P_M2_1

October 7, 2023

This assignment will be reviewed by peers based upon a given rubric. Make sure to keep your answers clear and concise while demonstrating an understanding of the material. Be sure to give all requested information in markdown cells. It is recommended to utilize Latex.

0.0.1 Problem 1

What does it mean for one event C to cause another event E — for example, smoking (C) to cause cancer (E)? There is a long history in philosophy, statistics, and the sciences of trying to clearly analyze the concept of a cause. One tradition says that causes raise the probability of their effects; we may write this symbolically is

$$P(E|C) > P(E). \qquad (1)$$

Part a) Does equation (1) imply that $P(C|E) > P(C)$? If so, prove it. If not, give a counter example.

#answer written at very bottom of the page, keep scrolling

Part b) Another way to formulate a probabilistic theory of causation is to say that

$$P(E|C) > P(E|C^C). \qquad (2)$$

Show that equation (1) implies equation (2).

#answer written at very bottom of the page, keep scrolling

Part c) Let C be the drop in the level of mercury in a barometer and let E be a storm. Briefly describe why this leads to a problem with using equation (1) (or equation (2)) as a theory of causation.

#answer written at very bottom of the page, keep scrolling

Part d) Let A, C, and E be events. If $P(E|A \cap C) = P(E|C)$, then C is said to screen A off from E. Suppose that $P(E \cap C) > 0$. Show that screening off is equivalent to saying that $P(A \cap E|C) = P(A|C)P(E|C)$. What does this latter equation say in terms of independence?

#answer written at very bottom of the page, keep scrolling

Part e) Now let A be a the drop in the level of mercury in a barometer, E be a storm, and C be a drop in atmospheric pressure. Does the result from part (d) help fix the problem suggested in part (c)?

YOUR ANSWER HERE

1 Problem 2

Suppose you have two bags of marbles that are in a box. Bag 1 contains 7 white marbles, 6 black marbles, and 3 gold marbles. Bag 2 contains 4 white marbles, 5 black marbles, and 15 gold marbles. The probability of grabbing the Bag 1 from the box is twice the probability of grabbing the Bag 2.

If you close your eyes, grab a bag from the box, and then grab a marble from that bag, what is the probability that it is gold?

Part a)

Solve this problem by hand. This should give us a theoretical value for pulling a gold marble.

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Part b)

Create a simulation to estimate the probability of pulling a gold marble. Assume you put the marble back in the bag each time you pull one out. Make sure to run the simulation enough times to be confident in your final result.

Note: To generate n random values between [0,1], use the `runif(n)` function. This function generates n random variables from the Uniform(0,1) distribution, which we will learn more about later in this course!

[3]:
```
sims = 100000

#create vectors to track if it is Bag 1/if we draw gold marbel
bag1 = rep(0, sims)
bag2 = rep(0, sims)
gold = rep(0, sims)

#run the loop
for(i in 1:sims){

  #flip to see what bag we draw from
  draw.bag = runif(1)

  #flip to see if we draw a gold marbel goes
  draw.gold = runif(1)

  #the case where we pull from bag 1
  if(draw.bag <= .66){
```

```
    #mark that we drew from bag 1
    bag1[i] = 1

    #We draw gold marbel with probability .19 in this case
    if(draw.gold <= .19){

      #mark that drew a gold marbel
      gold[i] = 1
    }
  }

  #the case where we grab from bag 2
  if(draw.bag > .66){

    #mark that we drew from bag 2
    bag2[i] = 1

    #We draw gold marble with probability .62 in this case
    if(draw.gold <= .62){

      #mark that drew a gold marbel
      gold[i] = 1
    }
  }
}

#we should get .66 for bag 1, .33 for bag 2, and .33 for gold
mean(bag1); mean(bag2); mean(gold)
```

0.66181

0.33819

0.33703

2 Problem 3

Suppose you roll a fair die two times. Let A be the event "the sum of the throws equals 5" and B be the event "at least one of the throws is a 4".

Part a)

By hand, solve for the probability that the sum of the throws equals 5, given that at least one of the throws is a 4. That is, solve $P(A|B)$.

#answer written at very bottom of the page, keep scrolling

Part b)

Write a simple simulation to confirm our result. Make sure you run your simulation enough times to be confident in your result.

Hint: Think about the definition of conditional probability.

