More Examples

34 (page 74):
Summary of the problem: day shift: 20 workers; swing shift: 15 workers; graveyard shift: 10 works.
Total number of workers: 20+15+10=45.
6 will be selected for interview
Note that the order of the selected workers does not matter, so combination rule should be applied here
a. The selected workers can be from any of the shifts, but we only want to count the possibilities that all of them are from the day shift. We know there are 20 workers in the day shift. Any selections from these 20 workers will generate those special panels, meaning that all of them are from the day shift. hence, it is equal to select 6 out of these 20, that is

$$\binom{20}{6} = 38,760$$

To calculate the probability, we need calculate the number of all possible outcomes in the sample space, meaning that all possible collections of 6 workers from 45 workers, that is,

$$\binom{45}{6} = 8,145,060$$

Now

$$P(\text{all from day shift}) = \frac{38,760}{8,145,060} = 0.0048$$

b. In this event, we only care whether the selected workers are from the same shift. So, they can be selected from the day shift, (we have calculated the number of possibilities already in a), the swing shift, and the graveyard shift. In order to get the total numbers, we need to calculate the numbers of possibilities in the remaining two cases.
all from the swing shift (select 6 from 15)

$$\binom{15}{6}$$

all from the graveyard shift (select 6 from 10):

$$\binom{10}{6}$$

Hence, the total number is

$$\binom{20}{6} + \binom{15}{6} + \binom{10}{6}$$

And

$$P(\text{all from the same shift}) = \frac{\binom{20}{6} + \binom{15}{6} + \binom{10}{6}}{\binom{45}{6} = 8,145,060}$$

=0.0054

c.

There are two ways to solve this problem. We want the selected panel to include at least two swifts. So, they may represent two shifts or three shifts. For the two-shift case, the represented shifts could be (day, swing), (day, graveyard) and (swing, graveyard). Even we fixed the two shifts, for example, (day, swing), the numbers from day and swing can vary.....It seems very tedious to figure out all possible selections in this event (let me denote it by C). For this problem, if you are careful and patient enough, you still can get it right. But it takes too much time and effort. So, is there any other ways? Thanks to a nice simple formula that is

$$P(C) + P(C') = 1.$$

The formula provides us an opportunity to get $P(C)$ through $P(C')$ where C' is the complement of C. We wish that C' is a much simpler event so that we can get the number of outcomes in it easily. Now, let see what C' is? In fact, C'= the selected workers only represent one shift, in other words, the selected workers are from same shift, which is exactly the event we have studied already in b. Hence,

$$P(C') = .0054$$

Now

$$P(\text{the selected workers represent at least two shifts}) = 1 - P(C') = 0.9946$$

d. Let D= at least one of the shift unrepresented

Unfortunately, D and D' both are complicated. So the magic in c. does not work here. So, sometime, we have to work hard because we don't have choice, like almost no one likes exams, but almost everyone works hard during exam. Since D is too complex, we have to think if we can use simpler events to reperesent D. And it is the right approach. In D, the unrepresented shift is not specified, it could any of the three, day, swing, and graveyard. So, why not just specify them to see how they are related to D, let

D_1= the day shift unrepresented

D_2= the swing shift unrepresented

D_3= the graveyard shift unrepresented

I claim that

$$D = D_1 \bigcup D_2 \bigcup D_3 (\text{why?}).$$

Now we can use another nice formula to help us

$$P(D) = P(D_1 \bigcup D_2 \bigcup D_3) = P(D_1) + P(D_2) + P(D_3)$$

$$-P(D_1 \bigcap D_2) - P(D_1 \bigcap D_3) - P(D_2 \bigcap D_3) + P(D_1 \bigcap D_2 \bigcap D_3)$$

Since D_1=the day shift unrepresented = all the selected workers are from the swing shift or the graveyard shift. $\#D_1$ (the number of possible outcomes in D_1) is equal to select 6 out of 15 + 10 (why??). So,

$$P(D_1) = \frac{\begin{pmatrix} 25 \\ 6 \end{pmatrix}}{\begin{pmatrix} 45 \\ 6 \end{pmatrix}}$$

Similarly, you can calculate $P(D_2)$ and $P(D_3)$. Now What is $D_1 \bigcap D_2$? it is the day shift unrepresented plus the swing shift unrepresented = all the selected workers are from the graveyard shift, the number of possibilities is select 6 out of 10.

$$P(D_1 \bigcap D_2) = \frac{\begin{pmatrix} 10 \\ 6 \end{pmatrix}}{\begin{pmatrix} 45 \\ 6 \end{pmatrix}}$$

Similarly we can calculate $P(D_1 \bigcap D_3)$ and $P(D_2 \bigcap D_3)$. (I encorage you to derive them by yourself). Now how about $P(D_1 \bigcap D_2 \bigcap D_3)$? I will just give you the final answer here

$$P(D) = 0.2885$$