

# Homework 17

Joe Baker, Brett Schreiber, Brian Knotten

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Show that  $(NP \cup \text{co-NP}) \subseteq PP$ .

Proof:

We will show that Boolean Satisfiability  $\in PP$ . Since Boolean Satisfiability is NP-complete, then any  $L \in NP \rightarrow L \in PP$ .

Let  $M$  be a probabilistic Turing machine which takes a Boolean formula  $F$  as input. Then does the following:

1. Pick a random assignment of variables for  $F$
2. Verify if  $F$  is satisfied
3. If  $F$  is satisfied, then accept
4. If  $F$  is not satisfied, accept with  $P(1/2)$  or reject with  $P(1/2)$

First note that each step can clearly be done in polynomial time.  $P(M \text{ accepts}) = 1/2 + P(\text{random assignment is satisfied})$  and since  $P(\text{random assignment is satisfied}) > 0$ , then  $P(M \text{ accepts}) > 1/2$  and  $P(M \text{ rejects}) < 1/2$ . Thus Boolean Satisfiability  $\in PP$  since  $M$  is a Turing machine which meets the requirements for PP. So NP is contained in PP.

Since PP is closed under complement and contains NP, it must also contain co-NP. Finally,  $(NP \cup \text{co-NP}) \subseteq PP$ .