Homework 9

CS1675
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Part I:

1

Want $P(F=0|D=0)$ i.e. the probability that the tank is empty given that the driver observed the gauge showing empty.
We will use Bayes' Theorem: $P(w|D) = \frac{P(D|w)P(w)}{P(D)}$ and $P(D) = \Sigma_w P(D|w)P(w)$

So: $P(F=0|D=0) = \frac{P(D=0|F=0)P(F=0)}{P(D=0)}$. This corresponds to figure 8.54 with $a=B$, $b=F$, $c=G$, and $d=D$.

$P(F=0)$ is given on page 377 in Bishop as 0.1

$P(D=0) = P(D=0|G=0)\cdot P(G=0) + P(D=0|G=1)\cdot P(G=1)$
$= 0.9\cdot 0.315 + 0.1\cdot 0.685$ ($G=0 and G=1$ given on page 377)
$= 0.352$

$P(D=0|F=0) = \Sigma_{B,G} P(D=0|G)\cdot P(G|B,F=0)\cdot P(B)$
$= 0.9\cdot 0.81 + 0.1\cdot 0.19$ ($P(G=0|F=0)$ given by 8.31, $P(G=1|F)$ given on page 377)
$= 0.748$

Combine with $P(F=0)$ to get $P(F=0|D=0) = \frac{0.748\cdot 0.1}{0.352} = 0.2125$

Put together with Bayes' Theorem we get:
$P(F=0|D=0,B=0) = \frac{P(D=0|F=0,B=0)P(F=0)}{P(D=0)}$
$P(D=0|F=0,B=0) = \Sigma_G P(D=0|G)\cdot P(G|B=0,F=0)$
$= 0.9\cdot 0.9 + 0.1\cdot 0.1$
$= 0.81 + 0.01 = 0.82$
$P(F=0) = 0.1$
$P(D=0) = \Sigma_{GF} P(D=0|G)\cdot P(G|B=0,F)\cdot P(F)$
$= (0.9\cdot 0.9\cdot 0.1) + (0.1\cdot 0.1\cdot 0.1) + (0.9\cdot 0.8\cdot 0.9) + (0.1\cdot 0.2\cdot 0.9)$

So, $P(F=0|D=0,B=0) = \frac{0.82\cdot 0.1}{0.748} = 0.109626$

2

Using both versions of Bayes' Theorem we can construct:
$P(Y|x_1,x_2,...,x_n) = \frac{P(Y)\Pi_{i=1}^{n} P(x_i|Y)}{\Sigma_j P(Y=y_j)\Pi_i P(x_i|Y=y_j)}$

a

Want $P(J=1|W=1,B=0,C=0,R=1)$

$= \frac{P(J=1) \cdot P(W=1|J=1) \cdot P(B=0|J=1) \cdot P(C=0|J=1) \cdot P(R=1|J=1)}{(P(J=0) \cdot P(W=1|J=0) \cdot P(B=0|J=0) \cdot P(C=0|J=0) \cdot P(R=1|J=0)) + (P(J=1) \cdot P(W=1|J=1) \cdot P(B=0|J=1) \cdot P(C=0|J=1) \cdot P(R=1|J=1))}$

$= \frac{0.3 \cdot 0.8 \cdot 0.8 \cdot 0.3 \cdot 0.5}{(0.7 \cdot 0.3 \cdot 0.5 \cdot 0.7 \cdot 0.4) + (0.3 \cdot 0.8 \cdot 0.8 \cdot 0.3 \cdot 0.5)}$

$= \frac{0.0288}{0.0582}$

$= 0.4948$

b

Want $P(J = 1|W = 1, B = 1, C = 1, R = 1)$

$= \frac{P(J=1) \cdot P(W=1|J=1) \cdot P(B=1|J=1) \cdot P(C=1|J=1) \cdot P(R=1|J=1)}{(P(J=0) \cdot P(W=1|J=0) \cdot P(B=1|J=0) \cdot P(C=1|J=0) \cdot P(R=1|J=0)) + (P(J=1) \cdot P(W=1|J=1) \cdot P(B=1|J=1) \cdot P(C=1|J=1) \cdot P(R=1|J=1))}$

$= \frac{0.3 \cdot 0.8 \cdot 0.2 \cdot 0.7 \cdot 0.5}{(0.7 \cdot 0.3 \cdot 0.5 \cdot 0.3 \cdot 0.4) + (0.3 \cdot 0.8 \cdot 0.2 \cdot 0.7 \cdot 0.5)}$

$= \frac{0.0168}{0.0294}$

$= 0.5714$

Part III:

- Sentence 1: “john ate”
- Probability of occurrence: 0.0256787500
- Sentence 2: “john saw the cat”
- Probability of occurrence: 0.0002161767969
- Sentence 3: “cat saw the john”
- Probability of occurrence: 0.00004636805
- Sentence 4: “john saw the saw”
- Probability of occurrence: 0.0002810662834
- Sentence 5: “ john ate the cat”
- Probability of occurrence: 0.0001916131906
- Sentences in order of likelihood:
    - Sentence 1: “john ate”
    - Sentence 4: “john saw the saw”
    - Sentence 2: “john saw the cat”
    - Sentence 5: “john ate the cat”
    - Sentence 3: “cat saw the john”

All five sentences start with nouns, and the four most likely begin with proper nouns, so it makes sense that they are more likely than the only sentence that begins with “cat” (Sentence 3), which is rarely a proper noun and is therefore usually preceded by a (missing) determiner. Sentence 5 (“john ate the cat”) and Sentence 2 (“john saw the cat”) are within 0.00002 of each other, which makes sense as they are both the pronoun-verb-determiner-noun pattern and differ only by the verb. Sentence 4 (“john saw the saw”) is 0.00007 more likely than Sentence 2 (“john saw the cat”), which is interesting as they are both the same p-v-d-n pattern and again differ only by a word, this time a noun. The difference in probability between these two (Sentences 4 and 2) is particularly interesting as Sentence 4 is a very odd sentence to actually say and is uncommon when compared to Sentence 2. Sentence 1 (“john ate”) being the most common sentence makes sense, at least intuitively, because of its brevity and the fact that its last word is a verb. Sentences terminating with a verb being more common than those that do not would also explain why Sentence 4 is (relatively) far more common than Sentences 2, 4, and 3 – “saw” is more likely to be interpreted as a verb than a noun.

Something I found interesting when completing this assignment is that, unless I’m seriously misinterpreting the purpose of this HMM and the results, short sentences are judged as far more likely than longer sentences. This makes sense, at least to me, when comparing a four-word sentence to a ten-word sentence, but not when comparing a one word sentence to a two word sentence.