Homework 19

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Here, we will reduce standard matrix multiplication to multiplication of two lower triangular matrices. Assume we have an algorithm $multTri(A, B)$ that solves multiplication of two lower triangular matrices in $O(n^2)$ time. Given as input to standard matrix multiplication two matrices A and B, both of which are $n \times n$, we can construct two new matrices that will be passed into $multTri(A', B')$ by doing the following:

$$A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & A & 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 & 0 & 0 \\ B & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Where 0 represents an $n \times n$ matrix of all zeros. Both of these matrices will be $3n \times 3n$. Both transformations take n^2 time, as it would take constant time to construct a $3n \times 3n$ matrix of zeros, then n^2 time to copy over the elements of A and B into the correct places in A' and B'. Multiplying A' and B' gives us the $3n \times 3n$ array:

$$A' \times B' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ AB & 0 & 0 \end{bmatrix}$$

We can then output the bottom left $n \times n$ elements in the matrix, which takes n^2 time. These elements are exactly the values of the elements of $A \times B$. Thus, we have converted the input from matrix multiplication to input for $multTri(A, B)$ in n^2 time, and converted the output from $multTri(A, B)$ to the output for standard matrix multiplication in n^2 time, so if an $O(n^2)$ algorithm exists for $multTri(A, B)$, so does one exist for standard matrix multiplication.