ELEN3007 - Probabilistic Systems Analysis Assignment

October 26, 2017

1 ELEN3007 ICAO-Codes-Weather-analysis

This Script imports the weather data for the Chulman Airport (UELL) weather station, in Russia and then answers question outlined. Each question that has a discussion of results is analised in the respective question markdown block before the code block.

The Git repo, storing this code, can be seen here:

https://github.com/SoIidarity/ICAO-Weather-Analysis-Python This main file, showing all code execution, can be seen here:

https://github.com/SoIidarity/ICAO-Weather-Analysis-Python/blob/master/MainFile.ipynb

Both a PDF version and HTML version of this report are included in this submission. For an optimal viewing experience, please view the HTML version as the formatting is persistent from the Jupyter notebook. Additionally, the PDF version has HTML elements stripped out, such as some images. Alternativly, view the second Git link above to view the notebook online, through Github.

First, Import libraries and such needed for program execution.

```
In [10]: import numpy as np
         import seaborn as sns

         import pandas as pd
         from statsmodels.graphics.tsaplots import plot_acf

         import scipy.stats
         import matplotlib.pyplot as plt
         from IPython.display import HTML, display

         from io import BytesIO
         from base64 import b64encode
         import scipy.misc as smp

         from mpl_toolkits.mplot3d import Axes3D
         from matplotlib import cm
         from matplotlib.ticker import LinearLocator
         from scipy.stats import norm

         plt.rcParams['figure.figsize'] = (26, 13)
```

First, define some useful functions. These are used in the printing of data later on. Also used for rendering images to the HTML page.

```
In [2]: def printMatrix(data):  #used to print matricies to HTML
            display(HTML(
                '<table><tr>{}</tr></table>'.format(
                    '</tr><tr>'.join(
                        '<td>{}</td>'
                        .format('</td><td>'
                        .join(str(_) for _ in row)) for row in data)
                )
            ))

        def printText(text):
            display(HTML('<p>' + text + '<p>'))

        def displayHTML(html):
            display(HTML(html))

        def drawImg(img):
            b = BytesIO()
            img.save(b, format='png')
            displayHTML("<img src='{0}'/>"
                        .format(b64encode(b.getvalue()).decode('utf-8')))
```

1.1 Question 1

Import data from text files. These are stored as csvs in the "Data" directory in the repo. This CSV is formatted as: {Unit Timestamp, max Temp, avg Temp, min Temp}. Each CSV is read into a matrix. These matricies are then added to a vector so they can be itterated through later on.

```
In [3]: w1995 = np.genfromtxt('Data/1995.csv', delimiter=',')
        w2000 = np.genfromtxt('Data/2000.csv', delimiter=',')
        w2005 = np.genfromtxt('Data/2005.csv', delimiter=',')
        w2010 = np.genfromtxt('Data/2010.csv', delimiter=',')
        w2015 = np.genfromtxt('Data/2015.csv', delimiter=',')

        weatherData = [w1995, w2000, w2005, w2010, w2015]
```

1.2 Question 2

Next, identify the minimum, maximum, mean and standard deviation for each year. As the matricies are in a vector, we can do this sequentially in a loop. These outputs are produced in a matrix, where the columns are the years (1995,2000,2005,2010,2015) and the rows are the minimum, maximum, mean and standard deviation for each year. This can be seen in the table below.

The mean is calculated with the standard numpy mean equation that calculates the following.

This is the average of the data set:

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Standard deviation is used to quantify variance in the data, as defined by:

$$s = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N-1}}$$

1.2.1 Comment on findings

The minimum, maximum, mean and standard deviations are very similar for each year. this is to be expected as they were taken from the same location. However, as time went on over the past 20 years, it seems that the averages became walmer with both the maximums and minimums increasing over time. There are outlyers in this trend as seen in 2010's minimum tempreture. This trend is also verified by looking at the mean where each year is shown to be getting walmer.

