

Homework 7
CS1675
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Part I:

$x = [11], y = [00], \eta = 0.3, \delta_j = z_j(1 - z_j)\Sigma_k W_{kj}\delta_k$
Activation = $\frac{1}{1+e^{-a}}, a_j = \Sigma_{i=1}^D w_{ji}^{(1)} x_i + w_{j0}^{(1)}$
All weights set to 0.05

Forward Activations:

$$z_1 = \frac{1}{1+e^{-a}}$$

$$\begin{aligned} -a &= x_0 w_{10}^{(1)} + x_1 w_{11}^{(1)} + x_2 w_{12}^{(1)} \\ &= 1 \cdot 0.05 + 1 \cdot 0.05 + 1 \cdot 0.05 \\ &= 0.15 \end{aligned}$$

$$z_1 = \frac{1}{1+e^{-0.15}} = 0.5374$$

$$z_2 = \frac{1}{1+e^{-a}}$$

$$\begin{aligned} -a &= x_0 w_{20}^{(1)} + x_1 w_{21}^{(1)} + x_2 w_{22}^{(1)} \\ &= 1 \cdot 0.05 + 1 \cdot 0.05 + 1 \cdot 0.05 \\ &= 0.15 \end{aligned}$$

$$z_2 = \frac{1}{1+e^{-0.15}} = 0.5374$$

$$y_1 = \frac{1}{1+e^{-a}}$$

$$\begin{aligned} -a &= z_0 w_{10}^{(2)} + z_1 w_{11}^{(2)} + z_2 w_{12}^{(2)} \\ &= 1 \cdot 0.05 + 0.5374 \cdot 0.05 + 0.5374 \cdot 0.05 \\ &= 0.1037 \end{aligned}$$

$$y_1 = \frac{1}{1+e^{-0.1037}} = 0.5259$$

$$y_2 = \frac{1}{1+e^{-a}}$$

$$\begin{aligned} -a &= z_0 w_{20}^{(2)} + z_1 w_{21}^{(2)} + z_2 w_{22}^{(2)} \\ &= 1 \cdot 0.05 + 0.5374 \cdot 0.05 + 0.5374 \cdot 0.05 \\ &= 0.1037 \end{aligned}$$

$$y_2 = \frac{1}{1+e^{-0.1037}} = 0.5259$$

Backpropogation:

$$\delta_k = y_k(1 - y_k)(y_k - t_k), t_k = \text{desired } y_k \text{ output} = 0, y_k = y_1 = y_2 = 0.5259$$

$$\frac{\partial E}{\partial w_{kj}^{(2)}} = \delta_k z_j, w^{(\tau+1)} = w^{(\tau)} - \eta \nabla E(w^{(\tau)})$$

$$\delta_{y1} = 0.5259 \cdot (1 - 0.5259) \cdot (0.5259 - 0) = 0.1311$$

$$\delta_{y2} = 0.5259 \cdot (1 - 0.5259) \cdot (0.5259 - 0) = 0.1311$$

$$z_0 = 1, z_1 = z_2 = 0.5374, \eta = 0.3, w^{(\tau)} = 0.05 \quad \forall w_{kj}$$

$$w_{10}^{(2)} = w_{10}^{(2)} - 0.3 \cdot \delta_{y1} \cdot z_0 = 0.05 - 0.3 \cdot 0.1311 \cdot 1 = 0.01067$$

$$w_{11}^{(2)} = w_{11}^{(2)} - 0.3 \cdot \delta_{y1} \cdot z_1 = 0.05 - 0.3 \cdot 0.1311 \cdot 0.5374 = 0.02886$$

$$w_{12}^{(2)} = w_{12}^{(2)} - 0.3 \cdot \delta_{y1} \cdot z_2 = 0.05 - 0.3 \cdot 0.1311 \cdot 0.5374 = 0.02886$$

$$w_{20}^{(2)} = w_{20}^{(2)} - 0.3 \cdot \delta_{y2} \cdot z_0 = 0.05 - 0.3 \cdot 0.1311 \cdot 1 = 0.01067$$

$$w_{21}^{(2)} = w_{21}^{(2)} - 0.3 \cdot \delta_{y2} \cdot z_1 = 0.05 - 0.3 \cdot 0.1311 \cdot 0.5374 = 0.02886$$

$$w_{22}^{(2)} = w_{22}^{(2)} - 0.3 \cdot \delta_{y2} \cdot z_2 = 0.05 - 0.3 \cdot 0.1311 \cdot 0.5374 = 0.02886$$

