

P_M4_1

October 24, 2023

1 Module 4 Peer Review Assignment

2 Problem 1

A continuous random variable with cumulative distribution function F has the median value m such that $F(m) = 0.5$. That is, a random variable is just as likely to be larger than its median as it is to be smaller. A continuous random variable with density f has the mode value x for which $f(x)$ attains its maximum. For each of the following three random variables, (i) state the density function, (ii) compute the median, mode and mean for the random variable, and (iii) Provide at least one graph for the density function using values of the parameter(s) that you select. Indicate the median, mode, and mean values on your graph. (The purpose of this problem is to see the relative locations of the median, mode, and mean for the different random variables).

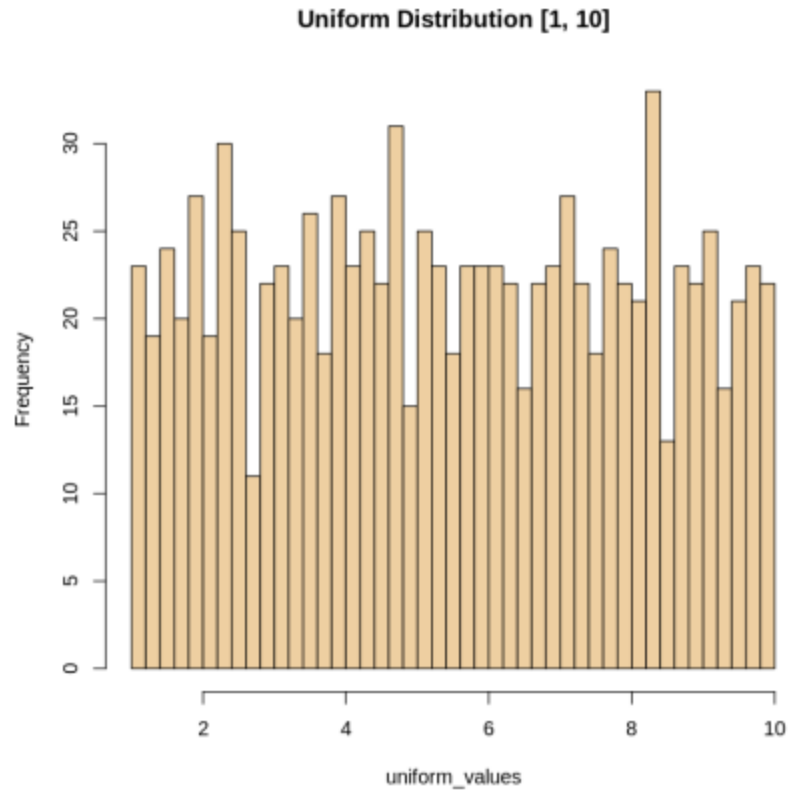
a) W which is uniformly distributed over the interval $[a, b]$, for some value $a, b \in \mathbb{R}$.

```
[73]: set.seed(123)
      uniform_ <- runif(1000, min = 1, max = 10)

      # Density plot
      density_plot <- density(uniform)

      # Histogram
      hist(uniform_values,
           breaks = 50,
           col = "navajowhite2",
           main = "Uniform Distribution [1, 10]")
```

PDF, Mean, Median, and Mode, written at bottom of page.



YOUR ANSWER HERE

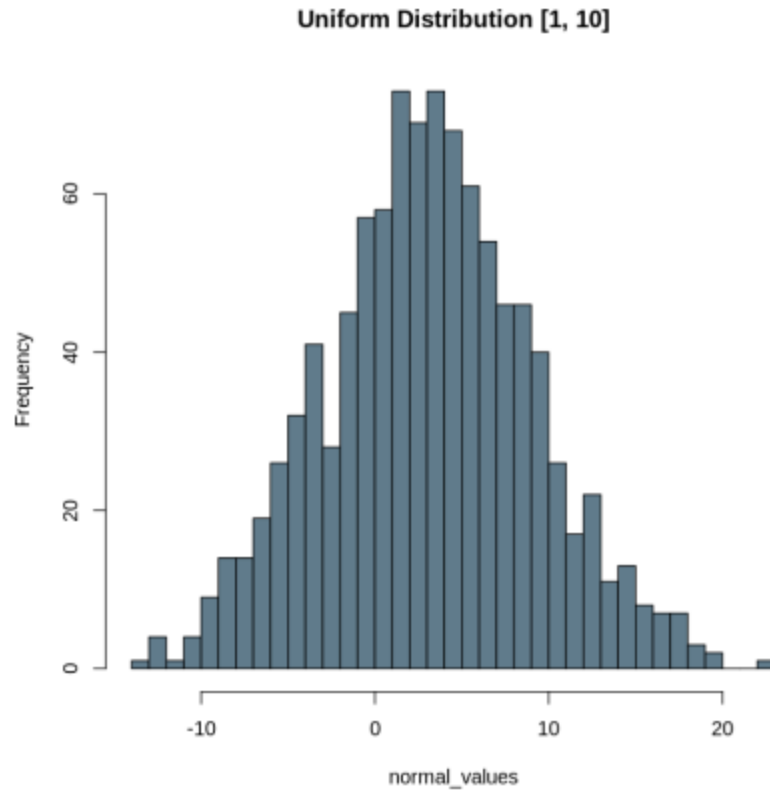
b) X which is normal with parameters μ and σ^2 , for some value $\mu, \sigma^2 \in \mathbb{R}$.

```
[75]: set.seed(123)
normal <- rnorm(1000, 3, 6)

density_plot <- density(normal)

hist(normal_values,
      breaks = 50,
      col = "lightskyblue4",
      main = "Uniform Distribution [1, 10]")
```