The high standard deviation seen is accounted for by the large tempreture changes during the course of the year. A delta between the maximum of 37 and minimum of -45 of 82 degrees is rather high.

```
In [4]: dataValues = np.zeros((4, 5))
        counter = 0;
        for year in weatherData:
            dataValues[0, counter] = year[:, 3].min()  #max of max values
            dataValues[1, counter] = year[:, 1].max()  #min of min values
            dataValues[2, counter] = round(year[:, 2].mean(), 2)  #average of average values
            dataValues[3, counter] = round(year[:, 2].std(), 2)  #Standard deviation of average
            counter = counter + 1;




        displayHTML(pd.DataFrame(dataValues,
                                 columns = ['1995', '2000', '2005', '2010', '2015'],
                                 index = ['Minimum', 'Maximum', 'Mean', 'Standard Deviation']).t
```

```
<IPython.core.display.HTML object>
```

1.3 Question 3

Each Years probability distribution functions are now plotted, for each year, on the same set of axes. To achive this, a fixed plot of univariate distributions is generated. This is done with the Seaborn displot function.

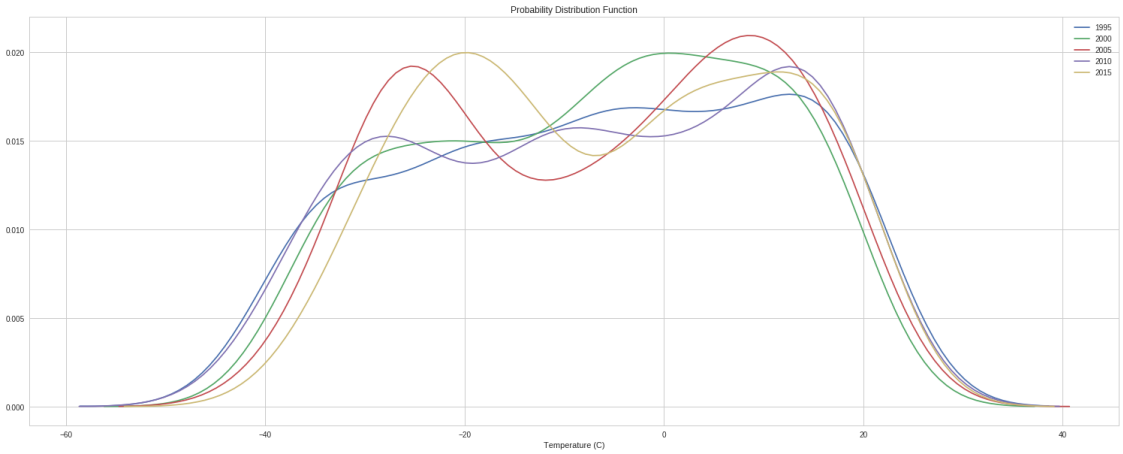
This value is calculated by using the seaborn distplot function that recreates the standard PDF equation as:

$$\mathrm{E}[X] = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

1.3.1 Comment on findings

Each year has a similar general probobility distribution function. They apear to be close to normal in shape. general average trends can also be seen through this graph such as 2005 had higher distribution of both highs and lows, with a lower distribution in the middel. This indicates more extreme weather during this year.

```
In [5]: sns.set_style('whitegrid')
        counter = 0;
        for year in weatherData:
            sns.distplot(year[:, 2], hist=False,
                         label=1995 + counter * 5,
                         axlabel="Temperature (C)")
            counter = counter + 1;
        plt.title('Probability Distribution Function')
        plt.show()
```



1.4 Question 4

The cross-correlation between each year's annual temperatures is now calculated. This is shown as a matrix output Note here that we are slicing the data on each year to ignore the leap year in 2000. This is done as the correlation needs matricies of the same dimension.

Additionally note that we need to normalise the data. This is the same as using the numpy corrfoef function.

TO calculate this, the numpy correlate function was used. This function makes use of this equation:

$$(f \star g)(\tau) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f^*(t)\, g(t+\tau)\, dt$$

1.4.1 Comment on findings

Through the cross-correlation of the presented data, one can see the relationship between each years weather. The closer these values are to 1, the more closely correlated the data is. The highest

correlation is seen between 2010 and 1995 intrestingly enough with a 94% correlation. The main diagonal of 1's indicates the full correlation, with each year corelated against its self.

```
In [6]: autoCorrelation = np.zeros((5, 5))
        for x in range(0, 5):
            for y in range(0, 5):
                a = weatherData[x][0:365, 2]
                b = weatherData[y][0:365, 2]
                a = (a - np.mean(a)) / (np.std(a) * len(a))
                b = (b - np.mean(b)) / (np.std(b))
                autoCorrelation[x, y] = np.correlate(a, b)
        columns = ['1995', '2000', '2005', '2010', '2015']
        index = ['1995', '2000', '2005', '2010', '2015']

        displayHTML(pd.DataFrame(autoCorrelation, columns, index).to_html())
```

```
<IPython.core.display.HTML object>
```

1.5 Question 5

The autocorrelation function for each year's data, where τ ranges from 0 to 364 is now generated.

