Q.4

Part 1

$$H(s) = \frac{s^2+5s+6}{s^3+6s^2+18s+40}$$

$$= \frac{s^2+5s+6}{s^2(s+4)+2s(s+4)+10(s+4)}$$

$$= \frac{s^2+5s+6}{(s+4)(s^2+2s+10)}$$

$$= \frac{s^2+5s+6}{(s+4)(s-(-1-3i))(s-(-1+3i))}$$

∴ The real poles of the system lie at $z=-4$ and there are 2 other complex poles $-1-3i, -1+3i$

- I have used Routh-Hurwitz criteria for stability. So in the characteristic eqn $s^3+6s^2+18s+40$ all coefficients are of same sign and all power of s from 3 to 0 are present thus it is a Hurwitz polynomial

Now creating the Routh's array

s^3	1	18	0
s^2	6	40	0
s	$b_1(34/3)$	$b_2(0)$	0
s^0	(40)	0	

$$b_1 = \frac{18(6)-40}{6} = 18-20/3 = 34/3, \quad b_2 = \frac{0-0}{6}$$

$$c_1 = \frac{\frac{34(40)}{3} - 0}{34/3} = 40$$

$$c_2 = \frac{0-0}{34/3} = 0$$

now in the first column of the array there are no zeros and there is no sign change thus by Hurwitz Routh criteria we can assume that the system is stable.

- with the pole analysis too we can see that all of them are in left half plane and thus system is stable

The presence of complex poles suggest that there are going to be oscillation in the system

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- $$H(s) = \frac{s^2+5s+6}{(s+4)(s^2+2s+10)} = \frac{A}{(s+4)} + \frac{Bs+C}{(s^2+2s+10)}$$ Part 2

$$s^2+5s+6 = A(s^2+2s+10) + (Bs+C)(s+4)$$

$s=-4$

$$16-20+6 = A(16-8+10) + 0$$

$$2 = A(18) \Rightarrow A = 1/9$$

$s=0$

$$6 = A(10) + (C)(4)$$

$$6 - 10/9 = 4C$$

$$\Rightarrow 44/9 = 4C \Rightarrow C = 11/9$$

$s=-1$

$$1-5+6 = A(1-2+10) + (C-B)(3)$$

$$\Rightarrow 2 = A(9) + \frac{11}{3} - 3B$$

$$\Rightarrow 1 - 11/3 = -3B \Rightarrow B = 8/9$$

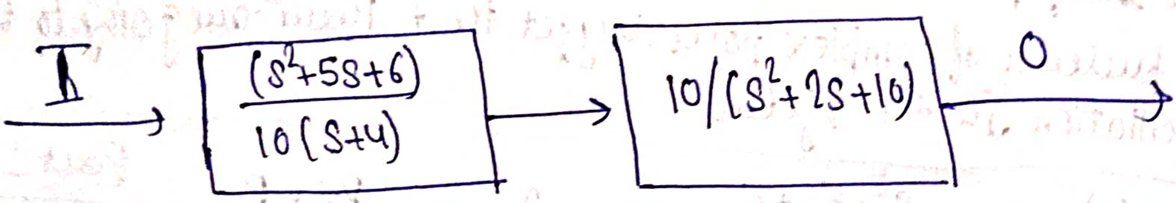
$$\therefore H(s) = \frac{1}{9(s+4)} + \frac{8s+11}{9(s^2+2s+10)}$$

~~Make the oscillation part~~

$$H(s) = \frac{s^2+5s+6}{(s+4)(s^2+2s+10)} = \frac{s^2+5s+6}{(s+4)}\left(\frac{1}{s^2+2s+10}\right)\frac{10}{10}$$

$$= \frac{10(s^2+5s+6)}{10(s+4)}\left(\frac{10}{s^2+2s+10}\right)$$

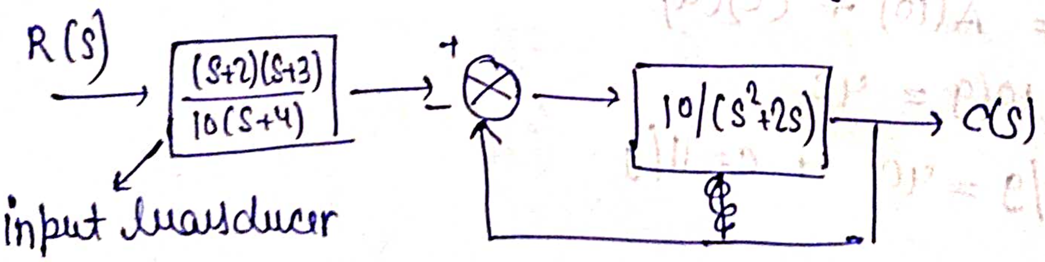
↓ Cascading of two system



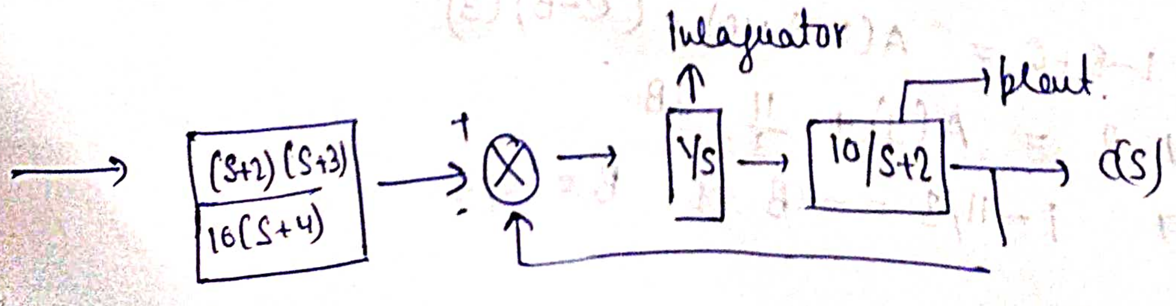
$$\frac{10}{s^2+2s+10} = \frac{10}{(s^2+2s)\left(1+10/s^2+2s\right)}$$

