Homework # 03
Power Systems Analysis II
EE 457 – Iowa State University

Instructor: Prof. Hugo N. Villegas Pico

Due date: March 03, 2020

The objective of this assignment is to practice basics of phasor calculations and rotor angle stability of synchronous machinery. Please, use the *publish* functionality of MATLAB to hand in any code related work. No screen captures of MATLAB code will be accepted. Code is expected to be carefully commented to earn credit. Please, upload your solutions to Canvas.

Warning: This homework is to be individually completed; no collaboration is permitted. Cheating will not be tolerated and reported to the Dean of Students Office.

Problem 1 (10 pts) Consider the following sets of balanced three-phase line-to-neutral voltages $v_{as}(t) = \sqrt{2}V\cos(\omega_e t + \phi_v)$, $v_{bs}(t) = \sqrt{2}V\cos(\omega_e t - 2\pi/3 + \phi_v)$, $v_{cs}(t) = \sqrt{2}V\cos(\omega_e t + 2\pi/3 + \phi_v)$ and line currents $i_a(t) = \sqrt{2}I\cos(\omega_e t + \phi_i)$, $i_b(t) = \sqrt{2}I\cos(\omega_e t - 2\pi/3 + \phi_i)$, $i_c(t) = \sqrt{2}I\cos(\omega_e t + 2\pi/3 + \phi_i)$. Assume the voltages and currents are measured at the same particular point of a grid with a solid neutral. Demonstrate that the instantaneous three-phase electric power:

$$P(t) = v_{as}(t)i_a(t) + v_{bs}(t)i_b(t) + v_{cs}(t)i_c(t) \tag{1}$$

at that point is constant, e.g., $P(t) = 3VI\cos(\phi_v - \phi_i)$. To that end, you may use the following trigonometric identity:

$$\cos(x)\cos(y) + \cos(x - 2\pi/3)\cos(y - 2\pi/3) + \cos(x + 2\pi/3)\cos(y + 2\pi/3) = 3/2\cos(x - y)$$

Problem 2 (10 pts) Using the information for Problem 1, write down the phasor abstractions of $v_{as}(t)$, $v_{bs}(t)$, and $v_{cs}(t)$ as well as of $i_a(t)$, $i_b(t)$, and $i_c(t)$. For example, $\tilde{V}_{as} = Ve^{j\phi_v}$ is the phasor abstraction of $v_{as}(t)$.

Problem 3 (10 pts) Using the information from Problem 2, calculate three-phase complex power, i.e.:

$$S = \tilde{V}_{as}\tilde{I}_a^* + \tilde{V}_{bs}\tilde{I}_b^* + \tilde{V}_{cs}\tilde{I}_c^* \tag{2}$$

Note that $(\cdot)^*$ represents the conjugate operator of a complex number. In addition, extract the real part of S, i.e., $P = \text{real}(S)$ which is commonly defined as active power. Discuss on similarities or differences between P of problem 3 and $P(t)$ of problem 1. Why?

Problem 4 (10 pts) Assume $V = 7.5\,\text{kV}$, $I = 1\,\text{kA}$, $\phi_v = 0$, and $\phi_i = -\pi/6\,\text{rad}$. What are $P(t)$ and P for these numerical values?

Problem 5 (10 pts) Assume a voltampere-base $S_{3b} = 100\,\text{MVA}$ and a line-to-neutral voltage base $V_b = 7.5\,\text{kV}$. Find the current base I_b. Also, by using this information per-unitize the phasors $\tilde{V}_{as}$ and $\tilde{I}_{as}$, e.g., $\tilde{V}_{as,u} = \tilde{V}_{as}/V_b$.

Problem 6 (10 pts) Calculate the per unit active power of one-phase $P_{u,1} = \text{real}(\tilde{V}_{as,u}\tilde{I}_{a,u}^*)$ from the results of Problem 5. Also, calculate the per-unit three-phase powers $P_u(t) = P(t)/S_{3b}$ and $P_u = P/S_{3b}$ using the results from problem 4. Are the numerical values of the per unit powers $P_{u,1}$, $P_u(t)$, P_u, equal or different? why?

