

Exercise: Veteran Data

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A study was conducted to compare the effects of two chemotherapy treatments in survival times for lung cancer patients. A total of 137 patients were randomly assigned to one of standard or test treatment group. The data include a number of covariates including tumor cell types, a Karnofsky performance score, the age of patient and time in months from diagnosis to randomization.

Table 1: Description of Variables in Veteran Lung Cancer Data.

| Variable | Description |
|----------|--|
| trt | 1 = standard 2 = test |
| celltype | 1 = squamous, 2 = smallcell, 3 = adeno, 4 = large |
| time | survival time |
| status | event status |
| karno | Karnofsky performance score. A scale of 10. |
| diagtime | the months from diagnosis of lung cancer to entry into the study |
| age | the age of the patient in years |
| prior | 0 = no prior therapy 10 = prior therapy |

```
library(survival)
attach(veteran)
?veteran
head(veteran)
```

```
##   trt celltype time status karno diagtime age prior
## 1   1 squamous  72      1    60        7  69     0
## 2   1 squamous 411      1    70        5  64    10
## 3   1 squamous 228      1    60        3  38     0
## 4   1 squamous 126      1    60        9  63    10
## 5   1 squamous 118      1    70       11  65    10
## 6   1 squamous  10      1    20        5  49     0
```

1. The Kaplan-Meier estimator

Now we calculate the Kaplan-Meier estimates of the survival function for each group using the `survfit` function in `library(survival)`.

`survfit` uses the Greenwood's formula ($\hat{\tau}^2(t)$ in Lecture note) for the variance calculation by default, which is $Var(\hat{S}(t)) = \hat{S}(t)^2 \sum_{j:t_j \leq t} \frac{d_j}{n_j(n_j - d_j)}$, where $\hat{S}(t) = \prod_{j:t_j \leq t} \left(1 - \frac{d_j}{n_j}\right)$.

Confidence limits for $S(t)$

Here we consider $100(1 - \alpha)\%$ pointwise confidence intervals for $S(t)$ for a particular specified time t .

`survfit` has several options for types of confidence intervals.

1. conf.type="plain"

$$\hat{S}(t) \pm z_{1-\alpha/2} \text{se}(\hat{S}(t))$$

2. conf.type="log" (default)

$$\hat{S}(t) \exp(\pm z_{1-\alpha/2} \text{se}(\log \hat{S}(t))),$$

where

$$\text{Var}(\log \hat{S}(t)) = \sum_{j:t_j \leq t} \frac{d_j}{n_j(n_j - d_j)}.$$

3. conf.type="log-log"

$$\{\hat{S}(t)\}^{\exp[\pm z_{1-\alpha/2} \text{se}(\log[-\log\{\hat{S}(t)\}])]},$$

where

$$\text{Var}[\log\{-\log \hat{S}(t)\}] = \frac{1}{\{\log \hat{S}(t)\}^2} \sum_{j:t_j \leq t} \frac{d_j}{n_j(n_j - d_j)}.$$

```
library(survival)
library(knitr)

# Kaplan-Meier curve for standard treatment
fit0.km <- survfit(Surv(time, status) ~ 1, data=veteran, subset=(trt==1))
summary(fit0.km)
```

```
## Call: survfit(formula = Surv(time, status) ~ 1, data = veteran, subset = (trt ==
##      1))
```

```
##
##   time n.risk n.event survival std.err lower 95% CI upper 95% CI
##   3      69      1  0.9855  0.0144   0.95771   1.000
##   4      68      1  0.9710  0.0202   0.93223   1.000
##   7      67      1  0.9565  0.0246   0.90959   1.000
##   8      66      2  0.9275  0.0312   0.86834   0.991
##  10      64      2  0.8986  0.0363   0.83006   0.973
##  11      62      1  0.8841  0.0385   0.81165   0.963
##  12      61      2  0.8551  0.0424   0.77592   0.942
##  13      59      1  0.8406  0.0441   0.75849   0.932
##  16      58      1  0.8261  0.0456   0.74132   0.921
##  18      57      2  0.7971  0.0484   0.70764   0.898
##  20      55      1  0.7826  0.0497   0.69109   0.886
##  21      54      1  0.7681  0.0508   0.67472   0.874
##  22      53      1  0.7536  0.0519   0.65851   0.862
##  27      51      1  0.7388  0.0529   0.64208   0.850
##  30      50      1  0.7241  0.0539   0.62580   0.838
##  31      49      1  0.7093  0.0548   0.60967   0.825
##  35      48      1  0.6945  0.0556   0.59368   0.812
##  42      47      1  0.6797  0.0563   0.57782   0.800
##  51      46      1  0.6650  0.0570   0.56209   0.787
##  52      45      1  0.6502  0.0576   0.54649   0.774
##  54      44      2  0.6206  0.0587   0.51565   0.747
##  56      42      1  0.6059  0.0591   0.50040   0.734
##  59      41      1  0.5911  0.0595   0.48526   0.720
##  63      40      1  0.5763  0.0598   0.47023   0.706
##  72      39      1  0.5615  0.0601   0.45530   0.693
```

