CS 1510 Homework 2

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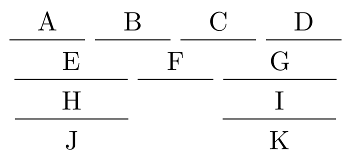
1

Proof by counterexample:
Consider the following input:



The given Least Overlaps algorithm will clearly select F as the first interval as it has the least number of overlaps (2) and B and C will be eliminated as the overlapping intervals. Next, the algorithm will arbitrarily select another interval as each of the remaining intervals overlaps 3 other intervals. Assuming A is selected, E, H, and J will be eliminated and the algorithm will proceed with selecting D, eliminating G, I, and K and emptying S.
Therefore, the solution generated by the algorithm on the above input is A, F, D:

However, the obvious solution given the above input is A, B, C, D:

Therefore the algorithm does not solve the problem of generating a maximum cardinality subset of intervals given a set of intervals.

2

a

b

Assume the algorithm given, A, does not solve the interval coloring problem. Let s be the maximum number of intervals that overlap at any point, and thus, the optimal output (minimum number of rooms after assigning all intervals). Then, A will output a value larger than s for some input, and thus, A does not solve the problem. We can conclude that in the case that A's output is greater than s, this means that A assigned a class to a new room when another room already had a space for it, as this would be the only case where A's output could be greater than s. Because, in this algorithm, classes already placed are never altered / assigned to another room, this is a contradiction. The algorithm checks the set R of rooms that already have a class scheduled in it, and places the class in the first room it finds that does not cause an overlap. Thus, the output of A cannot be larger than s.