

Homework 25

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Let's define the Toffoli gate on 3 input wires: a, b, c as the output a', b', c' , where $a' = a$, $b' = b$, and $c' = \neg c$ if $a = b = 1$, else $c' = c$.

The Toffoli gate is reversible, because every combination of outputs indicate what inputs were used. The input wires can be determined from the following procedure: given output wires a', b', c' , the input wires $a = a', b = b'$, and $c = \neg c'$ if $a = b = 1$, else $c = c'$.

The Toffoli gate is universal, because two Toffoli gates can be used to construct a NAND gate, which is universal. We will make a NAND gate for input wires x and y . Feed x , y , and 1 into a Toffoli gate such that $a = x, b = y, c = 1$. The output wire $c' = \neg 1 = 0$ if and only if $x = y = 1$, therefore $c' = \neg(x \wedge y)$. Thus a Toffoli gates can be used to construct a NAND gate, and so the Toffoli gate is universal.

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