| | | | | | | | |
|----|-----|----|---|--------|--------|---------|-------|
| ## | 82 | 38 | 1 | 0.5467 | 0.0603 | 0.44049 | 0.679 |
| ## | 92 | 37 | 1 | 0.5320 | 0.0604 | 0.42577 | 0.665 |
| ## | 95 | 36 | 1 | 0.5172 | 0.0605 | 0.41116 | 0.651 |
| ## | 100 | 34 | 1 | 0.5020 | 0.0606 | 0.39615 | 0.636 |
| ## | 103 | 32 | 1 | 0.4863 | 0.0607 | 0.38070 | 0.621 |
| ## | 105 | 31 | 1 | 0.4706 | 0.0608 | 0.36537 | 0.606 |
| ## | 110 | 30 | 1 | 0.4549 | 0.0607 | 0.35018 | 0.591 |
| ## | 117 | 29 | 2 | 0.4235 | 0.0605 | 0.32017 | 0.560 |
| ## | 118 | 27 | 1 | 0.4079 | 0.0602 | 0.30537 | 0.545 |
| ## | 122 | 26 | 1 | 0.3922 | 0.0599 | 0.29069 | 0.529 |
| ## | 126 | 24 | 1 | 0.3758 | 0.0596 | 0.27542 | 0.513 |
| ## | 132 | 23 | 1 | 0.3595 | 0.0592 | 0.26031 | 0.496 |
| ## | 139 | 22 | 1 | 0.3432 | 0.0587 | 0.24535 | 0.480 |
| ## | 143 | 21 | 1 | 0.3268 | 0.0582 | 0.23057 | 0.463 |
| ## | 144 | 20 | 1 | 0.3105 | 0.0575 | 0.21595 | 0.446 |
| ## | 151 | 19 | 1 | 0.2941 | 0.0568 | 0.20151 | 0.429 |
| ## | 153 | 18 | 1 | 0.2778 | 0.0559 | 0.18725 | 0.412 |
| ## | 156 | 17 | 1 | 0.2614 | 0.0550 | 0.17317 | 0.395 |
| ## | 162 | 16 | 2 | 0.2288 | 0.0527 | 0.14563 | 0.359 |
| ## | 177 | 14 | 1 | 0.2124 | 0.0514 | 0.13218 | 0.341 |
| ## | 200 | 12 | 1 | 0.1947 | 0.0501 | 0.11761 | 0.322 |
| ## | 216 | 11 | 1 | 0.1770 | 0.0486 | 0.10340 | 0.303 |
| ## | 228 | 10 | 1 | 0.1593 | 0.0468 | 0.08956 | 0.283 |
| ## | 250 | 9 | 1 | 0.1416 | 0.0448 | 0.07614 | 0.263 |
| ## | 260 | 8 | 1 | 0.1239 | 0.0426 | 0.06318 | 0.243 |
| ## | 278 | 7 | 1 | 0.1062 | 0.0400 | 0.05076 | 0.222 |
| ## | 287 | 6 | 1 | 0.0885 | 0.0371 | 0.03896 | 0.201 |
| ## | 314 | 5 | 1 | 0.0708 | 0.0336 | 0.02793 | 0.180 |
| ## | 384 | 4 | 1 | 0.0531 | 0.0295 | 0.01788 | 0.158 |
| ## | 392 | 3 | 1 | 0.0354 | 0.0244 | 0.00917 | 0.137 |
| ## | 411 | 2 | 1 | 0.0177 | 0.0175 | 0.00256 | 0.123 |
| ## | 553 | 1 | 1 | 0.0000 | NaN | NA | NA |

```
fit0.km$surv # est of S(t)
```

```
## [1] 0.98550725 0.97101449 0.95652174 0.92753623 0.89855072 0.88405797
## [7] 0.85507246 0.84057971 0.82608696 0.79710145 0.78260870 0.76811594
## [13] 0.75362319 0.75362319 0.73884626 0.72406934 0.70929241 0.69451549
## [19] 0.67973856 0.66496164 0.65018471 0.62063086 0.60585394 0.59107701
## [25] 0.57630009 0.56152316 0.54674623 0.53196931 0.51719238 0.51719238
## [31] 0.50198084 0.48629394 0.47060704 0.45492014 0.42354634 0.40785944
## [37] 0.39217253 0.39217253 0.37583201 0.35949149 0.34315097 0.32681044
## [43] 0.31046992 0.29412940 0.27778888 0.26144836 0.22876731 0.21242679
## [49] 0.21242679 0.19472456 0.17702232 0.15932009 0.14161786 0.12391563
## [55] 0.10621339 0.08851116 0.07080893 0.05310670 0.03540446 0.01770223
## [61] 0.00000000
```

```
fit0.km$std.err # se of H(t) or -log(S(t))
```

```
## [1] 0.01459893 0.02079951 0.02566635 0.03364887 0.04045094 0.04359689
## [7] 0.04956207 0.05242734 0.05523682 0.06073767 0.06344892 0.06614507
## [13] 0.06883324 0.06883324 0.07162522 0.07442000 0.07722374 0.08004229
## [19] 0.08288126 0.08574611 0.08864223 0.09454997 0.09757262 0.10064876
## [25] 0.10378438 0.10698580 0.11025969 0.11361316 0.11705384 0.11705384
## [31] 0.12080094 0.12490369 0.12913636 0.13351264 0.14275820 0.14766315
```

```
## [37] 0.15278373 0.15278373 0.15860158 0.16471413 0.17115855 0.17797809
## [43] 0.18522359 0.19295532 0.20124545 0.21018135 0.23044473 0.24207288
## [49] 0.24207288 0.25724509 0.27434639 0.29389974 0.31664798 0.34369039
## [55] 0.37673945 0.41864776 0.47462190 0.55551713 0.68939535 0.98755554
## [61]      Inf
```

```
fit0.km$lower # lower 95% CI for S(t)
```

```
## [1] 0.957708167 0.932225845 0.909594044 0.868338214 0.830062311 0.811654129
## [7] 0.775917611 0.758494913 0.741324068 0.707641998 0.691093573 0.674720630
## [13] 0.658511386 0.658511386 0.642076214 0.625797382 0.609666523 0.593676439
## [19] 0.577820910 0.562094548 0.546492680 0.515646755 0.500396160 0.485256867
## [25] 0.470226669 0.455303714 0.440486479 0.425773747 0.411164597 0.411164597
## [31] 0.396151404 0.380698046 0.365373756 0.350178113 0.320173165 0.305365110
## [37] 0.290688219 0.290688219 0.275417708 0.260305707 0.245354942 0.230568888
## [43] 0.215951849 0.201509057 0.187246807 0.173172619 0.145626048 0.132177179
## [49] 0.132177179 0.117612453 0.103396054 0.089557640 0.076135425 0.063179515
## [55] 0.050757251 0.038962286 0.027931171 0.017876902 0.009167347 0.002555167
## [61]      NA
```

```
fit0.km$upper # upper 95% CI for S(t)
```

```
## [1] 1.0000000 1.0000000 1.0000000 0.9907700 0.9726901 0.9629206 0.9423023
## [8] 0.9315478 0.9205416 0.8978703 0.8862423 0.8744391 0.8624724 0.8624724
## [15] 0.8502009 0.8377734 0.8251982 0.8124826 0.7996327 0.7866541 0.7735514
## [22] 0.7469894 0.7335368 0.7199734 0.7063015 0.6925229 0.6786393 0.6646519
## [29] 0.6505618 0.6505618 0.6360820 0.6211794 0.6061491 0.5909916 0.5602952
## [36] 0.5447555 0.5290868 0.5290868 0.5128563 0.4964706 0.4799275 0.4632241
## [43] 0.4463568 0.4293212 0.4121120 0.3947232 0.3593758 0.3413989 0.3413989
## [50] 0.3223949 0.3030764 0.2834252 0.2634203 0.2430389 0.2222596 0.2010720
## [57] 0.1795093 0.1577634 0.1367327 0.1226413      NA
```

```
# Kaplan-Meier curve for test treatment
```

```
fit1.km <- survfit(Surv(time, status) ~ 1, data=veteran, subset=(trt==2))
summary(fit1.km)
```

```
## Call: survfit(formula = Surv(time, status) ~ 1, data = veteran, subset = (trt ==
##      2))
```

```
##
##      time n.risk n.event survival std.err lower 95% CI upper 95% CI
##      1      68      2   0.9706  0.0205    0.93125    1.000
##      2      66      1   0.9559  0.0249    0.90830    1.000
##      7      65      2   0.9265  0.0317    0.86647    0.991
##      8      63      2   0.8971  0.0369    0.82766    0.972
##     13      61      1   0.8824  0.0391    0.80900    0.962
##     15      60      2   0.8529  0.0429    0.77278    0.941
##     18      58      1   0.8382  0.0447    0.75513    0.930
##     19      57      2   0.8088  0.0477    0.72056    0.908
##     20      55      1   0.7941  0.0490    0.70360    0.896
##     21      54      1   0.7794  0.0503    0.68684    0.884
##     24      53      2   0.7500  0.0525    0.65383    0.860
##     25      51      3   0.7059  0.0553    0.60548    0.823
##     29      48      1   0.6912  0.0560    0.58964    0.810
##     30      47      1   0.6765  0.0567    0.57394    0.797
##     31      46      1   0.6618  0.0574    0.55835    0.784
##     33      45      1   0.6471  0.0580    0.54289    0.771
```

