Homework 22

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Asking whether

$$\exists_x \forall_y \exists_z (x \vee y \vee \neg z) \wedge (\neg x \vee \neg y \vee z)$$

is satisfiable is equivalent to asking if:

$$\Sigma_{x=0}^{1} \Pi_{y=0}^{1} \Sigma_{z=0}^{1} (1-(1-x)(1-y)z)(1-xy(1-z) \neq 0$$

Merlin will construct a function $s(x)$ as follows:

$$\begin{aligned}
s(x) &= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} (1-(1-x)(1-y)z)(1-xy(1-z)) \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} 1 - z(1-x)(1-y) - xy(1-z) + xyz(1-x)(1-y)(1-z) \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} 1 - (z-xz)(1-y) - xy + xyz + (xyz - x^2yz)(1-y)(1-z) \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} 1 - (z-xz)(1-y) - xy + xyz + (xyz - x^2yz)(1-y)(1-z) \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} 1 - (z-xz) - y(z-zx) - xy + xyz + (xyz - x^2yz)(1-y)(1-z) \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} 1 - z - xz - yz + xyz - xy + xyz + xyz - x^2yz - y(xyz - x^2yz))(1-z) \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} 1 - z - xz - yz + 2xyz - xy - x^2yz - (xy^2z + x^2y^2z)(1-z) \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} 1 - z - xz - yz + 2xyz - xy - x^2yz - (xy^2z + x^2y^2z - z(xy^2z + x^2y^2z)) \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} 1 - z - xz - yz + 2xyz - xy - x^2yz - (xy^2z + x^2y^2z - xy^2z^2 - x^2y^2z^2) \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} 1 - z - xz - yz + 2xyz - xy - x^2yz - xy^2z - x^2y^2z + xy^2z^2 + x^2y^2z^2 \\
&= \Pi_{y=0}^{1} \Sigma_{z=0}^{1} 1 - z - xy - xz - yz + 2xyz - x^2yz - xy^2z - x^2y^2z + xy^2z^2 + x^2y^2z^2
\end{aligned}$$