P_M5_1

November 5, 2023

1 Module 5 Peer Review Assignment

2 Problem 1

Roll two six-sided fair dice. Let X denote the larger of the two values. Let Y denote the smaller of the two values.

a) Construct a table that gives the joint probability mass function for X and Y. (Note: "X is the larger value and Y is the smaller value in a two dice roll" means that for any two dice roll, X will be greater than or equal to Y).

3 All answers written at bottom of page

b) What is $P(X \geq 3, Y = 1)$?

c) What is $P(X \geq Y + 2)$?

YOUR ANSWER HERE

d) Are X and Y independent? Explain.

YOUR ANSWER HERE

4 Problem 2

Let (X, Y) be continuous random variables with joint PDF:

$$f(x, y) = \begin{cases} cxy^2 & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{else} \end{cases}$$

Part a)

Solve for c. Show your work.

YOUR ANSWER HERE

Part b)

Find the marginal distributions $f_X(x)$ and $f_Y(y)$. Show your work.

YOUR ANSWER HERE

Part c)

Solve for $E[X]$ and $E[Y]$. Show your work.

YOUR ANSWER HERE

Part d)

Using the joint PDF, solve for $E[XY]$. Show your work.

YOUR ANSWER HERE

Part e)

Are X and Y independent?

YOUR ANSWER HERE

Problem 1

a)

	$y=1$	$y=2$	$y=3$	$y=4$	$y=5$	$y=6$	← $P(X=x)$
$x=1$	$1/36$	0	0	0	0	0	$1/36$
$x=2$	$2/36$	$1/36$	0	0	0	0	$3/36$
$x=3$	$2/36$	$2/36$	$1/36$	0	0	0	$5/36$
$x=4$	$2/36$	$2/36$	$2/36$	$1/36$	0	0	$7/36$
$x=5$	$2/36$	$2/36$	$2/36$	$2/36$	$1/36$	0	$9/36$
$x=6$	$2/36$	$2/36$	$2/36$	$2/36$	$2/36$	$1/36$	$11/36$
$P(Y=y)$ →	$11/36$	$9/36$	$7/36$	$5/36$	$3/36$	$1/36$	1

b) $P(X\geq 3, Y=1) = P(3,1) + P(4,1) + P(5,1) + P(6,1)$

$= 2/36 + 2/36 + 2/36 + 2/36$

$= 8/36 = \boxed{2/9}$

c) $P(X \geq Y+2) = P(3,1) + P(4,1) + P(5,1) + P(6,1) = 2/9$

$+ \; P(4,2) + P(5,2) + P(6,2) = 1/6$

$+ \; P(5,3) + P(6,3) = 4/36 = 1/9$

$+ \; P(6,4) = 2/36 = 1/18$

$\longrightarrow = 2/9 + 1/6 + 1/9 + 1/18 = \boxed{5/9}$

d). X and Y are not independent because their conditional probabilities are different from their single event probabilities.

i.e $P(X=1) = 1/36$ $\quad P(X=1 \mid Y=2) = 0$

Cont'd

Solve for C, Cov(X, Y)

d) Solve for $E[XY]$ → ① $\iint xy \cdot f(x,y)\,\partial x\,\partial y$

② $= \iint xy \cdot cxy^2$

$= \iint cx^2y^3 = c\iint x^2y^3\,\partial x\,\partial y$ ③

④ $c\int_0^1\int_0^1 x^2y^3\,\partial x\,\partial y = c\int_0^1 \frac{x^3y^3}{3}\Big|_{x=0}^{x=1} = c\int_0^1 \frac{y^3}{3}\,\partial y$ ⑤

⑥ $\frac{c}{3}\int_0^1 y^3\,\partial y = \cancel{\frac{c}{12}\int}$ ⑦ $\frac{c}{12}y^4\Big|_{y=0}^{y=1} = \boxed{\frac{c}{12}}$

e) are X and Y independent?

ind. iff $f(x,y) = f_X(x) \cdot f_Y(y)$

$cxy^2 \neq \frac{c}{3}x \cdot \frac{c}{2}y^2$

not independent

Problem 2

Let (X, Y) be continuous random variables with joint PDF:

$$f(x,y) = \begin{cases} cxy^2 & \text{if } 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0 & \text{else} \end{cases}$$

a) Solve for c.

① $1 = \int_0^1 \int_0^1 cxy^2 \, \partial x \, \partial y$

② $1 = \int_0^1 \int_0^1 cxy^2 \, \partial y \, \partial x$ ③ $= \frac{1}{3} \int_0^1 cxy^3 \, \partial x \Big|_{y=0}^{y=1} = $ ④ $\frac{1}{3} \int_0^1 cx \, \partial x$

⑤ $\rightarrow \frac{1}{6} cx^2 \Big|_{x=0}^{x=1} = $ ⑥ $\frac{1}{6} c = 1$

$\boxed{c = 6}$

b) Find marginal distributions $f_x(x)$ and $f_Y(y)$

$f_x(x) \rightarrow \int_0^1 cxy^2 \partial y = c\left(x \frac{y^3}{3}\right)\Big|_0^1 = \boxed{c \frac{x}{3}}$

$f_Y(y) \rightarrow \int_0^1 cxy^2 \partial x = c\left(y^2 \frac{x^2}{2}\right)\Big|_0^1 = \boxed{c \frac{y^2}{2}}$

c) Solve for $\mathcal{E}[X]$ and $\mathcal{E}[Y]$

$\mathcal{E}X = \int_{-\infty}^{\infty} x \cdot f_x(x) \partial x$ $\qquad \mathcal{E}Y = \int_{-\infty}^{\infty} y \cdot f_Y(y) \partial y$

$= \int_0^1 x \cdot \frac{c}{3} \partial x = \frac{c}{3} \frac{x^2}{2} \Big|_0^1 = \frac{c}{3} \cdot \frac{1}{2} = \boxed{\frac{c}{6}}$

$= \frac{c}{2} \int_0^1 y^2 \partial y = \frac{c}{6} y^3 \Big|_0^1 = \boxed{\frac{c}{6}}$