| | | | | | | | |
|----|-----|----|---|--------|--------|---------|-------|
| ## | 36 | 44 | 1 | 0.6324 | 0.0585 | 0.52754 | 0.758 |
| ## | 43 | 43 | 1 | 0.6176 | 0.0589 | 0.51230 | 0.745 |
| ## | 44 | 42 | 1 | 0.6029 | 0.0593 | 0.49717 | 0.731 |
| ## | 45 | 41 | 1 | 0.5882 | 0.0597 | 0.48216 | 0.718 |
| ## | 48 | 40 | 1 | 0.5735 | 0.0600 | 0.46724 | 0.704 |
| ## | 49 | 39 | 1 | 0.5588 | 0.0602 | 0.45244 | 0.690 |
| ## | 51 | 38 | 2 | 0.5294 | 0.0605 | 0.42313 | 0.662 |
| ## | 52 | 36 | 2 | 0.5000 | 0.0606 | 0.39423 | 0.634 |
| ## | 53 | 34 | 1 | 0.4853 | 0.0606 | 0.37993 | 0.620 |
| ## | 61 | 33 | 1 | 0.4706 | 0.0605 | 0.36573 | 0.606 |
| ## | 73 | 32 | 1 | 0.4559 | 0.0604 | 0.35163 | 0.591 |
| ## | 80 | 31 | 2 | 0.4265 | 0.0600 | 0.32373 | 0.562 |
| ## | 84 | 28 | 1 | 0.4112 | 0.0597 | 0.30935 | 0.547 |
| ## | 87 | 27 | 1 | 0.3960 | 0.0594 | 0.29509 | 0.531 |
| ## | 90 | 25 | 1 | 0.3802 | 0.0591 | 0.28028 | 0.516 |
| ## | 95 | 24 | 1 | 0.3643 | 0.0587 | 0.26560 | 0.500 |
| ## | 99 | 23 | 2 | 0.3326 | 0.0578 | 0.23670 | 0.467 |
| ## | 111 | 20 | 2 | 0.2994 | 0.0566 | 0.20673 | 0.434 |
| ## | 112 | 18 | 1 | 0.2827 | 0.0558 | 0.19203 | 0.416 |
| ## | 133 | 17 | 1 | 0.2661 | 0.0550 | 0.17754 | 0.399 |
| ## | 140 | 16 | 1 | 0.2495 | 0.0540 | 0.16326 | 0.381 |
| ## | 164 | 15 | 1 | 0.2329 | 0.0529 | 0.14920 | 0.363 |
| ## | 186 | 14 | 1 | 0.2162 | 0.0517 | 0.13538 | 0.345 |
| ## | 201 | 13 | 1 | 0.1996 | 0.0503 | 0.12181 | 0.327 |
| ## | 231 | 12 | 1 | 0.1830 | 0.0488 | 0.10851 | 0.308 |
| ## | 242 | 10 | 1 | 0.1647 | 0.0472 | 0.09389 | 0.289 |
| ## | 283 | 9 | 1 | 0.1464 | 0.0454 | 0.07973 | 0.269 |
| ## | 340 | 8 | 1 | 0.1281 | 0.0432 | 0.06609 | 0.248 |
| ## | 357 | 7 | 1 | 0.1098 | 0.0407 | 0.05304 | 0.227 |
| ## | 378 | 6 | 1 | 0.0915 | 0.0378 | 0.04067 | 0.206 |
| ## | 389 | 5 | 1 | 0.0732 | 0.0344 | 0.02912 | 0.184 |
| ## | 467 | 4 | 1 | 0.0549 | 0.0303 | 0.01861 | 0.162 |
| ## | 587 | 3 | 1 | 0.0366 | 0.0251 | 0.00953 | 0.140 |
| ## | 991 | 2 | 1 | 0.0183 | 0.0180 | 0.00265 | 0.126 |
| ## | 999 | 1 | 1 | 0.0000 | NaN | NA | NA |

```
fit1.km$surv # est of S(t)
```

```
## [1] 0.97058824 0.95588235 0.92647059 0.89705882 0.88235294 0.85294118
## [7] 0.83823529 0.80882353 0.79411765 0.77941176 0.75000000 0.70588235
## [13] 0.69117647 0.67647059 0.66176471 0.64705882 0.63235294 0.61764706
## [19] 0.60294118 0.58823529 0.57352941 0.55882353 0.52941176 0.50000000
## [25] 0.48529412 0.47058824 0.45588235 0.42647059 0.42647059 0.41123950
## [31] 0.39600840 0.38016807 0.36432773 0.33264706 0.33264706 0.29938235
## [37] 0.28275000 0.26611765 0.24948529 0.23285294 0.21622059 0.19958824
## [43] 0.18295588 0.16466029 0.14636471 0.12806912 0.10977353 0.09147794
## [49] 0.07318235 0.05488676 0.03659118 0.01829559 0.00000000
```

```
fit1.km$std.err # se of H(t) or -log(S(t))
```

```
## [1] 0.02111002 0.02605251 0.03416334 0.04107993 0.04428074 0.05035372
## [7] 0.05327267 0.05895707 0.06174655 0.06451389 0.07001400 0.07827804
## [13] 0.08105994 0.08386446 0.08669683 0.08956222 0.09246584 0.09541300
## [19] 0.09840915 0.10145993 0.10457124 0.10774928 0.11433239 0.12126781
## [25] 0.12488854 0.12862394 0.13248465 0.14063028 0.14063028 0.14525711
```

```
## [31] 0.15008041 0.15553391 0.16125257 0.17361790 0.17361790 0.18894108
## [37] 0.19739986 0.20650224 0.21635583 0.22709414 0.23888544 0.25194545
## [43] 0.26655631 0.28664155 0.30992300 0.33750468 0.37110502 0.41358466
## [49] 0.47016196 0.55171152 0.68633248 0.98541984          Inf
```

```
fit1.km$lower # lower 95% CI for S(t)
```

```
## [1] 0.931249696 0.908298321 0.866466452 0.827663163 0.809003685 0.772783638
## [7] 0.755127288 0.720558806 0.703600421 0.686835340 0.653830589 0.605482979
## [13] 0.589644975 0.573935858 0.558350781 0.542885532 0.527536456 0.512300383
## [19] 0.497174586 0.482156725 0.467244822 0.452437226 0.423129867 0.394227263
## [25] 0.379926606 0.365726282 0.351626537 0.323731062 0.323731062 0.309351156
## [31] 0.295090839 0.280275370 0.265603502 0.236700873 0.236700873 0.206727978
## [37] 0.192032858 0.177540994 0.163261032 0.149203443 0.135380898 0.121808777
## [43] 0.108505870 0.093885637 0.079731407 0.066093677 0.053041093 0.040669842
## [49] 0.029120832 0.018614438 0.009531672 0.002651890          NA
```

```
fit1.km$upper # upper 95% CI for S(t)
```

```
## [1] 1.0000000 1.0000000 0.9906301 0.9722730 0.9623525 0.9414131 0.9304900
## [8] 0.9079002 0.8962798 0.8844663 0.8603146 0.8229296 0.8101908 0.7973233
## [15] 0.7843323 0.7712217 0.7579955 0.7446567 0.7312081 0.7176520 0.7039907
## [22] 0.6902256 0.6623896 0.6341520 0.6198839 0.6055165 0.5910496 0.5618156
## [29] 0.5618156 0.5466859 0.5314386 0.5156634 0.4997475 0.4674848 0.4674848
## [36] 0.4335639 0.4163223 0.3988859 0.3812478 0.3633997 0.3453319 0.3270328
## [43] 0.3084889 0.2887877 0.2686849 0.2481584 0.2271866 0.2057597 0.1839115
## [50] 0.1618398 0.1404700 0.1262226          NA
```

```
par(mfrow=c(1,2))
plot(fit0.km, xlab="TIME(DAYS) SINCE RANDOMIZATION",
     ylab="ESTIMATED PROBABILITY OF SURVIVAL", cex.lab=.7, cex.axis=0.5)
mtext("STANDARD TREATMENT", side=3, line=0.5, cex=.8)
plot(fit1.km, xlab="TIME(DAYS) SINCE RANDOMIZATION",
     ylab="ESTIMATED PROBABILITY OF SURVIVAL", cex.lab=.7, cex.axis=0.5)
mtext("TEST TREATMENT", side=3, line=0.5, cex=.8)
```

