

3)

According to US standard model the density of air is varying with temperature and altitude is formulated as

$$\rho(h) = \rho_0 \left(\frac{T_0}{T(h)} \right)^{g/L}$$

T_0 = temperature at sea level

ρ_0 = Density at sea level

$T(h)$ = temperature at altitude h from ground.

$g = 9.8 \text{ ms}^{-2}$

L = temperature lapse rate $\& L = -dT/dh$

Now the force of drag on the rockets can be given as the drag equation :-

$$F = \frac{1}{2} \rho V^2 A C_d = \frac{1}{2} \rho_0 V^2 A C_d \left(\frac{T_0}{T(h)} \right)^{g/L}$$

Thus the eqⁿ shows that at higher altitudes the air density being low drag force would be lower and would increase as the rocket will descent.

The performance can be analyzed in following phases:-

High Altitude Phase

At the highest altitudes of rocket's trajectory, the air density will be really low leading to less deceleration. As at those altitudes velocity won't be high, but the density term involving an exponential will dominate and thus the force exerted on the air brakes will be less.

This will lead to difficulty in controlling speed and descent rate at those altitudes.

→ wind at these heights can be really high and thus wind drift can be significant.

Transition Phase

When the rocket transitions from higher altitudes to lower ones the air density increases substantially leading to an increase in the breaking force and better deceleration. In this phase gaining precise control over rocket is ^{relatively} easier, but

→ But it is to be kept in mind that density will increase suddenly leading to a sudden deceleration may affect rocket's orientation and trajectory

→ Entering in lower altitude causes an increase in air friction, leading to increased heating of rocket, which lead to damage of electrical components.

Low Altitude Phase

This phase is characterized by highest air density and thus highest drag forces are experienced here by the rocket.

→ In this phase the rocket due to highest levels of air friction experiences highest amount of heating

→ The drag force significantly reducing the rocket's descent velocity helps to achieve control over the rocket.

→ The drag force being really high can lead to high stress on rocket's structure. The design and structural strength of the rocket is required to withstand the stresses.