Homework 17

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$\rho = \frac{1}{\pi}$
The complete binary representation of $\frac{1}{\pi}$ is infinitely long. Any finite representation is not completely accurate, since $\frac{1}{\pi}$ is irrational.

Since $\frac{1}{\pi}$ is infinitely long, the probability that it contains an encoding of a computation history as a substring is 1.

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Show that (NP $\cup$ co-NP) $\subseteq$ PP.

Proof:
We will show that Boolean Satisfiability $\in$ PP. Since Boolean Satisfiability is NP-complete, then any $L \in$ NP $\rightarrow L \in$ PP.
Let M be a probabilistic Turing machine which takes a Boolean formula F as input. Then does the following:

1. Pick a random assignment of variables for F
2. Verify if F is satisfied
3. If F is satisfied, then accept
4. If F is not satisfied, accept with $P(1/2)$ or reject with $P(1/2)$

First note that each step can clearly be done in polynomial time. P(M accepts) $= 1/2 + P$(random assignment is satisfied) and since P(random assignment is satisfied) > 0, then P(M accepts) $> 1/2$ and P(M rejects) $< 1/2$. Thus Boolean Satisfiability $\in$ PP since M is a Turing machine which meets the requirements for PP. So NP is contained in PP.

Since PP is closed under complement and contains NP, it must also contain co-NP. Finally, (NP $\cup$ co-NP) $\subseteq$ PP.