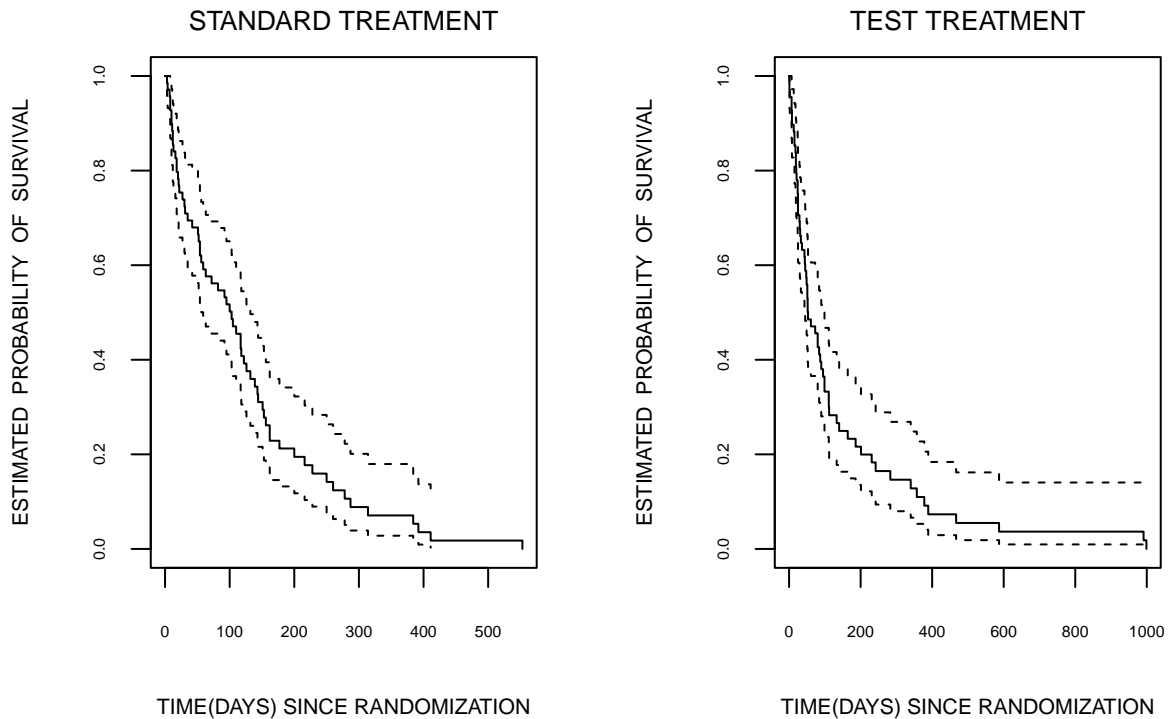


Figure 1: Pointwise 95% interval estimates for $S(t)$

```
library(ggsurvfit)

## Loading required package: ggplot2

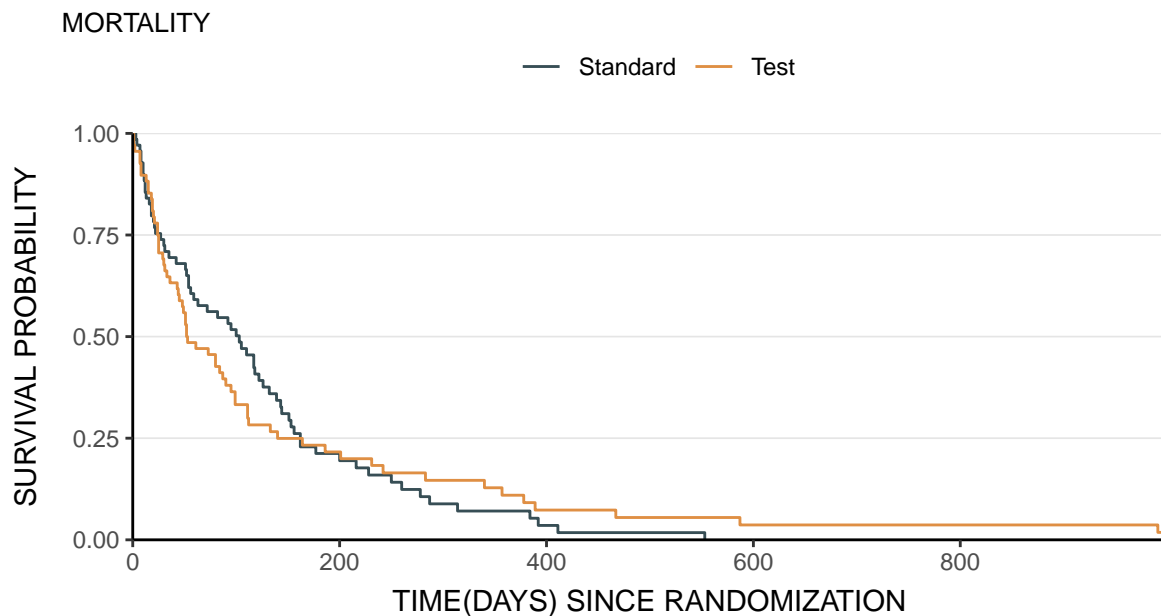
library(ggplot2)
library(ggsci)

veteran$trt2 <- factor(veteran$trt, 1:2, c("Standard", "Test"))

p1 <- survfit2(Surv(time, status) ~ trt2, data = veteran) %>%
  ggsurvfit(type="survival",
    theme = theme_classic() +
      theme(legend.position = "top") +
      theme(panel.grid.major.y = element_line(color = "gray90", size = 0.3))) +
  scale_x_continuous(breaks=c(0, 200, 400, 600, 800, 1000), expand=c(0,0))+
  scale_y_continuous(limits=c(0, 1), expand=c(0,0)) +
  labs(x = "TIME(DAYS) SINCE RANDOMIZATION", y = "SURVIVAL PROBABILITY",
    title = "MORTALITY") +
  theme(axis.title.y = element_text(vjust = -0.1),
    axis.title.x = element_text(vjust = -0.1),
    plot.title = element_text(hjust = -0.08, vjust=-3, size=10))+
  scale_color_jama()+
  add_risktable(times = c(0, 200, 400, 600, 800, 1000),
    risktable_stats = c("n.risk"),
    risktable_group = c("risktable_stats"),
    risktable_height = 0.2, # Adjusts the height of the risk table (default is 0.25),
    stats_label = "No. at risk",
    size =3, hjust= 0)
```

```
## Warning: The `size` argument of `element_line()` is deprecated as of ggplot2 3.4.0.
## i Please use the `linewidth` argument instead.
```

p1



| | | | | | |
|-------------|----|----|---|---|---|
| No. at risk | | | | | |
| Standard | 69 | 12 | 2 | 0 | 0 |
| Test | 68 | 13 | 4 | 2 | 2 |

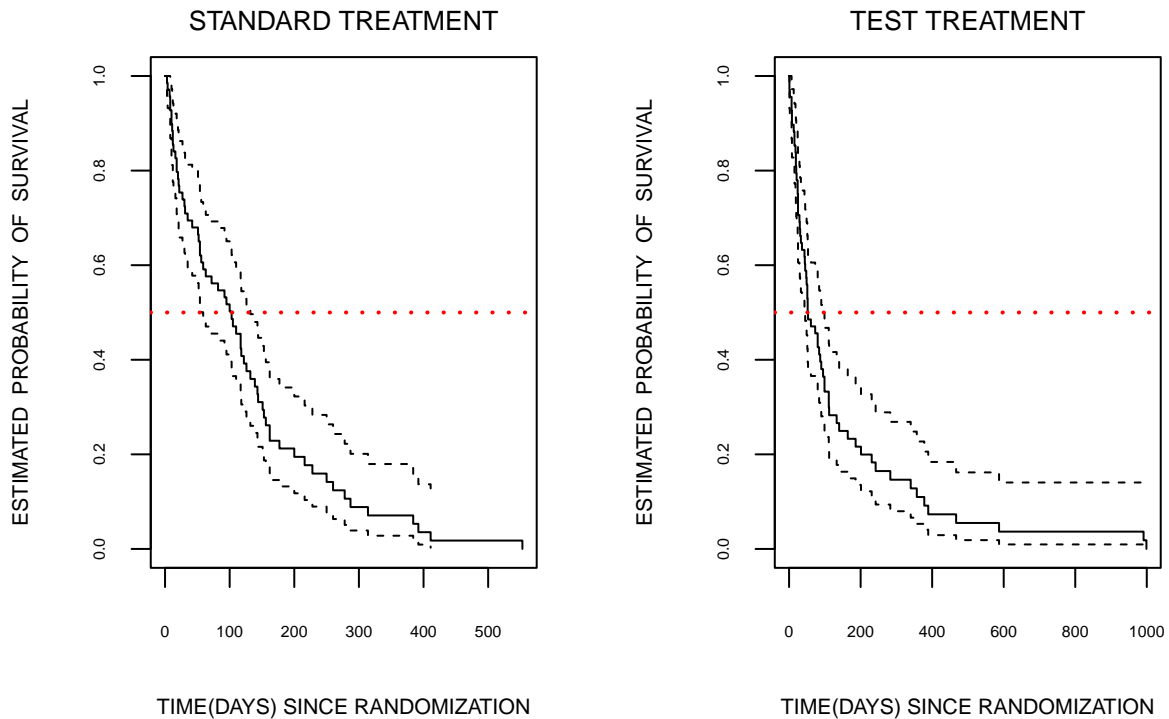
Figure 2: Estimated survival function stratified by treatment group

Median Survival Time

Suppose we construct the confidence limit of $S(t)$ based on the log-transformation. From the graphical method, the median survival time for the standard treatment group is 103 days and the corresponding 95% confidence intervals are (59, 132). In addition, the median survival time for the test treatment group is 52 days and the corresponding 95% confidence intervals are (44, 95).

```
par(mfrow=c(1,2))
plot(fit0.km, xlab="TIME(DAYS) SINCE RANDOMIZATION",
      ylab="ESTIMATED PROBABILITY OF SURVIVAL", cex.lab=0.7, cex.axis=0.5)
abline(h=0.5, lty=3, lwd=2, col="red")
mtext("STANDARD TREATMENT", side=3, line=0.5, cex=.8)

plot(fit1.km, xlab="TIME(DAYS) SINCE RANDOMIZATION",
      ylab="ESTIMATED PROBABILITY OF SURVIVAL", cex.lab=0.7, cex.axis=0.5)
abline(h=0.5, lty=3, lwd=2, col="red")
mtext("TEST TREATMENT", side=3, line=0.5, cex=.8)
```

3. Log-rank test

Now, we want to test $H_0 : S_1(t) = S_2(t)$ for all t .

```
library(KMsurv)
survdif(Surv(time, status)~factor(trt), data = veteran)
```

```
## Call:
## survdif(formula = Surv(time, status) ~ factor(trt), data = veteran)
##
##              N Observed Expected (O-E)^2/E (O-E)^2/V
## factor(trt)=1 69         64    64.5   0.00388   0.00823
## factor(trt)=2 68         64    63.5   0.00394   0.00823
##
##  Chisq= 0   on 1 degrees of freedom, p= 0.9
```

```
1-pchisq(0.00823, 1)
```

```
## [1] 0.9277156
```

The test statistic is

$$\frac{(O - E)^2}{V} \sim \chi_1^2 \quad \text{under } H_0$$

The p-values is $P(\chi_1^2 > 0.00823) = 0.9277156$. Therefore we do not reject the null hypothesis, therefore, the survival function is not different between the standard treatment group and the test treatment group.

