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1  /*
2    A)
3    The DES model is implemented using a tick/clock based simulation where we use ↗
4    bernouli trials to approximate a poisson distribution.
5    Each event (bike arrival, client(class1,2,3)) has its own random number ↗
6    generator. The interval selected for the bernouli approximation
7    is 100000 per 1 poisson interval, meaning that for 1 poisson time interval we ↗
8    run 100000 bernouli trials. The reason is to make
9    the approximation more accurate. We start off the simulation with  $X[0] = 10$ , ↗
10   and the totalMoney at  $(K1*r1 + K2*r2)$ . When a bike arrives
11   we increment the total number of bikes, and then distribute to anyone ↗
12   waiting. When a new client arrives, if a bike is available we give
13   a bike to the client, otherwise we put the client onto the queue and apply ↗
14   the penalty (0 for class 3). We replicate the simulation
15   100 times, namely because it takes such a long time to run, and then we ↗
16   averaged the totalMoney at the end of each run.
17 */
18
19 #include <iostream>
20 #include <random>
21 #include <queue>
22 #include <time.h>
23
24 struct Client
25 {
26     int type;
27 };
28
29 int main()
30 {
31     const int T = 120;
32     int X[121] = { 0 }; //There are T+1 events
33     const double bikeArrivalRate = 6;
34     //clients have rate r1 = 3, r2 = 1, r3 = 4
35     const double clientRates[4] = { 0, 3.0, 1.0, 4.0 };
36
37     //client class 1/2 pay annually ( $K1 = 0.5$ ,  $k2 = .1$ ), total amount is  $(K1*r1 + ↗$ 
38      $K2*r2)$ 
39     //class 3 pays per ride amount  $k3 = 1.25$ 
40
41     //when annual members (class 1/2) arrive at empty station, there is penalty ↗
42      $c1 = 1.0$ ,  $c2 = 0.25$ ,  $c3 = 0$ 
43     const double clientPenalty[4] = { 0, -1.0, -0.25, 0 };
44
45     //create and seed the generator
46     std::default_random_engine generator;
47     generator.seed(time(0));
48
49     //We can use a bernouli distribution with paramter  $p = \text{lambda} / ↗$ 
50     bernouliInterval
51     int bernouliInterval = 100000;
52     std::bernoulli_distribution randomVariableGenerator[4];
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43     randomVariableGenerator[0] = std::bernoulli_distribution(bikeArrivalRate /
        bernoulliInterval);
44     randomVariableGenerator[1] = std::bernoulli_distribution(clientRates[1] /
        bernoulliInterval);
45     randomVariableGenerator[2] = std::bernoulli_distribution(clientRates[2] /
        bernoulliInterval);
46     randomVariableGenerator[3] = std::bernoulli_distribution(clientRates[3] /
        bernoulliInterval);
47
48     const int numberOfTrials = 100;
49     double averageMoneyAmount = 0;
50
51     std::cout << "Starting the trials" << std::endl;
52
53     for (int t = 0; t < numberOfTrials; t++)
54     {
55         //client queue
56         std::queue<Client> line;
57
58         //we can assume total money starts at 0 + the deterministic annual
            prorated charge of clients classes 1 and 2
59         double totalMoney = (0.5 * clientRates[1]) + (0.1 * clientRates[2]);
60         X[0] = 10; //we start with 10 bikes at X(0)
61
62         for (int i = 1; i <= T; i++)
63         {
64             X[i] = X[i - 1]; //new time interval starts with bike amount from
                prev interval
65
66             for (int q = 0; q < bernoulliInterval; q++)
67             {
68                 //see if a bike has arrived
69                 if (randomVariableGenerator[0](generator)) X[i]++;
70
71                 //distribute the bikes to any clients waiting
72                 while (!line.empty() && X[i] > 0)
73                 {
74                     auto client = line.front();
75                     line.pop(); //remove from queue
76
77                     X[i]--; //decrement bike count
78                 }
79
80                 //see if a client of class 1-3 has arrived
81                 for (int j = 1; j <= 3; j++)
82                 {
83                     if (randomVariableGenerator[j](generator))
84                     {
85                         if (j == 3) totalMoney += 1.25; // class 3 pays per ride
86
87                         //if a client arrives and there are no bikes
88                         if (X[j] == 0)

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89         {
90             //add the client into the queue
91             line.emplace(Client{ j });
92             //we apply a penalty, for class3 penalty is 0
93             totalMoney += clientPenalty[j];
94         }
95         else
96         {
97             X[j]--; //otherwise just give the client a bike
98         }
99     }
100 }
101 }
102 }
103
104 std::cout << "Total Money at the end of experiment " << totalMoney << "\n";
105     std::endl;
106     averageMoneyAmount += totalMoney;
107 }
108
109 std::cout << "Average amount of money over " << numberOfTrials << "
110     iterations" << " : "
111     << (averageMoneyAmount / numberOfTrials) << std::endl;
112
113 return 0;
114 }
```