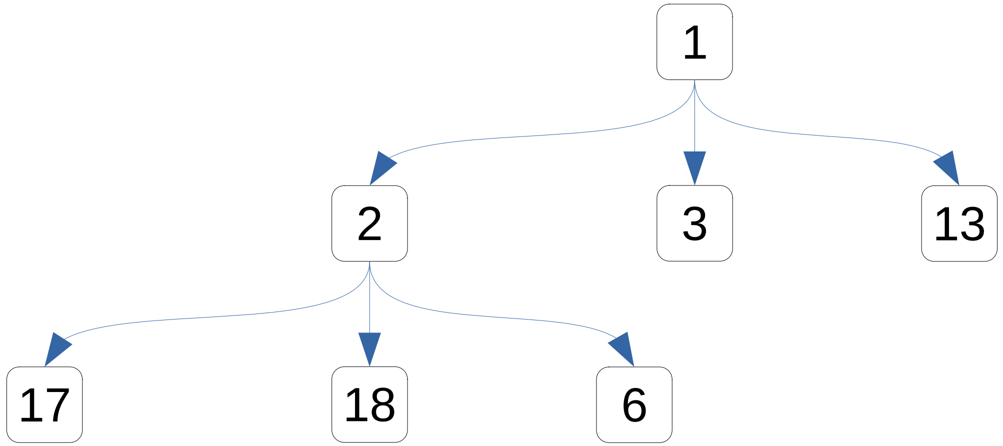
Hauteur ?
Complexité insertion ?

Tas ternaire
[3,6,1,13,17,18,2]



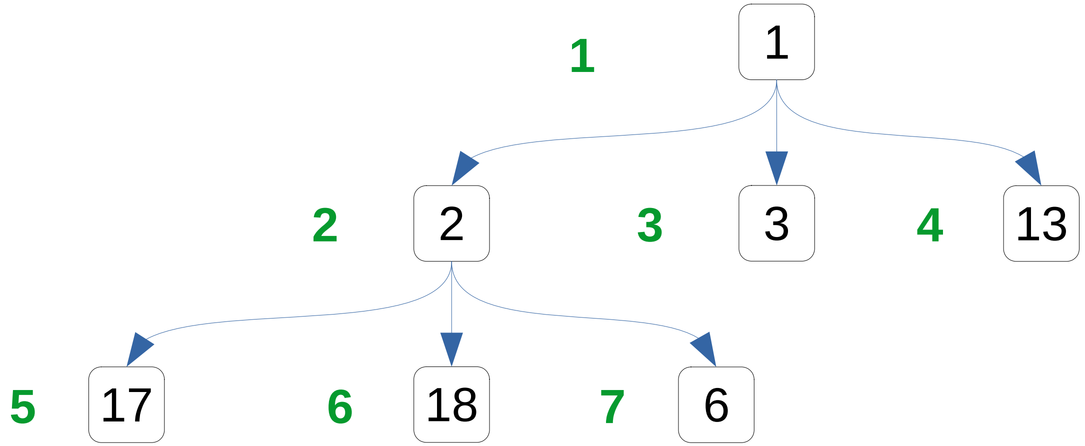
Hauteur ? **$h = \lfloor \log3(2n - 1) \rfloor$**
Complexité insertion ?

Tas ternaire
[3,6,1,13,17,18,2]