PDF, Mean, Median, and Mode, written at bottom of page.



YOUR ANSWER HERE

c) Y which is exponential with rate $\lambda \in \mathbb{R}$.

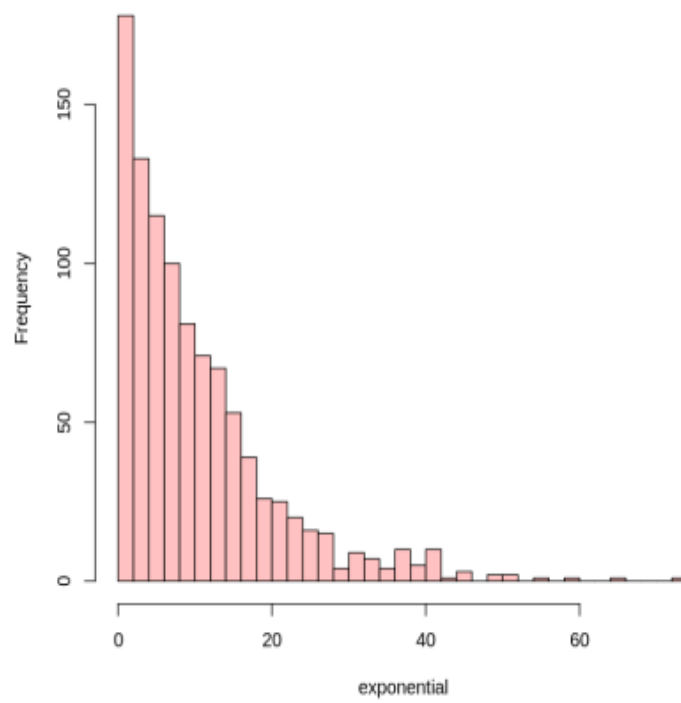
```
[74]: set.seed(123)
      exponential <- rexp(1000, rate = 0.1)

      density_plot <- density(exponential)

      hist(exponential,
            breaks = 50,
            col = "rosybrown1",
            main = "Exponential Distribution (Rate = 0.1)")
```

PDF, Mean, Median, and Mode, written at bottom of page.

Exponential Distribution (Rate = 0.1)



a) W which is uniformly distributed over the interval $[1, 10]$ for some value $1, 10 \in \mathbb{R}$.

$$(i) \text{ PDF} = \frac{1}{b-a} = \frac{1}{10-1} = \frac{1}{9}$$

$$(ii) \text{ Mean} : \frac{a+b}{2} = \frac{1+10}{2} = 5.5$$

Median: same as mean for uniform distribution $\rightarrow 5.5$

Mode: Any value between $[1, 10]$

b) X which is normal with parameters $\mu=3$ and $\sigma^2=6$ for some value $\mu, \sigma^2 \in \mathbb{R}$

$$(i) \text{ PDF} = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-(x-\mu)^2/2\sigma^2}$$

$$\hookrightarrow \frac{1}{\sqrt{2\pi \cdot 6}} e^{-\frac{(x-3)^2}{2 \cdot 6}}$$

(ii) mean: 3 median: 3 mode: Any value between $[1, 10]$

c) Y which is exponential with rate $\lambda = .10 \in \mathbb{R}$

$$(i) \text{ PDF} = f(x) = \lambda e^{-\lambda x} = .10 e^{-.10x}$$

$$(ii) \text{ mean: } \frac{1}{\lambda} = \frac{1}{.10} = 10 \quad \text{median: } \frac{\ln(2)}{\lambda} = \frac{\ln(2)}{.10} = 6.93$$

$$\text{mode: } 0$$

3 Problem 2

For this problem, we're going to visualize what's happening when we go between different normal distributions.

