SUB. – CS430 INFORMATION & CODING THEORY LAB.

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING,

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QUESTIONS FOR ICT - LAB PRACTICE

1. **Implement a random number generator.**
   Solution

2. **Given a source that emits a sequence of symbols following a specific probability distribution from some finite set of alphabets, WAP that computes the self information of each symbol and the entropy of the source.**
   Solution

3. **Write code to calculate entropy of each symbol from an input file.**
   Solution

4. **WAP to check if a given code is**
   a. **uniquely decodable or not**
   b. **instantaneous or not**

   Solution

5. **WAP to check if a given code satisfies Kraft's inequality.**
   Solution

6. **WAP to implement Run length coding.**

7. **WAP to implement Huffman coding. Also compute the average code length and the efficiency of the code generated using Huffman algorithm.** Solution
8. **WAP to implement Arithmetic Coding.**
   Solution

9. **WAP to implement Shannon Fano coding.**
   Solution

10. **WAP to implement Lempel Ziv Welch encoding.**
    Solution

11. **Implement Lempel-Ziv Decoder and combine Huffman and Lempel-Ziv encoders to form a combined encoder.**
    Solution

12. **Implement Linear Block Code Generator. Read the Generator Matrix from a file.**
    Solution

13. **WAP to implement a syndrome-decoding scheme for (n, k) binary linear code.**
    Solution

14. **WAP to find generator matrix G over GF(2) if the parity check matrix is given.**
    Solution

15. **WAP to find the minimum distance of a binary linear block code for a given generator matrix for that code.**
    Solution

16. **WAP to implement Cyclic Redundancy Check (CRC) codes.**
    Solution

17. **WAP to implement Hamming Codes.**
    Solution

18. **WAP to find the minimum distance of a cyclic code over GF(2) for a given generator polynomial for that code.**
    Solution

19. Given symbols and their probabilities, Huffman source coding was done on those symbols and code efficiency was found to be 0.977195454. Now apply Shannon-Fano source encoding and find the code-efficiency of this algorithm on the same given example. Observe the differences between the both source encoding schemes.

Symbols and their Probabilities –

A	B	C	D	E
0.4	0.19	0.16	0.15	0.1

Given –

Efficiency for Huffman Encoding = 0.977195454

H(x) = Source Entropy = 2.14983

Solution

20. Huffman coding. Consider the random variable X = x1 x2 x3 x4 x5 x6 x7 [0.50 0.26 0.11 0.04 0.04 0.03 0.02]

(a) Find a binary Huffman code for X.

(b) Find the expected code length for this encoding.

(c) Extend the Binary Huffman method to Ter-nary (Alphabet of 3) and apply it for X.

Solution

21. WAP to calculate the minimum hamming distance, and validate that minimum hamming distance is equal to minimum hamming weight.

(Solution same as Q15. Validation part left for the reader)

22. Implement randomized Shannon Fano and compare it with normal Shannon Fano.

(In randomized shannon-fano, we randomize the splitting point of the probability array at each step. Rest is the same as in normal shannon-fano. The comparison part is left for the reader.)

23. Given the values of k, n and generator matrix for binary code. A bit stream input is given to the program as channel encoded data. Decode this data and obtain uncoded data.

(Same as Q13)

24. **Let G** $¿\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ **be the generator matrix for [2, 3] ternary code (C). Find all the possible encoded messages**

(Q-24) $G = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ for a (2,3) ternary code

Here,

$k = 2$, $n = 3$

For a ternary code, $q = 3$

∴ Possible Uncoded Messages,

~~M = {0 0 0, 0 0 1~~

$M = \{00, 01, 10, 11, 02, 20, 22, 21, 12\}$

∴ Possible encoded messages,

$C = mG$

$C = \{000, 012, 210, 222, 021, 120, 111, 102, 201\}$

How is it done –

Let $m = [2 \ 2]$

∴ Corresponding Codeword,

$$C = [2 \ 2] \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} = [(4+0)\%3 \ \ (2+2)\%3 \ \ (0+4)\%3]$$

$$= [1 \ 1 \ 1]$$

25. Derive a generator matrix for (2, 2) ternary code, and then find all possible uncoded and the encoded messages.

Q 25) A generator matrix of (2, 2) ternary code could be

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

For $k = 2$,
Possible Uncoded Messages,

$M = \{00, 01, 10, 11, 02, 20, 22, 21, 12\}$

Encoded messages can be found by applying

$C = mG$

$\therefore$ Possible encoded messages :-

$C = \{00, 01, 10, 11, 02, 20, 22, 21, 12\}$

26. Let C=(2,2) be a cyclic code over F(2). Try to detect and correct single-bit errors for this code structure.

Q26) $C = (2,2)$ cyclic code over $F(2)$

For $C = (2,2)$, $n = 2$, $k = 2$

So, the modulo polynomial is:

$f(x) = (x^2 - 1) = (1+x)(1+x) \cdot 1$

$\therefore$ We have 3 choices for $g(x)$,

$g_1(x) = 1$

$g_2(x) = 1+x$

$g_3(x) = (1+x)^2$

We know,

degree of $g(x) = n-k = 2-2 = 0$

$\therefore$ We can only choose $g(x) = 1$

With $g(x) = 1$, no coding is done.

