Homework 27

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a

Let the state of $v = \alpha_{00}\left|00\right\rangle + \alpha_{01}\left|01\right\rangle + \alpha_{10}\left|10\right\rangle + \alpha_{11}\left|11\right\rangle$ where $Prob[\text{register's state is measured as } b_1 b_2] = |\alpha_{b_1 b_2}|^2$.

The coefficients can be rewritten to reflect the probabilities of each individual bit as x_i and y_i such that $Prob[b_1 = i] = x_i^2$ and $Prob[b_2 = i] = y_i^2$.

Using the multiplicative rule the α coefficients can be rewritten as follows:

$$\begin{aligned} Prob[b_1 = i \cap b_2 = j] &= Prob[b_1 = i] * Prob[b_2 = j] \\ |\alpha_{ij}|^2 &= x_i^2 y_j^2 \\ \alpha_{ij} &= x_i y_j \end{aligned}$$

So the overall equation for $v = x_0 y_0 \left|00\right\rangle + x_0 y_1 \left|01\right\rangle + x_1 y_0 \left|10\right\rangle + x_1 y_1 \left|11\right\rangle$.

b

The probability of measuring the second bit as a certain value given that the first bit measured to be a certain value can be described as follows:

$$\begin{aligned} Prob[b_2 = j | b_1 = i] &= \frac{Prob[b_2 = j \cap b_1 = i]}{Prob[b_1 = i} \\ &= \frac{x_i^2 y_j^2}{x_i^2} \\ &= y_j^2 \end{aligned}$$

So the probability of measuring the second bit to be a certain value is independent of the first bit, so the equation stays as $v = x_0 y_0 \left|00\right\rangle + x_0 y_1 \left|01\right\rangle + x_1 y_0 \left|10\right\rangle + x_1 y_1 \left|11\right\rangle$.

c

The probability of measuring the first bit as a certain value given that the second bit measured to be a certain value can be described as follows:

$$\begin{aligned} Prob[b_1 = i | b_2 = j] &= \frac{Prob[b_1 = i \cap b_2 = j]}{Prob[b_2 = j} \\ &= \frac{x_i^2 y_j^2}{y_j^2} \\ &= x_i^2 \end{aligned}$$

So the probability of measuring the second bit to be a certain value is independent of the first bit, so the equation stays as $v = x_0y_0 |00\rangle + x_0y_1 |01\rangle + x_1y_0 |10\rangle + x_1y_1 |11\rangle$. Thus all three methods of measurement have the same probabilities.

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a

b

c