Homework 31

Joe Baker, Brett Schreiber, Brian Knotten

April 5, 2018

59

Step 1: Show that MAX-3SAT reduces to the problem P of finding the maximum number of linear equations in n with coefficients that are rational numbers that can be satisfied.

Consider a MAX-3SAT instance of the following form:

$$(x_0 \vee x_1 \vee x_2) \wedge$$
$$(x_0 \vee \overline{x_1} \vee x_2) \wedge$$
$$(\overline{x_0} \vee x_1 \vee \overline{x_2})$$

All clauses satisfiable when all three variables are assigned true. Convert each clause into a separate equation of the form $x + y + z = 1$, where each variable is either itself, x, or if it is negated, is of the form $1 - x$. So the equations are:

$$\begin{aligned} x_0 + x_1 + x_2 &= 1 \\ x_0 + (1 - x_1) + x_2 &= 1 \\ (1 - x_0) + x_1 + (1 - x_2) &= 1 \end{aligned}$$

This system of equations can be simplified to:

$$\begin{aligned} x_0 + x_1 + x_2 &= 1 \\ x_0 - x_1 + x_2 &= 0 \\ -x_0 + x_1 - x_2) &= -1 \end{aligned}$$

Now, an oracle machine for solving the problem P can determine the approximate maximum subset of satisfying equations with assignments to the variables such that if a variable is 0, then the corresponding SAT assignment is false, and if it's nonzero, then the SAT variable is assignmened true.