HW9 Individual

CS40 Fall'21

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Due: Monday, Dec 6, 2021 at 10:00PM on Gradescope

In this assignment,

You will have more practice with counting, binary relations, and proof strategies learned so far.

For all HW assignments:

Please see the instructions and policies for assignments on the class website and on the writeup for HW1. In particular, these policies address

- Collaboration policy
- Where to get help
- Typing your solutions
- Expectations for full credit

You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "HW9-Individual".

Assigned questions

1. We define the following function:

$$f : \{0,1\}^3 \to \{0,1\}^3,$$

where the output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, $f(001) = 101$ and $f(110) = 110$. Indicate whether the f is onto, one-to-one, neither or both. If the function is not onto or not one-to-one, give an example showing why.

Answer: .

$\{0,1\}^3 = \{000, 001, 011, 100, 101, 010, 110, 111\}$
$f(000) = 100$
$f(001) = 101$
$f(011) = 111$
$f(101) = 101$
$f(010) = 110$
$f(111) = 111$
$f(110) = 110$

Since f(000) and f (100) both equal 100, the function is not one-to-one. Since there is no preimage for 000, the function is also not onto.

2. Count the number of different one-to-one functions $f : \{0,1\}^7 \to \{0,1\}^7$. Justify your answer.

Answer: .

Since $\{0,1\}^7$ is the set of all 7 long binary strings, the total number of elements is $2^7 = 128$.
Since $f : \{0,1\}^7 \to \{0,1\}^7$ should be a one-to-one function, each element in $\{0,1\}^7$ must be mapped to a unique element in $\{0,1\}^7$.
That means that we now have 2^7 choices for the first term, $2^7 - 1$ choices for the second term, and so on.
Therefore, the total number of choices is $128! * (2^7)!$

3. Recall that in a movie recommendation system, each user's ratings of movies is represented as a n-tuple (with the positive integer n being the number of movies in the database), and each component of the n-tuple is an element of the collection $\{-1, 0, 1\}$.

Assume there are five movies in the database, so that each user's ratings can be represented as a 5-tuple. Let R be the set of all ratings, that is, the set of all 5-tuples where each component of the 5-tuple is an element of the collection $\{-1, 0, 1\}$.

Consider the following two binary relations on R:

$$A_1 = \{(u, v) \in R \times R \mid \text{users } u \text{ and } v \text{ agree about the first movie in the database}\}$$

$$A = \{(u, v) \in R \times R \mid \text{users } u \text{ and } v \text{ don't care or haven't seen the same number of movies}\}$$

Binary relations that satisfy certain properties (namely, are reflexive, symmetric, and transitive) can help us group elements in a set into categories.

(a) **True** or **False**: The relation A_1 holds of $u = (1, 1, 1, 1, 1)$ and $v = (-1, -1, -1, -1, -1)$.

Answer: True

(b) **True** or **False**: The relation A holds of $u = (1, 0, 1, 0, -1)$ and $v = (-1, 0, 1, -1, -1)$.

(c) **True** or **False**: A_1 is reflexive; namely, $\forall u \in R \ (\ (u, u) \in A_1 \ )$

(d) **True** or **False**: A_1 is symmetric; namely, $\forall u \in R \ \forall v \in R \ (\ (u, v) \in A_1 \to (v, u) \in A_1 \ )$

(e) **True** or **False**: A_1 is transitive; namely, $\forall u \in R \ \forall v \in R \ \forall w \in R(\ ((u, v) \in A_1 \wedge (v, w) \in A_1) \to (u, w) \in A_1 \ )$

(f) **True** or **False**: A is reflexive; namely, $\forall u \in R \ (\ (u, u) \in A \ )$

(g) **True** or **False**: A is anti-symmetric; namely, $\forall u \in R \ \forall v \in R \ (\ (\ (u, v) \in A \wedge (v, u) \in A \ ) \to (u = v) \ )$

(h) **True** or **False**: A is transitive; namely, $\forall u \in R \ \forall v \in R \ \forall w \in R(\ ((u, v) \in A \wedge (v, w) \in A) \to (u, w) \in A \ )$

4. In the previous question select any one of parts (c) to (h) that evaluated to True and provide a formal proof using the strategies you have learned in CS40

5. *No justifications are required for credit for this question. It's a good idea to think about how you would explain how you arrived at your examples.* Given the relations A_1 and A in Q4 answer the following questions:

   (a) Give two distinct examples of elements in $[\ (1, 0, 0, 0, 0) \ ]_{A_1}$

   Answer: (1,1,0,0,0) (1,-1,0,0,0)

   (b) Give two distinct examples of elements in $[\ (1, 0, 0, 0, 0) \ ]_A$

   Answer: (0,1,0,0,0) (0,0,1,0,0)

   (c) Find examples $u, v \in R$ where $[u]_{A_1} \neq [v]_{A_1}$ but $[u]_A = [v]_A$

   Answer: $u = (1, 0, 0, 0, 0)$ and $v = (-1, 0, 0, 0, 0)$

   (d) Find examples $u, v \in R$ (different from the previous part) where $[u]_{A_1} = [v]_{A_1}$ but $[u]_A \neq [v]_A$

   Answer: $u = (1, 0, 0, 0, 1)$ and $v = (1, 0, 0, 1, 1)$

6. **Bonus - not for credit (but much appreciated):** Please complete the course ESCI and TA evaluations by Dec 3 (Friday).