Cox Proportional Hazard Regression Model

Estimation

We fit a Cox regression model including the treatment indicator (`trt`), cell types (`celltype`), performance status (`karno`), the months from diagnosis of lung cancer to entry into the study (`diagtime`), prior therapy (`prior`), and age(`age`).

```

library(survival)
attach(veteran)

## The following objects are masked from veteran (pos = 8):
##
##   age, celltype, diagtime, karno, prior, status, time, trt
veteran$celltypef. <- factor(veteran$celltype, levels=c("large", "squamous",
                                                       "smallcell", "adeno"))
fit1 <- coxph(Surv(time, status)~ factor(trt) + celltypef. + karno + diagtime
              + factor(prior) + age , data=veteran, method="breslow")
summary(fit1)

## Call:
## coxph(formula = Surv(time, status) ~ factor(trt) + celltypef. +
##       karno + diagtime + factor(prior) + age, data = veteran, method = "breslow")
##
##   n= 137, number of events= 128
##
##              coef exp(coef)  se(coef)      z Pr(>|z|)
## factor(trt)2      0.289936  1.336342  0.207210  1.399  0.16174
## celltypef.squamous -0.399628  0.670570  0.282663 -1.414  0.15742
## celltypef.smallcell  0.456859  1.579106  0.266273  1.716  0.08621 .
## celltypef.adeno     0.788672  2.200471  0.302668  2.606  0.00917 **
## karno              -0.032622  0.967905  0.005505 -5.926 3.11e-09 ***
## diagtime           -0.000092  0.999908  0.009125 -0.010  0.99196
## factor(prior)10     0.072327  1.075006  0.232133  0.312  0.75536
## age                -0.008549  0.991487  0.009304 -0.919  0.35816
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##              exp(coef) exp(-coef) lower .95 upper .95
## factor(trt)2      1.3363      0.7483      0.8903      2.0058
## celltypef.squamous  0.6706      1.4913      0.3853      1.1669
## celltypef.smallcell  1.5791      0.6333      0.9370      2.6611
## celltypef.adeno     2.2005      0.4544      1.2159      3.9824
## karno              0.9679      1.0332      0.9575      0.9784
## diagtime           0.9999      1.0001      0.9822      1.0180
## factor(prior)10     1.0750      0.9302      0.6821      1.6943
## age                0.9915      1.0086      0.9736      1.0097
##
## Concordance= 0.736 (se = 0.021 )
## Likelihood ratio test= 61.41 on 8 df,  p=2e-10
## Wald test              = 61.65 on 8 df,  p=2e-10
## Score (logrank) test = 65.92 on 8 df,  p=3e-11

library(knitr)
xname <- c("Treatment", "Cell Type", "", "", "performance status",
           "Time from Diagnosis to entry", "Prior Therapy", "Age")
compare <- c("Test vs. Standard", "Squamous vs. Large", "Small vs. Large",
            "Adeno vs. Large", "1 unit increase", "1 month increase",
            "Yes vs. No", "1 year increase")
res <- data.frame(xname, compare, summary(fit1)$coef[,2],
                  paste("(", round(exp(confint(fit1))[,1], 3), ", ",
                        round(exp(confint(fit1))[,2], 3), ")", sep=""), summary(fit1)$coef[,c(4,5)])

```

```
colnames(res) <- c("Covariates", "Comparison", "RR", "95% CI", "z", "p-value")
rownames(res) <- NULL

kable(res,
  digits = c(3, 3, 3, 4),
  caption="The results of Cox regression modeling for lung cancer patient survival")
```

Table 2: The results of Cox regression modeling for lung cancer patient survival

| Covariates | Comparison | RR | 95% CI | z | p-value |
|------------------------------|--------------------|-------|----------------|--------|---------|
| Treatment | Test vs. Standard | 1.336 | (0.89, 2.006) | 1.399 | 0.162 |
| Cell Type | Squamous vs. Large | 0.671 | (0.385, 1.167) | -1.414 | 0.157 |
| | Small vs. Large | 1.579 | (0.937, 2.661) | 1.716 | 0.086 |
| | Adeno vs. Large | 2.200 | (1.216, 3.982) | 2.606 | 0.009 |
| performance status | 1 unit increase | 0.968 | (0.958, 0.978) | -5.926 | 0.000 |
| Time from Diagnosis to entry | 1 month increase | 1.000 | (0.982, 1.018) | -0.010 | 0.992 |
| Prior Therapy | Yes vs. No | 1.075 | (0.682, 1.694) | 0.312 | 0.755 |
| Age | 1 year increase | 0.991 | (0.974, 1.01) | -0.919 | 0.358 |

```
# drop treatment
fit5 <- coxph(Surv(time, status)~ celltypef. + karno, data=veteran, method="breslow")
summary(fit5)
```

```
## Call:
## coxph(formula = Surv(time, status) ~ celltypef. + karno, data = veteran,
##       method = "breslow")
##
##      n= 137, number of events= 128
##
##              coef exp(coef)  se(coef)      z Pr(>|z|)
## celltypef.squamous -0.325143  0.722424  0.276694 -1.175  0.23996
## celltypef.smallcell  0.387005  1.472565  0.261005  1.483  0.13814
## celltypef.adeno     0.825659  2.283384  0.293329  2.815  0.00488 **
## karno               -0.030904  0.969569  0.005179 -5.968 2.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##              exp(coef) exp(-coef) lower .95 upper .95
## celltypef.squamous    0.7224    1.3842    0.4200    1.2426
## celltypef.smallcell    1.4726    0.6791    0.8829    2.4561
## celltypef.adeno       2.2834    0.4379    1.2850    4.0575
## karno                 0.9696    1.0314    0.9598    0.9795
##
## Concordance= 0.734 (se = 0.023 )
## Likelihood ratio test= 58.77 on 4 df,  p=5e-12
## Wald test               = 60.58 on 4 df,  p=2e-12
## Score (logrank) test = 63.22 on 4 df,  p=6e-13
```

We drop non-significant variables by backward elimination, which leads to the model including only cell type

and performance status variables with the form of

$$\begin{aligned} h(t|\mathbf{z}_i) &= h_0(t) \exp(\mathbf{z}_i' \boldsymbol{\beta}) \\ &= h_0(t) \exp(z_{i1}\beta_1 + z_{i2}\beta_2 + z_{i3}\beta_3 + z_{i4}\beta_4), \end{aligned}$$

where $\mathbf{z}_i = (I(\text{cell-type} = \text{Squamous}), I(\text{cell-type} = \text{Small cell}), I(\text{cell-type} = \text{Adeno cell}), \text{performance status})'$, $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4)'$.

```
xname <- c("Cell Type", "", "", "performance status")
compare <- c("Squamous vs. Large", "Small vs. Large", "Adeno vs. Large", "1 unit increase")
res <- data.frame(xname, compare, summary(fit5)$coef[,2],
                  paste("(", round(exp(confint(fit5))[,1], 3), ", ",
                        round(exp(confint(fit5))[,2], 3), ")", sep=""), summary(fit5)$coef[,c(4,5)])
colnames(res) <- c("Covariates", "Comparison", "RR", "95% CI", "z", "p-value")
rownames(res) <- NULL

kable(res,
      digits = c(3, 3, 3, 4),
      caption="The results of the final model using a Cox regression for lung cancer patient survival")
```

Table 3: The results of the final model using a Cox regression for lung cancer patient survival

| Covariates | Comparison | RR | 95% CI | z | p-value |
|--------------------|--------------------|-------|----------------|--------|---------|
| Cell Type | Squamous vs. Large | 0.722 | (0.42, 1.243) | -1.175 | 0.240 |
| | Small vs. Large | 1.473 | (0.883, 2.456) | 1.483 | 0.138 |
| | Adeno vs. Large | 2.283 | (1.285, 4.058) | 2.815 | 0.005 |
| performance status | 1 unit increase | 0.970 | (0.96, 0.979) | -5.968 | 0.000 |

Here we have $\hat{\beta}_1 = -0.325$, $s.e.(\hat{\beta}_1) = 0.276$ and the 95% CI for β_1 is $-0.325 \pm 1.96 \cdot 0.276 = (-0.867, 0.217)$. The estimated relative risk (RR) is $\exp(\hat{\beta}_1) = 0.722$ and the 95% CI for RR is $\exp(-0.325 \pm 1.96 \cdot 0.276) = (0.420, 1.243)$, which is interpreted as the relative risk of death for individuals with squamous cell type versus large cell type given a fixed value of performance status.

