Homework 27

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March 27, 2018

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a

$v = \frac{1}{\sqrt{4}}\left|00\right\rangle + \frac{1}{\sqrt{4}}\left|01\right\rangle + \frac{1}{\sqrt{4}}\left|10\right\rangle + \frac{1}{\sqrt{4}}\left|11\right\rangle$ Since the coefficients are the square-root of the probability, the probability of measuring any of $00, 01, 10, 11$ is $\frac{1}{4}$.

b

Since $\left|xy\right\rangle$ can be rewritten as $\left|x\right\rangle\left|y\right\rangle$, we can simplify the problem into:

$$
\begin{aligned}
&= \frac{1}{\sqrt{4}}\left|00\right\rangle + \frac{1}{\sqrt{4}}\left|01\right\rangle + \frac{1}{\sqrt{4}}\left|10\right\rangle + \frac{1}{\sqrt{4}}\left|11\right\rangle \\
&= \frac{1}{\sqrt{4}}\left|0_1\right\rangle\left|0_2\right\rangle + \frac{1}{\sqrt{4}}\left|0_1\right\rangle\left|1_2\right\rangle + \frac{1}{\sqrt{4}}\left|1_1\right\rangle\left|0_2\right\rangle + \frac{1}{\sqrt{4}}\left|1_1\right\rangle\left|1_2\right\rangle \\
&= \frac{1}{2}\left|0_1\right\rangle\left|0_2\right\rangle + \frac{1}{2}\left|0_1\right\rangle\left|1_2\right\rangle + \frac{1}{2}\left|1_1\right\rangle\left|0_2\right\rangle + \frac{1}{2}\left|1_1\right\rangle\left|1_2\right\rangle \\
&= \frac{1}{\sqrt{(2)}}(\left|0_1\right\rangle + \left|1_1\right\rangle)(\left|0_2\right\rangle + \left|1_2\right\rangle)
\end{aligned}
$$

Measuring the first qubit would have a 1/2 probability of resulting in a 0 and a 1/2 probability of resulting in a 1, and the equation becomes either:
$v = 0_1 + \frac{1}{\sqrt{2}}(\left|0_2\right\rangle + \left|1_2\right\rangle)$
or:
$v = 1_1 + \frac{1}{\sqrt{2}}(\left|0_2\right\rangle + \left|1_2\right\rangle)$
both with probability 1/2.

And now the second qubit has a 1/2 chance of being either a 0 or 1 in both cases. So v has a uniform 1/4 probability of being any of the following:
$v = 0_1 0_2$
$v = 0_1 1_2$
$v = 1_1 0_2$
$v = 1_1 1_2$

c

The same proof as in b, but measure the second bit first. The probabilities will be the same due to the commutivity of multiplying the coefficient.

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a

b

c