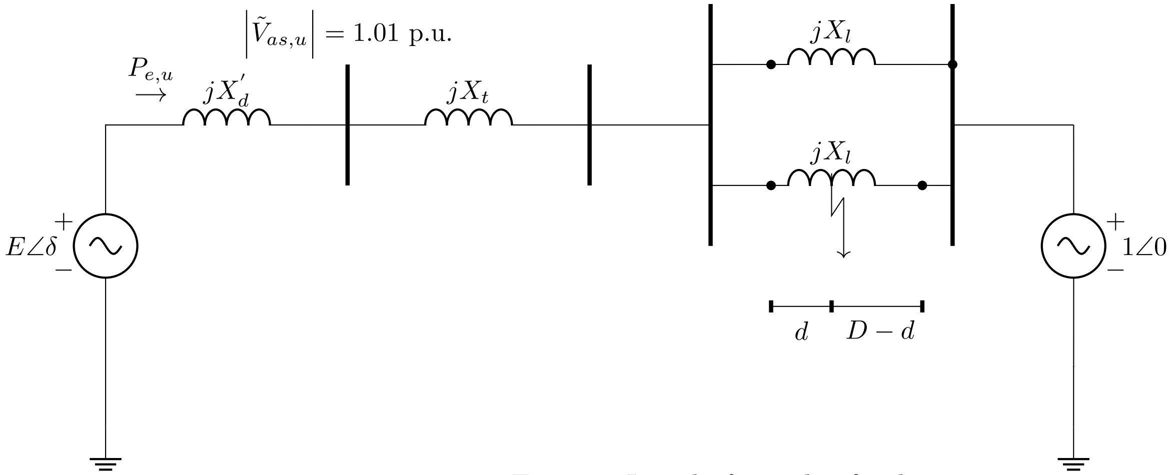
Problem 7 (40 pts) Consider a synchronous machine connected to a stiff grid represented by a generator with infinite inertia and fixed terminal voltage, i.e., $1.0e^{j0}$ p.u. The voltage magnitude at the terminals of the synchronous machine is assumed to be equal to $\left|\tilde{V}_{as,u}\right| = 1.01$ p.u. just before a fault occurs. The per-phase circuital illustration of this interconnection is shown in Fig. 1. Relevant per-unit parameters of the system are $H = 5\,\text{s}$ (machine inertia constant), $X_d' = 0.2$ p.u. (machine transient reactance), $X_t = 0.1$ p.u. (transformer reactance), and $X_l = 0.4$ p.u. (transmission line reactance.)

You are the best power plant engineer and you are in charge of elucidating whether the machine will remain stable for different loading conditions and different locations of a solid three-phase fault as shown in Fig. 1. Consider that the electric power out of the machine could be $P_{e,u} \in \{0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$ p.u. The distance $d \in \{0.1, 0.2, 0.3, \ldots, 0.9\}$ models a percentage of the physical distance from the sending end bus to the fault location and $D = 1$. The fault is cleared in all instances in 15 cycles of 60 Hz. During a solid three-phase fault, the circuit can be modeled as the one illustrated in Fig. 2. Note that the transmission line is split into two reactances as function of distance. Assume the fault is cleared by opening the circuit breakers at both ends of the line, hence resulting in the circuit of Fig. 3.