Confidence intervals are drawn as a cone. By default, this is set to a 95% confidence interval, suggesting that correlation values outside of this code are very likely a correlation and not a statistical fluke.

First, each Autocorrelation is drawn on its own graph. After, they are overlayed.

Autocorrelation is found through the use of the plot_acf function from the tsaplots statistics library. From a signal processing point of view, this can be seen as:

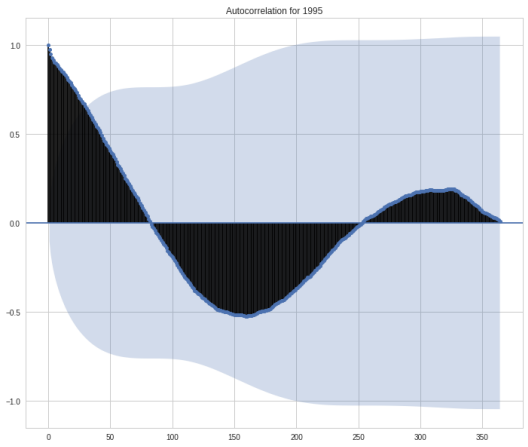
$$R_{ff}(\tau) = (f * g_{-1}(\overline{f}))(\tau) = \int_{-\infty}^{\infty} f(u+\tau)\overline{f}(u)\,\mathrm{d}u = \int_{-\infty}^{\infty} f(u)\overline{f}(u-\tau)\,\mathrm{d}u$$

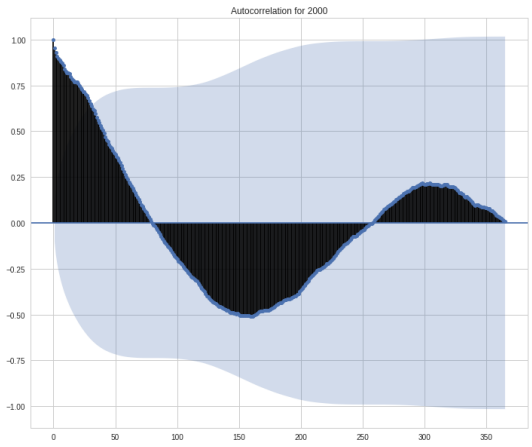
1.5.1 Comment on findings

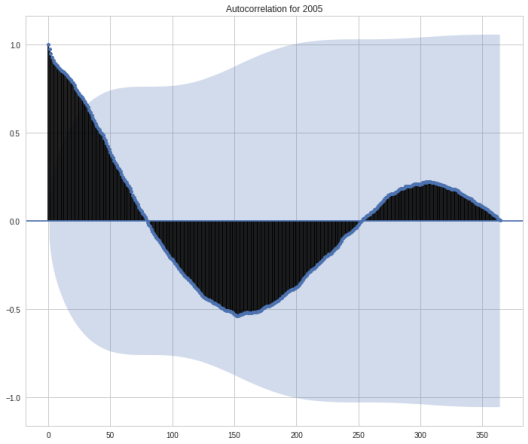
This autocorrelation shows how each year is related to a time-shifted version of itself. This can be seen as an effective convolution process. From this, one can see how each part of the year is related to the rest of the year for a given shift amount τ. For example, at τ=0, the magnitude is 1. At this time, no shifting has occurred. At approximately τ=150, there is the highest negative correlation seen. This is due to the correlation between the middle of summer and winter of the two data sets.

