

# Homework 29

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## 53

### a

Alice will do one of the following operations on  $a$  depending on the values of  $x$  and  $y$ .

If  $x = 0$  and  $y = 0$ , leave  $a$  as is, producing  $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

If  $x = 0$  and  $y = 1$ , rotate  $a$  by  $\frac{\pi}{2}$ , producing  $-\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

If  $x = 1$  and  $y = 0$ , rotate  $a$  by  $-\frac{\pi}{2}$ , producing  $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

If  $x = 1$  and  $y = 1$ , rotate  $a$  by  $\pi$ , producing  $-\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

So these operations will put  $a$  into one of the four Bell states.

### b

If  $x = 0$  and  $y = 0$ , then  $a = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$  and  $b = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

If  $x = 0$  and  $y = 1$ , then  $a = -\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$  and  $b = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

If  $x = 1$  and  $y = 0$ , then  $a = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$  and  $b = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

If  $x = 1$  and  $y = 1$ , then  $a = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$  and  $b = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

### c

Bob will perform a Bell measurement on  $a$ ,  $H(a)$ . There are four possible outcomes depending on what  $a$  is.

When  $a = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ :

$$H(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

When  $a = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$ :

$$H(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

When  $a = -\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ :

$$H(-\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle$$

When  $a = -\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$ :

$$H(-\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -|0\rangle$$

Each of these results corresponds to a rotation of the form  $\sin(\theta) + \cos(\theta)$ . Bob can then compare this result to his bit  $b$  and see what the difference in angle is. This will indicate the quadrant that the original value of  $a$  was in. Since there are 4 quadrants, each could represent combinations of  $x$  and  $y$  such that QI  $\Rightarrow x = 0, y = 0$ , QII  $\Rightarrow x = 0, y = 1$ , QIII  $\Rightarrow x = 1, y = 1$ , and QIV  $\Rightarrow x = 1, y = 0$ . So Bob can determine the values of  $x$  and  $y$ .

## 54

The final state works by mapping the two classical particles  $z_0, z_1$  to indicate the quadrant that the Bell measurement of  $a, x$  ends up in. Then the rotation from  $b$  to the quadrant indicated by  $z_0, z_1$  is measured. Finally, that difference in rotation is added to  $b$  to make a qubit identical to  $x$ .

At first,  $x = \alpha|0\rangle + \beta|1\rangle$ , and  $a =$  either 0 or 1, since it's already been measured. Then,  $x$  will become its negative,  $x = \beta|0\rangle + \alpha|1\rangle$ , if and only if  $a = 1$ . Then, the Hadamard operation is performed on  $a$ , which becomes  $a = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$ . Then,  $x$  and  $a$  are both measured to be either 0 or 1. At this point, consider these measured values to be  $x$  and  $y$  described in problem 53, and do the procedure described in the solution to 54 to get the original state of  $x$ .