Conjecturing's Conjecture 1. *Let B be an arbitrary chomp board, $n(B)$ be the number of cookies in that board and $c(B)$ be the number of columns of that board. If B is a P position, $n(B) \geq 2 \cdot c(B) - 1$.*

Proof. Suppose $n(B) \geq 2 \cdot c(B) - 1$ for all P positions with less than k columns. Let B be an arbitrary P board with k columns. Any move yields an N position, B'. So suppose the first player takes a cookie in the last column in the first row. An arbitrary number, t cookies, are removed. Then, the resulting board is an N position, so there is at least one reachable P position. Note, however, the second player cannot reach a P position by taking any cookie in the first row, as this move could have been made by the first player, yet we assume the first player was handed a P position, and thus had no reachable P positions. Then, the second player must take a cookie on some other row to reach a P position. This move results in some arbitrary number, l cookies, removed. We will call this new P position B''.

$$\begin{aligned} n(B'') &= n(B) - t - l \\ c(B'') &= c(B) - 1 \end{aligned}$$

By the inductive hypothesis,

$$\begin{aligned} n(B'') &\geq 2 \cdot c(B'') - 1 \\ n(B) - t - l &\geq 2(c(B) - 1) - 1 \\ n(B) &\geq 2 \cdot c(B) - 3 + t + l \end{aligned}$$

t and l are both at least one (otherwise, the moves to take those cookies would not have been legal).

$$\begin{aligned} n(B) &\geq 2 \cdot c(B) - 3 + 1 + 1 \\ n(B) &\geq 2 \cdot c(B) - 1 \end{aligned}$$

As desired. $\square$