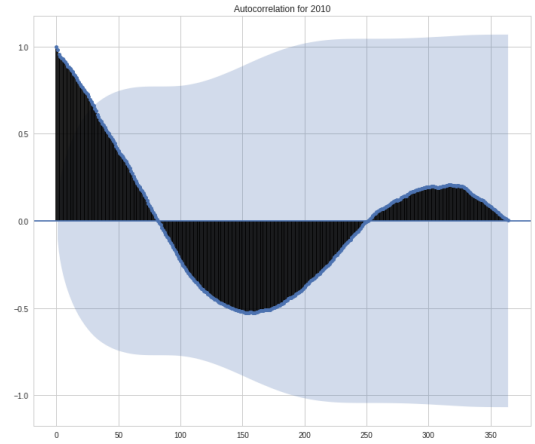
```
In [7]: counter = 0;
        for year in weatherData:

            if (counter == 0):  #on the first loop, generate the subplots
                fig, axs = plt.subplots(1, 2)

            if counter == 4:  # if we are at the last figure, we want it to be on its own line
                plot_acf(year[:, 2],
```

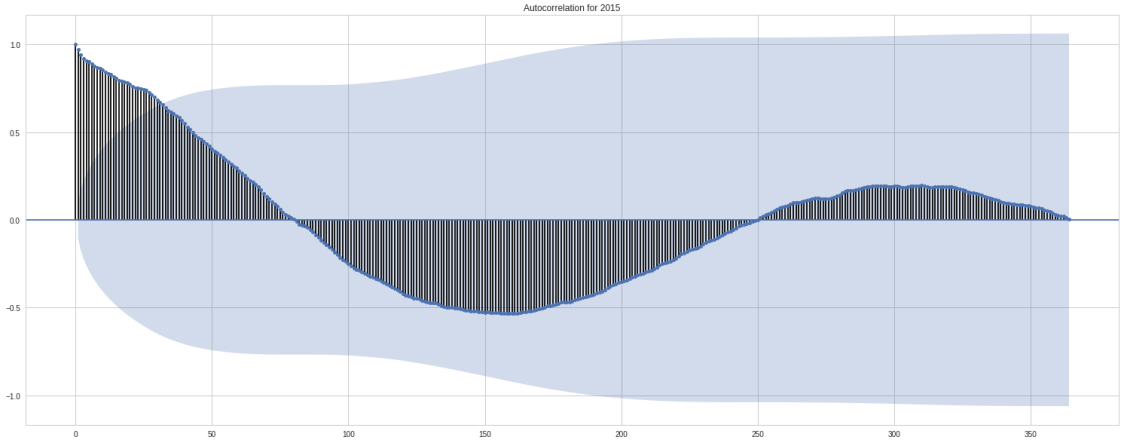
```
                title = "Autocorrelation for {}"
                .format(1995 + 5 * counter))
        plt.show()
    else:
        plot_acf(year[:, 2],
                title = "Autocorrelation for {}"
                .format(1995 + 5 * counter),
                ax=axs[counter % 2])
    if counter % 2 == 1:  # every two figures, we need to generate a new row
        plt.show()
        if (counter < 2):  #a new sub plot is needed on second row
            fig, axs = plt.subplots(1, 2)
    counter = counter + 1

counter = 0;
fig, axs = plt.subplots(1, 1)
for year in weatherData:

    plot_acf(year[:, 2],
             title = "Autocorrelation All 5 years together"
             .format(1995 + 5 * counter),
            ax=axs)

plt.show()
```

