HW6 Individual

CS40 Fall'21

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Due: Monday, Nov 15, 2021 at 10:00PM on Gradescope

Integrity reminders for individual homeworks

- "Individual homeworks" must be solely your own work.
- You may not collaborate on individual homeworks with anyone or seek help from online tutors or entities outside the class.
- You may ask questions about the homework in office hours (of the instructor, TAs, and/or tutors) and on Piazza. However, the staff will only answer clarifying questions on these homeworks. You *cannot* use any online resources about the course content other than the text book and class material from this quarter.
- Do not share written solutions or partial solutions for homework with other students. Doing so would dilute their learning experience and detract from their success in the class.

You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "HW4-Individual".

Practice three useful proof strategies for comparing two sets:

Statement to Prove	Logic Form	Proof Strategy
$A \subseteq B$	$\forall x(x \in A \to x \in B)$	For arbitrary $x \in A$, show that $x \in B$
$A \nsubseteq B$	$\exists x(x \in A \land x \notin B)$	Find an element $x \in A$, such that $x \notin B$
$A = B$	$\forall x(x \in A \leftrightarrow x \in B)$	Show that $A \subseteq B$ and $B \subseteq A$

Assigned Questions

1. Show that $A \nsubseteq B$, where A and B are defined as follows:

$$A = \{3k + 1 \mid k \in \mathbb{N}\},$$
$$B = \{4k + 1 \mid k \in \mathbb{N}\}.$$

Answer: .
$k \in \mathbb{N}$
So by substituting in the first 4 values of k, we get:
$A = \{4, 7, 10, 13, ...\}$
$B = \{5, 9, 13, 17, ...\}$
Since there are some values in A that are not in B, we can conclude that $A \nsubseteq B$.

2. Show that $A = B$, where A and B are defined as follows:

$$A = \{x \in \mathbb{Z} \mid x \text{ is a prime number and } 12 \leq x \leq 18\},$$
$$B = \{x \in \mathbb{Z} \mid x = 4k + 1 \text{ for some } k \in \{3, 4\}\}.$$

Answer: .
The only primes in between 12 and 18 are 13 and 17.
Therefore, $A = \{13, 17\}$.
Since $k \in \{3, 4\}$, we can simplify B as:
$B = \{4(3) + 1, 4(4) + 1\}$
$B = \{13, 17\}$
From this we can see that $A = B$.

3. Let $A = \{6k + 5 \mid k \in \mathbb{N}\}$ and $B = \{3k + 2 \mid k \in \mathbb{N}\}$. Show that $A \neq B$.

Answer: .
$k \in \mathbb{N}$
So by substituting in the first 4 values of k, we get:
$A = \{11, 17, 23, 29, ...\}$
$B = \{5, 8, 11, 14, ...\}$
Since there are some values in A that are not in B, we can conclude that $A \neq B$.

4. Let $W = \mathcal{P}(\{1, 2, 3, 4, 5\})$. Consider the following statement and attempted proof:

$$\forall A \in W \, \forall B \in W \;\; (\, ((A \cap B) \subseteq A) \to (A \subseteq B) \,)$$

(1) Towards a universal generalization argument, **choose arbitrary** $A \in W, B \in W$.

(2) We need **to show** $((A \cap B) \subseteq A) \to (A \subseteq B)$.

(3) Towards a proof of the conditional by the contrapositive, **assume** $A \subseteq B$. We need **to show** that $(A \cap B) \subseteq A$.

(4) By definition of subset inclusion, this means we need **to show** $\forall x\ (x \in A \cap B \to x \in A\ )$.

(5) Towards a universal generalization, **choose an arbitrary** x; we need **to show** that $x \in A \cap B \to x \in A$.

(6) Towards a direct proof, **assume** $x \in A \cap B$. We need **to show** $x \in A$.

(7) By definition of set intersection, we have that $x \in A \wedge x \in B$.

(8) By the definition of conjunction, $x \in A$, which is what we needed to show.

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(a) Demonstrate that this statement is invalid by providing and justifying a **counterexample** (disproving the statement).

Answer: .

Let $A = \{2, 3\}$
Let $B = \{2\}$
Then, we get that $A \cap B = \{2\}$
Therefore $A \cap B \subseteq A$ but $A \subsetneq B$

(b) Explain why the attempted proof is invalid by identifying in which step a definition or proof strategy is used incorrectly, and describing how the definition or proof strategy was misused.

Answer: The third step is incorrect. The contrapositive of $p \to q$ should be $\neg q \to \neg p$. The given proof is assuming q in order to prove p.

5. Give a recursive definition for each of the following sets S. Each set S will be a subset of the set containing all binary strings. A string x belongs to the recursively defined set S if and only if x has each of the following properties (provide a different definition for each part). Note that in each case, you may provide multiple rules for the recursive step of your definition.

(a) The set S consists of all strings (including the empty string) that have an even number of 1's but may have an even or odd number of zeros.

Answer: If x belongs to S then x11 or 11x also belongs to S.

(b) The set S consists of all strings (including the empty string) that have the same number of 0's and 1's.

Answer: If x belongs to S, then 0S1 or 1S0 also belongs to S.

6. Apply your recursive definitions for S for each part of the previous question to construct all elements of S with length less than or equal to 4. Provide your answer in roster notation.

Answer: .

$S = \{0, 11, 101, 011\}$