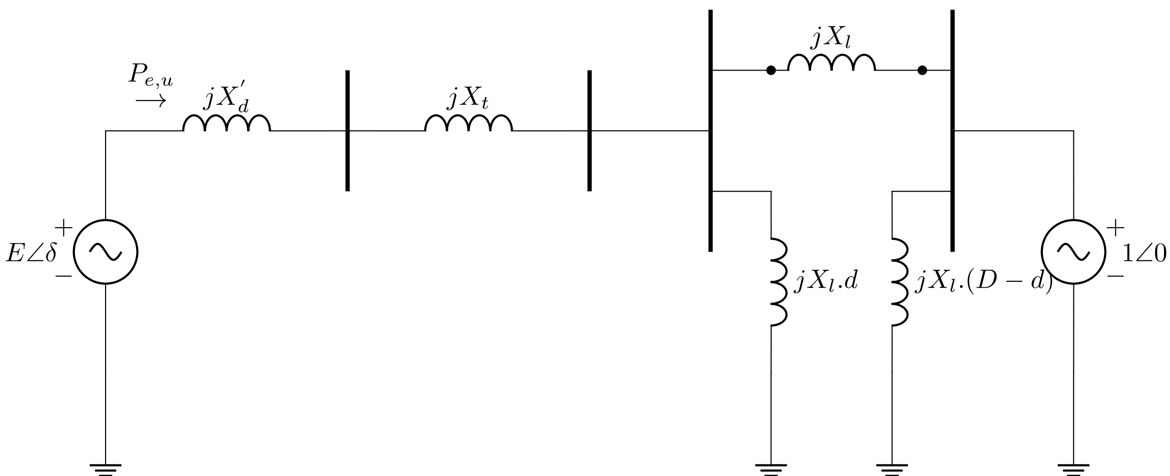
Table 1: Rotor Angle Stability Report

	$P_{e,u} = 0.5$ p.u.	$P_{e,u} = 0.6$ p.u.	$\dots$	$P_{e,u} = 1.0$ p.u.
$d = 0.1$	S or U?			
$d = 0.2$				
$\vdots$				
$d = 0.9$				

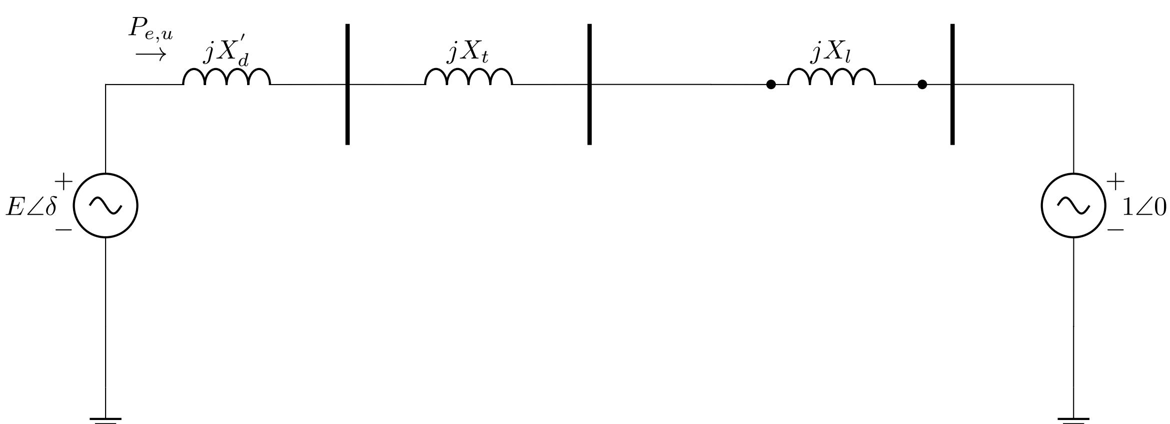
Using the aforementioned information, assemble a dynamic model and MATLAB simulation code to study the rotor speed and relatively rotor angle dynamics. Use this to conduct transient stability analysis of all possible scenarios for the machine loading $P_{e,u}$ and fault distance d. To earn credit construct a two entry table as shown in Table 1 which specifies whether a particular combination $(d, P_{e,u})$ yielded S: Stable or U: Unstable transient simulation in the sense of Lyapunov. Also depict the simulation results of one case you consider is the worst (according to your judgment). Illustrate the circle with radius ϵ you used to ascertain stability in the sense of Lyapunov, use your judgement to decide the center and radius of the circle. Please, think of how you may program the computer so that it makes the job for you.



$Fig\ 1:\ Just\ before\ the\ fault$



$Fig\ 2:\ During\ the\ fault$



$Fig\ 3:\ After\ the\ fault\ is\ cleared$