Since the performance status increase by 10 units, we now estimate the effect of 10-unit change in performance status on the risk of death, controlling for cell type, which is $10\hat{\beta}_4$.

$10\hat{\beta}_4 = -0.309$, $s.e.(10\hat{\beta}_4) = 10 * s.e.(\hat{\beta}_4) = 0.052$ and the 95% CI for $10\beta_4$ is $-0.309 \pm 1.96 \cdot 0.052 = (-0.411, -0.208)$. Then, the estimated RR associated with a 10 unit increase in performance status is 0.734 and its 95% CI is $\exp(-0.309 \pm 1.96 \cdot 0.052) = (0.663, 0.813)$.

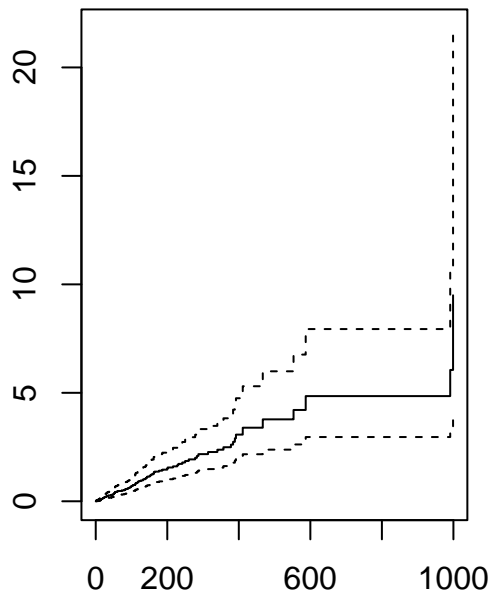
We now plot the estimated cumulative hazard function and the survival function. In R, these estimators are calculated for a hypothesis person with the average covariates. Therefore, it is strongly recommended to use `newdata` to specify covariate values. For example, we are interested in estimating the cumulative hazard function and the survival function for the individuals with squamous cell type and performance status of 50.

```
# The breslow estimates
H0 <- basehaz(fit5)

# The cumulative hazard and survival function for the individual with specified covariates
mydata <- with(veteran, data.frame(celltypef.="squamous", karno=50))
H <- survfit(fit5, newdata=mydata, type="aalen")

par(mfrow=c(1,2))
plot(H, fun="cumhaz", main="Estimated Cumulative Hazard")
plot(H, main="Estimated Survival Function")
```

Estimated Cumulative Hazard



Estimated Survival Function

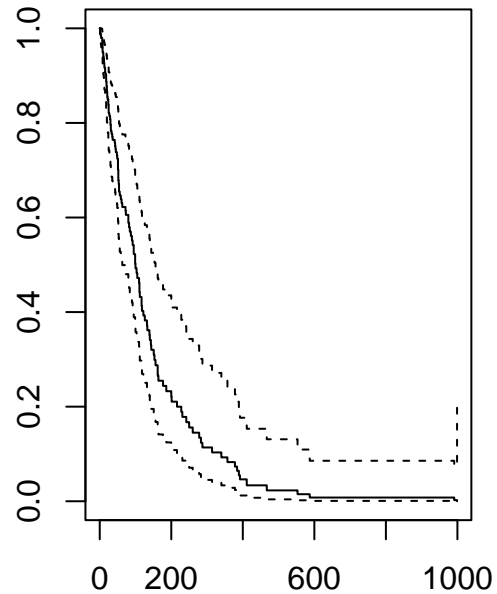


Figure 3: The estimated cumulative hazard function and the survival function for the individuals with cell type of squamous and performance status of 50.

Model diagnostics

Here, we check the proportional hazard assumptions.

```
# Cox-Snell Residuals
coxsnell <- veteran$status - resid(fit5, type="martingale")
fit.coxsnell <- survfit(Surv(coxsnell, veteran$status)~1)

plot(log(fit.coxsnell$time), log(fit.coxsnell$cumhaz), ylab="log Cumulative Hazard of Cox-snell Residuals",
     abline(0, 1, lty=2, lwd=1.5))
```

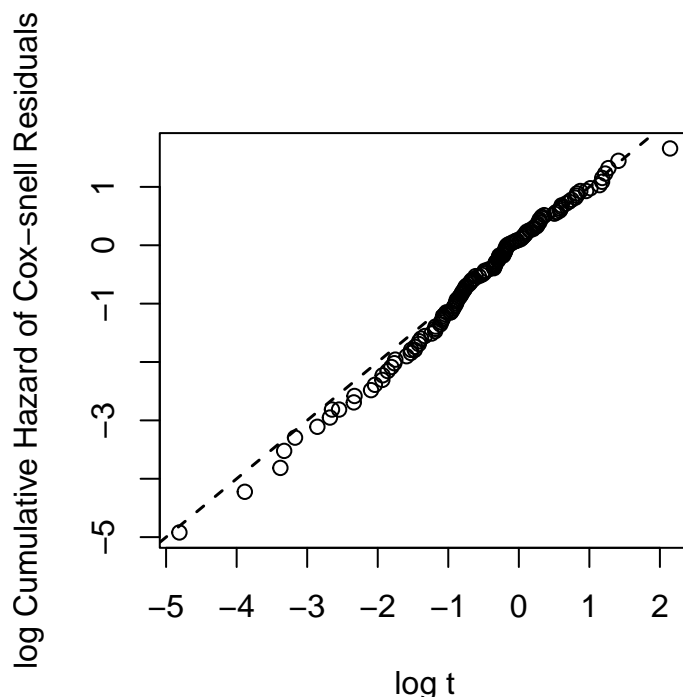


Figure 4: The Cox-Snell residual plots

Although the Cox-Snell residual plots in Figure 2 show no evidence for the misspecified model, we will conduct the formal test for the PH assumption. Note that the Cox-snell residual plots are not very informative.

`cox.zph` function is used for checking the PH assumptions with the argument `transform` for a functional form of $g(t)$. `transform = "identity"` corresponds to the identity transform $g(t) = t$, `transform = "log"` to $\log(t)$, `transform = "rank"` to the rank of the event times and `transform = "km"` to the Kaplan-Meier estimates $\hat{S}(t)$. In the output of `cox.zph`, `chisq` gives the test statistics with `df`, degrees of freedom and `p` gives the p-value. The last row of GLOBAL gives the global test of proportional hazards over all `p` Covariates.

```
zph.id.fit5 <- cox.zph(fit5, transform = 'identity') #  $g(t) = t$ 
zph.log.fit5 <- cox.zph(fit5, transform = 'log') #  $g(t) = \log$ 
zph.km.fit5 <- cox.zph(fit5, transform = 'km') #  $g(t) = \hat{S}(t)$  from the Kaplan-Meier - default
```

```
zph.id.fit5
```

```
##           chisq df      p
## celltypef.  17.8  3 0.00048
## karno       7.0  1 0.00814
## GLOBAL     20.7  4 0.00037
```

```
zph.log.fit5
```

```
##           chisq df      p
## celltypef.  12.6  3 0.00554
## karno      11.2  1 0.00081
## GLOBAL     21.4  4 0.00026
```

```
zph.km.fit5
```

```
##           chisq df      p
## celltypef.  13.9  3 0.00305
## karno      14.1  1 0.00017
```

```
## GLOBAL      23.3  4 0.00011
```

```
plot(zph.id.fit5[1])
```

