

Homework 22

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Asking whether

$$\exists x \forall y \exists z (x \vee y \vee \neg z) \wedge (\neg x \vee \neg y \vee z)$$

is satisfiable is equivalent to asking if:

$$\Sigma_{x=0}^1 \Pi_{y=0}^1 \Sigma_{z=0}^1 (1 - (1 - x)(1 - y)z)(1 - xy(1 - z)) \neq 0$$

a

i

Merlin will construct a function $s(x)$ as follows:

$$\begin{aligned} s(x) &= \Pi_{y=0}^1 \Sigma_{z=0}^1 (1 - (1 - x)(1 - y)z)(1 - xy(1 - z)) \\ &= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - z(1 - x)(1 - y) - xy(1 - z) + xyz(1 - x)(1 - y)(1 - z) \\ &= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - (z - xz)(1 - y) - xy + xyz + (xyz - x^2yz)(1 - y)(1 - z) \\ &= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - (z - xz)(1 - y) - xy + xyz + (xyz - x^2yz)(1 - y)(1 - z) \\ &= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - (z - xz) - y(z - xz) - xy + xyz + (xyz - x^2yz)(1 - y)(1 - z) \\ &= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - z - xz - yz + xyz - xy + xyz + xyz - x^2yz - y(xyz - x^2yz)(1 - z) \\ &= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - z - xz - yz + 2xyz - xy - x^2yz - (xy^2z + x^2y^2z)(1 - z) \\ &= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - z - xz - yz + 2xyz - xy - x^2yz - (xy^2z + x^2y^2z - z(xy^2z + x^2y^2z)) \\ &= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - z - xz - yz + 2xyz - xy - x^2yz - (xy^2z + x^2y^2z - xy^2z^2 - x^2y^2z^2) \\ &= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - z - xz - yz + 2xyz - xy - x^2yz - xy^2z - x^2y^2z + xy^2z^2 + x^2y^2z^2 \\ &= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - z - xy - xz - yz + 2xyz - x^2yz - xy^2z - x^2y^2z + xy^2z^2 + x^2y^2z^2 \end{aligned}$$

b

i

Merlin will construct a linearized function $s(x)$ as follows:

$$\begin{aligned}
s(x) &= \Pi_{y=0}^1 \Sigma_{z=0}^1 (1 - (1-x)(1-y)z)(1 - xy(1-z)) \\
&= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - z(1-x)(1-y) - xy(1-z) + xyz(1-x)(1-y)(1-z) \\
&= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - (z - xz)(1-y) - xy + xyz + (xyz - xyz)(1-y)(1-z) \\
&= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - (z - xz)(1-y) - xy + xyz + (0)(1-y)(1-z) \\
&= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - (z - xz)(1-y) - xy + xyz \\
&= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - (z - xz) - y(z - zx) - xy + xyz \\
&= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - z + xz - yz + xyz - xy + xyz \\
&= \Pi_{y=0}^1 \Sigma_{z=0}^1 1 - z + xz - yz - xy + 2xyz
\end{aligned}$$