4.3 Suppose the error involved in making a certain measurement is a continuous r.v. with pdf

$$f(x) = \begin{cases} 0.09375(4 - x^2) & -2 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

a. Sketch the graph of $f(x)$.
b. Compute $P(X > 0)$.
c. Compute $P(-1 < X < 1)$.
d. Compute $P(X < -.5 \text{ or } X > .5)$

a.

b.

$$P(X > 0) = \int_0^2 .09375(4 - x^2)dx = .09375(4x - \frac{x^3}{3}) \mid_0^2 = .5$$

(Is there a shortcut to get the answer?)

c.

$$P(-1 < X < 1) = \int_{-1}^1 .09375(4 - x^2)dx = .6875$$

d

$$P(X < -.5 \textbf{ or } X > .5) = 1 - P(-.5 \leq X \leq .5) =$$
$$1 - \int_{-.5}^{.5} .09735(4 - x^2)dx = 1 - .3672 = .6328$$

4.4 Let X denote the vibratory stress (psi) on a wind turbine blade at a particular wind speed in a wind tunnel. The article "Blade Fatigues Life Assessment with Application to VAWTS" proposes the Rayleigh distribution, with pdf

$$f(x;\theta) = \begin{cases} \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

as a model of the distribution of X
a. Verify that $f(x;\theta)$ is a legitimate pdf
b. Suppose θ is 100, what is the probability that X is at most 200? less than 200? At least 200?
c. What is the probability that X is between 100 and 200?
d. Give the expression for $P(X \leq x)$

a.

$$\int_{-\infty}^{+\infty} f(x;\theta) = \int_0^{+\infty} \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} dx = -e^{-x^2/2\theta^2} \Big|_0^{+\infty} = 0 - (-1) = 1$$

b.

$$P(X \leq 200) = \int_0^{200} \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} dx = -e^{-x^2/2\theta^2} \Big|_0^{200} = -.1353 - (-1) = .8647$$

c.

$$P(100 \leq X \leq 200) = \int_{100}^{200} \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} dx = -e^{-x^2/2\theta^2} \Big|_{100}^{200} = .4712$$

d. For $x \leq 0$, $P(X \leq x) = 0$. For $x > 0$,

$$P(X \leq x) = \int_0^{x} \frac{y}{\theta^2} \cdot e^{-y^2/(2\theta^2)} dy = -e^{-y^2/2\theta^2} \Big|_0^{x} = 1 - e^{-x^2/2\theta^2}$$