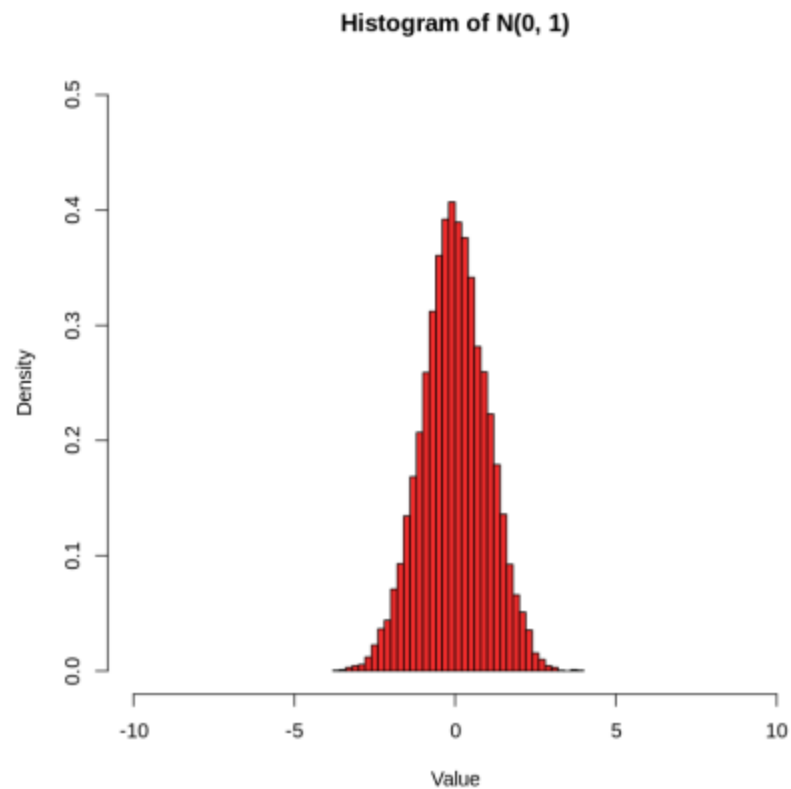
Part A)

Draw at least 10000 samples from the standard normal distribution $N(0, 1)$ and store the results. Make a density histogram of these samples. Set the x -limits for your plot to $[-10, 10]$ and your y -limits to $[0, 0.5]$ so we can compare with the plots we'll generate in **Parts B-D**.

```
[76]: set.seed(1)

# Generate random samples from Normal distribution (0, 1)
results <- rnorm(10000)

# Create a density histogram
hist(results,
      breaks = 50,
      xlim = c(-10, 10),
      ylim = c(0, 0.5),
      main = "Histogram of N(0, 1)",
      xlab = "Value",
      ylab = "Density",
      col = "firebrick2",
      prob = TRUE)
```