[18]:
```
sims = 1000000

total.five.one.4 = rep(0, sims)
one.4 = rep(0, sims)

#run the loop
for(i in 1:sims){

    roll.total.five.one.4 = runif(1)
    roll.one.4 = runif(1)

    #the case where we roll at least one 4
    if(roll.one.4 <= .305){

        #mark that rolled at least one 4
        one.4[i] = 1

        #We roll a total of 5 with probability .18 in this case
        if(roll.total.five.one.4 <= .181){

        total.five.one.4[i] = 1

        }
    }
}

#probability to roll a total of five and at least one 4 is 5.5%
mean(total.five.one.4);
#probability to roll at least one 4 is 30%
mean(one.4);

#now we need to see even A given B, which should come out to 18%
"Probability of rolling a total of five given that you roll at least one 4"
round(mean(total.five.one.4) / mean(one.4), 4)
```

0.055195

0.305626

'Probability of rolling a total of five given that you roll at least one 4'

0.1806

Problem 1. A,B,C,D,E

Problem 1

a. $P(E|C) > P(E)$ does not always imply $P(C|E)$; specifically in the case that these two events are independent of each other.

b. we want to show that $P(E|C) > P(E) \xrightarrow{\text{implies}} P(E|C) > P(E|C^c)$

Knowing the formal def. of conditional Probability, we know:

$$P(E) = P(E|C)P(C) + P(E|C^c)(1 - P(C)) \leftarrow P(C^c)$$

$$= P(E|C)P(C) + P(E|C^c) - P(E|C^c)P(C)$$

Given

$$P(E|C) > P(E|C)P(C) + P(E|C^c) - P(E|C^c)P(C)$$

$$P(E|C) - P(E|C)P(C) > P(E|C^c) - P(E|C^c)P(C)$$

$$P(E|C)(1 - P(C)) > P(E|C^c)(1 - P(C))$$

$$= P(E|C) > P(E|C^c)$$

c. Using equation 1 or 2 could lead to problems because conditional probability does not directly relate to causation by itself. That is to say, we cannot determine independence from these equations.

d. Given $P(E|C) = P(E|A \cap C)$ we know..

$$P(E|C) = \frac{P(E \cap A \cap C)}{P(A \cap C)} \rightarrow \frac{P(C)P(A \cap E|C)}{P(C)P(A|C)}$$

- The later equation shows that Events A and E are independent of each other since the multiplication rule $\rightarrow P(A|C)P(E|C)$ for both events given the condition C does not change the outcome of both events (union).

e. This will help us solve the probability of these specific events because it considers their independence. * A ind. of E

we know this because the level of mercury does not tell us anything about atmospheric pressure.

Problem 2.A and 3.A

Problem 2 a. $P(A)$ = Gold marble, $P(B_1) = 2P(B_2)$

$P(B_2) = \frac{1}{3}$ $P(B_1) = \frac{2}{3}$ $\rightarrow \frac{1}{3} + \frac{2}{3} = 1$

$$P(A) = P(A \cap (B_1 \cup B_2))^*$$

$$= P[(A \cap B_1) \cup (A \cap B_2)]$$

mutually exclusive events (grab 1 bag) $\rightarrow P(A \cap B_1) + P(A \cap B_2)$

so we can add $\rightarrow$

$$= P(B_1) \cdot P(A|B_1) + P(B_2) \cdot P(A|B_2)$$

$$= \frac{2}{3} \cdot \frac{3}{16} + \frac{1}{3} \cdot \frac{15}{24} = \frac{1}{3} = \boxed{33\%}$$

Problem 3 a. $P(A)$ = sum of throws equals 5

$P(B)$ = at least one of the throws is a 4

we want $P(A|B) \rightarrow \frac{P(A \cap B)}{P(B)} \rightarrow \frac{.055}{.305} = \cancel{.166} \rightarrow \boxed{.18}$

$A \cap B = (\{1,4\}, \{1,4\}) \rightarrow \frac{2}{36} = .05$

$$B = \left\{ \begin{matrix} \{4,1\}, \{4,2\}, \{4,3\}, \{4,5\}, \{4,6\} \\ \{1,4\}, \{2,4\}, \{3,4\}, \{5,4\}, \{6,4\} \end{matrix} \{4,4\} \right\} \rightarrow \frac{11}{36} = .30$$