HW5 Individual

CS40 Fall'21

Bharat Kathi (5938444)

Due: Monday, Nov 8, 2021 at 10:00PM on Gradescope

Integrity reminders for individual homeworks

- "Individual homeworks" must be solely your own work.
- You may not collaborate on individual homeworks with anyone or seek help from online tutors or entities outside the class.
- You may ask questions about the homework in office hours (of the instructor, TAs, and/or tutors) and on Piazza. However, the staff will only answer clarifying questions on these homeworks. You *cannot* use any online resources about the course content other than the text book and class material from this quarter.
- Do not share written solutions or partial solutions for homework with other students. Doing so would dilute their learning experience and detract from their success in the class.

You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "HW4-Individual".

Assigned Questions

1. Determine the value of n based on the given information. Show your calculation steps.

   (a) n **div** $8 = 10$, n **mod** $8 = 5$

   Answer: .

   $n = dq + r$
   n **div** $d = q$ and n **mod** $d = r$
   $n = 8(10) + r$ and $n = 8(q) + 5$
   $q = 10$ and $r = 5$
   $n = 8(10) + 5$
   $n = 85$

   (b) n **div** $3 = -10$, n **mod** $3 = 2$

   Answer: .

   $n = dq + r$
   n **div** $d = q$ and n **mod** $d = r$
   $n = 3(-10) + r$ and $n = 3(q) + 2$
   $q = -10$ and $r = 2$
   $n = 3(-10) + 2$
   $n = -28$

2. Suppose that $(A72D781CC52)_{16} = x$. Express the value of $(A72D781CC527)_{16}$ in terms of x. Show your steps to justify your answer.

   Answer: .

   $(A72D781CC527)_{16}$
   $= A(16)^11 + 7(16)^10 + 2(16)^9 + ... + 5(16)^2 + 2(16)^1 + 7$
   $= 16(A(16)^10 + 7(16)^9 + 2(16)^8 + ... + 5(16)^1 + 2(16)^0) + 7$
   $= 16((A72D781CC52)_{16}) + 7$
   $= 16x + 7$

3. Some numbers and their prime factorizations are given below.

   - $140 = 2^2 \cdot 5 \cdot 7$
   - $175 = 5^2 \cdot 7$
   - $1083 = 3 \cdot 19^2$
   - $25480 = 2^3 \cdot 5 \cdot 7^2 \cdot 13$

   Use these prime factorizations to compute the following quantities and justify your answers.

   (a) $gcd(1083, 140)$

   Answer: .

   $140 = 2^2 * 5 * 7$
   $1083 = 3 * 19^2$
   Since there is no common prime factors among prime factors of both, the GCD is 1.

   (b) $gcd(25480, 175)$

Answer: .

$175 = 5^2 * 7$

$25480 = 2^3 * 5 * 7^2 * 13$

The common prime factors of both are 5 and 7, so the GCD is 35.

Euclid's algorithm in pseudocode

```
procedure gcd(x, y: positive integers)
a := x
b := y
if a > b
  swap a and b
while a ≠ 0
  r := b mod a
  b := a
  a := r
return b {gcd(x, y) = b}
```

Template of table to trace Euclid's algorithm

x	y	r	a	b	$a \neq 0?$

$gcd(x, y) =?$

4. For each of the following inputs: (a) Use the template of the table provided above to trace Euclid's algorithm, (b) find the gcd of the two numbers using the algorithm (c) check whether the multiplicative inverse exists and if it does, find the inverse of x mod y.

   (a) x=52 and y=77

   Answer: .

x	y	r	a	b	$a \neq 0?$
52	77	25	25	52	T
52	77	2	2	25	T
52	77	1	1	2	T
52	77	0	0	1	F

   $gcd(52, 77)$ is 1.
   Inverse is 40.

   (b) x=630 and y=147

   Answer: .

x	y	r	a	b	$a \neq 0?$
630	147	42	42	147	T
630	147	21	21	42	T
630	147	0	0	21	F

   $gcd(52, 77)$ is 21.
   Inverse does not exist.

5. Below is the pseudocode for a greedy algorithm for making change given a total amount n and r coins of denomination $c_1, c_2, \ldots, c_r$.

Change making algorithm in pseudocode

```
procedure change(c1, c2, ..., cr: values of denominations of coins, where c1 > c2 > ... > cr; n: a positive integer)
```

```
for i := 1 to r
  d_i := 0 {d_i counts the number of coin of denomination c_i used}
  while n ≥ c_i
    d_i := d_i + 1 {Add a coin of denomination c_i}
    n := n − c_i

return d_1, d_2, ..., d_r {d_i the number of coins of denomination c_i in the change for i=1, 2, ..., r}
```

(a) In the particular case where the four denominations are quarters, dimes, nickels, and pennies, we have $c_1 = 25$, $c_2 = 10$, $c_3 = 5$, and $c_4 = 1$. Trace the algorithm by filling the table below for $\mathit{change}(25, 10, 5, 1; 67)$

n	i	d_1	d_2	d_3	d_4	$n \geq c_i$?
67	1	0				T
42	1	1				T
17	1	2				F
17	2		0			T
7	2		1			F
7	3			0		T
2	3			1		F
2	4				0	T
1	4				1	T
0	4				2	F

$\mathit{change}(25, 10, 5, 1; 67) = (2, 1, 1, 2)$

(b) Write the pseudocode of an algorithm covered in lecture that uses the same approach as the one presented above but solves a different problem. Describe the main differences between the two algorithms.

6. Devise an algorithm that finds the first term of a sequence of positive integers that is less than the immediately preceding term of the sequence. Please present the algorithm in pseudocode (similar to Q5).

Location of first term less than next term algorithm

```
procedure getlocation(b_1, b_2, ..., b_n : list of n positive integers)
location := 0
i := 2
while i <= n and location := 0
    if b_i < b_{i-1}
        location := i
    else
        i := i + 1
return location
```

7. For positive integers a, b, and c prove that if $gcd(a, b) = 1$ and $a \mid bc$, then $a \mid c$.

Answer: .

Since $gcd(a, b) = 1$, there exists two integers x, y such that $1 = ax + by$.
$c = cax + cby$
$c = (ac)x + (bc)y$
Since $a \mid bc$, then $a = (bc)y$ for some integer y.
Since $a \mid ac$, then $a = (ac)x$ for some integer x.
Combining these two, we can get $a \mid ((ac)x + (bc)y)$.
This then simplifies to $a \mid c$.