Homework 22

Joe Baker, Brett Schreiber, Brian Knotten

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Asking whether

$$\exists_x \forall_y \exists_z (x \lor y \lor \neg z) \land (\neg x \lor \neg y \lor z)$$

is satisfiable is equivalent to asking if:

$$\Sigma_{x=0}^{1} \Pi_{y=0}^{1} \Sigma_{z=0}^{1} (x + y + (1 - z))((1 - x) + (1 - y) + z \neq 0$$

Merlin will construct a function $s(x)$ as follows:

$$\begin{aligned}
s(x) &= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} (x + y + (1 - z))((1 - x) + (1 - y) + z \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} x(1 - x) + x(1 - y) + xz + y(1 - x) + y(1 - y) + yz + (1 - z)(1 - x) + (1 - z)(1 - y) + z(1 - z) \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} x - x^2 + x - xy + xz + y - xy + y - y^2 + yz + 1 + xz - x - z + yz - y - z + 1 + z - z^2 \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} x - x^2 - 2xy + 2xz + y - y^2 + 2yz + 2 - z - z^2 \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} - x^2 + x - 2xy + 2xz + 2yz - y^2 + y - z - z^2 + 2 \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} - (x + y - z - 2)(x + y - z + 1)
\end{aligned}$$