$$= \frac{\frac{10}{(s^2+2s)}}{1+\frac{10}{s^2+2s}}$$

Thus according to rules of block diagram analysis we can do



⇓ further breakdown



• Thus we analyse the second order system as

$$\frac{10}{s^2+2s+10} = \frac{\omega_n^2}{(s^2+2\zeta\omega_n s+(\omega_n)^2)}$$

where ω_n = natural frequency

ζ = damping ratio

$\therefore$ $\omega_n = \sqrt{10}$

$\zeta = 1/\sqrt{10} < 1 \Rightarrow$ The system is under damped.

$\therefore$ peak time $= \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \dfrac{3.14}{\sqrt{10}\sqrt{1-\frac{1}{10}}} =$ ~~0.95 sec~~ 1.046 sec.

settling time $\approx 4/\omega_n\zeta = \dfrac{4}{\sqrt{10}\left(\frac{1}{\sqrt{10}}\right)} = 4$ sec. or $\dfrac{-\ln(0.02)}{\omega_n\zeta} = 3.91$ sec

% overshoot $= e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 35.11\%$

So we can see that the overshoot % is quite high than the prescribed 10-20%.

Maximum settling time = 3s

Maximum overshooting % = 15

Part 3

from the eqn of control system we know

$$\text{Settling time} = \frac{-\ln(0.02)}{\zeta\omega_n} \leq 3$$

and $e^{-\zeta\pi/\sqrt{1-\zeta^2}} \leq 0.15$

when ζ = damping co-efficient, ω_n = natural frequency

$$\therefore e^{-\zeta\pi/\sqrt{1-\zeta^2}} \leq 0.15$$

$$\Rightarrow \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \leq \ln(0.15) \Rightarrow \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \leq -1.89$$

$$\Rightarrow \frac{\zeta\pi}{\sqrt{1-\zeta^2}} \geq 1.89 \Rightarrow \frac{\zeta}{\sqrt{1-\zeta^2}} \geq 0.6$$

$$\Rightarrow \zeta^2 \geq 0.36 - 0.36\zeta^2$$

$$\Rightarrow 1.36\zeta^2 \geq 0.36$$

$$\Rightarrow \zeta^2 \geq 0.264$$

$$\Rightarrow \zeta \geq 0.513$$

- $\frac{-\ln(0.02)}{3\omega_n} \leq \zeta$

$$\Rightarrow \frac{-\ln(0.02)}{3\omega_n} \leq 0.513$$

$$\Rightarrow \frac{3.912}{3(0.513)} \leq \omega_n \Rightarrow \boxed{\omega_n \geq 2.54 \text{ rad}}$$

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$$\text{settling time} \approx 4/\omega_n\zeta = \frac{4}{\sqrt{10}\left(\frac{1}{\sqrt{10}}\right)} = 4\,sec.\ \text{or}\ \frac{-\ln(0.02)}{\omega_n\zeta} = 3.91\,sec$$

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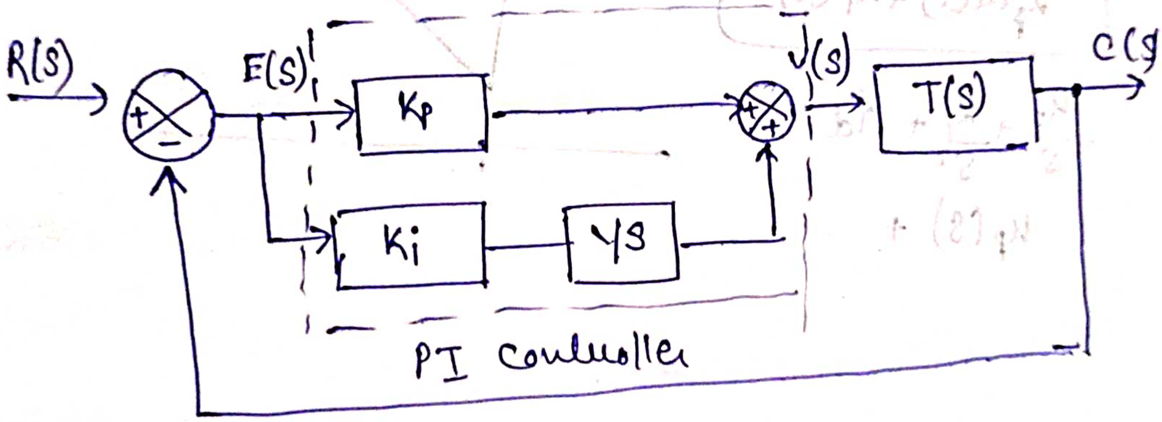
- The ~~H(S)~~ transfer f^n for a PI ~~f^n~~ controller looks as

$$\Rightarrow \left(K_p + \frac{K_i}{S}\right) = H(s)$$

K_p = Proportional gain
K_i = integrator gain.

The K_p and K_I are tuned via the Simulink PID tuner

- A schematic diagram of the ~~~~ system is given below



The Phase gain obtained is = 45.9°
K_p = 3.7765
K_I = 27.4096.