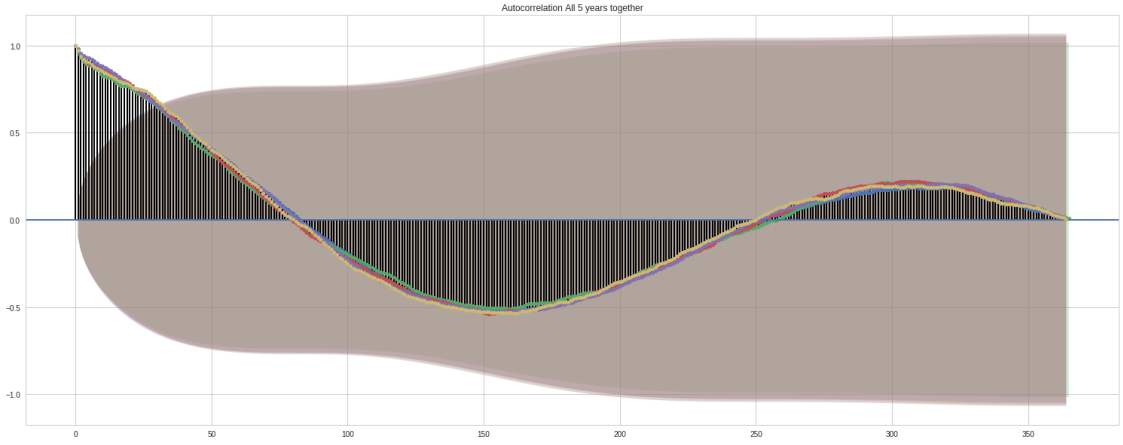





Autocorrelation for 2005
1.0
0.5
0.0
−0.5
−1.0
0
50
100
150
200
250
300
350


Autocorrelation for 2010
1.0
0.5
0.0
−0.5
−1.0
0
50
100
150
200
250
300
350


Autocorrelation for 2015
1.0
0.5
0.0
−0.5
−1.0
0
50
100
150
200
250
300
350


Autocorrelation All 5 years together
1.0
0.5
0.0
−0.5
−1.0
0
50
100
150
200
250
300
350

1.6 Question 6.a &6.b

Next, each year, temp is broken down into subdivisions in the range:

$$[minimum - 0.1 = t_0, maximum + 0.1 = t_10]$$

into ten equal intervals. This is then used to generate 10 intervals, as:

$$[[t_0, t_1], ..., [t_9, t_10]]$$

1. binCorours are predefined RGB values, given in the question
2. The linspace function below is used to generate a set of numbers with equal distributions between the min and max data for each year.
3. Digitisation is used to put the data into the respective buckets. This can then be used later on when the graphs are drawn
4. generate a matrix to store the image colour values
5. iterate over each day of the year*2. double as we need to have two rows for each day(high and low)
6. set the current iteration-1 row of pixels to the low digitised index value from colour matrix provided.
7. same as above, but for the current iterator for the high values provided
8. convert matrix to image
9. draw image to screen

The output below this code shows first the bins for each year, for question 6.a. Then, the blankets are drawn for question 6.b

```
In [8]: binColours = [[0.139681, 0.311666, 0.550652], [0.276518, 0.539432, 0.720771],
                     [0.475102, 0.695344, 0.802081], [0.670448, 0.803486, 0.824645],
                     [0.809791, 0.848259, 0.777550], [0.861927, 0.803423, 0.673050],
                     [0.830690, 0.667645, 0.546349], [0.742023, 0.475176, 0.424114],
                     [0.613033, 0.281826, 0.306352], [0.450385, 0.157961, 0.217975]]
        counter = 0;
        for year in weatherData:
            num = 10
            binsLow = np.linspace(year[:, 3].min() - 0.1, year[:, 3].max() + 0.1, num)
            binsHigh = np.linspace(year[:, 1].min() - 0.1, year[:, 1].max() + 0.1, num)

            digitizedLow = np.digitize(year[:, 3], binsLow)  #put the data into the bins
            digitizedHigh = np.digitize(year[:, 3], binsHigh)

            rows = 2 * len(year[:, 3])
            image = np.zeros((rows, 451, 3))  #make a matrix to store the image
        #itterate over each year's values from the above values and set pixels colours
            for x in range(0, rows, 2):
                image[x - 1, 0:451] = binColours[int(digitizedLow[int(x / 2)])]
                image[x, 0:451] = binColours[int(digitizedHigh[int(x / 2)])]
            printText("Tempreture blanket for year: {}".format(1995 + 5 * counter))

            displayHTML(pd.DataFrame(np.column_stack((binsLow.reshape(10, 1),
```

```
                                                    (binsHigh.reshape(10, 1))),
                                        columns=["Low bin", "High bin"]).to_html())

            outputImage = smp.toimage(image)
            outputImage.save('WeatherBlancketsOutput/{}.png'.format(1995 + 5 * counter))
            drawImg(outputImage)  #Draw image to screen, using custom draw function to put outpu
            counter = counter + 1;
```

