

Part IV: Problem 1.3

We will use Bayes' Theorem to solve both parts of this problem:

$$p(w|D) = \frac{p(D|w)p(w)}{p(D)} \text{ and } p(D) = \sum_w p(D|w)p(w)$$

So, by Bayes' Theorem the probability of selecting an apple $p(a)$ is given by:

$$p(a) = p(a|r)p(r) + p(a|b)p(b) + p(a|g)p(g) = \frac{3}{10} \cdot 0.2 + \frac{1}{2} \cdot 0.2 + \frac{3}{10} \cdot 0.6 = 0.34$$

That is, the probability of selecting an apple $p(a)$ is given by the probability that you choose to select from the red box $p(r)$ times the probability you select an apple from the red box $p(a|r)$, plus the probability that you choose to select from the blue box $p(b)$ times the probability you select an apple from the blue box $p(a|b)$, plus the probability that you choose to select from the green box $p(g)$ times the probability you select an apple from the green box $p(a|g)$.

Also by Bayes' Theorem, the probability of having selected from the green box given that you selected an orange $p(g|o)$ is given by:

$$p(g|o) = \frac{p(o|g)p(g)}{p(o)}$$

Where the probability of selecting an orange in general $p(o)$ is given by:

$$p(o) = p(o|r)p(r) + p(o|b)p(b) + p(o|g)p(g) = \frac{4}{10} \cdot 0.2 + \frac{1}{2} \cdot 0.2 + \frac{3}{10} \cdot 0.6 = 0.36$$

That is, the probability of selecting an orange $p(o)$ is given by the probability that you choose to select from the red box $p(r)$ times the probability you select an orange from the red box $p(o|r)$, plus the probability that you choose to select from the blue box $p(b)$ times the probability you select an orange from the blue box $p(o|b)$, plus the probability that you choose to select from the green box $p(g)$ times the probability you select an orange from the green box $p(o|g)$.

So the probability of having selected from the green box given that you selected an orange $p(g|o)$ is:

$$p(g|o) = \frac{p(o|g)p(g)}{p(o)} = \frac{\frac{3}{10} \cdot 0.6}{0.36} = 0.5$$

That is, the probability of having selected from the green box given that you selected an orange $p(g|o)$ is given by the probability you choose to select from the green box $p(g)$ times the probability you select an orange from the green box $p(o|g)$, divided by the probability you choose an orange in general $p(o)$.