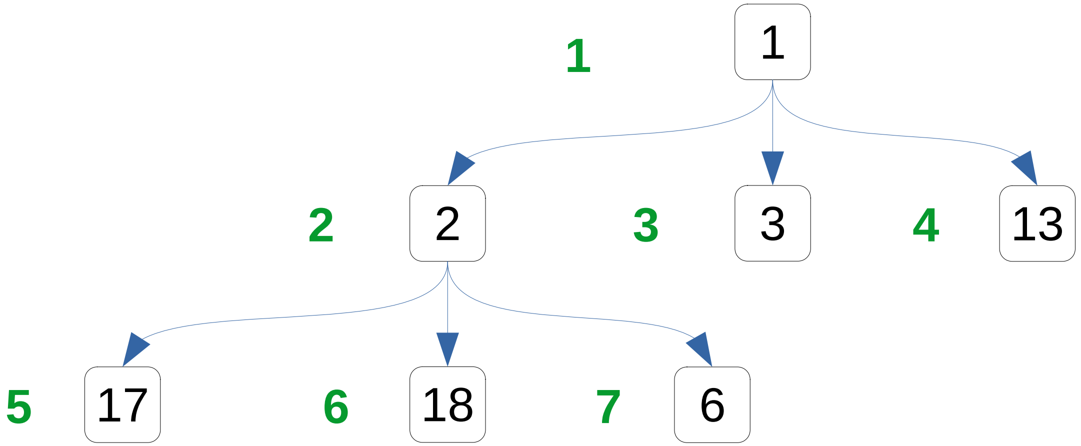
Hauteur ? **h = ⌊log3(2n - 1)⌋**
Complexité insertion ? **O(log3(n))**
Est-ce plus rapide ? **Non car O(log2(n)) ≡ O(log3(n))**

Tas ternaire
[3,6,1,13,17,18,2]



fg =
fm =
fd =
pere =

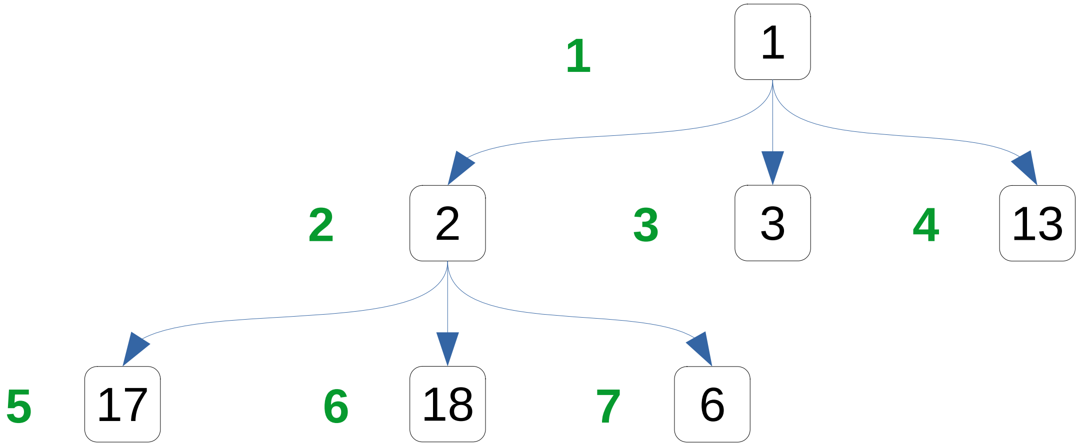
Tas ternaire
[3,6,1,13,17,18,2]



fg   = 3 * i – 1
fm   = 3 * i
fd   = 3 * i + 1
pere = round(i/3)

Stockage efficace ?

Tas ternaire
[3,6,1,13,17,18,2]



fg = 3 * i – 1
fm = 3 * i
fd = 3 * i + 1
pere = round(i/3)

Stockage efficace ? Tableau

Tas à 2 clés

- Quelle structure ?

Tas à 2 clés

- Quelle structure ? Utiliser un deuxième tableau allant de 0 à K tel que chaque case contienne l’emplacement de l’élément (f,i) du tableau codant le tas.
- Quelle structure pour (f,c) ?

Tas à 2 clés

- Quelle structure ? Utiliser un deuxième tableau allant de 0 à K tel que chaque case contienne l'emplacement de l'élément (f,i) du tableau codant le tas.
- Quelle structure pour (f,c) ? Table de hachage basée sur la valeur de c. Cependant les opérations passent en O(n) dans le pire cas.