Homework 15

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a

Problem 6.5: Show for every $k > 0$ that PH contains languages whose circuit complexity is $\Omega\left(n^k\right)$.

Proof:
Let C be a circuit with complexity at of at least n^k. We know that such a circuit must exist by Theorem 6.22 from the book. We can construct a boolean formula F from the gates of C with k quantifiers over the boolean formula. Now let L be the language of all variable assignments for k which satisfy F. Since $|C|$ is polynomial, C can decide if an input is valid for F in polynomial time. So there must exist a TM M with k advice tapes (from the quantifiers) that can decide the input with its advice tapes in polynomial time. Thus for each $k > 0$, there is a language in PH whose circuit complexity is $\Omega\left(n^k\right)$.

b

Problem 6.6: Show for every $k > 0$ that Σ_2^p contains languages whose circuit complexity is $\Omega\left(n^k\right)$.

Proof:
Consider the circuit C from problem 6.5. Now construct the same formula F, but only use two quantifiers. One $\exists$ over the tuple of some of the variables and one $\forall$ for the remaining tuple of variables. Again let L be the language of all variable assignments for k which satisfy F. Since $|C|$ is polynomial, C can decide if an input is valid for F in polynomial time. So there must exist a TM M with 2 advice tapes (from the quantifiers) that can decide the input with its advice tapes in polynomial time. Thus for each $k > 0$, there is a language in Σ_2^p whose circuit complexity is $\Omega\left(n^k\right)$.

c

Problem 6.7: Show that if P = NP, then there is a language in EXP that requires circuits of size $\frac{2^n}{n}$.

Proof:
Assume P = NP and let $L \in$ EXP. If L is decidable by a circuit of complexity n^k, then because the polynomial hierarchy is collapsible and by problem 6.6, L is in P. So any L in EXP that is not in P must have a circuit with complexity $\frac{2^n}{n}$.