

# P\_M3\_1

October 15, 2023

## 1 Module 3 Peer Review Assignment

### 2 Problem 1

You work at a factory that manufactures light bulbs. You have determined that 5% of light bulbs that are produced are defective. For each of the scenarios below:

1. Define an appropriate random variable and distribution.
2. State the values that the random variable can take on.
3. State any assumptions that you need to make.
4. Find the probability that the random variable you defined takes on the value  $X = 4$ .

#### Part a)

Out of 30 lightbulbs,  $k$  are defective.

#answers written at the bottom

#### Part b)

You test each lightbulb as it comes off the line. The  $k^{th}$  light bulb is the first defective light bulb you find.

#answers written at the bottom

#### Part c)

You find your second defective light bulb after observing  $k$  light bulbs in all.

YOUR ANSWER HERE

### 3 Problem 2

Consider a loaded six-sided die that is twice as likely to roll an even number as an odd number. Let  $X$  be random variable for value that is rolled from the die.

#### Part a)

What is the Probability Mass Function for  $X$ . Write this out as a table.

#answers written at the bottom

**Part b)**

What is the Cumulative Distribution Function for  $X$ ?

#answers written at the bottom

**Part c)**

What is  $E[X]$ ?

#answers written at the bottom

## 4 Problem 3

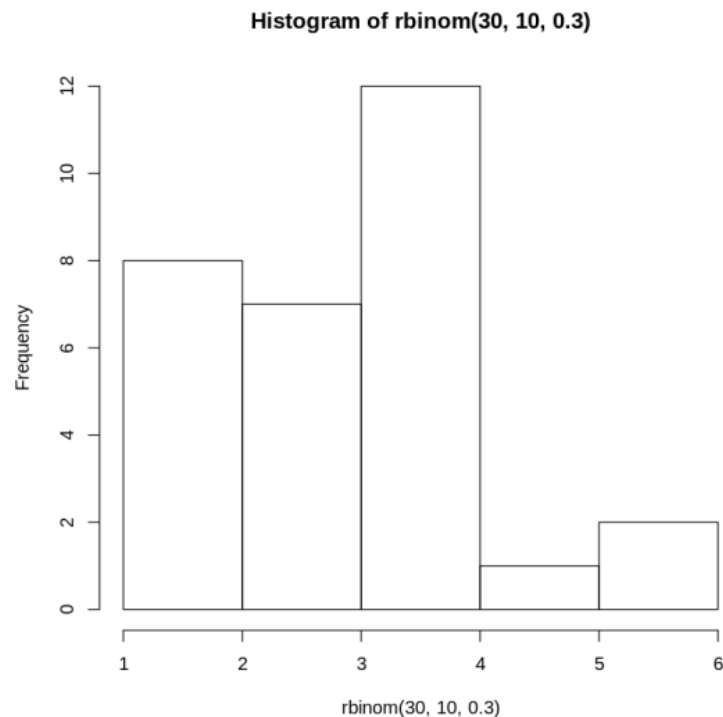
How would we simulate variables from these distributions in R? It'll turn out that the method is fairly similar across all these distributions so, for simplicity, let's just say we want to simulate  $X \sim \text{Bin}(n, p)$ . Take a look at the official documentation for this function [here](#). Not extremely clear, is it? Let's go through it one step at a time.

**Part a)**

What if we want a random variable from this distribution? That is, we know some underlying distribution and we want to simulate many results from that distribution. Then we would use the "random generation" function `rbinom()`.

Play around with this function, with different `size` and `prob` parameters to get a feel for how it works. Finally, generate 30 results from a  $\text{Bin}(10, 0.3)$  distribution and plot a histogram of the results.

```
[18]: hist(rbinom(30, 10, .3))
```



### Part b)

What if we have some value  $k$  and we want to know what's probability of generating  $k$ ? That is, we're solving the Probability Mass Function  $P(X = k)$ . Then we would use the "density" function `dbinom()`.

Let  $X \sim \text{Bin}(15, 0.4)$ . By hand, solve  $P(X = 4)$ . Then use the `dbinom()` function to confirm your result.

YOUR ANSWER HERE

```
[41]: dbinom(4, 15, .4)
```

```
0.12677580324864
```

### Part c)

What if we wanted to solve for some value of the Cumulative Density Function? That is, we know  $k$  and want to find  $P(X \leq k) = p$ . Then we would use the "distribution function" `pbinom()`.