So, we cannot detect or correct errors

27.

11 Let C be the $(5,3)$ linear code with generating matrix

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

Assign letters to the words in K^3 as follows:

000	100	010	001	110	101	011	111
A	B	E	H	M	R	T	W

Using the generating matrix $\mathbf{G}$, encode the message BETHERE.

Q27)

$C(5,3)$

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$n = 5,\ k = 3,\ q = 2$

Also, we have source encoding as:

$A \longrightarrow 000$
$B \longrightarrow 100$
$E \longrightarrow 010$
$H \longrightarrow 001$
$M \longrightarrow 110$
$R \longrightarrow 101$
$T \longrightarrow 011$
$W \longrightarrow 111$

We have, message,
$m =$ BETHERE

So, the corresponding source code for this message is:

$$C = \underbrace{100}_{B}\underbrace{010}_{E}\underbrace{011}_{T}\underbrace{001}_{H}\underbrace{010}_{E}\underbrace{101}_{R}\underbrace{010}_{E}$$

For channel coded data, we have
$k = 3$
So, we will break this message in chunks of 3 bits and encode them to get the channel coded message.

$\therefore c = 100010011001010101010$

First Chunk,

$c_1 = 100$

Corresponding channel code

$C_1 = 10110 = c_1 G$

Second Chunk,

$c_2 = 010$

Corresponding channel code

$C_2 = 01011 = c_2 G$

Similarly we can apply this process to all chunks and make the final channel code,

$$C = 1011001011011100010101011$$
$$1001101011$$

This is our final channel code.

It can be visualized as,

$$\underbrace{10110}_{100}\underbrace{01011}_{010}\underbrace{01110}_{011}\underbrace{00101}_{001}\underbrace{01011}_{010}\underbrace{10011}_{101}\underbrace{01011}_{010}$$

28.

12 Suppose C is a $(7,4)$ linear code with generating matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Calculate the codeword corresponding to the message word 0111. Which message word corresponds to the codeword 1011000?

Q28. $C = (7,4)$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

For $m = [0\ 1\ 1\ 1]$

The corresponding codeword,

$C = mG$

$C = 0111001$

From the generator matrix, the parity check matrix can be obtained, as the generator matrix is in systematic form, $G = [I \mid P]$

$\therefore H = [P^T \mid I]$

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

For the given codeword, $C = 1011000$

If this codeword has no errors, then $CH^T = 0$

Let's check,

$$CH^T = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$CH^T = \begin{bmatrix} (1+1+0)\%2 & (0+1+1)\%2 & (1+0+1)\%2 \end{bmatrix}$$

$$CH^T = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} = 0$$

$\therefore$ This codeword is error free

The uncoded message can be obtained by selecting the first k-bits of the codeword, as the generator matrix is systematic in form $G = [I \,\vdots\, P]$

$\therefore$ For $C = 1011000$, $k = 4$, $n = 7$

The uncoded message is: $\underline{m = 1011}$

29.

13. Let $C = \{0000, 0011, 1100, 1111\}$, so C is a $(4, 2)$ linear code. Verify that

$$\mathbf{H} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$$

is a parity check matrix for C.

Q ca) $C = \{0000, 0011, 1100, 1111\}$

It is a $(4,2)$ linear code

Given, $H = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

If H is the correct parity check matrix of C, then

$$C_i H^T = 0 \quad \forall \; C_i \in C$$

Let's check,

$$CH^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

(Here, we are ~~consi~~ considering all codewords at once, instead of multiplying one by one).

$$CH^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence, H is a correct parity check matrix for C.

30.

14. Find a parity check matrix for the code $C = \{000000, 101010, 010101, 111111\}$.

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Given,

$C = \{000000, 101010, 010101, 111111\}$

$q = 2$

For finding a parity check matrix, we must find a systematic generator matrix for C.

Here, $n(C) = 4$ = Cardinality of C

$\therefore k = \log_q(n(C))$

$\Rightarrow k = \log_2 4 = \underline{2}$

Also, length of each codeword in C, $n = 6$

$\therefore$ The dimensions of generator matrix are: $k \times n = 2 \times 6$

So, we must find 2 linearly independent codewords from C

Let's choose, $C_1 = 101010$, $C_2 = 010101$

This choice is made because it makes our generator matrix systematic

$$G = \left[\begin{array}{cc|cccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right]$$

$$\quad\ \ I \qquad\quad P$$

So, corresponding parity check matrix is:

$$H = [P^T \mid I] = \left[\begin{array}{cc|cccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\qquad\qquad\quad P^T \qquad\quad I$$