Computational Solid Mechanics Final Project Report

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1. Problem Description

   Strong form:

   Given $f_i : \Omega \to \mathbb{R}$, $g_i : \Gamma_{g_i} \to \mathbb{R}$, $h_i : \Gamma_{h_i}$

   determine $u_i : \bar{\Omega} \to \mathbb{R}$ s.t.

   $$\sigma_{ij,j} + f_i = 0$$

   $$u_i = g_i \quad \text{on } \Gamma_{g_i}$$

   $$\sigma_{ij} n_j = h_i \quad \text{on } \Gamma_{h_i}$$

   Weak form

   $\mathcal{S}_i := \{u_i : u_i \in H^1 \cdot u_i = g_i \text{ on } \Gamma_g)$

   $\mathcal{V}_i := \{w_i : w_i \in H^1, w_i = 0 \text{ on } \Gamma_g\}$

   Given ..., find that $u_i \in \mathcal{S}_i$, s.t.

   $$a(\vec{w}, \vec{u}) = (\vec{w}, \vec{f}) + (\vec{w}, \vec{h})_{\Gamma_h}$$

   where $a(\vec{w}, \vec{u}) := \int_\Omega w_{i,j} \sigma_{ij} \, d\Omega$

   $$(\vec{w}, \vec{f}) = \int_\Omega w_i f_i \, d\Omega$$

   $$(\vec{w}, \vec{h})_{\Gamma_h} = \int_\Gamma w_i h_i \, d\Gamma$$

   Galerkin formulation

Galerkin formulation and implement

$$\mathcal{V}_i^h = \{ w_i^h : w_i^h(x) = \sum_{A \in \eta - \eta_g} c_{iA} N_A(x) \}.$$

↑ i-th component

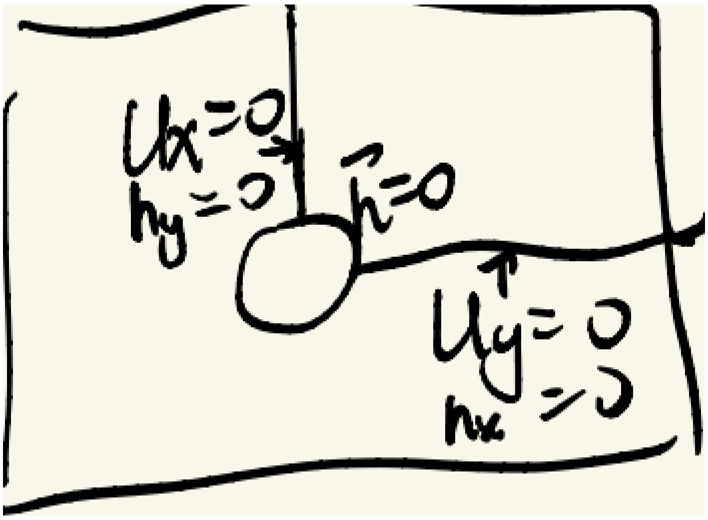
$$\mathcal{S}_i^h = \{ u_j^h = v_j^h + g_j^h,\ v_j^h = \sum_{B \in \eta - \eta_g} d_{jB} N_B,\ g_j^h = \sum_{B \in \eta_g} g_{jB} N_B \}.$$

$$\sum_{A \in \eta - \eta_g} c_A \{ a(N_A \vec{e}_i, u^h) = (N_A \vec{e}_i, f) + (N_A \vec{e}_i, h)_\Gamma \}$$

$$\Rightarrow a(N_A \vec{e}_i, N_B \vec{e}_j) d_{jB} = \int_\Omega N_A f_i \, d\Omega + \int_{\Gamma_h} N_A h_i \, da - a(N_A \vec{e}_i, \sum_{B \in \eta_g} g_{jB} N_B \vec{e}_j)$$

Boundary condition

For the outer surface, there are Dirichlet BC (e.g. g = 0) and Neumann BC (e.g. h = T). For inner hole, the BC is h = 0. For symmetry surface, the BC is like this



2. The implementation of the element stiffness matrix
   I choose $B_a^T D B_b$ implementation. Calculate B matrix first, then get k^e by matrix calculation.
3. Manufactured solution
   With given Tx = 10kpa, I calculate 3 stresses at each nodes as manufactured solution, then transfer the polar coordinates into Cartesian coordinates with equations below

$$\sigma_{rr}(r,\theta) = \frac{T_x}{2}\left(1 - \frac{R^2}{r^2}\right) + \frac{T_x}{2}\left(1 - 4\frac{R^2}{r^2} + 3\frac{R^4}{r^4}\right) cos2\theta,$$

$$\sigma_{\theta\theta}(r,\theta) = \frac{T_x}{2}\left(1 + \frac{R^2}{r^2}\right) - \frac{T_x}{2}\left(1 + 3\frac{R^4}{r^4}\right) cos2\theta,$$

$$\sigma_{r\theta}(r,\theta) = -\frac{T_x}{2}\left(1 + 2\frac{R^2}{r^2} - 3\frac{R^4}{r^4}\right) sin2\theta.$$

$$\sigma_x = \frac{\sigma_r + \sigma_\theta}{2} + \frac{\sigma_r - \sigma_\theta}{2}\cos 2\theta - \tau_{r\theta}\sin 2\theta$$

$$\sigma_y = \frac{\sigma_r + \sigma_\theta}{2} - \frac{\sigma_r - \sigma_\theta}{2}\cos 2\theta + \tau_{r\theta}\sin 2\theta$$

$$\tau_{xy} = \frac{\sigma_r - \sigma_\theta}{2}\sin 2\theta + \tau_{r\theta}\cos 2\theta$$

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sigma_rr = @(Tx,R,r,theta) Tx./2.*(1 - R.^2./r.^2) + Tx./2.*(1 - 4.*R.^2./r.^2 + 3.*R.^4./r.^4).*cos(2.*theta);
sigma_tt = @(Tx,R,r,theta) Tx./2.*(1 + R.^2./r.^2) - Tx./2.*(1 + 3.*R.^4./r.^4).*cos(2.*theta);
sigma_rt = @(Tx,R,r,theta) -Tx./2.*(1 + 2.*R.^2./r.^2 - 3.*R.^4./r.^4).*sin(2.*theta);
sigma_rr = sigma_rr(Tx,R,r,theta);
sigma_tt = sigma_tt(Tx,R,r,theta);
sigma_rt = sigma_rt(Tx,R,r,theta);
% f
sigma_11 = (sigma_rr + sigma_tt)./2 + (sigma_rr - sigma_tt)./2.*cos(2.*theta) - sigma_rt.*sin(2.*theta);
sigma_22 = (sigma_rr + sigma_tt)./2 - (sigma_rr - sigma_tt)./2.*cos(2.*theta) + sigma_rt.*sin(2.*theta);
sigma_12 = (sigma_rr - sigma_tt)./2.*sin(2.*theta) + sigma_rt.*cos(2.*theta);
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4. The codes are in driver.m