Let  $X \sim \text{Bin}(15, 0.4)$ . By hand, solve  $P(X \leq 4)$ . Then use the `pbinom()` function to confirm your result.

YOUR ANSWER HERE

```
[5]: #problem handwritten below
```

```
round(pbinom(4, 15, .4), 4)
```

```
0.2173
```

### Part d)

Finally, we have the "quantile" function `qbinom()`. This function is the reverse of the `pbinom()` function, in that it takes a probability  $p$  as an argument and returns the value  $k$  of the CDF that results in that much probability.

Use the `qbinom()` function to confirm your results from **Part c**. That is, plug in the probability you got from **Part c** and see if you get the same  $k$ .

```
[13]: qbinom(c(.2172, .2173), 15, .4)
```

```
1. 4 2. 5
```

Nearly every distribution has these four functions, and they will be very useful for our future calculations and simulations.

### Problem 1

1a) Out of 30 light bulbs,  $K$  are defective

1. Random variable = defective bulb,  $X \sim \text{Bin}(30, .05)$

2.  $K = 0, 1, 2, 3, 4, \dots, 30$

3. Assume independence - No bulb affects the other

4.  $X \sim \text{Bin}(30, .05)$ ,  $P(X=4)$

$$= \binom{n}{i} p^i (1-p)^{n-i} = \binom{30}{4} (.05)^4 (.95)^{30-4} = 0.045$$

1b) Test light bulbs...  $K^{\text{th}}$  bulb is first defective.

1. Random variable = How many bulbs inspected until first defective

$X \sim \text{Geo}(.05)$

2.  $K = 1, 2, 3, 4, \dots$

3. Assume independence again - No bulb affects the other

4.  $X \sim \text{Geo}(.05)$ ,  $P(X=4)$

$$= p(1-p)^{i-1} \quad \text{for } i=1, 2, 3, \dots$$

$$\hookrightarrow (.05)(.95)^3 \approx 0.043$$

1c) Find second defective bulb after observing  $K$  bulbs

1. Random variable = How many bulbs were inspected before 2<sup>nd</sup> defective

$X \sim \text{Neg Bin}(2, .05)$

$\uparrow$   
# of defective bulbs

2.  $K = 2, 3, 4, \dots$

$\uparrow$   
we need at least

2 inspected

3. Again assuming independence within each light bulb.  
Defective light bulbs do not affect non-defect.

4.  $X \sim \text{Neg Bin}(2, .05)$ ,  $P(X=4)$

$$* p(X=n) = \binom{n-1}{k-1} p^k (1-p)^{n-k}$$

$$\hookrightarrow P(X=4) = \binom{4-1}{2-1} (0.05)^2 (1-0.05)^{4-2} = 0.0068$$



### Problem 2

a) PMF Table  $\rightarrow$  6 sided die  $P(\text{roll even}) = 2P(\text{roll odd})$

x	P(x)
1	$\frac{1}{3} \cdot \frac{1}{3} = .11$
2	$\frac{2}{3} \cdot \frac{1}{3} = .22$
3	$\frac{1}{3} \cdot \frac{1}{3} = .11$
4	$\frac{2}{3} \cdot \frac{1}{3} = .22$
5	$\frac{1}{3} \cdot \frac{1}{3} = .11$
6	$\frac{2}{3} \cdot \frac{1}{3} = .22$

b) CDF

x	P(x)	F(x)
1	.11	.11
2	.22	.33
3	.11	.44
4	.22	.66
5	.11	.77
6	.22	1

$$c) E(X) = (1 \cdot .11) + (2 \cdot .22) + (3 \cdot .11) + (4 \cdot .22) + (5 \cdot .11) + (6 \cdot .22)$$

$$\hookrightarrow 3.6667 = \frac{33}{9}$$

### Problem 3

Part c)  $X \sim \text{Bin}(n, p)$ ,  $P(X \leq x) = \sum_{i=0}^x \binom{n}{i} p^i (1-p)^{n-i}$   $i=0, 1, 2, \dots, x$

$P(X \leq 4)$   $X \sim \text{Bin}(15, .4)$

$$\hookrightarrow P(X=0) = \binom{15}{0} .4^0 (1-.4)^{15} = .0005$$

$$P(X=1) = \binom{15}{1} .4^1 (1-.4)^{14} = .0047$$

$$P(X=2) = \binom{15}{2} .4^2 (1-.4)^{13} = .029$$

$$P(X=3) = \dots = .0634$$

$$P(X=4) = \dots = .1268$$

$$P(X \leq 4) = .2173$$