Homework 18

Joe Baker, Brett Schreiber, Brian Knotten

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$BPNP$ is the set of languages which can probablistically reduce to 3SAT with a probability of $\frac{2}{3}$. Let L be a language in $BPNP$.

L can be probablistically reduced to 3SAT with probability of failure $\frac{1}{2^n}$ using a circuit C, this is possible, because C can be produced by running a reducer TM R enough times such that the probability of failing to reduce is $\frac{1}{2^n}$.

Let the randomized reduction to 3SAT be conducted by a random bitstring of length m, which is functionally dependent on n. There exist 2^m possible random reductions, which produces a 3SAT instance that accurately represents

There are 2^m possible reductions given m. For any input x of size n bits, there are at most $\frac{2^m}{2^{n+1}}$ reductions that are not correct. By the union bound over all inputs, there are at most $2^n * \frac{2^m}{2^{n+1}} = \frac{2^m}{2}$ reductions which are not correct out of the total 2^m reductions. By the probablistic method, there must be at least one reduction which is correct for all inputs. Hard-code this string m, along with the NP/poly machine that solves 3SAT, and you will have a circuit that solves L. Therefore, $BPNP \subseteq NP/poly$.

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Let L be a language in BPL. This means that a TM for L correctly decides with $\frac{2}{3}$ probability on input x with a logarithmic amount of space.

Let N be a TM in P with the following behavior:
On input $< M, x >$, where M is a BPL Turing Machine:
Let M have a probability $\frac{1}{2}$ to take one of two transitions on each configuration.
Let m be the maximum number of steps M takes on input x.
Enumerate 2^m different bitstrings such that each bit represents a choice at each transition.
Count the total number of accepting and rejecting outcomes, and accept if the majority of the random bitstrings cause M to accept. Otherwise, reject.

Since the language of $M \in$ BPL, the majority of probablistic outcomes will result in M accepting if x is in the language. So N, a P TM, can deterministically decide membership for a probablistic TM M.