Homework 9

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February 2, 2018

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Definition 1: $TIME(T(n))$ is the set of all languages L such that $\exists$ TM M such that (1) M accepts x iff $x \in L$ and (2) $\forall$ but finitely many x, M on x halts in $T(|x|)$ steps.
Definition 2: $TIME(T(n))$ is the set of all languages L such that $\exists$ TM N and a number b such that (1) N accepts x iff $x \in L$ and (2) $\forall x$, N on x halts within $b * T(|x|)$ steps.

First, show $M \Rightarrow N$:
Given M, there exists a finite number of strings X such that M does not halt on each $x \in X$ in $T(|x|)$ steps.
Consider the running time of M on each $x_i \in X$. Let s_i be the number of steps M takes to accept x_i.
Let $b_i = \frac{s_i}{|x_i|}$. Therefore $s_i = b_i * |x_i|$.
Let b be the largest b_i for all $x_i \in X$.
Now construct N as a Turing machine which does the following:
On input x:

    Run M on x while counting the number of steps.
    If M accepts x in $b * T(|x|)$ steps, then accept.

$N \in TIME(T(n))$ because it accepts x within some number $b * T(|x|)$ steps. Therefore $M \Rightarrow N$.

Second, show $N \Rightarrow M$:
We will use N that accepts x in $b * T(|x|)$ steps to build M that accepts x in $T(|x|)$ steps with finitely many exceptions.
Given N, we will construct M's alphabet γ' and set of states Q' from N's alphabet γ and set of states Q as follows:
$\gamma' = \gamma \cup \gamma^b$
$Q' = Q \times b \times \gamma^{3b} \cup Q \times \gamma^i$ where $i = 1, ..., m-1$
Define M as follows:

- Copy the input from the read tape to a new tape in chunks of size b using the symbols from γ' that correspond to the chunks.
- As each symbol in a chunk of b is read M will transition to a new state in $Q \times \gamma^i$ in order to remember the symbols in the chunk it has read.
- When the end of the input is encountered, pad the rest of the chunk with spaces until the chunk has size b
- M will then perform four steps by moving to the left once, right twice, and to the left once again. During each of these steps M will move to a state in $Q \times b \times \gamma^{3b}$ to track the next b steps of N.
- When M encounters an accepting state of N, accept.

M executes every b number of steps of N in a constant number of steps, so M runs in $T(|x|)$ time and accepts iff N accepts.

We have shown that given M we can construct N and therefore $M \Rightarrow N$ and that given N we can construct M and therefore $N \Rightarrow M$. Thus, $M \Leftrightarrow N$ and the definitions are equivalent.