$$\delta_j = z_j(1 - z_j) \sum_k w_{kj} \delta_k, z_j = z_j \text{ from activation } (0.5374), w_{kj} = \text{weight from before update } (0.05), \frac{\partial E}{\partial w_{ji}^{(1)}} = \delta_j x_i$$

$$\delta_{z1} = z_1 \cdot (1 - z_1) \cdot (w_{11}^{(2)} \cdot \delta_{y1} + w_{21}^{(2)} \cdot \delta_{y2})$$

$$= 0.5374 \cdot (1 - 0.5374) \cdot (0.05 \cdot 0.1311 + 0.05 \cdot 0.1311)$$

$$= 0.003259$$

$$\delta_{z2} = z_2 \cdot (1 - z_2) \cdot (w_{12}^{(2)} \cdot \delta_{y1} + w_{22}^{(2)} \cdot \delta_{y2})$$

$$= 0.5374 \cdot (1 - 0.5374) \cdot (0.05 \cdot 0.1311 + 0.05 \cdot 0.1311)$$

$$= 0.003259$$

$$w_{10}^{(1)} = w_{10}^{(1)} - 0.3 \cdot \delta_{z1} \cdot x_0 = 0.05 - 0.3 \cdot 0.003259 \cdot 1 = 0.04902$$

$$w_{11}^{(1)} = w_{11}^{(1)} - 0.3 \cdot \delta_{z1} \cdot x_1 = 0.05 - 0.3 \cdot 0.003259 \cdot 1 = 0.04902$$

$$w_{12}^{(1)} = w_{12}^{(1)} - 0.3 \cdot \delta_{z1} \cdot x_2 = 0.05 - 0.3 \cdot 0.003259 \cdot 1 = 0.04902$$

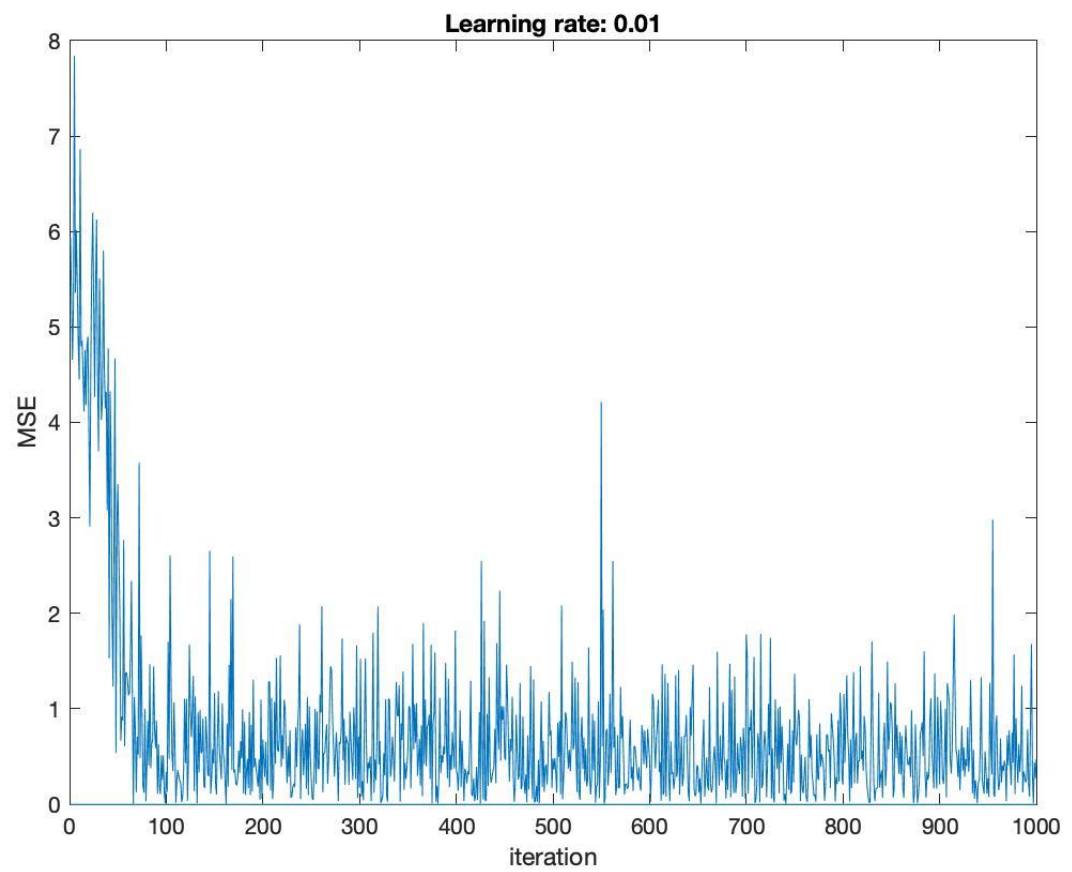
$$w_{20}^{(1)} = w_{20}^{(1)} - 0.3 \cdot \delta_{z2} \cdot x_0 = 0.05 - 0.3 \cdot 0.003259 \cdot 1 = 0.04902$$

