Homework 19

Joe Baker, Brett Schreiber, Brian Knotten

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Problem 7.8 from the text: Show that if $\overline{SAT} \in BPNP$, then PH collapses to Σ_3^p.
Hint: Recall the proof that BPP is in Σ_2^p.

Start of the proof:
$\overline{SAT} \in BPNP \Rightarrow PH \subseteq \Sigma_3^p$
Assume $\overline{SAT} \in BPNP$.

End of the proof:
$\Sigma_3^p = \Pi_3^p$
Therefore $PH \subseteq \Sigma_3^p$, since $\exists ... \forall \exists \forall \exists P = \exists ... \forall \Sigma_3^p = \exists ... \forall \Pi_3^p = \exists ... \Pi_3^p ... = \Pi_3^p = \Sigma_3^p$.

Brett's note: Proving that $\overline{SAT} \leq SAT$ would imply that $NP = coNP$, which implies that $PH = NP = \Sigma_1^p$, which is a stronger conclusion than the homework. Since it's stronger, it is probably harder to prove, so I would avoid this route.