HW1 Propositional Logic

CS40 Spring'22

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Due: on Gradescope

Assigned Questions

1. Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

   (a) How tall is Storke Tower?

   Answer: No, this is a question.

   (b) Storke Tower is as tall as 33 people stacked on top of each other.

   Answer: Yes, this is a proposition. The negation would be "Storke Tower is not as tall as 33 people stacked on top of each other".

2. Express each English statement using the logical operations $\vee$, $\wedge$, $\neg$ and the propositional variables t, n, and m defined below. The use of the word "or" means inclusive or.

   - t: My first flight was delayed.
   - n: I had a snack at the airport.
   - m: I made my final connection.

   (a) I never snack at the airport.

   Answer: $\neg n$

   (b) Even though my first flight was delayed, I still made my final connection.

   Answer: $t \wedge m$

   (c) Whenever I have a snack at the airport, I miss my final connection.

   Answer: $n \wedge \neg m$

   (d) I miss my final connection only if I have a snack in the airport, or my flight was delayed.

   Answer: $(n \vee t) \leftrightarrow \neg m$

3. Practice finding the truth values for conditional statements in English. Which of the following conditional statements are true and why?

(a) If 51 is an even number, then the sky is green.

Answer: $F \to F = T$, because 51 is an even number (F) and the sky is not green (F). Since our statement is that F implies F, this is True.

(b) If 51 is an odd number, then the sky is blue.

Answer: $T \to T = T$, because 51 is an odd number (T) and the sky is blue (T). Since our statement is that T implies T, this is True.

4. In this question, practice expressing English colloquialisms using logical operations.

Consider the following situations:

- i: It is raining.
- j: It is pouring.
- k: It is shining.
- l: We are having a parade.

Write a logical expression to capture each of the following colloquialisms[1]:

(a) It either rains or it pours.

Answer: $i \vee j$

(b) It is raining on our parade.

Answer: $i \wedge l$

(c) We are having this parade, come rain or shine.

Answer: $l \wedge (i \vee k)$

5. Give an English sentence in the form "If...then...." that is equivalent to each sentence.

(a) Rafael can ride the elephant only if he is not afraid of heights.

Answer: If Rafael can ride the elephant, then he is not afraid of heights.

(b) Rafael can ride the elephant if he is not afraid of heights.

Answer: If Rafael is not afraid of heights, then he can ride the elephant.

(c) Being able to swim is a necessary skill needed for Tyra to learn to surf.

Answer: If Tyrna can learn how to surf, they can swim.

(d) Being able to swim is a sufficient skill needed for Tyra to learn to surf.

Answer: If Tyrna can swim, they can learn how to surf.

6. Use the laws of propositional logic listed in Section 1.5, Table 1.5.1 of the book to prove the following. Explicitly specify which laws are being used.

(a) $\neg p \to q \equiv \neg q \to p$

Answer: .

$p \vee q \equiv \neg q \to p$ (Conditional Law)
$q \vee p \equiv \neg q \to p$ (Commutative Law)
$\neg q \to p \equiv \neg q \to p$ (Conditional Law)

[1]Inclusive or is assumed unless explicitly stated otherwise.

(b) $\neg p \to (q \wedge \neg q) \equiv p$

Answer: .

$\neg p \to (F) \equiv p$ (Complement Law)
$p \vee F \equiv p$ (Conditional Law)
$p \equiv p$ (Identity Law)

(c) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \equiv p \wedge \neg r$

Answer: .

$(p \wedge \neg r \wedge q) \vee (p \wedge \neg r \wedge \neg q) \equiv p \wedge \neg r$ (Commutative Law)
$(p \wedge \neg r) \wedge (q \vee \neg q) \equiv p \wedge \neg r$ (Distributive Law)
$(p \wedge \neg r) \wedge (T) \equiv p \wedge \neg r$ (Complement Law)
$p \wedge \neg r \equiv p \wedge \neg r$ (Identity Law)

7. Use any technique of your choice to show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology

Answer: Using the truth table below we can see that the statement is True for every value of p, q, and r. Therefore, it is a tautology.

p	q	r	$((p \to q) \wedge (q \to r)) \to (p \to r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

8. There are three kinds of people on an island: knaves who always lie, knights who always tell the truth, and spies who can either lie or tell the truth. You encounter three people, Andromeda, Brunhilda, and Clytemnestra. You know one of these people is a knight, one is a knave, and one is a spy. Each of the three people knows the identity of the other two. Andromeda says "Clytemnestra is the knave," Brunhilda says "Andromeda is the knight," and Clytemnestra says "I am the spy." Determine whether or not a unique solution exists, and if so, state who the knave, knight, and spy are. If there is no unique solution, list all possible solutions or state that there are no solutions.

Answer: We first define the following:

p: Andromeda is a knight
q: Brunhilda is a knight
r: Clytemnestra is a knight
s: Andromeda is a knave
t: Brunhilda is a knave
u: Clytemnestra is a knave

Andromeda says: u
Brunhilda says: p
Clytemnestra says: $\neg(r \wedge u)$

Now we can create the compound proposition to figure out whether a solution exists:

$u \wedge p \wedge \neg(r \wedge u)$

p	q	r	$u \wedge p \wedge \neg(r \wedge u)$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

From the above truth table, we can see that there is a unique solution: Andromeda is a knight, Brunhilda is a spy, and Clytemnestra is a knave

Looking at the truth table, we know that p is true and u is true. p states that Andromeda is a knight, and u states that Clytemnestra is a knave. That means we know that Brunhilda must be a spy.

9. Use the Disjunctive Normal form to construct a proposition Q whose truth table is as below:

p	q	r	Q
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T

Answer: The proposition using disjunctive normal form is $(p \wedge q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$