$$w_{21}^{(1)} = w_{21}^{(1)} - 0.3 \cdot \delta_{z2} \cdot x_1 = 0.05 - 0.3 \cdot 0.003259 \cdot 1 = 0.04902$$

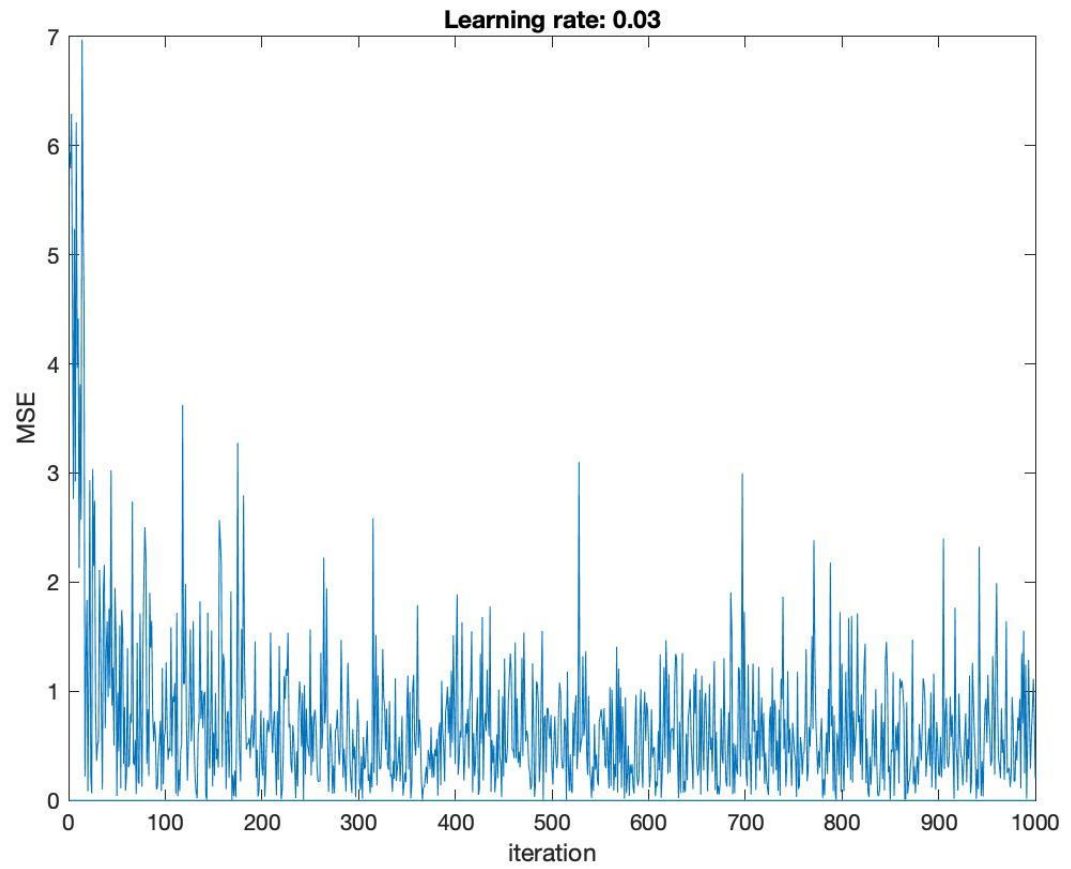
$$w_{22}^{(1)} = w_{22}^{(1)} - 0.3 \cdot \delta_{z2} \cdot x_2 = 0.05 - 0.3 \cdot 0.003259 \cdot 1 = 0.04902$$

Part III:

- Root mean squared error between the predicted and ground truth labels with a learning rate of 0.01: 0.708557



- Root mean squared error between the predicted and ground truth labels with a learning rate of 0.03: 0.716838



- Root mean squared error between the predicted and ground truth labels with a learning rate of 0.05: 0.955722