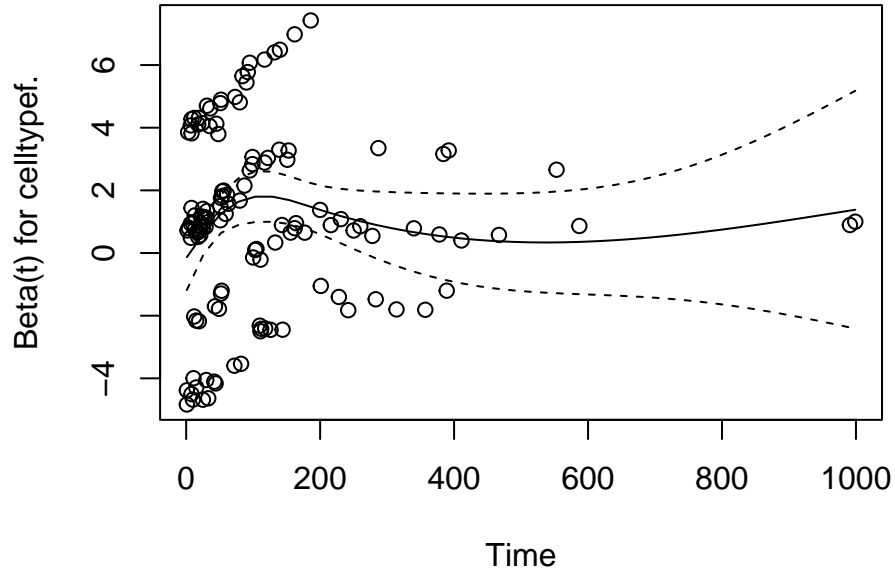


Figure 5: The scaled Schoenfeld residual plot of the cell types in the model with the Kaplan-Meier transformation (i.e. $\hat{\beta}_l + r_{lj}^*(\hat{\beta})$ versus t_j , $j=1,2$.)

```
plot(zph.id.fit5[2])
```

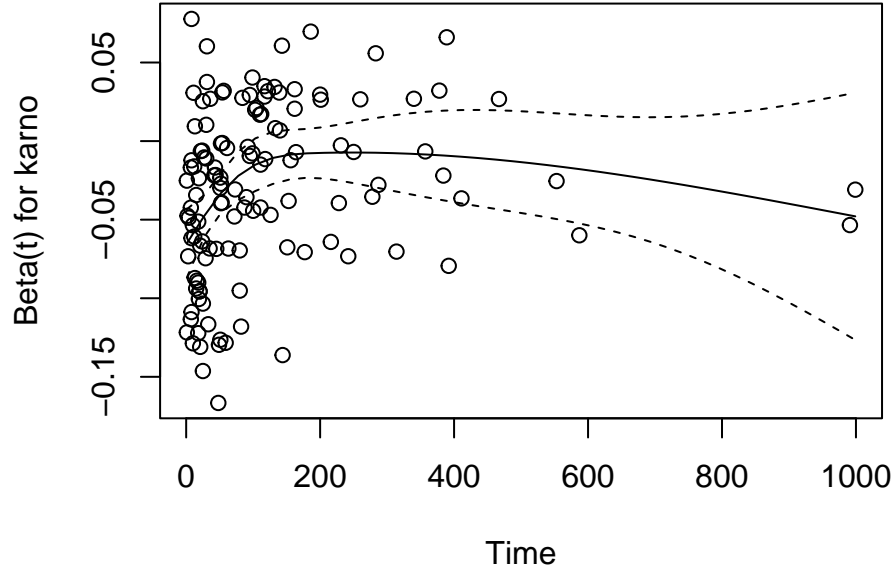


Figure 6: The scaled Schoenfeld residual plot of the performance status in the model with the Kaplan-Meier transformation (i.e. $\hat{\beta}_l + r_{lj}^*(\hat{\beta})$ versus t_j , $l=1,2$.)

Based on the identity, log, and the Kaplan-Meier scale, there is some evidence for nonproportionality for the cell types and performance status, which are the significant predictor in the Cox model. In particular, the left penal of Figure 4 shows that the upward trend ends around 100 days for the scaled Schoenfeld residual

plot of the performance status, $\hat{\beta}_l + r_{lj}^*(\hat{\beta})$ versus t_j , which identifies 100 days as a point to allow the hazard ratio to change.

Stratification

To remove the problem of non-proportionality, first we use a cell-type variable as a stratifying variable. Then, a new model has the form of

$$h_k(t|\mathbf{z}_{ki}) = h_{k0}(t) \exp(z_{ki4}\beta_1), \quad k = 1, 2, 3, 4,$$

which allows the cell-type specific baseline hazard functions.

strata is used to specify a stratifying variable in coxph.

```
fit5.str <- coxph(Surv(time, status) ~ karno + strata(celltypef.), data=veteran, method="breslow")
km <- survfit(fit5.str)
km
```

```
## Call: survfit(formula = fit5.str)
##
##           n events median 0.95LCL 0.95UCL
## large      27      26    111      100     200
## squamous   35      31    118       82     283
## smallcell  48      45     54       30      95
## adeno      27      26     52       36      92
```

#summary(km) # gives the Kaplan-Meier estimates with 95% CI

```
library(survminer)
```

```
## Loading required package: ggpubr
```

```
##
```

```
## Attaching package: 'survminer'
```

```
## The following object is masked from 'package:survival':
```

```
##
```

```
##      myeloma
```

Survival function versus time stratified by cell types

```
ggsurvplot(km, data = veteran, risk.table = TRUE, conf.int=FALSE, risk.table.height = 0.3)
```

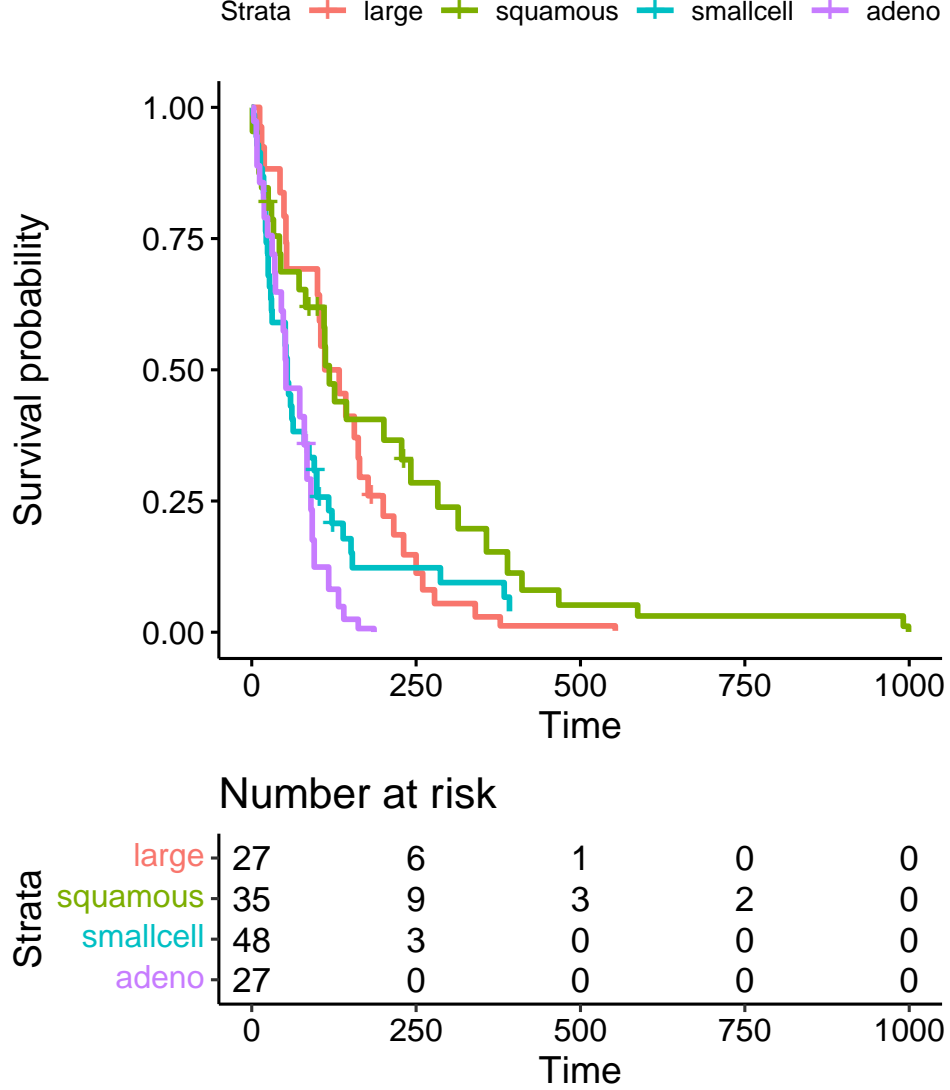



Figure 7: Survival functions versus time stratified by cell types for the individual with performance status of the mean of the performance status

Time-dependent Covariate

Second, we create a time-dependent covariate to allow the effect of performance status to depend on time through the interaction term with time. Here, we let $g(t) = I(t > 100)$.