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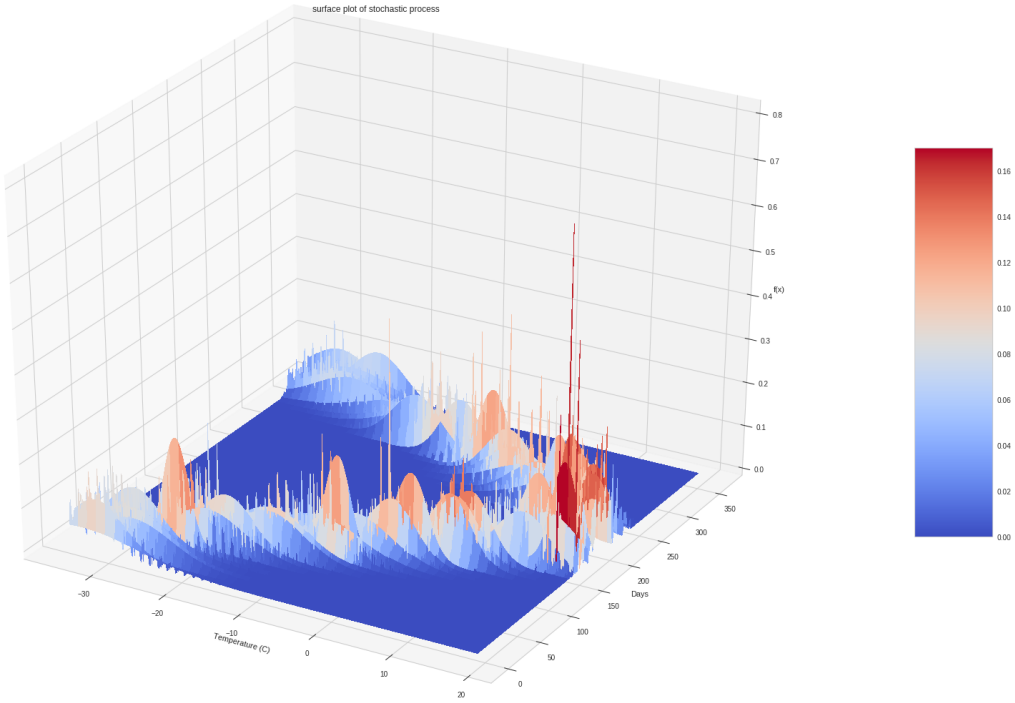
1.7 Question 7

Next, a surface plot of a stochastic process is generated. For this, avarages are generated over all 5 sets of data for each year, as well as a standard diviation for each day. Next, a linear space is generated representing the posible tempreture ranges. finally, the PDF is found then plotted in 3D.

```
In [11]: plt.rcParams['figure.figsize'] = (30, 20)  #Make the figure for this question bigger

         #define the variables to store the mean, std deviation,
         # range for each day(temp), pdfs and a vector for number of days
         meanDay = np.zeros((365, 1))
         stdDay = np.zeros((365, 1))
         dayArray = np.zeros((5, 1))
         pdf = np.zeros((365, 365))
         dayRange = range(0, 365, 1)

         for day in dayRange:
             for inx, year in enumerate(weatherData):
                 dayArray[inx, 0] = year[:, 2][day]
             meanDay[day] = dayArray.mean()  #calculate the mean
             stdDay[day] = dayArray.std()  # calculate the standard deviation

         #generate a linear space of all days in the region, for the min to max temp
         tempretureSpace = np.linspace(meanDay.min(),
                                       meanDay.max(),
                                       len(weatherData[0][:, 2]))

         #itterate over the days again, now generating the pdf
         for day in dayRange:
             pdf[day, :] = norm.pdf(tempretureSpace,
                                    meanDay[day],
                                    stdDay[day])

         #convert the values to a meshgrid
         # (return coordinate matrices from coordinate vectors)
         tempretureSpace, dayRange = np.meshgrid(tempretureSpace, dayRange)

         #finally, plot it as a 3d surf
         fig = plt.figure()
         ax = fig.gca(projection='3d')
         surf = ax.plot_surface(tempretureSpace,
                                dayRange,
                                pdf,
                                cmap=cm.coolwarm,
                                linewidth=0,
                                antialiased=False)

         ax.set_xlabel('Temperature (C)')
```

```
ax.set_ylabel('Days')
ax.set_zlabel('f(x)')

#add a colour bar on the side to see magnitudes
fig.colorbar(surf, shrink=0.5, aspect=5)
plt.title('surface plot of stochastic process')
plt.show()
```



In [ ]: