Homework 29

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a

Alice will do one of the following operations on a depending on the values of x and y.

If $x = 0$ and $y = 0$, leave a as is, producing $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
If $x = 0$ and $y = 1$, rotate a by $\frac{\pi}{2}$, producing $-\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
If $x = 1$ and $y = 0$, rotate a by $-\frac{\pi}{2}$, producing $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$
If $x = 1$ and $y = 1$, rotate a by π, producing $-\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

So these operations will put a into one of the four Bell states.

b

If $x = 0$ and $y = 0$, then $a = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ and $b = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
If $x = 0$ and $y = 1$, then $a = -\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ and $b = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
If $x = 1$ and $y = 0$, then $a = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$ and $b = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
If $x = 1$ and $y = 1$, then $a = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$ and $b = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

c

Bob will perform a Bell measurement on a, $H(a)$. There are four possible outcomes depending on what a is.
When $a = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$:

$H(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}1\\ 0\end{bmatrix} = |0\rangle$

When $a = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$:

$H(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}0\\ 1\end{bmatrix} = |1\rangle$

When $a = -\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$:

$H(-\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}-\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}0\\ -1\end{bmatrix} = -|1\rangle$

When $a = -\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$:

$H(-\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}-\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}-1\\ 0\end{bmatrix} = -|0\rangle$

Each of these results corresponds to a rotation of the form $\sin(\theta) + \cos(\theta)$. Bob can then compare this result to his bit b and see what the difference in angle is. This will indicate the quadrant that the original value of a was in. Since there are 4 quadrants, each could represent combinations of x and y such that QI $\Rightarrow x = 0, y = 0$, QII $\Rightarrow x = 0, y = 1$, QIII $\Rightarrow x = 1, y = 1$, and QIV $\Rightarrow x = 1, y = 0$. So Bob can determine the values of x and y.

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The final state works by mapping the two classical particles z_0, z_1 to indicate the quadrant that the Bell measurement of a, x ends up in. Then the rotation from b to the quadrant indicated by z_0, z_1 is measured. Finally, that difference in rotation is added to b to make a qubit identical to x.

At first, $x = \alpha |0\rangle + \beta |1\rangle$, and $a =$ either 0 or 1, since it's already been measured. Then, x will become its negative, $x = \beta |0\rangle + \alpha |1\rangle$, if and only if $a = 1$. Then, the Hadamard operation is performed on a, which becomes $a = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$. Then, x and a are both measured to be either 0 or 1. At this point, consider these measured values to be x and y described in problem 53, and do the procedure described in the solution to 54 to get the original state of x.