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$P_0 = 1100\,kPa$ $\gamma = 1.4$
$T_0 = 400K$
$A_e/A_t = 2.5$
$P_{amb} = 100\,kPa$

• i) given the area ratio of 2.5, the Mach no. the nozzle is designed to work considering no enthalpic flow condition = 2.45, when the nozzle is chocked

$$A_e/A^* = 2.5, \quad M = 2.45$$

i) initial ambient pressure = 100 kPa
assuming the condition to be chocked

$$\frac{A_E}{A_T} = \frac{1}{M}\left[\frac{1 + \frac{\gamma-1}{2}M^2}{1+(\gamma-1)/2}\right]^{(\gamma-1)/2(\gamma+1)} = 2.5$$

$$\frac{1}{M}\left[1 + \frac{\gamma-1}{2}M^2\right]^{0.083} = 2.53$$

$$\left[1 + \frac{(\gamma-1)}{2}M^2\right]^{0.083} = M(2.53)$$

$$\left[1 + 0.2M^2\right]^{0.083} = M(2.53)$$

$\boxed{M = 0.3962}$ (Solved this eqn online)

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given the exit flow is subsonic the back pressure = exit pressure.

$$\frac{P_E}{P_0} = \left(1 + \frac{(\gamma-1)}{2} M^2\right)^{\gamma/1-\gamma}$$

$$P_E = P_0 \left(1 + \frac{(\gamma-1)}{2} M^2\right)^{\gamma/1-\gamma}$$

$$= P_0 (0.900)$$

$$= 1100 \text{ kPa} (0.9)$$

$$P_E = 990 \text{ kPa}.$$

$$P_{0\,exit} = P_{static} + \frac{1}{2}\rho V^2$$

$$= 990 \text{ KPa} + \frac{1}{2}(1 \text{Kgm}^{-3})(0.39\, V_{sound})^2$$

$$\text{Speed of sound at exit} = \sqrt{\frac{\gamma P}{\rho}}$$

$$= \sqrt{\frac{(1.4)(100\times10^3)}{(1)}} = 374.165 \text{ ms}^{-1}$$

$$= 990 \text{ KPa} + \frac{1}{2}(1)(0.39 \times 374.165)^2$$

$$= 990 \text{ KPa} + 10.646 \text{ kPa}$$

$$= 1000.646 \text{ kPa}.$$

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(ii) when final shockwave appears, the flow will be ~~be~~ supersonic with a shockwave standing at the exit, the mach no upstream

$= 2.45$

$\therefore$ for $\gamma = 1.4$ and M_1 (Mach ~~down stream~~ upstream) $= 2.45$

~~M_1/M_1~~ M_2 (mach ~~up st~~ down stream) $= 0.518$

$\therefore$ Exit pressure can be found from the iso entropic relations as

$$\frac{p_{E1}}{p_{01}} = \left(1 + \frac{(\gamma-1)M_1^2}{2}\right)^{\gamma/1-\gamma} = \left(1 + 0.2(2.45)^2\right)^{1.4/-0.4}$$

$$p_{E1} = 0.063\,(1100\ KPa)$$
$$= 69.59\ KPa.$$

$$p_{0exit} = P_{E1} + \frac{1}{2}\rho V^2$$

$$V = 2.45\left(\sqrt{\frac{\gamma P}{\rho}}\right) = \overset{0.518}{\cancel{2.45}}\sqrt{\frac{\gamma R T}{M_0}}$$

$$\cancel{\frac{T_2}{T_1}} \neq \frac{T_2}{T_0} = 2.04$$

$$T_2 = 2.04(400K) = 816\ K$$

$$M_0 = 0.028\ Kgmol^{-1}$$

$$V = 0.518\sqrt{\frac{1.4(8.314)(816)}{0.028}} = \cancel{1426.92\,ms^{-1}}\ 301.69\,ms^{-1}$$

$$\frac{\rho_2}{\rho_0} = 3.27$$

$\rho_2 = 3.27\,Kgm^{-3}$ $(\rho_0 = 1kgm^{-3})$

$$P_{02} = 69.59\,KPa + \frac{1}{2}(3.27)(301.69)^2$$

$$= 218.402\,KPa$$

(iii) $A/A_{throat} = 1.5$

for the Normal shock at this area ratio the

$M_1 = 1.86$

and for 1.86

$M_2 = 0.6036.$

$$\frac{p_{exit}}{p_0} = \left(1 + \frac{(\gamma-1)M_1^2}{2}\right)^{\gamma/1-\gamma}$$

$$\Rightarrow p_{exit} = p_0\left(1 + \frac{(\gamma-1)M_1^2}{2}\right)^{\gamma/1-\gamma}$$

$$= 174.607\,KPa$$

$$p_0 = P_{exit} + \frac{1}{2}\rho V^2$$

$\rho_2/\rho_1 = 2.45$

$\rho_2 = 2.45\,kgm^{-3}$

$174.607\,kPa + \frac{1}{2}(2.45)(V^2_{\text{speed of sound}})$

$V = \sqrt{\frac{\gamma RT}{M_0}}$ $\qquad T_2/T_0 = 1.57$

$T_2 = 1.57(400)$

$\therefore 174.607\,kPa + \frac{1}{2}(2.45) = 309.78\,kPa + 174.607$

$= 484.38\,kPa.$