Homework 9

CS1675
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Part I:

1

Want $P(F = 0|D = 0)$ i.e. the probability that the tank is empty given that the driver observed the gauge showing empty.
We will use Bayes' Theorem: $P(w|D) = \frac{P(D|w)P(w)}{P(D)}$ and $P(D) = \Sigma_w P(D|w)P(w)$

So: $P(F = 0|D = 0) = \frac{P(D=0|F=0)P(F=0)}{P(D=0)}$. This corresponds to figure 8.54 with $a = B$, $b = F$, $c = G$, and $d = D$.

$P(F = 0)$ is given on page 377 in Bishop as 0.1

$P(D = 0) = P(D = 0|G = 0) \cdot P(G = 0) + P(D = 0|G = 1) \cdot P(G = 1)$
$= 0.9 \cdot 0.315 + 0.1 \cdot 0.685$ ($G = 0 and G = 1$ given on page 377)
$= 0.352$

$P(D = 0|F = 0) = \Sigma_{B,G} P(D = 0|G) \cdot P(G|B, F = 0) \cdot P(B)$
$= 0.9 \cdot 0.81 + 0.1 \cdot 0.19$ ($P(G = 0|F = 0)$ given by 8.31, $P(G = 1|F)$ given on page 377)
$= 0.748$

Combine with $P(F = 0)$ to get $P(F = 0|D = 0) = \frac{0.748 \cdot 0.1}{0.352} = 0.2125$

Put together with Bayes' Theorem we get:
$P(F = 0|D = 0, B = 0) = \frac{P(D=0|F=0,B=0)P(F=0)}{P(D=0)}$
$P(D = 0|F = 0, B = 0) = \Sigma_G P(D = 0|G) \cdot P(G|B = 0, F = 0)$
$= 0.9 \cdot 0.9 + 0.1 \cdot 0.1$
$= 0.81 + 0.01 = 0.82$
$P(F = 0) = 0.1$
$P(D = 0) = \Sigma_{GF} P(D = 0|G) \cdot P(G|B = 0, F) \cdot P(F)$
$= (0.9 \cdot 0.9 \cdot 0.1) + (0.1 \cdot 0.1 \cdot 0.1) + (0.9 \cdot 0.8 \cdot 0.9) + (0.1 \cdot 0.2 \cdot 0.9)$

So, $P(F = 0|D = 0, B = 0) = \frac{0.82 \cdot 0.1}{0.748} = 0.109626$

2

Using both versions of Bayes' Theorem we can construct:
$P(Y|x_1, x_2, ..., x_n) = \frac{P(Y)\Pi_{i=1}^{n} P(x_i|Y)}{\Sigma_j P(Y=y_j)\Pi_i P(x_i|Y=y_j)}$

a

Want $P(J = 1|W = 1, B = 0, C = 0, R = 1)$

$= \frac{P(J=1)\cdot P(W=1|J=1)\cdot P(B=0|J=1)\cdot P(C=0|J=1)\cdot P(R=1|J=1)}{(P(J=0)\cdot P(W=1|J=0)\cdot P(B=0|J=0)\cdot P(C=0|J=0)\cdot P(R=1|J=0))+(P(J=1)\cdot P(W=1|J=1)\cdot P(B=0|J=1)\cdot P(C=0|J=1)\cdot P(R=1|J=1))}$

$= \frac{0.3\cdot 0.8\cdot 0.8\cdot 0.3\cdot 0.5}{(0.7\cdot 0.3\cdot 0.5\cdot 0.7\cdot 0.4)+(0.3\cdot 0.8\cdot 0.8\cdot 0.3\cdot 0.5)}$

$= \frac{0.0288}{0.0582}$

$= 0.4948$

b

Want $P(J=1|W=1, B=1, C=1, R=1)$

$= \frac{P(J=1)\cdot P(W=1|J=1)\cdot P(B=1|J=1)\cdot P(C=1|J=1)\cdot P(R=1|J=1)}{(P(J=0)\cdot P(W=1|J=0)\cdot P(B=1|J=0)\cdot P(C=1|J=0)\cdot P(R=1|J=0))+(P(J=1)\cdot P(W=1|J=1)\cdot P(B=1|J=1)\cdot P(C=1|J=1)\cdot P(R=1|J=1))}$

$= \frac{0.3\cdot 0.8\cdot 0.2\cdot 0.7\cdot 0.5}{(0.7\cdot 0.3\cdot 0.5\cdot 0.3\cdot 0.4)+(0.3\cdot 0.8\cdot 0.2\cdot 0.7\cdot 0.5)}$

$= \frac{0.0168}{0.0294}$

$= 0.5714$