Then, our model has the form of

$$h_k(t|\mathbf{z}_{ki}) = h_{k0}(t) \exp(z_{ki4}\beta_1 + z_{ki4} * I(t > 100)\beta_2), \quad k = 1, 2, 3, 4 \quad (1)$$

The interpretation of regression coefficients is give in the following table.

```
# Code to create a dataframe with a time-dependent covariate, It,
# where It = 0 if time <= 100 days; 1 if > 100 days months
timedepeff.f <- function(indata, cutpoint) {
  outdata <- NULL
  for (i in 1:nrow(indata)) {
    time <- indata$time[i]
```

| Period | Cell type | performance status | Hazard | Relative Risk |
|------------------|-----------|--------------------|---|-------------------------------|
| (0, 100] | k | $10 + c$ | $h_{k0}(t) \exp((10 + c)\beta_1)$ | $\exp(10\beta_1)$ |
| | k | c | $h_{k0}(t) \exp(c\beta_1)$ | |
| (100, ∞] | k | $10 + c$ | $h_{k0}(t) \exp((10 + c)(\beta_1 + \beta_2))$ | $\exp(10(\beta_1 + \beta_2))$ |
| | k | c | $h_{k0}(t) \exp(c(\beta_1 + \beta_2))$ | |

```

status <- indata$status[i]
if ( time <= cutpoint) {
  estart <- 0
  estop <- time
  estatus <- status
  It <- 0
} else{
  estart <- c(0, cutpoint)
  estop <- c(cutpoint, time)
  estatus <- c(0, status)
  It <- c(0, 1)
}
nlen <- length(estart)
karno <- rep(indata$karno[i], nlen)
id <- rep(i, nlen)
celltypef. <- rep(indata$celltypef[i], nlen)
outdata <- rbind(outdata, data.frame(id, estart, estop, estatus, It, karno, celltypef.))
}
outdata <- data.frame(outdata, row.names=c(1:nrow(outdata)))
return(outdata)
}

# adding a time-dependent covariate
veteran.tdeff <- timedepeff.f(indata=veteran, cutpoint=100)
dimnames(veteran.tdeff)[[2]] <- c("id", "estart", "estop", "estatus", "timecut", "karno", "celltypef.")

# adding interaction with karno*timecut and stratify by celltype
final.fit <- coxph(Surv(estart, estop, estatus) ~ karno + karno:factor(timecut)
+ strata(celltypef.), data=veteran.tdeff, method="breslow")
summary(final.fit)

## Call:
## coxph(formula = Surv(estart, estop, estatus) ~ karno + karno:factor(timecut) +
## strata(celltypef.), data = veteran.tdeff, method = "breslow")
##
## n= 190, number of events= 128
##
##
##          coef exp(coef) se(coef)      z Pr(>|z|)
## karno      -0.044308  0.956660  0.006305 -7.028  2.1e-12 ***
## karno:factor(timecut)1  0.043203  1.044150  0.013867  3.116  0.00184 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##          exp(coef) exp(-coef) lower .95 upper .95
## karno          0.9567      1.0453    0.9449    0.9686

```

```
## karno:factor(timecut)1    1.0441    0.9577    1.0162    1.0729
##
## Concordance= 0.695 (se = 0.027 )
## Likelihood ratio test= 50.43 on 2 df,  p=1e-11
## Wald test              = 49.4 on 2 df,  p=2e-11
## Score (logrank) test = 55.39 on 2 df,  p=9e-13

zph.id.final.fit <- cox.zph(final.fit, transform="identity")
zph.log.final.fit <- cox.zph(final.fit, transform="log")
zph.km.final.fit <- cox.zph(final.fit, transform="km")
zph.id.final.fit

##                chisq df    p
## karno          0.875  1 0.35
## karno:factor(timecut) 1.509  1 0.22
## GLOBAL         2.099  2 0.35

zph.log.final.fit

##                chisq df    p
## karno          0.094  1 0.76
## karno:factor(timecut) 1.798  1 0.18
## GLOBAL         2.243  2 0.33

zph.km.final.fit

##                chisq df    p
## karno          0.0982  1 0.75
## karno:factor(timecut) 1.9684  1 0.16
## GLOBAL         2.7463  2 0.25

plot(zph.id.final.fit[1])
```

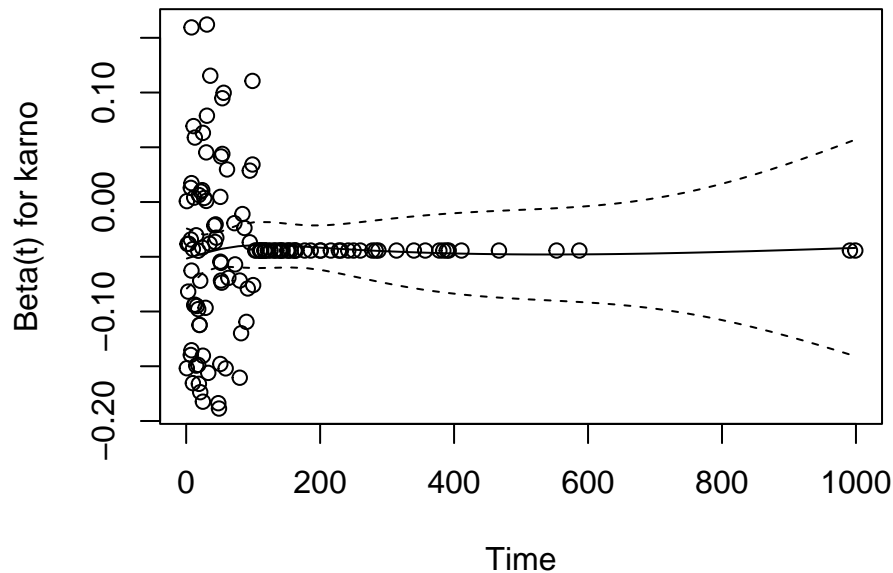


Figure 8: The scaled Schoenfeld residual plot of the performance status in the model with the Kaplan-Meier transformation (i.e. $\hat{\beta}_1 + r_{1j}^*(\hat{\beta})$ versus t_j .)

The p-values from the marginal and global tests do not give evidence against the PH assumption and the

plot of the scaled Schoenfeld residuals do not suggest any time trend in the coefficients. Therefore, we choose the model (1) as our final model.

```
# Relative risk for a 10 unit increase for the first 100 days
rr1 <- exp(10*final.fit$coefficients[1])
rr.CI.1 <- exp(10*(final.fit$coefficients[1] + c(-1.96, 1.96)*sqrt(final.fit$var[1,1])))
rr1

##      karno
## 0.6420584

rr.CI.1

## [1] 0.5674259 0.7265073

# Relative risk for a 10 unit increase for after 100 days
est2<- final.fit$coefficients[1] + final.fit$coefficients[2]
sd2 <- sqrt(final.fit$var[1,1] + 2*final.fit$var[1,2] + final.fit$var[2,2])

rr2 <- exp(10*est2)
rr.CI.2 <- exp(10*(est2 + c(-1.96, 1.96)*sd2))
rr2

##      karno
## 0.9890129

rr.CI.2

## [1] 0.7763699 1.2598975

z <- est2/sd2 # z-value; H_0: beta1 + beta2 = 0
2*(1-pnorm(abs(z))) # p-value

##      karno
## 0.9287244
```

For the first 100 days, the estimated RR associated with a 10 unit increase in performance status is 0.642 and its 95% CI is (0.567, 0.727) with p-value = 0.002. However, there is only 1.1% reduction in the risk of death for a 10 unit increase in performance status after 100 days (RR=0.989, 95% CI = (0.776, 1.260); $p = 0.929$). This suggests that performance status is more beneficial to reducing the risk of death of a lung cancer patient during the early days after the randomization.