Homework 25

Joe Baker, Brett Schreiber, Brian Knotten

March 25, 2018

46

47

Let's define the Toffoli gate on 3 input wires: a, b, c as the output a', b', c', where $a' = a$, $b' = b$, and $c' = \neg c$ if $a = b = 1$, else $c' = c$.

The Toffoli gate is reversible, because every combination of outputs indicate what inputs were used. The input wires can be determined from the following procedure: given output wires a', b', c', the input wires $a = a', b = b'$, and $c = \neg c'$ if $a = b = 1$, else $c = c'$.

The Toffoli gate is universal, because two Toffoli gates can be used to construct a NAND gate, which is universal. We will make a NAND gate for input wires x and y. Feed x, y, and 1 into a Toffoli gate such that $a = x, b = y, c = 1$. The output wire $c' = \neg 1 = 0$ if and only if $x = y = 1$, therefore $c' = \neg(x \wedge y)$. Thus a Toffoli gates can be used to construct a NAND gate, and so the Toffoli gate is universal.

48

a

0. The photon starts in super-positional state:

$$a\,|H\rangle + b\,|V\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

1. Then a half-silvered mirror (Hadamard Operation):

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{a+b}{\sqrt{2}} \\ \frac{a-b}{\sqrt{2}} \end{bmatrix}$$

2. Then a full-silvered mirror (Not Operation):

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{a+b}{\sqrt{2}} \\ \frac{a-b}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{a-b}{\sqrt{2}} \\ \frac{a+b}{\sqrt{2}} \end{bmatrix}$$

3. Then a half-silvered mirror (Hadamard Operation):

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{a-b}{\sqrt{2}} \\ \frac{a+b}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} a \\ -b \end{bmatrix}$$

So the outgoing state of the photon is $a\,|H\rangle - b\,|V\rangle$

b

The probability of the observer seeing the photon come out of the mirror horizontally is: a^2

c

The probability of the observer seeing the photon come out of the mirror vertically is: $(-b)^2 = b^2$