Homework 10

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February 4, 2018

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Consider the language $L = \{< M > | < M >$ is a Turing Machine that loops forever$\}$. One of the strings in L can effectively written as a valid Java program containing the line:

while(true) {}

Mini Java's looping mechanism must be given a finite number describing how many times to loop. So all Mini Java programs cannot simulate an infinite loop. Therefore, L is a language that Java can simulate, but Mini Java cannot.

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a

A TM M can be defined as follows so that $L(M) = A$:
On input x:
    Instantiate $c = 0$ on the working tape.
    for each character $x_i \in x$:
        if $x_i = $ '(', then increment c.
        else, if $x_i = $ ')', then decrement c.
        if $c < 0$ reject. (There is a right paren before a left one).
    Accept if an only if the final value of $c = 0$.
$L(M) = A$ because $L(M)$ only accepts when the number of left parentheses matches the number of right parentheses. M runs in logspace because the in the worst case, the input x to M will be n number of left parentheses. So the working tape has to count up to n. But by using the standard base-2 binary encoding of n, the working tape will only use a maximum of $\log(n)$ cells.

b

A TM N can be defined as follows so that $L(N) = B$:
Assume N has two work tapes.
On input x:
    First pass: For each character x_i, x_{i+1} in x:
        If $x_i, x_{i+1} = $ '(', ']', reject,
        or if $x_i, x_{i+1} = $ '[', ')', reject.

    Set $c_1 = 0$ on the first working tape.
Set $c_2 = 0$ on the second working tape.
    Second pass: For each character x_i in x:
        If $x = $ '(', increment c_1
        or if $x = $ ')', decrement c_1
        or if $x = $ '[', increment c_2
        or if $x = $ ']', decrement c_2

If $c_1 < 0$ or $c_2 < 0$, then immediately reject.
If $c_1 = 0$ and $c_2 = 0$, then accept. Otherwise, reject.

N runs in logspace, even though it uses two work tapes, since $2 \log(n) = O(\log(n))$.