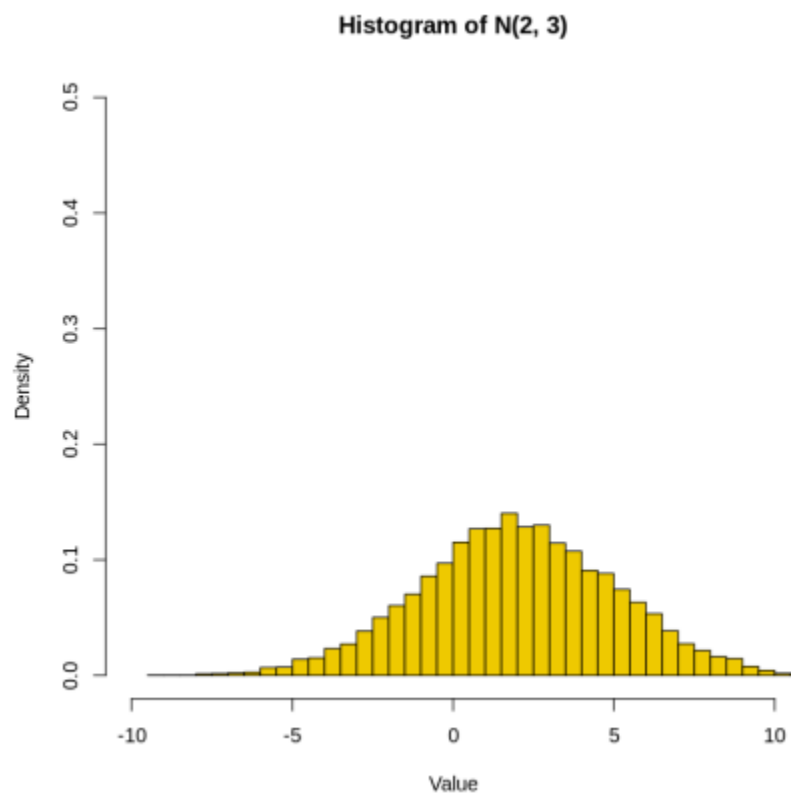


Part b) Now generate 10000 samples from a $N(2,3)$ distribution and plot a histogram of the results, with the same x -limits and y -limits. Does the histogram make sense based on the changes to parameters?

Note: Be careful with the parameters for `rnorm`. It may help to check the documentation.

```
[43]: set.seed(1)
samples <- rnorm(10000, 2, sd = 3)

# Create a density histogram
hist(samples,
      breaks = 50,
      xlim = c(-10, 10),
      ylim = c(0, .5),
      main = "Histogram of N(2, 3)",
      xlab = "Value",
      ylab = "Density",
      col = "gold2",
      prob = TRUE)
```



Part c)

Suppose we are only able to sample from the standard normal distribution $N(0, 1)$. Could we take those samples and perform a simple transformation so that they're samples from $N(2, 3)$? Try this, and plot another histogram of the transformed data, again with the same axes. Does your histogram based on the transformed data look like the histogram from **Part B**?

```
[10]: set.seed(1)
data <- rnorm(10000)

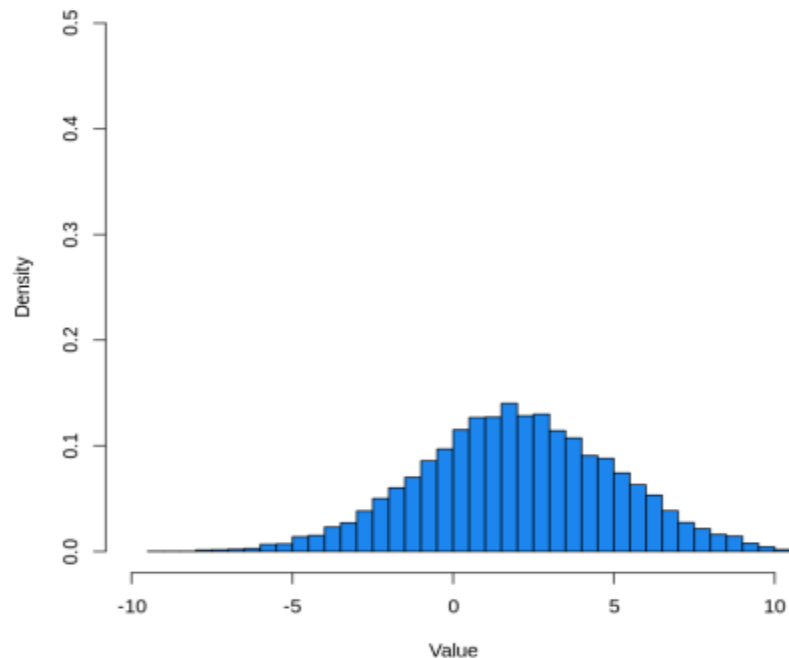
# Transform the samples to N(2, 3)
transformed_data <- 3 * data + 2

# Create a density histogram of the transformed data
```

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```
hist(transformed_data,
      breaks = 50,
      xlim = c(-10, 10),
      ylim = c(0, 0.5),
      main = "Histogram of Transformed Data (N(2, 3))",
      xlab = "Value",
      ylab = "Density",
      col = "dodgerblue2",
      prob = TRUE)
```

Histogram of Transformed Data (N(2, 3))



But can you go back the other way? Take the $N(2, 3)$ samples from **Part B** and transform them into samples from $N(0, 1)$? Try a few transformations and make a density histogram of your transformed data. Does it look like the plot of $N(0, 1)$ data from **Part A**?

```
[25]: set.seed(1)

data <- rnorm(10000, 2, 3)

# Transform the samples to N(0, 1)
transformed_data <- (data-2)/3

# Create a density histogram of the transformed data
hist(transformed_data,
      breaks = 50,
      xlim = c(-10, 10),
      ylim = c(0, 0.5),
      main = "Histogram of Transformed Data (N(0, 1))",
      xlab = "Value",
      ylab = "Density",
      col = "violetred1",
      prob = TRUE)
```

Histogram of Transformed Data (N(0, 1))

